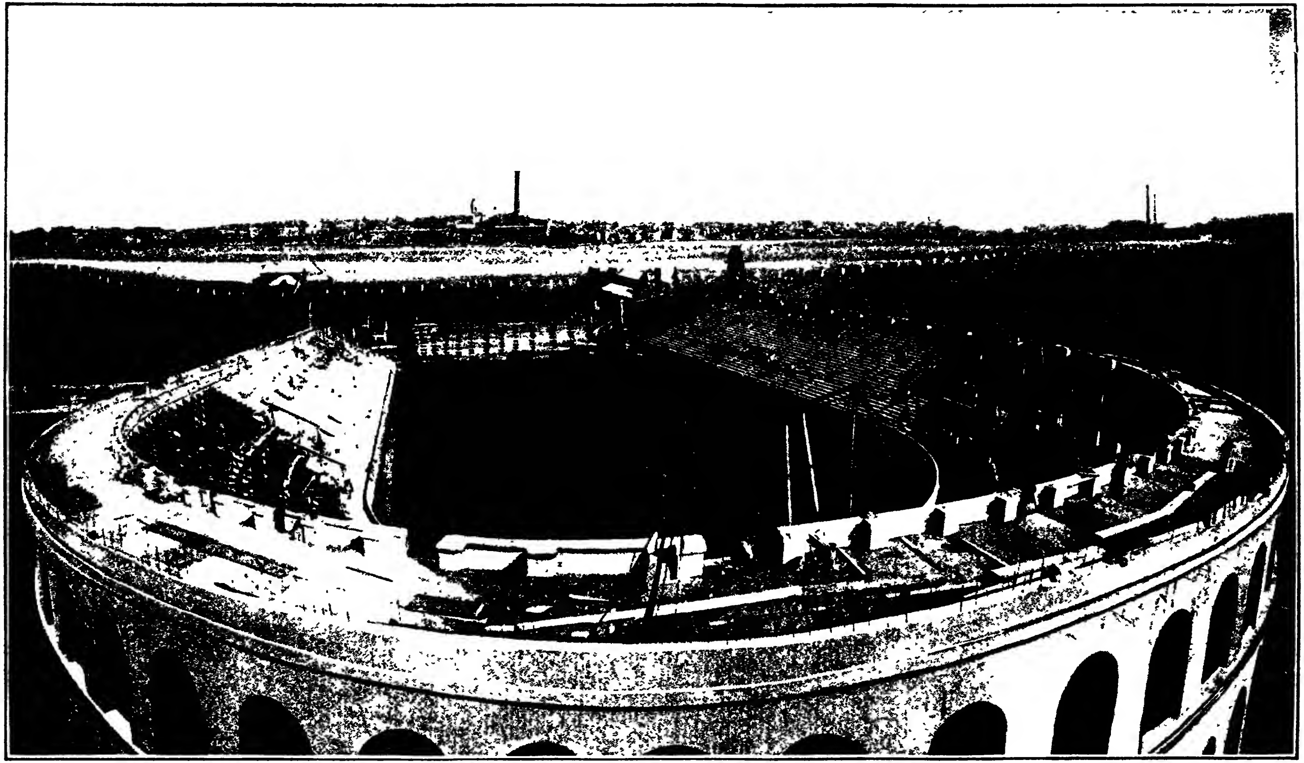

Frontispiece

Harvard Stadium

A TREATISE

ON

CONCRETE

PLAIN AND REINFORCED

MATERIALS, CONSTRUCTION, AND DESIGN OF
CONCRETE AND REINFORCED CONCRETE

WITH CHAPTERS BY
R. FERET, WILLIAM B. FULLER, FRANK P. McKIBBEN AND
SPENCER B. NEWBERRY

BY

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PREFACE TO FIRST EDITION

This treatise is designed for practicing engineers and contractors, and also for a text and reference book on concrete for engineering students.

To broaden the scope of the work and avoid personal inaccuracies, each chapter has been submitted for critcism to at least one, and, in some cases, to three or four specialists in the particular line treated. We have aimed to refer by name to all authorities quoted, and where the data is taken from books or periodicals, to give the original publication, so that each subject may be investigated further. Proof clippings have also been submitted for approval to those whose names are mentioned. Numerous cross references will be found as well as many repetitions, inserted for the purpose of emphasizing important facts.

The chapters are arranged for convenience in reference, and therefore are not always in logical order.

The Concrete Data in Chapter I presents a list of definitions of words and terms relating distinctively to cement and concrete; a summary of the most important facts and conclusions, with references to the pages discussing them; data on concrete labor, and conversion ratios.

The Elementary Outline of the Process of Concreting, Chapter II, is designed, not for the civil engineer, but for those seeking simple directions as to the exact procedure in laying a small quantity of concrete. Most of the subjects there treated are discussed at length in subsequent chapters.

The Specifications for Cement in Chapter III include the latest recommendations of committees of our national societies, with incidental changes to adapt them for direct use in purchase specifications. The Concrete Specifications have been prepared by the authors to represent standard practice. Specifications for First-class or High Steel, drawn up by Mr. Taylor, are, we believe, the first recommendations which have been made to safely adapt this important material to reinforced concrete construction.

In Chapter IV the Choice of Cement is considered in an elementary fashion, which will serve as a guide to the constructor. Classification of Cements, Chapter V, distinguishes the various cements and limes manufactured in the United States and Europe.

Mr. Spencer B. Newberry, an international authority on the subject treated, has very kindly written for us Chapter VI on the Chemistry of Hydraulic Cement, discussing this complex subject in such a clear and practical manner that it will be of interest not only to the scientist, but also to the general reader and to the cement manufacturer. Mr. Newberry has also criticised Chapter V.

Chapters VII and VIII give the latest information on the testing of cement. Chapter IX presents practical rules for selecting sand for mortar, and the effect of different sands and of foreign ingredients upon its quality. Characteristics of the Aggregate are further treated, and practical data in regard to it are given in Chapter X.

The subject of Proportioning Concrete has been treated, at our request, by Mr. William B. Fuller, the concrete expert, and his practical use of mechanical analysis is fully discussed.

The tables of Quantities of Materials for Concrete and Mortar, in Chapter XII, and the diagram of curves, will be found useful in estimating materials.

The Strength of Concrete, Chapter XX, is taken up from a practical standpoint so that the data may be directly employed in design.

The theory and design of reinforced concrete are as yet in an elementary stage, but the rules and tables in Chapter XXI represent the most advanced knowledge on the subject.

Practical methods of Mixing and Laying Concrete are treated in Chapters XIII, XIV and XV.

Mr. René Feret, of Boulogne-sur-Mer, France, whose extended researches enable him to speak with authority, has kindly written for us Chapter XVI, entitled The Effect of Sea Water.

Chapters XVII, XVIII and XIX, on Freezing, Fire and Rust Protection, and Water-Tightness are of practical interest to the contracting engineer.

Plain and Reinforced Concrete Structures are treated in as much detail as space permits in Chapters XXIII to XXVIII inclusive. The designs are taken mostly from original drawings redrawn by the authors. They have been selected, not as extraordinary productions, but because the data in regard to them may be of use in designing similar structures.

Methods of Cement Manufacture in its modern types are described in detail in Chapter XXX.

The References in Chapter XXXI will be found especially valuable to one pursuing more extended investigations than can be presented in a volume of this size.

They have been selected from the large number contained in the authors' index, as those which it may be to the advantage of the reader to consult.

NOTE: The chapter numbers have been changed to agree with the Second Edition.

The articles are usually described by their subject-matter rather than by their titles verbatim.

Appendix I gives the method of chemically analyzing cement and cement materials according to the recommendations of the American Chemical Society.

Additional formulas for reinforced concrete beams, too complicated for insertion in the body of the book, are given in Appendix II, these having been kindly compiled by Prof. Frank P. McKibben for this treatise.

The authors desire to express their sincere appreciation of the various kindnesses extended to them while compiling the work. It has been necessary, because of the lack of authoritative information on many fundamental questions, not only to conduct numerous original investigations, but also to correspond with the most prominient engineers in this country, and with experts in England, France, and Austria.

Mr. Feret, besides writing the chapter on The Effect of Sea Water, has kindly criticised Chapter IX, and made numerous suggestions which have been incorporated.

Mr. Fuller has examined and criticised all the chapters on practical construction, and Prof. McKibben has rendered material assistance in the line of investigations and criticisms relating to the theories of reinforced concrete.

The authors are indebted to many gentlemen for careful criticism of chapters or portions of chapters, for drawings, or for replies to questions, and take this opportunity to express their sincere appreciation of all such assistance. Among those to whom especial acknowledgment is due are the following:

Messrs. Earle C. Bacon, David B. Butler (England), Harry T. Buttolph, Howard A. Carson, Edwin C. Eckel, William E. Foss, George B. Francis, John R. Freeman, Charles S. Gowen, Allen Hazen, Rudolph Hering, James E. Howard, Richard L. Humphrey, A. L. Johnson, George A. Kimball, Robert W. Lesley, Alfred Noble, William Barclay Parsons, Henry H. Quimby, George W. Rafter, Ernest L. Ransome, Clifford Richardson, Thomas F. Richardson, A. E. Schutté, W. Purves Taylor, Edwin Thacher, Leonard C. Wason, George S. Webster, Robert Spurr Weston, Joseph R. Worcester; and Professors Ira O. Baker, Lewis J. Johnson, Edgar B. Kay, Gaetano Lanza, Charles L. Norton, Charles M. Spofford, George F. Swain, Arthur N. Talbot.

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Machine Works, Fairbanks Co., Falkenau-Sinclair Machine Co., Farrel Foundry and Machine Co., Iroquois Iron Works, Kent Mill Co., Link-Belt Engineering Co., McKelvey Concrete Machinery Co., W. F. Mosher & Son, Tinius Olsen and Co., Philadelphia Pneumatic Tool Co , Thos. Prosser and Son, Ransome Concrete Machinery Co., Riehlé Bros. Testing Machine Co., Robins Conveying Belt Co., Sherburne and Co., T. L. Smith, Henry Troemner, Tucker and Vinton.

FREDERICK W. TAYLOR.
SANFORD E. THOMPSON.

February, 1905.

The writer wishes to state that the investigation and study necessary for the writing of this book were done by his colleague, Mr. Thompson, and desires that full credit for this should be given to him.

FREDERICK W. TAYLOR.

PREFACE TO SECOND EDITION

The second edition aims to cover the developments in the design and construction of reinforced concrete since the issue of the first edition. To accomplish this, more than 200 pages of entirely new and original text and tables have been added, giving to the constructing engineer, the architect, and the contractor data for design and for building, and to the student a comprehensive and practical text and reference book.

One of the principal objects also in writing and in revising the book has been to make it useful to those men who are practically engaged in this class of work and yet who are unable to devote enough of their time to make either a profound or an original study of it. Attention is directed to the new Chapter I, in which many of the essentials of concrete construction are pointed out and the reader is warned against the serious errors that have frequently been made in this field.

The chapter on Reinforced Concrete Design, which is increased from 51 to 131 pages, includes a comprehensive statement of the details of design. Features of special interest in this chapter are the treatment of column design; the discussion of shear and diagonal tension; the design of the supports of beams and girders; the treatment of bending moments; the design of flat plates; the most recent tests on hooked bars; the analysis of shrinkage and temperature reinforcement; and careful notes relating to many smaller though not less important details. Tables and diagrams for design covering over 20 pages are prepared for office use. A complete example of floor design gives the mathematical computations in detail for all the parts of the several members.

In subsequent chapters are treated the designs of retaining walls, footings, culverts, and chimneys.

Prof. Frank P. McKibben has kindly prepared the chapter on Arches, which presents the design of the arch by the elastic theory and gives a complete example with all the steps to be followed.

In Chapter XXIX brief reference is made to a variety of structures in which concrete is employed as the building material.

Prominent among the changes in the first part of the book, which is devoted to plain concrete, are the revised Specifications for Cement and Con-

crete in Chapter III; Chapter IX on Proportioning; the enlargement of Chapters XIV and XV on Mixing and Depositing; the addition on pages 236 and 237 of tables for quantities of materials for rubble concrete; and the insertion of the most recent tests and conclusions on the strength and permeability of concrete. The list of references in Chapter XXXI has been increased over fifty per cent, new references having been carefully selected from the immense quantity of current literature published since the first issue of our book.

The large increase in the quantity of material has necessitated a rearrangement of the matter and beyond page 235 the pages have been renumbered. To simplify the formulas, the demonstrations have been placed as far as possible in footnotes or appendices. By the use of a thinner but higher quality of paper the book is increased but slightly in size.

The authors desire to express their appreciation of assistance rendered in the work of revision. Special acknowledgment is due to Messrs. E. D. Boyer, R. D. Bradbury, William B. Fuller, Frank P. McKibben, Spencer B. Newberry, George F. Swain, Arthur N. Talbot, and Joseph R. Worcester; also to Mr. Edward Smulski for his original studies for the matter on Reinforced Concrete Design.

September, 1909

FREDERICK W. TAYLOR.
SANFORD E. THOMPSON.

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A Treatise on Concrete

CHAPTER I.

ESSENTIAL ELEMENTS IN CONCRETE CONSTRUCTION

The forming of concrete structures is essentially a manufacturing operation, and requires more close attention to detail both in the design and the building than most other classes of construction. For the benefit of those who are not thoroughly experienced, a number of the most essential elements are recorded below with references to pages upon which more detailed information may be obtained

General properties of materials and of concrete are outlined in Chapter Ia on Concrete Data, and Chapter II, page 11, gives in elementary form an outline of the process of concreting

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CHAPTER I*a*

CONCRETE DATA

DEFINITIONS

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Aggregate is the inert material, such as sand, broken stone, etc., with which the cement or other adhesive material is mixed to form concrete or mortar. The term is sometimes erroneously applied to the coarse material, such as broken stone, only.

Akron Cement is a Natural cement from the vicinity of Akron, N. Y. 49

Beton is the French word for concrete.

Beton-Coignet is a mixture of hydraulic lime, cement, and sand 42

Concrete* is an artificial stone made by mixing cement, or some similar material — which after mixing with water will set or harden so as to adhere to inert material, -- and an aggregate composed of hard, inert particles of varying size, such as a combination of sand or broken stone screenings, with gravel, broken stone, cinders, broken brick, or other coarse material.

Concrete Rubble is masonry of large stones, usually of derrick size, with joints of concrete instead of mortar 296

Density represents the ratio of the sum of the volumes or mass of the particles, or absolutely solid substance, of a material contained in a measured unit volume to the total measured unit volume.. 138*a*

Granolithic is concrete consisting of Portland cement and fine broken stone or sand troweled to form a wearing surface............ 600

Grappiers Cement (*Ciment de grappiers*) is made in France from particles which have escaped disintegration in the manufacture of hydraulic lime.. 50

Hydrated Lime is specially prepared powdered slaked lime........ 53

Hydraulic Lime contains lime and clay in such proportions that it hardens under water 52

James River Cement is a Natural cement from the James River Valley 49

Laitance is decomposed cement formed in the presence of an excess of water 302

Laitier Cement (*Ciment de laitier*) is the French name for Puzzolan or slag cement... 50

*Also applied to mixtures of an aggregate with a material such as asphalt -- which liquefies on application of heat.

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Lime of Teil (*Chaux du Teil*) is a celebrated hydraulic lime of France 52

Louisville Cement is a Natural cement from the vicinity of Louisville, Ky. 49

Mortar is a mixture of cement or lime and sand or other fine aggregate having water added so as to make it like a paste.

Natural Cement is made from natural rock containing the required constituents in approximately uniform proportions.......... 49

Parker's Cement is a term sometimes used in England for Natural or Roman cement 49

Paste is a mixture of neat, *i.e.*, pure, cement or lime with water.

Portland Cement is made from an artificial mixture of materials containing lime and clay.......... 48

Puzzolan Cement is a mechanical mixture of slaked lime with blast furnace slag, or with natural puzzolanic matter, such as volcanic ash.......... 50

Reinforced Concrete is concrete in which steel is imbedded to increase its strength.

Roman Cement is the English name for Natural cement.......... 49

Rosendale Cement is a Natural cement from the Rosendale District in eastern New York State 49

Rubble Concrete is concrete in which large stones are placed....... 296

Sand Cement or **Silica Cement** is a mechanical mixture of Portland cement and fine sand.......... 42

Slag Cement is the name sometimes given to Puzzolan cement.... 50

Vassy Cement (*Ciment de Vassy*) is a common French Natural cement 49

Voids are the spaces throughout a mass of concrete, mortar, or paste that are filled with air or water. 135

WEIGHTS AND VOLUMES

Portland Cement weighs per barrel, net..........	376	lb.	29
" " " " bag "	94	"	29
Natural Cement weighs per barrel, net..........	282	"	31
" " " " bag, net	94	"	31
Cement Barrel weighs from 15 to 30 lb., averaging about	20	"	
Portland Cement is assumed in standard proportioning to weigh per cubic foot..........	100	"	217
Packed Portland Cement, as in barrels, averages per cubic foot about	115	"	219
Packed Portland Cement based on 3.5 cubic feet barrel contents weighs per cubic foot	108½	"	

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Loose Portland Cement averages per cubic foot about ...	92 lb.	219
Volume of Cement Barrel, if cement is assumed to weigh 100 lbs. per cubic foot	3.8 cu.ft.	217
American Portland Cement Barrel averages between heads about	3.5 " "	218
Foreign Portland Cement Barrel averages between heads about	3.25 " "	219
Natural Cement Barrel averages between heads about	3.75 " "	
Weight of Paste of neat Portland cement averages per cubic foot about	137 lb.	376
Volume of Paste made from 100 lb. of neat Portland cement averages about	0.86 cu.ft.	229
Volume of Paste made from one barrel of neat Portland cement averages about............................	3.2 " "	229
Weight of Portland Cement Mortar in proportions 1:2½ averages per cubic foot	135 lb.	
Weight of Concrete and Mortar varies with the proportions as well as with the materials of which it is composed		362
Weight of Portland Cement Concrete per cubic foot....		611
Cinder Concrete averages	112 "	
Conglomerate Concrete averages................	150 "	
Gravel Concrete averages......................	150 "	
Limestone Concrete averages..................	148 "	
Sandstone Concrete averages	143 "	
Trap Concrete averages	155 "	
Loose Unrammed Concrete is 5% to 25% lighter than concrete in place, varying with the consistency		277

CEMENT TESTING FOR SMALL PURCHASERS

Soundness. A sound cement will not go to pieces on the work. The test is therefore of greatest importance, and is often the only one necessary. Take about ½ pound, or one cupful, of Portland cement and mix by kneading 1½ minutes with sufficient water to form a paste of a consistency like putty. Press portions of the paste on to 3 pieces of window glass 4 inches square, so as to make 3 pats each about 3 inches in diameter and ½ inch thick at center tapering to a thin edge, and place in moist air for 24 hours. Then keep one pat in air at moderate temperature (about 60° or 70° Fahr.) for 28 days, keep second pat in water for 28 days, and place third pat in loosely closed vessel over boiling water and keep there for five hours. Reject cement if any pats show radial cracks or curl or crumble. The air

pat should not change color. Portland cements may be accepted on the steam test alone if time is limited. Natural cements should be subjected to water and air but not to steam. (See p. 79.)

Fineness. The finer the cement of a certain class the higher is its value. Sift 5 ounces of dry cement containing no lumps through a sieve about 6 to 8 inches diameter with 100 meshes per linear inch. Not more than ½ ounce of either Portland or Natural cement should remain on sieve. To compare quality of two brands otherwise similar, sift through a 200-mesh sieve and choose the finer cement. (See p. 67.)

Setting. A quick-setting cement is difficult to handle on the work and a too slow setting cement delays removal of forms. If a Vicat needle cannot be obtained for testing, use the Gillmore needles, — two steel rods, one, one-twelfth inch diameter at its end, loaded to weigh ¼ pound, the other, one-twenty-fourth inch diameter loaded to weigh 1 pound. A pat of pure Portland cement paste made like the soundness pat must not be able to support the weight of the lighter needle until 30 minutes after mixing, and must support the heavier needle in less than 10 hours. A paste or mortar or concrete has reached its final set when it will support a pressure of the thumb without indenting. (See p. 70.)

Purity. "Provide a glass-stoppered bottle of muriatic acid, two shallow white bowls or two ½-inch by 6-inch test tubes, a glass rod, and a pair of rubber gloves. Put in a bowl or a tube as much cement as can be taken on a nickel 5-cent piece; moisten it with half a teaspoonful of water; cover with clear muriatic acid poured slowly upon the cement while stirring it with the glass rod. Pure Portland cement will effervesce slightly, and will give off some pungent gas and will gradually form a bright yellow jelly without any sediment. Powdered limestone or powdered cement-rock mixed with the pure cement will cause a violent effervescence, the acid boiling and giving off strong fumes until all the carbonate of lime has been consumed, when the bright yellow jelly will form. Powdered sand or quartz or silica mixed with cement will produce no other effect than to remain undissolved as a sediment at the bottom of the yellow jelly. Reject cement which has either of these adulterants."* (See p. 65.)

Tensile Strength. The tensile test is frequently unnecessary with a standard brand of cement employed in ordinary construction. Neat Portland cement should test at least 500 pounds in 7 days and 600 pounds in 28 days. Mixed with three parts standard sand by weight, it should test at least 150 pounds in 7 days and 200 pounds in 28 days. (See p. 30.)

*Judson's City Roads and Pavements, 1902.

Specific Gravity. The test requires delicate apparatus and is seldom necessary. Specific gravity of Portland cement should exceed 3.1. (See p. 30.)

Magnesia must not exceed 4%. (See p. 30.)

Sulphuric Anhydride must not exceed 1.75%. (See p. 30.)

Color is no indication of quality. (See p. 113.)

Weight is no indication of quality. (See p. 114.)

PROPERTIES OF SAND AND SCREENINGS

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Sharpness of grain is not necessary 154a

Quality of sand is chiefly dependent upon the coarseness and relative coarseness of its grains 147

Clay or **Loam** in sand is sometimes injurious to mortars because introducing too much fine material, while in other cases it may be beneficial because the fine material is needed 154a

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Voids in sand cannot be accurately determined by pouring water into it, but can be found by weighing the sand and finding its moisture 165

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Coarse Sand requires less water than fine sand, and when mixed with cement makes a denser mortar 216

Fine Sand with grains of uniform size weighs nearly the same when dry and has nearly the same percentage of voids as screened coarse sand. Fine sand with ordinary moisture is, on the other hand, lighter and more porous than coarse sand 170

Mixed Sand usually weighs more and contains a smaller volume of voids than coarse or fine sand 171

PROPERTIES OF COARSE OR MIXED AGGREGATE

Equal Spheres if symmetrically piled in the theoretically most compact manner would have 26% voids, but by experiment it is found that in practice it is impossible to pile them so as to get below 44% voids 169

Voids are approximately equal in the different portions of a dry material which has been screened to uniform sizes 170

Smallest Percentage of Voids occurs in a mixture of sizes so graded that the voids of each size are filled with the largest particles which will enter them 171

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STRENGTH OF CONCRETE AND MORTAR

With the same Aggregate the strength and water-tightness of a concrete or mortar increases as the percentage of cement in a unit volume of mortar or concrete is increased 133

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Sharp Sand produces but slightly stronger mortar than rounded sand 154*a*

Coarse Sand produces stronger and usually more impervious mortar than fine sand 147

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Fine Sand always produces mortars of lower strength than coarse sand 147

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Clay or **Loam** in the sand is apt to weaken rich mortars and strengthen lean mortars 154a

Gravel vs. Broken Stone Concrete. The difference in quality is so slight that usually the cheaper material may be selected. Gravel concrete, because of the smooth, rounded surfaces, appears from tests to be weaker than broken stone concrete if the sizes of particles in the two cases are alike, but a gravel mixture may require less cement because of better gradation of sizes of particles 387

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High Working Strength in Steel permits low percentage of steel.. 519

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Ultimate Strength of Portland cement concrete and mortar appears to be but slightly, if at all, affected by freezing............... 321
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Protection of Steel requires ½ inch to 2 inches of concrete	333
Cinders do not corrode metal	329

DATA ON HANDLING CONCRETE

Average load of broken stone or gravel for wood wheelbarrow	2.4	cu. ft.
" " " sand for wood wheelbarrow	2.5	" "
Large load of broken stone or gravel for iron wheelbarrow on short haul in concrete work	3.0	" "
Large load of sand for iron wheelbarrow on short haul in concrete work	3.5	" "
Average load of ordinary concrete* for iron wheelbarrow	1.9	" "
Large " " " " " " "	2.2	" "
Number of shovelfuls of concrete per barrow in average load	13	
" " " " " " " " large "	15	
Average net time of one man filling wheelbarrow with concrete,	1⅓	min.
Quick " " " " " " " " "	1	"
Average quantity concrete* mixed, wheeled 50 ft., and rammed, per man, per day of 10 hours†	2.2	cu. yd.
Large quantity concrete* mixed, wheeled 50 ft. and rammed, per man, per day of 10 hours†	3	" "
Average quantity concrete* laid as above with a gang of 15 men per day of 10 hours†	33	" "
Large quantity concrete* laid as above with a gang of 15 men per day of 10 hours†	47	" "
Approximate average quantity of concrete* leveled and rammed in 6-inch layers, per man, per day of 10 hours	11	" "
Approximate large quantity of concrete* leveled and rammed in 6-inch layers, per man, per day of 10 hours	16	" "
Approximate average surface of rough braced plank form built and removed by one carpenter per day of 10 hours	25	sq. "

CHANGING FOREIGN TO AMERICAN MEASURES

To convert values of kilograms per square centimeter to pounds per square inch, multiply the former by 14.2 (more exactly 14.2234).

To convert values of pounds per square inch to kilograms per square centimeter, multiply the former by 0.07 (more exactly, 0.07031).

*All measurements of concrete are reduced to terms of quantity in place after ramming.
†Note that the leveling and ramming, but not the labor on form, are included in this item.

To convert values of pounds per square inch to tons (2,000 lb.) per square foot, divide the former by 14 (more exactly 13.89)

To convert Centigrade to Fahrenheit temperatures, multiply the former by 1.8 and add 32° to the product.

To convert Fahrenheit to Centigrade temperature, deduct 32° from the former and divide by 1.8.

One millimeter = 0.0394 inch
One centimeter = 0.3937 "
One meter = 39.37 inches or 3.281 feet
One square centimeter = 0.155 square inch
One " meter = 10.764 square feet or 1.196 square yards
One cubic centimeter = 0.061 cubic inch
One " meter = 35.31 cubic feet, or 1.308 cubic yards
One liter = 61.02 cubic inches or 0.0353 cubic foot, or 1.057 U. S. liquid quarts or 0.2642 U. S. liquid gallon
One gram = 0.0353 avoirdupois ounce
500 grams = 1.1 pounds avoirdupois
One kilogram = 2.2046 pounds avoirdupois
One tonne or metric ton = 2204.62 pounds or 1.1023 tons (of 2,000 lb.)
One English penny = $0.0203
One " shilling = $0.2433
One " pound = $4.8665
One French franc = $0.193
One German mark = $0.238

CHAPTER II

ELEMENTARY OUTLINE OF THE PROCESS OF CONCRETING

This chapter is not written for experienced civil engineers and contractors, nor for those who desire to make a scientific study of methods and principles. On the contrary, it is merely an elementary outline, indicating to the inexperienced the various steps which must be taken with this class of masonry. In subsequent chapters the various divisions of the subject are treated in detail.

The question as to whether concrete is preferable to some other form of masonry may often resolve itself into a question of cost. The cost, in turn, is dependent upon the character of the structure, the rate of labor and the price of the various materials entering into the work. Portland cement concrete has been laid in large masses at as low a price as $3 per cubic yard, while for thin walls built under disadvantageous conditions the cost of constructing molds may cause it to run as high as $30 per cubic yard, and in the case of ornamental work even above this. Before estimating the cost in any case, the materials must be chosen and the relative proportions of the ingredients determined from a consideration of the design of the structure.

WHERE CONCRETE MAY BE USED

By far the largest proportion of Portland cement concrete is laid in heavy foundation work and in other structures, such as tunnels and subways, below the surface of the ground. It is peculiarly adapted for foundations of engines or machinery, heavy walls, piers, etc. In the former the concrete is often carried all the way up to the base of the engine or machine, instead of being topped with brick or stone. It is widely used for sidewalks or floors upon the ground level, and for suspended floors. When suitably reinforced with steel, it furnishes probably the most economical and effective material for fire-proof construction. Its use for walls of buildings is largely increasing, but on account of the very indefinite time required in the building and moving of forms the cost may largely exceed the original estimate unless the builder is experienced in this class of work. Under favorable conditions, however, a 6-inch wall of concrete will cost no more, and usually less, than a 12-inch wall of brick work, and will be

stronger, more durable, and fire-proof. The strength of concrete columns and beams is readily calculated by means of formulas.

Concrete is destined to be used to a large extent in the construction of tanks and vats for holding various liquids which attack wood and iron. Their construction is comparatively simple, but the work must be carefully performed if the result is to be permanent and satisfactory. Concrete is especially suitable for all kinds of arches, because the stresses therein are chiefly compressive. Other classes of work for which concrete is largely employed are dams, retaining walls, penstocks, bridges, abutments, sewer and water conduits, and reservoirs. For ornamental work developments are constantly being made, and it is noteworthy that concrete or mortar can be cast in molds in a somewhat similar manner to that in which plaster of Paris is run for interior decoration.

SELECTION OF MATERIALS

Concrete is ordinarily composed of cement, sand, gravel or crushed stone, or both, and water. The selection of each of these materials is largely dependent upon local conditions, and no unalterable rule can be laid down in regard to it, but certain general conditions may serve as a guide to the inexperienced.

Cement. It is a wise rule to use Portland cement for nearly all classes of concrete, and the remarks in this chapter are based entirely upon this material. Portland cement is more uniform and therefore more reliable, while its strength is so much higher than Natural cement that by mixing it with larger proportions of sand and stone, properly graded, it will usually yield better results at less cost than Natural cement.

If the job is small and unimportant, it is generally safe to select in the market a brand of Portland cement of American manufacture which has a first-class reputation, and to use it without testing. As a precaution, however, it is usually advisable that samples from a few of the packages of every shipment be tested for soundness. This can be done after a little practice with scarcely any apparatus. (See p. 79.) For very important concrete construction complete specifications should be prepared before purchasing the cement, and a small laboratory established for conducting tests to determine whether it is fulfilling the requirements. (See p. 28.)

Aggregate. The sand and broken stone or gravel are termed the aggregate. The sand should be clean. One may obtain some idea of its cleanliness by placing it in the palm of one hand and rubbing it with the fingers of the other. If the sand is dirty, it will badly discolor the palm,

If the use of dirty sand is unavoidable, its effect upon the strength of the mortar should be investigated. Preference should be given to sand containing a mixture of coarse and fine grains. Extremely fine sand can be used alone, but it makes a weaker mortar than either coarse sand alone or a mixture of coarse and fine sand.

Either crushed stone or clean gravel, or both, is suitable for the coarse material of the aggregate. It is chiefly a question of which can be delivered upon the work at the least cost. If the gravel is chosen greater uniformity is attained by screening it over, say a ⅜-inch mesh screen, and then remixing the sand which falls through the screen with the coarser gravel in definite proportions, than by taking the run of the bank. If the gravel is dirty or clayey it should be washed with a hose, a little at a time, before it is shoveled on to the mixing platform.

Broken stone, if selected, may be used unscreened as it comes from the crusher, although it is preferable to screen out the dust and to use the latter as a portion of the sand. The maximum size is usually limited to 2½ inches. A smaller size than this, say one inch, will give, with less care, a finer surface. In a thick wall large sound stones may be placed by hand or derrick without detriment to the work, providing the consistency of the concrete is thin enough to properly imbed them.

PROPORTIONS

Accurate methods of proportioning the cement and aggregate in concrete are discussed in chapter XI, page 183, and if a large or very important mass is under consideration, or if the work must be water-tight, the correct proportioning requires more careful consideration than can be given it in this chapter. The method often adopted of pouring water into the coarser material to determine the percentage of voids, and thus finding the quantity of sand to use for filling them, is apt to be misleading, because so much depends upon the compactness of the stone, due to the method of handling it — that is, whether placed quietly, dropped, thrown, or shaken down — and because in the majority of cases the sand contains many grains so large that they will not enter the smaller voids of the coarser material. In a small job it is sufficiently accurate to select the proportion of cement to sand which will give the required strength to the concrete, and then use twice as much gravel or broken stone as sand. In figuring the capacities of the measures for the sand and stone it must be remembered that a barrel of Portland cement weighs 376 pounds, not including the barrel, and a bag of Portland cement 94 pounds, and we may assume for convenience

that a cement barrel holds 3.8 cubic feet. This is a fair average measurement of a heaped barrel, or a barrel with both heads removed—a convenient measure for sand.

As a rough guide to the selection of materials for various classes of work, we may take four proportions which differ from each other simply in the relative quantity of cement:

(*a*) **A Rich Mixture** for columns and other structural parts subjected to high stresses or requiring exceptional water-tightness: Proportions 1 : 1½ : 3; that is, one barrel (4 bags) packed Portland cement to 1½ barrels (5.7 cubic feet) loose sand to 3 barrels (11.4 cubic feet) loose gravel or broken stone.

(*b*) **A Standard Mixture** for reinforced floors, beams and columns, for arches, for reinforced engine or machine foundations subject to vibrations, for tanks, sewers, conduits, and other water-tight work: Proportions 1 : 2 : 4; that is, one barrel (4 bags) packed Portland cement to 2 bbl. (7.6 cu. ft.) loose sand to 4 barrels (15.2 cu. ft.) loose gravel or broken stone.

(*c*) **A Medium Mixture** for ordinary machine foundations, retaining walls, abutments, piers, thin foundation walls, building walls, ordinary floors, sidewalks, and sewers with heavy walls: Proportions 1 : 2½ : 5; that is, one barrel (4 bags) packed Portland cement to 2½ barrels (9.5 cu. ft.) loose sand to 5 barrels (19 cu. ft.) loose gravel or broken stone.

(*d*) **A Lean Mixture** for unimportant work in masses, for heavy walls, for large foundations supporting a stationary load, and for backing for stone masonry: Proportions 1 : 3 : 6; that is, one barrel (4 bags) packed Portland cement to 3 barrels (11.4 cu. ft.) loose sand to 6 barrels (22.8 cu. ft.) loose gravel or broken stone.

The above specifications are based upon fair average practice. If the aggregate is carefully graded and the proportions are scientifically fixed, smaller proportions of cement may be used for each class of work.

QUANTITIES OF MATERIAL

Inexperienced contractors have often lost money by assuming that the quantity of gravel plus the quantity of sand required will be equivalent to the volume of the finished concrete — that is, that 7½ cubic yards of concrete in the proportions of 1: 2½: 5 will require 2½ cubic yards of sand and 5 cubic yards of gravel. This is absolutely wrong, since the grains of sand fill, to a certain extent, the spaces between the larger pebbles. It is incorrect, on the other hand, to figure a quantity of gravel equal to the total

volume of the concrete, because the introduction of the mortar, which is always in excess of the actual voids, swells the bulk.

If gravel or stone having particles of uniform size is used it must be recognized that the work will cost from 5 to 10 per cent. more, on account of the additional quantity of material required to make a given volume of concrete. In measuring the gravel or stone before mixing there will be less solid matter in a measure, and consequently more sand and cement will be necessary to fill the spaces between the stones. This fact, which is often overlooked even by experienced men, is illustrated in a somewhat exaggerated fashion in Figs. 1 and 2. Here Fig. 1 illustrates

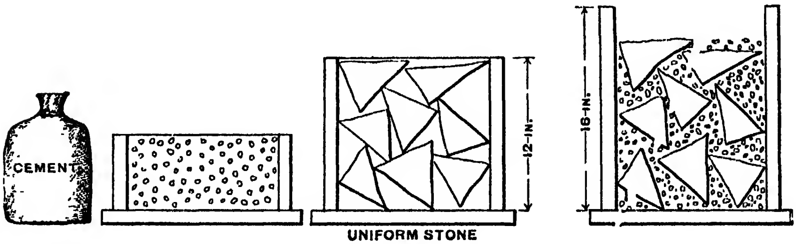


FIG. 1.—Diagram illustrating measurement of Dry Materials and the Mixture when Broken Stone is of uniform size. (*See p.* 15.)

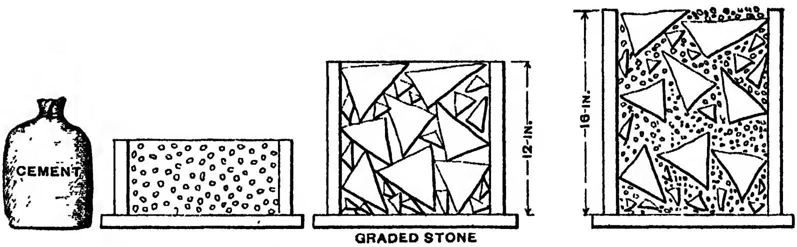


FIG. 2.—Dry Materials and Mixture when the Stone is of varying sizes. (*See p.* 15.)

the measurement of the dry materials and the mixture produced therefrom when the stone has been screened to one uniform size, while Fig. 2 shows the dry materials and the mixture when the stone is what is termed "crusher run" — that is, of varying sizes as it comes from the crusher.

It is obvious at a glance that the uniform stone measured in Fig. 1 contains less solid stone than the graded stone measured in Fig. 2. The spaces between the stones in the first case are very nearly equal to the volume of

the solid particles, and as the measure of the sand is one-half that of the stone, and the particles of cement fill the voids in the sand, this sand and cement mixes in between the stones, filling the spaces or voids, and resulting in a mixture but very slightly greater in volume than the stone alone. In the second case, Fig. 2, the spaces between the large stones in the stone measure are filled with graded smaller stones; so that there is a much smaller volume of spaces or voids. Hence, when the sand and cement, which are identical with that in Fig. 1, are mixed with it the volume of mixture becomes considerably larger than the original bulk of the stone. Consequently, if we start with definite proportions of materials, more concrete will be made with graded stone — such as "crusher run" broken stone, or gravel containing various sizes, ranging, say, from ¼ inch up to 2 inches — than if the stone has been screened to uniform size. If, on the other hand, the proportions of the materials are changed on account of the fewer voids in the mixed stone, and less sand and cement are used, a saving in these materials results.

Fuller's Rule For Quantities—The simplest rule for determining the quantities of materials for a cubic yard of concrete is one devised by William B. Fuller. Expressed in words, it is as follows:

Divide 11 by the sum of the parts of all the ingredients, and the quotient will be the number of barrels of Portland cement required for 1 cubic yard of concrete. The number of barrels of cement thus found, multiplied respectively by the "parts" of sand and stone, will give the number of barrels of each required for 1 cubic yard of concrete, and multiplying these values by 3.8 (the number of cubic feet in a barrel), and dividing by 27 (the number of cubic feet in a cubic yard), will give the quantities of sand and stone, in fractions of a cubic yard, needed for 1 cubic yard of concrete.

To express this rule in the shape of formulas:

Let

c = number of parts cement;

s = number of parts sand;

g = number of parts gravel or broken stone.

Then

$\frac{11}{c+s+g} = P$ = number of barrels Portland cement required for one cubic yard of concrete.

$P \times s \times \frac{3.8}{27}$ = number of cubic yards of sand required for one cubic yard of concrete.

$P \times g \times \frac{3.8}{27}$ = number of cubic yards of stone or gravel required for one cubic yard of concrete.

The following table is made up from Fuller's rule and represents fair averages of all classes of material. The first figure in each proportion represents the unit, or one barrel (4 bags), of packed Portland cement (weighing 376 pounds), the second figure, the number of barrels loose sand (3.8 cubic feet each) per barrel of cement, and the third figure, the number of barrels loose gravel or stone (of 3.8 cubic feet each) per barrel of cement:

Materials for One Cubic Yard of Concrete.

Proportions.	Cement, Barrels.	Sand, Cubic yards.	Gravel or stone, Cubic yards.
1:2:4	1.57	0.44	0.88
1:2½:5	1.29	0.45	0.91
1:3.6	1.10	0.46	0.93
1:4:8	0.85	0.48	0.96

If the coarse material is broken stone screened to uniform size it will, as is stated above, contain less solid matter in a given volume than an average stone, and about 5 per cent. must be added to the quantities of *all* the materials. If the coarse material contains a large variety of sizes so as to be quite dense, about 5 per cent. may be deducted from all of the quantities.

Example.—What materials will be required for six machine foundations, each 5 feet square at the ottom, 4 feet square at the top, and 8 feet high?

Answer. — Each pier contains 163 cubic feet, and the six piers therefore contain $\frac{6 \times 163}{27} = 36.2$ cubic yards. If we select proportions 1:2½:5, we find, multiplying the total volume by the quantities given in the table, that there will be requ ed, in round numbers, 47 barrels packed cement, 16½ cubic yards loose sa d, 33 cubic yards loose gravel.

TOOLS AND APPARATUS REQUIRED FOR CONCRETE WORK

The quantity of tools will, of course, vary with the size of the gang. The following schedule is based upon a small gang of eight or ten men, making concrete by hand:

Eight square pointed shovels, size No. 3, and such as illustrated in Fig. 3, page 18. (If a very wet mixture is used substitute small coal scoops.)

Three iron wheelbarrows, Fig. 4, page 18.

Two rammers, Figs. 99, 100, or 101, pages 281 and 282.

One mixing platform, about 15 feet square, built so substantially that it can be moved without coming to pieces, and having a 2 by 3-inch strip around the edge to prevent waste of materials and water. On a small job this may be of 1-inch stuff, resting on joists about 3 feet apart, provided it is stiffened by being tongued and grooved.

Fig. 3.—Square Pointed Shovel. (*See p.* 17.)

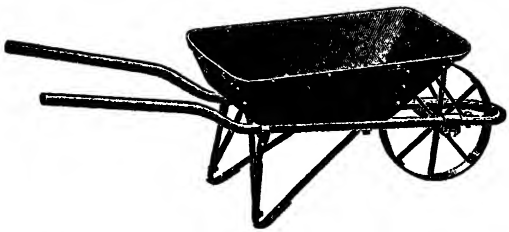


Fig. 4.—Concrete Wheelbarrow. (*See p.* 17.)

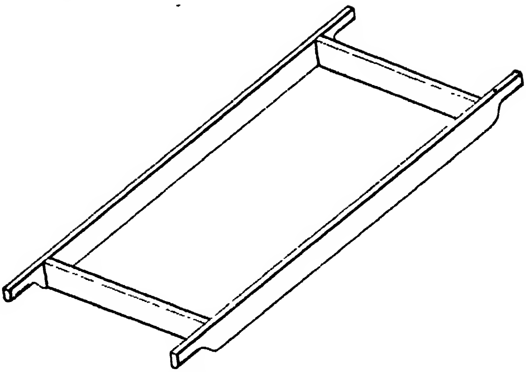


Fig. 5.—Measuring Box for Gravel. (*See p.* 18.)

One measuring box or barrel for sand, of a capacity for one batch of concrete. A convenient measure is a cement barrel, either whole or sawed in two, with both heads removed. It is filled and then lifted in such a manner as to spread the sand.

One measuring box for gravel (see Fig. 5) of a capacity for one batch of concrete.

Lumber for making and bracing forms.
Nails and, for some kinds of work, bolts, for forms.

CONSTRUCTION OF FORMS

Green spruce or fir lumber is suitable for forms. If a smooth face is required the surface of the boards or plank next to the concrete must be dressed and the edges tongued and grooved or beveled. The forms must be nearly water-tight. The sheeting, which is usually laid horizontal, may be 1 inch, 1½ inch or 2 inches thick, the distance apart of the studding being governed by the thickness selected. The studs must be placed not more than 2 feet apart for 1-inch sheeting nor more than 5 feet apart for 2-inch sheeting. They must be securely braced so as to withstand the pressure of the soft concrete and of the puddling or ramming.

The lower portion of a foundation wall in a trench excavated in earth so stiff as to stand nearly vertical may sometimes be laid with no form at all, and then narrowed in at the top to the required thickness, but if the sides of the trench are sloping it is generally cheaper to save concrete material by carrying the forms to the bottom. A thin wall may be

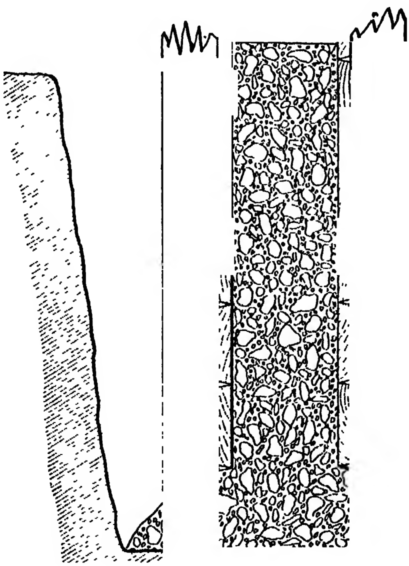


FIG. 6.—Construction of Form when Base of Wall is Spread. (See p. 19.)

greatly strengthened by spreading the base, which is readily accomplished by starting the boards or plank 6 or 8 inches above the bottom of the excavation and allowing the soft concrete to flow out under them on both sides of the wall so as to make footings, as shown in Fig. 6. The studs may run to the bottom, as indicated by the dotted lines, but should be tapered and greased so that they may be withdrawn without injury to the concrete.

For all walls under 9 or 10 inches in thickness, small steel rods $\frac{1}{4}$ or $\frac{3}{8}$ inch in diameter, spaced about 18 inches apart, will greatly increase the stiffness and add to the safety of the structure, especially while the concrete is hardening.

Forms must be left in place for three or four weeks if there is earth or water pressure against the wall. If, on the other hand, there is no strain upon it, 24 hours setting, or until the concrete will stand the pressure of the thumb without indentation, is sufficient.

Further descriptions of form construction and methods of facing are given in Chapter XV. Forms for special structures are described and illustrated in subsequent chapters treating of concrete design.

MIXING AND LAYING CONCRETE

The advisability of employing machinery for mixing the concrete depends chiefly upon the quantity to be laid. On a small job the first cost of mixing machinery and the running expenses, such as the labor of the engineman, which continue when the machine is idle, may bring the cost of machine-mixed concrete higher than hand mixed. The decision may be based entirely upon the cost per cubic yard of concrete laid, provided a first-class machine is employed, since good concrete can be made either by machine or by hand. The various types of concrete mixers and the methods of employing them are discussed in Chapter XIV.

The foreman for a gang of concrete mixers need not be of great intelligence, but must be one who will obey orders strictly, and know how to keep all of his men constantly busy. The amount of work turned out will depend to quite an extent on the arrangement of the gang, whether each man has certain definite operations to perform over and over again, and whether these operations fit into the work of the rest of the gang so that none of the men have idle moments.

A gang of at least 6 men besides the foreman is required even on small work, while as many as 23 men may be effectively employed. In addition to these, an inspector is generally necessary to watch the placing of the

concrete and see that the mixture is uniform and of proper consistency. Cheap laborers, as for instance Italians, make good men for mixing and transporting the concrete.

The materials for the concrete ought, of course, to be deposited as near the work as possible. The cement, whether it comes in bags or barrels, must be sheltered from the rain. Covering with plank is insufficient. Bags should be protected from moist atmosphere; a cellar is likely to be too damp. To keep the sand and stone as near the mixing platform as possible, it may be advantageous to haul the materials as they are required from day to day. If the sand or stone pile is at any time farther from the measuring boxes than a man can profitably throw with shovels without walking, say more than 8 or 10 feet, do not hesitate to have it loaded into wheelbarrows and dumped into the measuring boxes. Materials can be wheeled in barrows to a distance of 10 to 25 feet from the platform at about the same cost that they can be shoveled direct with a long throw.

There are many methods of mixing concrete by hand, as discussed in Chapter XIV, all of which with care produce good work. For the convenience of the inexperienced the following directions for the work of a small gang of six men with foremen may be useful. They are given merely for illustration, and must be more or less varied to suit local circumstances.

Directions for Mixing Concrete. Assume a gang of four men to wheel and mix the concrete, with two other men to look after the placing and ramming.

When starting a batch, two mixers shovel or wheel sand into the measuring box or barrel — which should have no bottom or top — level it and lift off the measure, leveling the sand still further if necessary. They then empty the cement on top of the sand, level it to a layer of even thickness, and turn the dry sand and cement with shovels three times, as described below, after which the mixture should be of uniform color.

While these two men are mixing sand and cement, the other two fill the gravel measure about half full, then the two sand men take hold with them, and complete filling it. The gravel measure is lifted, the gravel hollowed out slightly in the center, and the mixture of sand and cement shoveled on top in a layer of nearly even thickness.* A definite number of pails are filled with water, and poured directly on the top of these layers, greater uniformity being thus attained than by adding the water directly from a hose. After soaking in slightly the mass is ready for turning.

*Some engineers prefer to spread the stone on top of the sand and cement, while others prefer to mix the water with the sand and cement before adding them to the stone.

The method illustrated in Fig. 7 of turning with shovels materials which have already been spread in layers is as follows:

Two men, *a* and *b*, with square pointed shovels, stand facing each other at one end of the pile to be turned, one working right-handed and the other left-handed. Each man pushes his shovel along the platform under the pile, lifts the shovelful, turns with it, and then, turning the shovel completely over, and with a spreading motion drawing the shovel toward himself, deposits the material about 2 feet from its original position. Repetitions of this operation will form a flat ridge of the material, on a line with the pile as it originally lay, and flat enough so that the stones will not roll. As soon as, but not before, a single ridge is complete, two other men, *c* and *d*, should start upon this ridge, turning the materials for the second time, as shown in the illustration, and forming as before a flat ridge and finally a level pile which gradually replaces the last. A third mixing is accomplished in a similar way.

Fig. 7 gives the position of the piles as the concrete is being turned.

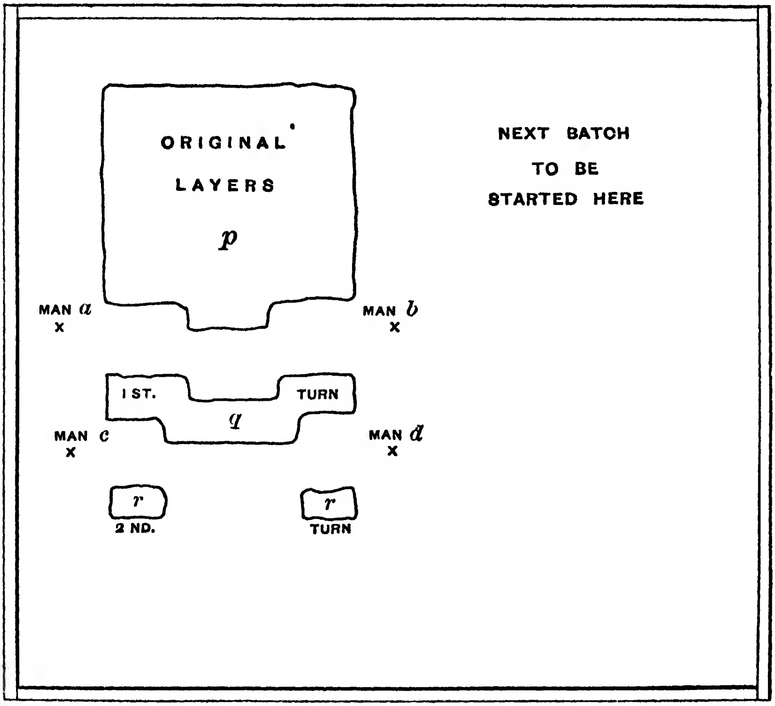


FIG. 7.—Position of Men and Concrete on Platform while Turning. (*See p.* 22.)

A portion of the original layers is shown at *p*, the ridge formed by men *a* and *b* shoveling from pile *p* is shown at *q*, and the beginning of the ridge formed by men *c* and *d* is shown at *rr*. The third turning is not shown.

The quantity of water used must be varied according to the moisture in the materials and the consistency required in the concrete. While the opinions of engineers regarding the proper consistency vary widely, it is advisable, the authors believe, for an inexperienced gang to use an excess of water. The rule may be made in hand mixing to use as much water as can be thoroughly incorporated with the materials. Concrete thus made will be so soft or "mushy" that it will fall off the shovel unless handled quickly.

After the material has been turned twice, as described, and as soon as the third turning has been commenced, two of the mixers who have finished turning may load the concrete into barrows and wheel to place. They should fill their own barrows, and after the mass has been completely turned for the third time by the other two men the latter should start filling the gravel measure for the next batch.

If the concrete is not wheeled over 50 feet, four experienced men ought to mix and wheel on the average about 10½ batches in ten hours. This figure is based on proportions 1 : 2½ : 5, and assumes that a batch consists of one barrel (four bags) Portland cement with 9.5 cubic feet of sand and 19 cubic feet of gravel or stone.

Assuming, as given on page 17, that 1.29 barrels of cement are required for 1 cubic yard of concrete, one barrel of cement — that is, one batch — will make 0.78 cubic yard of concrete; hence 10½ batches mixed and wheeled by four men in ten hours are equivalent to 8¼ cubic yards of concrete. This is for the very simplest kind of concreting and makes no allowance for the labor of supplying materials to the mixing platform or for building forms.

Placing Concrete. The concrete may be transported and handled by any means which will not cause the materials to separate. If mixed wet it may be dropped directly from shovels or barrows to place, or it may be run down an inclined pipe or chute. The layers should be about 6 inches thick. For a dry or a jelly-like mixture common square ended rammers are employed and the mass must be rammed until the mortar flushes to the surface. Wet concrete must be merely puddled or "joggled" to expel the air and surplus water. Before placing a fresh layer upon work which has set, the surface must be cleaned of dirt and scum, and thoroughly wet.

The placing of concrete and the kinds of rammers for different classes of work are discussed more at length in Chapter XV.

APPROXIMATE COST OF CONCRETE

The cost of concrete depends more upon the character of the construction and the conditions which govern it than upon the first cost of the materials. In a very general way, we may say that when laid in large masses or in a very heavy wall, so that the construction of the forms is relatively a small item, the cost per cubic yard in place is likely to range from $4 to $7. The lower figure represents contract work under favorable conditions with low prices for materials, and the higher figure small jobs and inexperienced men. Similarly, we may say that for sewers and arches, where centering is required, the price may range from $7 to $14 per cubic yard. Thin building walls under eight inches thick may cost from $10 to $20 per cubic yard, according to the character of construction and the finish which is given to the surface.

These ranges in price seem enormous for a material which is ordinarily supposed to be handled by unskilled labor, but it must be borne in mind that skilled workmen are required for constructing forms and centers, and often the labor upon these may be several times that of mixing and placing the concrete. As a rule, unless the job is a very small one or under the personal supervision of a competent engineer, it is cheaper and more satisfactory to employ an experienced contractor than day labor. Green men under an inexperienced foreman may not be counted upon to mix and lay over one-half the amount of concrete that will be handled by a skilled gang under expert superintendence.

A close estimate of cost may be reached, in cases where the conditions are known in advance, by taking up in detail and then combining the various units of the material and labor as outlined below.

Cost of Cement. As the price of Portland cement varies largely with the demand, it is necessary to obtain quotations from dealers for every purchase. It is such heavy stuff that the freight usually enters largely into the cost, and quotations should therefore be made f.o.b. the nearest point of delivery to the work. The cost of hauling by wagon may be readily estimated by assuming that a barrel of cement weighs 400 pounds (gross), and that a pair of horses will haul over an average country road a load of, say, 5 000 pounds, traveling in all a distance of 20 to 25 miles in a day, that is, 10 to 12½ miles with load. This assumes, of course, that the teams are good and properly handled.

Having found the cost of the cement per barrel, delivered, the approximate cost per cubic yard is at once obtained from the table on page 17. If, for example, the cost is \$2 per barrel and proportions 1: 2½ : 5 are selected, the cost of the cement per cubic yard of concrete will be 1.29 × \$2.00 = \$2.58.

Cost of Sand. The cost of sand depends chiefly upon the distance hauled. With labor at 15 cents per hour, the cost of loading (including the cost of the cart waiting at pit) may be estimated, if handled in large quantities, at 18 cents per cubic yard, or on a small job at 27 cents per cubic yard. For hauling add one cent for each 100 feet of distance from the pit. The additional cost of screening, if required, will vary with the coarseness of the material, but 15 cents per cubic yard may be called an average price for this, unless the sand is obtained by screening the gravel, when no allowance need be made. After finding the cost of one cubic yard of sand, the cost of the sand per cubic yard of concrete is readily figured from the table referred to. If, for example, the cost of sand screened, loaded and hauled 1 000 feet is 52 cents per cubic yard, the cost per cubic yard of concrete for proportions 1 : 2½ : 5 will be 0.45 × \$0.52 = \$0.23½.

Cost of Gravel or Broken Stone. If broken stone is used upon a small job for the coarse aggregate, it is usually purchased by the ton or cubic yard. A 2000 lb ton of broken stone may be considered as averaging approximately 0.9 cubic yards, although differences in specific gravity cause considerable variation. A two horse load is generally considered 1½ to 2 yards, the latter quantity requiring very high sideboards. The cost of screening gravel, if this is necessary, while a very variable item, may be estimated at 35 cents per cubic yard. The cost of loading gravel into double carts, with labor at 15 cents per hour, may be estimated on a small job at 38 cents per cubic yard. If handled in large quantities, 25 cents is an average cost. The cost of loading includes loosening and also the cost of the cart waiting at the pit. Hauling costs about one cent per cubic yard additional for each 100 feet of distance hauled under load. If, to illustrate, the cost of gravel picked, screened, loaded and hauled 1 000 feet is 83 cents per cubic yard, the cost of the gravel per cubic yard of concrete for proportions 1 : 2½ : 5 will be 0.91 × \$0.83 = \$0.75½.

For distances up to 300 feet both sand and gravel can be hauled more economically by wheelbarrows than by teams. The cost of loading wheelbarrows is about half the cost of loading carts, while the cost of hauling with barrows per 100 feet is about four times greater.

Cost of Labor. With an experienced gang working at the rate of 15

cents per hour, the cost of mixing and laying concrete, if shoveled directly to place from the mixing platform, will average about 80 cents per cubic yard, in addition to the work on forms. If, as is usually the case, the concrete is wheeled in barrows, 9 cents per cubic yard must be added to the above price for the first 25 feet that the barrows are wheeled under load, and $1\frac{1}{2}$ cents for each additional 25 feet wheeled. With other rates of wages, the cost may be considered as proportional. With a green gang, the cost will be nearly double the above figures, but as the men become worked in and the organization perfected, the cost should approximate more nearly the prices given.

In building construction where the material is mixed by machinery and hoisted to place, there are numerous incidental expenses and delays, so that it is not safe to figure the cost of labor for simply mixing and laying the concrete under ordinarily good conditions at less than $1.50 to $2.00 per cubic yard. The cost of materials must be added to this, so that the cost of the concrete itself laid in place but *not* including forms nor reinforcement is apt to be about $7.50 per cubic yard. Approximate costs per cubic foot of finished concrete are given in Chapter XXIV.

Cost of Forms. The labor on forms is not included in the above. This is an extremely variable item. The cost of rough plank forms, including labor and lumber for both sides of a 3-foot wall, may be as low as 50 cents per cubic yard of concrete, with other thicknesses of wall in inverse proportion. On elaborate work the price, which is really dependent upon the face area, will reach several dollars per cubic yard of concrete, the cost of the form work, in fact, usually exceeding the cost of the concrete. In building construction, such as a factory six stories in height of symmetrical design, the cost of materials and labor on forms may be estimated at from 9 to 12 cents per square foot of surface of forms. If forms are to be used only once, or if conditions are disadvantageous, these values may be doubled. The costs vary with the price of lumber, the design of the structure, the design of the forms, the character of the supervision, and the skill of the workmen.

Cost of Steel. The cost of bending and placing steel for reinforced concrete is apt to vary from $\frac{1}{2}$ to $1\frac{1}{2}$ ¢ per pound. If, therefore, the cost of the steel is about $40.00 per ton or 2¢ per pound, the cost in place may be estimated at 3¢ per pound.

THE STRENGTH OF CONCRETE

The strength of concrete varies (1) with the quality of the materials; (2) with the quantity of cement contained in a cubic yard of the concrete; and (3) with the density of the mixture.

We may say that the strongest and most economical mixture consists of an aggregate comprising a large variety of sizes of particles, so graded that they fit into each other with the smallest possible volume of spaces or voids, and enough cement to slightly more than fill all of these spaces or voids between the solids of the aggregate. It is obvious that with the same aggregate the strongest cement will make the strongest concrete.

On important construction the various materials to be used should be carefully tested, and specimens of the mixture selected made up in advance and subjected to test. As a guide to the loads which concrete will stand in compression, that is, under vertical loading where the height of the column or mass is not over, say, 12 times the least horizontal dimension, we may give the following approximate figures as safe strengths, after the concrete has set at least one month, for the proportions which have previously been selected in this article as typical mixtures.

The figures, compared with the results of recent experiments on long columns, allow with first-class construction a factor of safety of at least four at the age of one month, or about five and one-half at the age of six months, and are based on conservative practice. The relative strengths of the different mixtures are calculated from original investigations of the authors discussed in Chapter XX.

Safe Strength of Portland Cement Concrete in Direct Compression.

Proportions.	Pounds per square inch.	Tons per square foot.
1 : 1½ : 3	500	36
1 : 2 : 4	450	32
1 : 2½ : 5	400	29
1 : 3 : 6	360	26
1 : 4 : 8	290	21

With a large mass foundation, take values one-third greater.

With a vibrating or pounding load, take one-half these values.

The tensile strength of concrete is very much less than the compressive strength. Experiments made by the authors, with mixtures of average proportions, give the ultimate fiber stress in beams not reinforced as about one-eighth the breaking strength in compression. For this reason it is not safe to use concrete for beams unless reinforced with steel.

CHAPTER III

SPECIFICATIONS

In the following pages are given specifications for

Cement, in brief, for the small purchaser. (See p. 29.)

Portland cement, in full, for the large purchaser. (See p. 29.)

Natural cement, in full, for the large purchaser. (See p. 31.)

Concrete and Reinforced Concrete. (See p. 32.)

First class steel for reinforced concrete. (See p. 38.)

These specifications cover all ordinary concrete construction, and are adapted as far as possible for direct use in placing contracts for material and construction, although concrete specifications for structures of intricate design will require the insertion of additional paragraphs referring specifically to the particular work.

If sand, screenings, gravel, stone, or steel are purchased on separate contracts, paragraphs 3, 4, 5, or 7 (pp. 33 and 34) may be extracted from the concrete specifications.

The full specifications for cement are advised for important work, whether large or small, although the brief specifications which precede them may be sometimes useful.

Even when purchasing by the full specifications it may often be unnecessary actually to test the cement, except for set soundness and fineness, but the strict detail specifications are necessary so that if the cement is found to work unsatisfactorily samples may be subjected to complete tests on the ground, or sent to testing laboratories, and the remainder of the shipment or subsequent shipments rejected.

Printed specifications are frequently preceded by a "Notice to Contractors" stating the place and time of receiving bids, the amount of the check to be deposited with each bid and the bond required, and specifying that the contractor shall give references and shall state what work of a similar character he has performed. A "Form of Bid" is also sometimes inserted.

The specifications and contract are based upon the authors' practice supplemented by a careful study of the reports of the Joint Committee on Concrete and Reinforced Concrete, the Reinforced Concrete Committee of the National Cement Users Association and the specifications of the American Society for Testing Materials, of the American Railway Engineering & Maintenance-of-Way Association, of the City of Philadelphia, of the

United States Army, of the United States Navy, of the Massachusetts Metropolitan Commissions, of the New York Rapid Transit Commission, and others.

BRIEF SPECIFICATIONS FOR PURCHASE OF CEMENT

The cement shall be a first-class Portland† cement of a standard brand bearing a good reputation. It shall conform to the standard specifications of the American Society for Testing Materials. It shall be free from lumps and shall be packed in sound barreis.‡

FULL SPECIFICATIONS FOR PURCHASE OF PORTLAND CEMENT

1. **Packages.** Cement shall be packed in strong cloth or canvas sacks.§ Each package shall have printed upon it the brand and name of the manufacturer. Packages received in broken or damaged condition may be rejected or accepted as fractional packages.

2. **Weight.** Four bags shall constitute a barrel, and the average net weight of the cement contained in one bag shall be not less than 94 lb. or 376 lb. net per barrel. A cement bag may be assumed to weigh one pound. The weights of the separate packages shall be uniform.

3. **Requirements.*** Cement failing to meet the seven-day requirements may be held awaiting the results of the twenty-eight day tests before rejection.

4. **Tests.*** All tests shall be made in accordance with the methods proposed by the Committee on Uniform Tests of Cement of the American Society of Civil Engineers, presented to the Society January 21, 1903, and amended January 20, 1904, with all subsequent amendments thereto. (See Chapter VII, page 63.)

5. **Sampling.** Samples shall be taken at random from sound packages, one from every 10 barrels or 40 bags, and mixed. The total sample should weigh about 10 lb.

6.* The acceptance or rejection shall be based on the following requirements:

7. **Definition of Portland Cement.*** This term is applied to the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous‖ and calcareous¶ materials, and to which no addition greater than 3% has been made subsequent to calcination.

*Paragraphs designated by an asterisk are quoted from the Standard Specifications of the American Society for Testing Materials.

†Or Natural.

‡If stored in a dry place to be used immediately, it may be packed in stout cloth or canvas bags which are of course cheaper than barrels.

§If the cement is to be stored in a damp place or near the sea, it must be packed in well-made wooden barrels lined with paper.

‖Clayey.

¶Consisting chiefly of lime or calcium.

8. **Specific Gravity.*** The specific gravity of cement shall not be less than 3.10. Should the test of cement as received fall below this requirement, a second test may be made upon a sample ignited at a low red heat. The loss in weight of the ignited cement shall not exceed 4%.

9. **Fineness.*** It shall leave by weight a residue of not more than 8% on the No. 100, and not more than 25% on the No. 200 sieve.

10. **Time of Setting.*** It shall not develop initial set in less than thirty minutes; and must develop hard set in not less than one hour nor more than ten hours.

11. **Tensile Strength.*** The minimum requirements for tensile strength for briquettes one square inch in cross section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Neat Cement.

Age	Strength
24 hours in moist air	175 lb.
7 days (1 day in air, 6 days in water)	500 "
28 days (1 " " 27 " ")	600 "

One Part Cement, Three Parts Standard Ottawa Sand

Age	Strength
7 days (1 day in moist air, 6 days in water)	200 lb.
28 days (1 " " " 27 " ")	275 "

12. **Soundness or Constancy of Volume.*** Pats of neat cement about three inches in diameter, one-half inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of twenty-four hours.

(*a*) A pat is then kept in air at normal temperature, and observed at intervals for at least 28 days.

(*b*) Another pat is kept in water maintained as near 70° Fahr. as practicable, and observed at intervals for at least 28 days.

(*c*) A third pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel for five hours.

These pats to satisfactorily pass the requirements, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegration.

13. **Sulphuric Acid and Magnesia.** The cement shall not contain more than 1.75% of anhydrous sulphuric acid (SO_3), nor more than 4% of Magnesia (MgO).

*Paragraphs designated by an asterisk are quoted from the Standard Specifications of the American Society for Testing Materials.

FULL SPECIFICATIONS FOR THE PURCHASE OF NATURAL CEMENT

1. **Packages.** Cement shall be packed in strong cloth or canvas sacks.† Each package shall have printed upon it the brand or the name of the manufacturer. Packages received in broken or damaged condition may be rejected or accepted as fractional packages.

2. **Weight.** Three bags shall constitute a barrel, and the average net weight of the cement contained in one bag shall be not less than 94 lb., or 282 lb. net per barrel. A cement bag may be assumed to weigh one pound. The weights of the separate packages shall be uniform.

3. **Requirements.*** Cement failing to meet the seven-day requirements may be held awaiting the results of the twenty-eight day tests before rejection.

4. **Tests.*** All tests shall be made in accordance with the methods proposed by the Committee on Uniform Tests of Cement of the American Society of Civil Engineers, presented to the Society January 21, 1903, and amended January 20, 1904, with all subsequent amendments thereto. (See Chapter VII, p. 63.)

5. **Sampling.** Samples shall be taken at random from sound packages, and the cement from each package shall be tested separately.

6.* The acceptance or rejection shall be based on the following requirements:

7. **Definition of Natural Cement.*** This term shall be applied to the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

8. **Fineness.*** It shall leave by weight a residue of not more than 10% on the No. 100, and 30% on the No. 200 sieve.

9. **Time of Setting.*** It shall not develop initial set in less than ten minutes, and shall not develop hard set in less than thirty minutes, or in more than three hours.

10. **Tensile Strength.*** The minimum requirements for tensile strength for briquettes one square inch in cross section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

*Paragraphs designated by an asterisk are quoted from the Standard Specifications of the American Society for Testing Materials.

†If the cement is to be stored in a damp place or near the sea, it must be packed in well-made wooden barrels lined with paper.

Neat Cement.

Age	Strength
24 hours in moist air	75 lb.
7 days (1 day in air, 6 days in water)	150 "
28 days (1 " " 27 " ")	250 "

One Part Cement, Three Parts Standard Ottawa Sand.

Age	Strength
7 days (1 day in air, 6 days in water)	50 lb.
28 days (1 " " 27 " ")	125 "

11. **Constancy of Volume.*** Pats of neat cement about 3 inches in diameter, one-half inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat is then kept in air at normal temperature.

(*b*) Another pat is kept in water maintained as near 70° Fahr. as practicable.

These pats are observed at intervals for at least 28 days, and, to satisfactorily pass the tests, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

CONTRACT AND SPECIFICATIONS FOR PORTLAND CEMENT CONCRETE‡

(These specifications essentially embody the recommendations of the Joint Committee on Concrete and Reinforced Concrete (1909) and the Report of the Reinforced Concrete Committee (1909) of the National Association of Cement Users.)

This agreement made this day of ..... in the year 19

by and between (Name of party letting the contract.) of ..... ,

party of the first part, and (Name of accepted contractor.) ...of........ ..... ,

party of the second part.

Witnesseth: That the parties to these presents, each in consideration of the covenants and agreements on the part of the other, herein contained, have covenanted and agreed, and do hereby covenant and agree, for themselves and their heirs, executors, administrators, and assigns, and under the

*Paragraphs designated by an asterisk are quoted from the Standard Specifications of the American Society for Testing Materials.

‡For Natural cement concrete paragraphs 1, 11 and 14 must be slightly altered, and paragraphs 7 and 13c omitted.

penalty expressed in a bond bearing even date with these presents, and hereto annexed, as follows:

The contractor shall begin work within................days of the date of this contract, and shall, at his own proper cost and expense, provide and deliver all of the materials and perform all of the work called for by these specifications, and supply all implements, apparatus, and appliances needed in performing the work.

The entire work shall be completed on or before.................... 19.......*

1. **Cement.**† The cement shall be first-class Portland cement of reputable brand which shall conform in all respects to the cement specifications herewith annexed. The cement shall be stored in a building which will protect it from the weather. The floor upon which the cement is placed shall be at least 6 inches above the ground. It shall be stored so as to permit of easy access for inspection and identification of each shipment. A sufficient quantity shall be kept on hand at all times so that the Engineer may have opportunity and time to make tests sufficient to determine its quality. At least 12 days shall be allowed for inspection and necessary tests.

2. **Fine Aggregates.** The fine aggregate shall consist of sand, crushed stone or gravel screenings passing when dry a screen having ¼ inch diameter holes or a screen having four meshes to the linear inch. It shall be clean, coarse, and free from vegetable loam and other deleterious matter. A gradation of the size of grain is preferred. Mortars composed of one part Portland cement and three parts fine aggregate by weight when made into briquets shall show a tensile strength of at least 70% of the strength of 1:3 mortar of the same consistency made with the same cement and standard Ottawa sand. To avoid the removal of any coating on the grains which may affect the strength, bank sands shall not be dried before being made into mortar but shall contain natural moisture. The percentage of moisture may be determined upon a separate sample for correcting weight. From 10 to 40% more water may be required in mixing bank or artificial sands than for standard Ottawa sand to produce the same consistency.

*A premium and forfeiture clause may here be inserted, but a forfeiture clause without a premium in many cases cannot be legally enforced. The word "penalty" should never be employed.

†It is often advisable that the cement be furnished by the party letting the contract or, to prevent waste of cement, that it be sold by them to the contractor at an arbitrary price per barrel, say, about one-half the actual cost of the cement,—which price must be definitely stated in the contract.

3. **Coarse Aggregates.** The coarse aggregate shall consist of inert material such as crushed stone, or gravel, which is retained on a screen having ¼ inch diameter holes. The particles shall be clean, hard, durable, and free from all deleterious material. Aggregates containing soft, flat, or elongated particles, should be excluded from reinforced concrete. A gradation of sizes of the particles is advantageous. The maximum size of the coarse aggregate shall be such that it will not separate from the mortar in laying and will not prevent the concrete fully surrounding the reinforcement or filling all parts of the forms. Where concrete is used in mass, the size of the coarse aggregate may be such as to pass a 3 inch ring. For reinforced concrete a size to pass a 1 inch ring or a smaller size may be used.

4. **Gravel.*** The gravel shall be composed of clean pebbles free from sticks and other foreign matter and containing no clay or other material adhering to the pebbles in such quantity that it cannot be lightly brushed off with the hand or removed by dipping in water. It shall be screened† to remove the sand, which shall afterwards be remixed with it in the required proportions.

5. **Broken Stone.*** The broken or crushed stone shall consist of pieces of hard and durable rock, such as trap, limestone, granite, or conglomerate. The dust shall be removed by a sand screen, to be afterwards, if desired, mixed with and used as a part of the sand, except that if the product of the crusher is delivered to the mixer so regularly that the amount of dust, as determined by frequently screening samples, is uniform, the screening may be omitted and the average percentage of dust allowed for in measuring the sand.

6. **Water.** The water shall be free from oil, acid, strong alkalies, or vegetable matter.

7. **Reinforcing Steel.*‡** Steel for reinforcement shall have an "ultimate tensile strength of 55,000 to 65,000 pounds per square inch, an elastic limit of not less than one-half the ultimate strength (*i. e.* not less than 27,000 lb.) and a minimum elongation in 8 inches of 1,400,000 divided by the ulti-

*Omit paragraphs for materials which are not used. If two or more sizes of any aggregate are used, define them.

†In exceptional cases where the relation of pebbles to sand is very uniform, the mixture of sand and pebbles may be used without screening. Frequent tests shall then be made to see that the proportions of the coarse and fine grains are correct.

‡Specifications for high carbon steel are given in full on page 38. High carbon steel is distrusted by many, but may be safely employed if it fulfills the requirements there given, and owing to its greater strength will be more economical than ordinary merchant steel.

mate strength per cent."* The fracture shall be si ky Test specimens for bending shall be bent cold to 180° flat withou fracture.

8. **Proportions.** The proportions of the raw materials for the concrete shall be exactly determined from time to time by the Engineer in accordance with the relative coarseness of the aggregate, so as to attain as dense a concrete as is consistent with the terms of the specifications which follow. The unit of measure shall be the barrel, which shall be taken as containing 3.8 cubic feet. Four bags containing 94 pounds of cement each shall be considered the equivalent of one barrel. The following paragraphs designate the average relative volumes of material for each class of work.

For ....†, one barrel (376 lb.) cement to .... cubic feet sand‡ to ....cubic feet broken stone,‡ the cement to be measured as packed by the manufacturer, and the fine and coarse aggregate to be measured separately as loosely thrown into the measuring receptacle. If the coarse aggregate contains sand or other fine material, that which passes a sieve with ¼ inch round holes shall be considered as sand in measuring proportions. In general, the concrete on the work shall contain enough and only enough mortar to cover all particles of stone and fill the voids without an appreciable excess of mortar.

9. **Hand Mixing.**§ If the concrete is mixed by hand, the cement and aggregate shall be mixed and the water added on a tight platform large enough to provide space for the partially simultaneous mixing of two batches of not more than one cubic yard each. The sand and cement shall be spread in thin layers and mixed dry until of uniform color. This mixture may be spread upon the layer of stone or the stone shoveled upon it before adding the water, or it may be made into a mortar before spreading it with the stone. In the former method the materials shall be turned at least three times,—in addition to the mixing of the sand and cement already mentioned, the water being added on the first turning,—and in addition to the shoveling from the platform to place or into the vehicle for transportation. In the latter method, that is, if the sand and cement are first made into a mortar, the mass of mortar and stone shall be turned at least twice. Whatever method is employed, the number of turnings shall be sufficient to produce a resulting loose concrete of uniform color and

*Suggested for structural steel by the Committee on Boston Building Laws of the Boston Society of Civil Engineers.

†Insert a description of portion of structure. Repeat paragraph as required.

‡If other materials are selected for the aggregate alter the wording accordingly.

§With an experienced contractor this paragraph may be abbreviated to substantially the form of the final sentence.

appearance, with the cement uniformly distributed through the mass, the stones thoroughly incorporated into the mortar and the consistency uniform throughout, thus producing a concrete uniform in color and homogeneous.

10. **Machine Mixing.*** If the concrete is mixed in a machine mixer a machine shall be selected into which the materials, including the water, can be precisely and regularly proportioned, and which will produce a concrete of uniform consistency and color with the stones and water thoroughly mixed and incorporated with the mortar.

11. **Consistency.** (*a*) A medium or quaking mixture of a tenacious, jelly-like consistency, which quakes on ramming, shall be used for ordinary mass concrete, such as foundations, heavy walls, large arches, piers, and abutments.

(*b*) Wet or mushy concrete, so soft that it will flow when agitated, but not so wet as to produce a separation of the materials in transferring to the work, shall be used for rubble concrete, and for reinforced concrete, such as thin building walls, columns, doors, conduits, and tanks.

12. **Placing Concrete.** Concrete shall be conveyed to place in such a manner that there shall be no distinct separation of the different ingredients, or, in cases where such separation inadvertently occurs, the concrete shall be remixed before placing. It shall be placed in the work immediately after mixing and deposited and rammed or agitated by suitable tools in such a manner as to produce thoroughly compact concrete of maximum density. No concrete shall be placed until the reinforcing steel has been placed and firmly secured by wiring or other methods to prevent displacement. Concrete shall be frequently wet for several days to prevent too rapid drying out. Concrete shall not be placed in water, unless unavoidable. Where concrete must be placed under water, unusual care must be taken to prevent the cement from being floated away. This usually can be accomplished in still water by placing the concrete through a large pipe or tube, or in large work by means of a bottom dump concrete bucket.

Before placing fresh concrete, all shavings and debris of every nature must be removed and the old concrete surface thoroughly cleaned from all dirt and scum or laitance and drenched with water.† Noticeable voids or stone pockets discovered when the forms are removed shall be filled

*Mixing by machine is preferred because the most thorough and uniform consistency can be thus obtained.

†Tanks and other structures having thin walls to resist water pressure should be built preferably as monoliths, that is, with no interruption in the work, proceeding, if necessary, night and day.

immediately with mortar mixed in the same proportions as the mortar in the concrete. The lines and grades of the completed concrete shall accurately conform to the plan annexed to and forming a part of these specifications.

13. **Placing Reinforcement.** The reinforcement shall accurately conform in the finished structure to the plans annexed to and forming a part of these specifications. All reinforcement shall be free from rust, scale or coating of any character which would tend to reduce or destroy the bond Before placing concrete the reinforcement must be placed in the position required in the finished structure, and each piece or member so firmly fixed as to positively prevent any subsequent displacement.

14. **Freezing Weather.*** Concrete for reinforced concrete structures shall not be mixed or deposited at a freezing temperature, unless special precautions are taken to avoid the use of materials containing frost and to provide means for preventing the concrete from freezing after being placed in position and until it has thoroughly hardened.

15. **Forms.** The lumber for the forms and the design of the forms shall be adapted to the structure and to the kind of surface required on the concrete. For exposed faces the surface next to the concrete shall be dressed. Forms shall be substantially built and secured to prevent movement or deflection during concreting, and tight to prevent leakage of mortar. Before the removal of forms, the concrete shall be carefully inspected and its strength ascertained. Much care shall be given to this portion of the work, which is fraught with danger under incompetent direction. No exact time for the removal of forms can be safely prescribed because of the varying character of the work, the variations in the setting of different cements and the influence of atmospheric conditions. Forms shall be thoroughly cleaned before being used again.

16. **Joints.** Temperature changes and shrinkage during setting necessitate joints at frequent intervals or else effective reinforcement, depending upon the range in temperature and the design of the structure. In massive work, such as retaining walls, abutments, etc., built without reinforcement, joints shall be provided approximately every 30 feet throughout the length of the structure. Girders shall never be constructed over freshly formed columns without allowing a period of at least two hours to elapse to permit settlement in the columns. Before resuming work the top of the column shall be thoroughly cleansed of foreign matter and laitance. To obtain tight joints between old and new concrete the old surface shall be roughened,

*Natural cement concrete must *never* be exposed to frost until thoroughly hard and dry.

thoroughly cleaned of all foreign material and laitance or scum, drenched, and slushed with neat cement or a mortar not leaner than one part Portland cement to two parts fine aggregate. Joints in reinforced concrete shall be avoided when possible by casting the entire structure at one operation. In building construction, joints may be made in the columns flush with the lower side of the girders, and joints in members of a floor system in general shall be made at or near the center of the span. In all cases joints shall be at right angles to their surfaces.

17*a*.* **Ordinary Surface.** Surfaces shall have no special treatment further than care in placing the concrete to avoid noticeable voids or stone pockets. Forms shall be wet (except in freezing weather) before placing the concrete against them.

17*b*.* **Exposed Faces.** Faces exposed to view shall be made smooth by thrusting a spade or chisel through the concrete close to the form to force back the large stones and prevent stone pockets. The forms shall be thoroughly wet or greased with crude oil before placing the concrete against them. On removal of the forms, surfaces shall be....................†

17*c*.* **Mortar Surface.** Moldings, cornices, and other ornaments requiring mortar surface, shall be formed by spreading plastic mortar upon the interior of finely constructed molds, just as the concrete is being laid.

18. **Construction Details.** (Here may be placed descriptive paragraphs referring to special parts of the structure.)

19. **General Requirements.** Imperfect work or materials, or work or materials which may become damaged from any cause before its acceptance, shall be properly replaced to the satisfaction of the Engineer.

Foremen employed by the contractor shall be skilled in concrete mixing, and they shall receive and obey orders from the Engineer.

No claims for extra work shall be allowed unless made in writing previous to its performance and signed by both parties or by their authorized representatives.

In case of disagreement as to the meaning of the terms of the contract or as to the manner of its execution, one arbitrator shall be appointed by each party within one week after notification in writing by either party, and in case these cannot agree, a third arbitrator shall be selected by these two, and the decision of the majority of the arbitrators shall be final and binding on both parties. The cost of this arbitration shall be divided equally between the two parties to this contract.

*Select one or more paragraphs from 17a, 17b and 17c.

†State kind of finish desired, see page 288.

20. **Prices for Work.** The following prices shall be paid to the contractor as full compensation for the furnishing and use of all materials and implements required on the work and for all labor.

(Here shall be inserted all unit prices for all divisions of the work, or the lump sum for the entire work, or the lump sums for different divisions of the work, or for alternate proposals, followed by a paragraph stating the manner and time of payments and the amount withheld each month.)

In witness whereof the parties to these presents have affixed their hand and seals this................day of.............., 19.........
Signed in the presence of

......................(Seal)

....

......................(Seal)

....

BOND TO ACCOMPANY THE CONTRACT.*

Know all men by these presents, That we

....

as sureties, are held and firmly bound unto.............................
in the sum of..dollars
($..........), to be paid said..............................., for which payment, well and truly to be made, we bind ourselves, our heirs, executors and administrators, jointly and severally, firmly by these presents.

The condition of this obligation is such, that if the above bounden

....

heirs, executors, administrators or assigns, shall in all things stand to and abide by, and well and truly keep and perform, the covenants, conditions and agreements in the foregoing contract on his or their part to be kept and performed, at the time and in the manner therein specified, and shall indemnify and save harmless the said.................................
as therein stipulated, then his obligation shall become and be null and void; otherwise it shall be and remain in full force and virtue.

In witness whereof we hereunto set our hands and seals on this.........
..............day of............................in the year nineteen hundred and ....

...................(Seal)

...................(Seal)

Signed and sealed in presence of ....

....

*Form adopted by Metropolitan Commissioners, Mass.

SPECIFICATIONS FOR FIRST-CLASS STEEL TO BE USED IN REINFORCED CONCRETE.*

1. **Process of Manufacture.** Steel shall be made by the open hearth process.

2. **Chemical Properties.** Steel shall conform to the following limits in chemical composition:

Phosphorus shall not exceed 0.06.

Sulphur shall not exceed 0.06.

Manganese shall not exceed 0.80 or be below 0.40.

3. **Physical Properties.** The steel shall conform to the following physical qualities:

4. *Tensile Tests.* Tensile strength in pounds per square inch shall be not less than 85000

Yield point in pounds per square inch shall be not less than 52500

Elongation per cent. in eight inches shall be not less than........10

5. For material less than five-sixteenths inch ($\frac{5}{16}''$) and more than three-fourths inch ($\frac{3}{4}''$) in thickness the following modifications shall be made in the requirements for elongation:

(*a*) For each increase of one-eighth inch ($\frac{1}{8}''$) in thickness above three-fourths inch ($\frac{3}{4}''$) a deduction of one per cent. (1%) shall be made from the specified elongation.

(*b*) For material from $\frac{1}{4}$ inch to, but not including, $\frac{5}{16}$ inch thick the elongation shall be 8%.

For material from $\frac{3}{16}$ inch to, but not including, $\frac{1}{4}$ inch thick the elongation shall be 7%.

For material from $\frac{1}{8}$ inch to, but not including, $\frac{3}{16}$ inch thick the elongation shall be 6%.

For material less than $\frac{1}{8}$ inch thick the elongation shall be 5%

6. *Bending Test.* Test specimens for bending† shall be bent cold to the following angles without fracture on the outside of the bent portion:

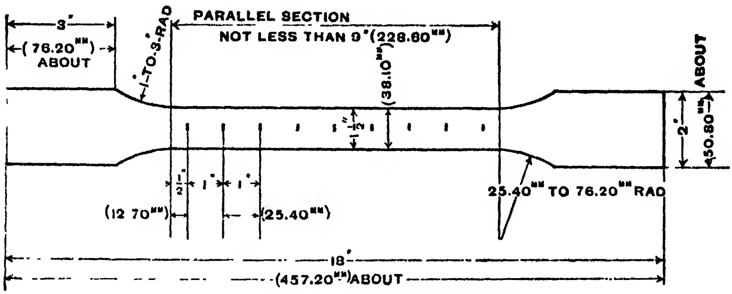
Around twice their own diameter.	*Around their own diameter.*
For specimens 1 inch thick 80°.	For specimens $\frac{1}{4}$ inch thick 130°.
For specimens $\frac{3}{4}$ inch thick 90°.	For specimens $\frac{3}{16}$ inch thick 140°.
For specimens $\frac{1}{2}$ inch thick 110°.	For specimens $\frac{1}{8}$ inch thick 180°.

*Steel of this hardness should not be used unless enough of it is to be bought to warrant the making of complete tests as per specifications. Ordinary mild steel may be purchased in the open market without specifications. In using steel bought in open market, it is not safe to count on a tensile strength greater than 55,000 lb. —FREDERICK W. TAYLOR.

†The most important test of all is the bending test, but any soft steel will stand the bending test, so that the tensile test is needed to secure a steel which is strong enough.

No steel which fails to pass the bending test shall under any circumstances be used.

7. **Test Pieces and Methods of Testing.** Where practicable the standard test specimen of eight-inch (8″) gaged length shall be used to determine the physical properties specified in paragraphs Nos. 4 and 5. The standard shape of the test specimen for sheared plates shall be as shown by the following sketch:



PIECE TO BE OF SAME THICKNESS AS PLATE

For material from which it is impracticable to obtain test specimens like those for sheared plates, the test specimen may be planed or turned parallel throughout its entire length, and in all cases where possible two opposite sides of the test specimen shall be the rolled surfaces. Small rolled bars of uniform section shall be tested full size as rolled.

8. All test specimens shall be cut from the finished material as it comes from the rolls, unless such materials are to be annealed, in which case the test specimens will be taken after the annealing process. In case several shapes are rolled from one heat, two test specimens will be taken from two different shapes, representing their class, for tension, and two for bending. When only one shape is rolled from a heat, two test specimens for tension and two for bending will be taken from each ten tons or fraction thereof.

9. Where practicable the bending test specimen shall be one and one-half inches ($1\frac{1}{2}''$) wide, and for material three-quarters inch ($\frac{3}{4}''$) and less in thickness, this specimen shall have the natural rolled surface on two opposite sides. For material more than three-quarters inch ($\frac{3}{4}''$) thick, the bending test specimen may be cut to one-half inch ($\frac{1}{2}''$) thick.

10. The bending test may be made by pressure or by blows.

11. In case a test specimen develops flaws or in case it breaks outside of the middle third of its gaged length, it may be discarded and another test specimen substituted therefor.

12. For the purposes of this specification, the yield point shall be determined by the careful observation of the drop of the beam, or halt in the gage, of the testing machine.

13. In order to determine if the material conforms to the chemical limitations prescribed in paragraph No. 2 herein, analysis shall be made of clean drillings taken from a small test ingot.

14. **Variation in Weight.** A variation in cross section or weight of more than $2\frac{1}{2}\%$ from that specified will be sufficient cause for rejection.

15. **Finish.** Finished material must be free from injurious seams, flaws, or cracks, and have a workmanlike finish.

16. **Annealing.** All bars which, owing to their shape or size, are liable to be under strain after cooling, must be reheated to a temperature not less than 1250° (Fahrenheit) nor more than 1375°, and this heating and subsequent cooling must be done in an approved manner.

CHAPTER IV

THE CHOICE OF CEMENT

When the construction under consideration is not of a grade to warrant the testing of different cements before making a selection, the question often arises as to whether, for example, Portland or Natural cement is most desirable fron the standpoint of economy, or whether common lime or a mixture of lime and cement is suitable for the purpose.

Although the decision must often depend upon local conditions, a few general rules may be formulated relating to the classes of construction for which different kinds of cement and lime are adapted, followed by illustrations of the methods for making a selection where there is a choice between two cements and between different brands of the same cement.

THE CLASS OF CEMENT

Portland Cement should be used in concrete and mortar for structures subjected to severe or repeated stresses; for structures requiring strength at short periods of time; for concrete building construction; for work laid under water or with which water will come in contact immediately after placing; for thin walls subjected to water pressure; for masonry exposed to wear or to the elements; and for all other purposes where its cost will be less than that of Natural cement concrete, or mortar of similar quality.

Natural Cement may be substituted for Portland in concrete, if economy demands it, for dry unexposed foundations where the load in compression can never exceed, say, 75 lb. per square inch (5 tons per sq. ft.) and will not be imposed until three months after placing; for backing or filling in massive concrete or stone masonry where weight and mass are the essential elements; for sub-pavements of streets, and for sewer foundations.

In mortar Natural cement is adapted for ordinary brickwork not subjected to high water pressure or to contact with water until, say, one month after laying, and for ordinary stone masonry where the chief requisite is weight and mass.

Natural cement concrete or mortar should never be allowed to freeze, should never be laid under water, in exposed situations, in columns, beams, floors or building walls, or in marine construction.

Mixtures of Portland and Natural Cements, unless mixed at the factory and sold as Improved Natural Hydraulic Cements, are not advised under any conditions.

Sand Cement* is recommended by the United States Army Engineers for grouting†, and it is sometimes employed as a substitute for Natural cement. Its use in place of pure Portland cement is often worth investigation and testing in combination with the aggregate.

Puzzolan or Slag Cements are limited to certain proper uses by the engineer officers of the U. S. Army‡ as follows:

> Puzzolan cement never becomes extremely hard like Portland, but Puzzolan mortars and concretes are tougher or less brittle than Portland.
> The cement is well adapted for use in sea water,§ and generally in all positions where constantly exposed to moisture, such as in foundations of buildings, sewers, and drains, and underground works generally, and in the interior of heavy masses of masonry or concrete.
> It is unfit for use when subjected to mechanical wear, attrition, or blows. It should never be used where it may be exposed for long periods to dry air, even after it has well set. It will turn white and disintegrate, due to the oxidation of its sulphides at the surface under such exposure.

Hydraulic Lime, which has the property of setting under water, is extensively employed on the continent of Europe, especially in France, when in the United States common lime would be used, and frequently in place of hydraulic cement. Beton Coignet is a mixture of hydraulic lime with cement and sand. Candlot‖ gives as the proportions most frequently employed, 1 cubic meter (35.3 cu. ft.) sand, 125 to 150 kilograms (276 to 331 lb.) lime, and 50 to 60 kilograms (110 to 132 lb.) cement. Hydraulic lime is not manufactured in the United States.

Common Lime is not suitable for a principal ingredient in concrete. It will not set in contact with water, sustain heavy loads, or resist wear.

The use of lime mortar, in the building laws of some cities, is limited to chimney construction in frame buildings, while other cities permit its use in walls of all except fireproof buildings. The Boston building laws (1896) limit the stresses on brick laid in lime mortar to 7 tons per square foot.

Lime and Natural Cement mortar is suitable for ordinary building brickwork, for light rubble foundations and for building walls.

Lime and Portland Cement mortar is adapted for the same purposes

*See page 48.
†Professional Papers No. 28.
‡Professional Papers No. 28.
§See Chapter XVI. by R. Feret.
‖Ciments et Chaux Hydrauliques, 1898, p. 289.

as mortars of lime and Natural cement, but are of superior quality and strength.

Hydrated Lime* is preferable to common lime paste or putty for use with Portland cement, because if properly manufactured it is more thoroughly slaked and is easily handled and measured.

Choice Determined by Cost. — When the character of the structure admits of either Portland or Natural cement, the choice is based upon the relative cost, which, in turn, is dependent upon the proportions that may be adopted in either case. The sand in Portland cement mortar is usually limited, by practical considerations of handling with the trowel, to proportions 1 : 3 in some instances and to 1 : 4 in others, while the most common proportions for Natural cement mortar are 1 : 2, that is, one part cement to two parts sand, by volume.

The relative cost, after assuming the proportions of the two substitute classes of mortar, is governed primarily by the quantity of cement in a cubic yard of mortar. For example, from table on page 229, 3.32 bbl. of cement (based on a barrel of 3.8 cu. ft.) are required per cubic yard of 1 : 2 mortar, while 2.48 bbl. are required for 1 : 3 mortar. Hence, if a decision lies between 1 : 2 Natural mortar and 1 : 3 Portland mortar, and the smaller item of quantity of sand is disregarded, the mortar produced from Natural cement at \$1.00 per barrel will cost the same as that produced from Portland cement at $(\$1.00 \times \frac{3.32}{2.48}) = \$1.34$ per barrel. Similarly, since 1 : 4 mortar requires 1.98 bbls. of cement per cubic yard, Portland cement mortar one part cement to 4 parts sand is equivalent in cost to 1 : 2 Natural cement mortar when Natural cement is \$1.00 per barrel and Portland cement is $(\$1.00 \times \frac{3.32}{1.98}) = \$1.68$ per barrel; that is, when Portland cement delivered on the job costs 68% more than Natural cement. Allowance for difference in quantity of sand brings the Portland values still lower, as shown in the table on page 45. With Portland and Natural cement mortars of equal cost, the Natural cement produces brickwork of lower cost because, a fact usually overlooked in estimates, a bricklayer can lay in a given time about 10% more brick with Natural cement mortar of proportions 1 : 2 than with Portland cement mortar of proportions, say, 1 : 3.

From the results of the comparatively few available tests, Portland cement concrete at the age of six months appears to be at least three times

*See S. Y. Brigham in *Engineering News*, Aug. 27, 1903, p. 177, and Charles Warner in *Rock Products*, Feb., 1904, p. 26.

as strong as Natural cement concrete in the same proportions, while at earlier periods the ratio is still larger. Since Portland cement concrete mixed 1 : 2 : 4 is only about 1½ times stronger than a 1 : 4 : 8 Portland mixture, it is very evident that the choice between Portland and Natural cement for concrete is determined, as in mortars, by practical considerations other than relative strength.

The following elementary example illustrates the method of estimating the comparative cost of Portland and Natural cement concrete:

Example. — What price can be paid per barrel for Portland cement to make a concrete 1 : 4 : 8 of equivalent cost to a 1 : 2 : 4 Natural cement concrete, if Natural cement costs $1.00 per barrel, sand $0.75 per cubic yard, and stone having 45% voids $1.50 per cubic yard?

Solution. — By reference to the table of quantities of materials on page 17, we find that the 1 : 2 : 4 Natural concrete will cost per cubic yard for materials only:

1.57 barrels cement at $1.00	$1.57
0.44 cubic yards sand " 0.75	0.33
0.88 " " stone " 1.50	1.32
Total materials	$3.22

The sand and stone for the 1 : 4 : 8 Portland mixture will cost, on the other hand, per cubic yard of concrete:

0.48 cubic yards sand at $0.75	$0.36
0.96 " " stone " 1.50	1.44
Cost of sand and stone	$1.80

Subtracting $1.80 from $3.22 leaves a difference of $1.42 which may be paid for the Portland cement in one cubic yard of concrete, and since by the quantity table 0.85 barrels of cement are required for a cubic yard of 1 : 4 : 8 concrete, the price for the Portland cement may be $1.42 ÷ 0.85 = $1.67 per barrel.

If the Natural cement had cost $1.25 per barrel, the price which could have been paid for Portland would have been approximately 25% higher or $2.09 per barrel.

This determination may be expressed in a formula:

$$x = \frac{am + bn + cr - (b'n + c'r)}{a'}$$

in which a, b, and c represent respectively the quantities of cement, sand, and stone required for a cubic yard of the Natural cement concrete, and m, n, and r their respective unit costs, while a', b', and c' represent similar

quantities for the Portland cement concrete, and *x* the required price per barrel of the Portland cement.

The following table is made out on this basis.

Prices of Portland Cement to produce Mortar or Concrete of equal cost to that from Natural Cement at $1.00 per barrel. (*See p. 44.*)

Proportions of Natural Cement Mortar	PROPORTIONS OF PORTLAND CEMENT MORTAR.							Proportions of Natural Cement Concrete.	PROPORTIONS OF PORTLAND CEMENT CONCRETE.				
	1:1	1:1½	1:2	1:2½	1:3	1:3½	1:4		1:2:4	1:2½:5	1:3:6	1:4:8	1:5:10
	$	$	$	$	$	$	$		$	$	$	$	$
1:1	1.00	1.23	1.46	1.69	1.92	2.15	2.38	1:2:4	1.00	1.15	1.32	1.67	2.01
1:1½		1.00	1.18	1.37	1.55	1.74	1.92	1:2½:5		1.00	1.14	1.44	1.72
1:2			1.00	1.15	1.30	1.46	1.61	1:3:6			1.00	1.26	1.51
1:2½				1.00	1.13	1.26	1.39						
1:3					1.00	1.12	1.23						

NOTE.—When the Natural cement is higher or lower than $1.00 per barrel, multiply its cost by the figures in the table to obtain approximate corresponding cost of Portland cement with which it is compared. Values make no allowance for difference in strength or labor of laying mortar.

The equivalent prices for Portland cement in mortars will be still nearer the price for Natural if allowance is made for the difference in the labor of laying brick, which in some cases may correspond to a difference of 30 cents per barrel of cement. It is evident from the table that for mortar Portland can rarely be substituted for Natural cement without increasing the cost of the work. A field still open for investigation is the employment as a substitute for Natural cement of Portland cement mixed with slaked lime or hydrated lime. The lime is so finely divided that it fills the voids and thus permits the use of more sand.

SELECTION OF THE BRAND

A precise comparison of costs of different brands of the same class of cement is impossible without thorough laboratory tests, described in Chapter VII, page 63. If the choice lies between two cements both of which have been found to be sound (see p. 77) and to set up properly, the degree of fineness, which may be readily ascertained with two sieves as described on page 67, is an aid to the decision. The finer cement will usually produce the stronger mortar.

The cheapest cement is not always the most economical. A method of comparing the relative economy of cements offered by bidders at different prices is illustrated in the following table for which the authors are indebted

to Mr. D. M. Andrews. Ten brands of Portland cement were submitted to the Government at prices ranging from \$2.77 to \$3.29.* Experiments showed that sample No. 5 was the strongest, with No. 4 a close second. The relative strength of the different brands in proportions 1:3, based on the strongest as 100.0, is given in the column headed Relative Strength of Mortar, and the column following this gives the product of the relative strength multiplied by the relative cheapness. In the case under consideration brand No. 5 was selected for purchase, because, although No. 4 gave higher economy, it appeared slightly unsound. Other data with reference to each brand was observed, including the volumes of the barrels, their gross net weights, the percentages of water used in mixing the pastes and mortar, the time of setting of the mortar, and the strength and relative economy of mortars with sand proportioned to price of cement, that is, for example, using 19% more sand with cement No. 10 than with No. 1, because the former's price was 19% greater.

*The price of Portland cement has since been materially lowered.

Relative Economy of Different Priced Portland Cements.

By D. M. Andrews.

No. of Sample Barrel.	Price per Barrel	Relative cheapness.	Fineness.		Time of Setting		Tensile Strength.						§Relative Strength of 1:3 Mortar	Relative Economy 1:3 Mortar: Strength × Cheapness	Remark
					Initial.	Final.	Neat.			1:3 Mortar.					
			No. 50 sieve.	No. 100 sieve.	Hours.	Hours.	7 days.	30 days.	60 days.	7 days.	30 days.	60 days.	60 days.	60 days.	
1	\$2.77	100.0	93.3	87.6	2	8	324	437	430	66	128	168	79.7	79.7	
2	2.79	99.3	99.3	87.3	8‡	8‡	282	429	468	62	103	124	59.1	58.7	
3	2.82	98.2	98.2	89.7	2	3¼	272	373	431	35	65	87	41.2	40.5	
4	2.82	98.2	100.0	90.6	5	9¼	369	460	564	144	184	209	99.4	97.7	Air pat cracked very slightly
5†	2.89	95.8	99.0	86.2	7¼	7¼	419	543	631	114	175	210	100.0	95.8	
6	2.90	95.5	94.6	77.0	¼	6¼	150	227	264	25	57	90	42.8	40.9	Lumpy and gritty on mixing
7	2.93	94.5	100.0	90.0	¼	8	440	588	568	127	156	202	96.2	90.9	
8	3.02	91.7	99.5	80.4	2¼	7	418	476	561	89	134	171	82.4	75.6	
9	3.05	90.8	98.5	91.2	3½	7¼	436	518	502	93	126	144	68.2	62.0	Pats cracked slightly.
10	3.29	84.2	99.4	92.7	2½	5	365	496	573	78	117	141	67.1	56.5	

†Accepted in preference to No. 4 because air pat slightly defective.
‡Cement not yet set
§Based on the highest, No. 5, as 100.0.

CHAPTER V

CLASSIFICATION OF CEMENTS.

From an engineering standpoint, limes and cements may be classified as

Portland cement.
Natural cement.
Puzzolan cement.
Hydraulic lime.
Common lime.

Typical analyses of each of these are presented in the following table The composition of Natural cement, even different samples of the same brand, is so extremely variable that their analyses cannot be regarded as characteristic of locality.

Typical Analyses of Cements.

	PORTLAND CEMENT		NATURAL CEMENT							COMMON LIME	
			AMERICAN		ENGL'H	FRENCH					
	Lehigh Valley[1] (mixed rock)	Western[2] (marl and clay)	Eastern Rosendale[3]	Western Louisville[3]	Roman[4]	Vassy[5]	Grappiers[6]	Puzzolan Cement[7]	Hydraulic Lime (Le Teil)[8]	Lime[9]	Magnesian Lime[10]
Silica $Si\,O_2$	21.31	21.93	18.38	20.42	25.48	22.60	26.5	28.95	21.70	1.03	1.12
Alumina $Al_2\,O_3$	6.89	5.98	15.20	4.76	10.30	8.90	2.5	11.40	3.19	1.27	c 68
Iron Oxide $Fe_2\,O_3$	2.53	2.35		3.40	7.41	5.30	1.5	0.54	0.66		
Calcium Oxide $Ca\,O$	62.89	62.92	35.84	46.64	44.54	52.69	63.0	50.20	60.70	97.02	58.51
Magnesian Oxide $Mg\,O$	2.64	1.10	14.02	12.00	2.92	1.15	1.0	2.96	0.85	0.68	39.69
Sulphuric Acid $S\,O_3$	1.34	1.54	0.93	2.57	2.61	3.25	0.5	1.37	0.60		
Loss on Ignition	1.39	2.91	3.73	6.75	3.68	6.11	5.0	3.39	12.20		
Other constituents	0.75		11.46	3.74	1.46			0.30	0.10		

[1]W. F. Hillebrand, Society of Chemical Industry, 1902, Vol. XXI.
[2]W. F. Hillebrand, Journal American Chemical Society, 1903, 25, 1180.
[3]Clifford Richardson, *Brickbuilder*, 1897, p 229
[4]Stanger & Blount, Mineral Industry, Vol. V, p. 69.
[5]Candlot, Ciments et Chaux Hydrauliques, 1898, p. 174.
[6]Le Chatelier, Annales des Mines, September and October, 1893, p. 36.
[7]Report of the Board of U. S. Army Engineers on Steel Portland Cement, 1900, p. 52.
[8]Candlot, Ciments et Chaux Hydrauliques, 1898, p. 24.
[9]Rockland-Rockport Lime Co.
[10]Western Lime and Cement Co.

PORTLAND CEMENT†

Portland cement is defined by Mr. Edwin C. Eckel of the U. S. Geological Survey as follows: "By the term Portland cement is to be understood the material obtained by finely pulverizing clinker produced by burning to semi-fusion an intimate artificial mixture of finely ground calcareous and argillaceous materials, this mixture consisting approximately of 3 parts of lime carbonate to 1 part of silica, alumina and iron oxide."

The definition is often further limited by specifying that the finished product must contain at least 1.7 times as much lime, by weight, as of silica, alumina, and iron oxide together.

The only surely distinguishing test of Portland cement is its chemical analysis and its specific gravity. (See pp. 64 and 65.) In the field it may often be recognized by its cold bluish gray color (see p. 113), although the color of Puzzolan and of some Natural cements is so similar that this is by no means a positive indication.

The term **Natural Portland Cement** arose from the discovery in Boulogne-sur-Mer, France, as early as 1846, of a natural rock of suitable composition for Portland cement. A similar discovery in Pennsylvania gave rise to the same term in America, but the manufacturers soon found it necessary to add to the cement rock a small percentage of purer limestone. Since the chemical composition of Portland cement, as defined above, is substantially uniform regardless of the materials from which it is made, in the United States the terms "natural" and "artificial" are meaningless.

In France, cements intermediate between Roman and Portland are called "natural Portlands."*

Sand Cement. Sand or silica cement is a mechanical mixture of Portland cement with a pure, clean sand very finely ground together in a tube mill or similar machine. For the best grades the proportions of cement to sand are 1 : 1, although as lean a mixture as 1 : 6 has been made to compete with Natural cements. The coarser particles in any Portland cement have little cementitious value, hence if a portion of the cement is replaced by inert matter and the whole ground extremely fine, its advocates maintain that the product is scarcely inferior to the unadulterated article. As made in the United States, the mixture is ground so fine that 95% of it will pass through a sieve having 200 meshes to the linear inch, and all of the 5% of residuum is said to be sand. In other words, all of the cement passes a No. 200 sieve.

† A subclassification of Portland cement is presented on page 53.

* Candlot's Ciments et Chaux Hydrauliques, 1898, p. 164.

NATURAL CEMENT

Natural cement is "made by calcining natural rock at a heat below incipient fusion, and grinding the product to powder."* Natural cement contains a larger proportion of clay than hydraulic lime, and is consequently more strongly hydraulic. Its composition is extremely variable on account of the difference in the rock used in manufacture.

Natural cements in the United States in numerous instances bear the names of the localities where first manufactured. For example, Rosendale cement, a term heard in New York and New England more frequently than Natural cement, was originally manufactured in Rosendale, Ulster County, N. Y. Louisville cement first came from Louisville, Ky. The James River, Milwaukee, Utica, and Akron are other Natural cements named for localities.

The United States produces a few brands of "Improved Natural Hydraulic Cement," intermediate in quality between Natural and Portland, by mixing inferior Portland cement with Natural cement clinker.

In England the best known Natural cement is called Roman cement. Occasionally one hears the term Parker's Cement, so called from the name of the discoverer in England.

LE CHATELIER'S CLASSIFICATION OF NATURAL CEMENTS

In France there are several classes of Natural cement. Mr. H. Le Chatelier† classifies Natural cements as those obtained "by the heating of limestone less rich in lime than the limestone for hydraulic lime. They may be divided into three classes:

"Quick-setting cements, such as Vassy and Roman (Ciments à prise rapide, Vassy, romain);

"Slow-setting cements (Ciments à prise demi-lente);

"Grappiers cement (Ciments de grappiers).

"**Vassy Cements** are obtained by the heating of limestone containing much clay, at a very low temperature, just sufficient to decarbonate the lime. They are characterized by a very rapid set, followed afterwards by an extremely slow hardening, much slower than that of Portland cements."

"They differ from Portland cements by containing a much higher percentage of sulphuric acid, which appears to be one of their essential elements, and a much lower percentage of lime.

*Professional Papers, No. 28, U. S. Army Engineers, p. 33.

†Procédés d'Essai des Matériaux Hydrauliques, Annales des Mines, 1893.

"**Slow-Setting Cements**, by the high temperature of calcination, approach Portland cements, but the natural limestones never possess the homogeneity of artificial mixtures, so that it is impossible to avoid in these cements the presence of a large quantity of free lime." The composition of these products varies from that of the Vassy cements to that of the real Portlands.

"**Grappiers Cements*** are obtained by the grinding of particles which have escaped disintegration in the manufacture of hydraulic limes. These grappiers are a mixture of four distinct materials, two of which, completely inert, are unburned limestone and the clinkers formed by contact with the siliceous walls of furnaces, and two of which, strongly hydraulic, are unslaked lime and true slow-setting cement. It is necessary that the latter should predominate in the grappiers for their grinding to give a useful product. The grappier of cement is obtained regularly only by the heating of a limestone but slightly aluminous and containing about three equivalents of carbonate of lime for one of silica; its production necessitates a heating at high temperature.

"These grappiers cements are even more apt to contain free lime than the Natural cements of slow set which are obtained by the heating of limestone containing much more alumina. Because of their constitution, also, the grappiers cements may vary greatly in composition since they are produced by the grinding of a mixture of grains of cement and of various inert materials. The cement grains have very nearly the composition of tricalcium silicate (SiO_2 3 CaO)."

PUZZOLAN OR SLAG CEMENT

Puzzolan cement is the product resulting from mixing and grinding together in definite proportions slaked lime and granulated blast furnace slag or natural puzzolanic matter (such as puzzolan, santorin earth, or trass obtained from volcanic tufa).

The ancient Roman cements belonged to the class of Puzzolans. They were made by mechanically mixing slaked lime with natural puzzolana formed from the fusion of natural rock found in the volcanic regions of Italy. In Germany, trass, a volcanic product related to Puzzolan, has been used with lime in the manufacture of cements.

Blast furnace slag is essentially an artificial puzzolana, formed by the combustion in a blast furnace, and the puzzolan or slag cements of the United States are ground mixtures of granulated blast furnace slag, of special composition, and slaked lime.

*Cements essentially of the Grappiers class in the United States are termed "Non-Staining Cements." These may closely approach Portland cement in strength.

A Board of Engineers officers, U. S. A., presented in 1901 the following conclusions,* based, undoubtedly, on the exhaustive studies upon the subject made by a previous Board† having the same chairman, Major W. L. Marshall:

This term (slag or Puzzolan cement) is applied to cement made by intimately mixing by grinding together granulated blast-furnace slag of a certain quality and slaked lime, without calcination subsequent to the mixing. This is the only cement of the Puzzolan class to be found in our markets (often branded as Portland), and as true Portland cement is now made having slag for its hydraulic base, the term "slag cement" should be dropped and the generic term Puzzolan be used in advertisements and specifications for such cements.

Puzzolan cement made from slag is characterized physically by its light lilac color; the absence of grit attending fine grinding and the extreme subdivision of its slaked lime element; its low specific gravity (2.6 to 2.8) compared with Portland (3 to 3.5); and by the intense bluish green color in the fresh fracture after long submersion in water, due to the presence of sulphides, which color fades after exposure to dry air.

The oxidation of sulphides in dry air is destructive of Puzzolan cement mortars and concretes so exposed. Puzzolan is usually very finely ground, and when not treated with soda sets more slowly than Portland. It stands storage well, but cements treated with soda to quicken setting become again very slow setting, from the carbonization of the soda (as well as the lime) element after long storage.

Puzzolan cement properly made contains no free or anhydrous lime, does not warp or swell, but is liable to fail from cracking and shrinkage (at the surface only) in dry air.

Mortars and concretes made from Puzzolan approximate in tensile strength similar mixtures of Portland cement, but their resistance to crushing is less, the ratio of crushing to tensile strength being about 6 to 7 to 1 for Puzzolan, and 9 to 11 to 1 for Portland. On account of its extreme fine grinding Puzzolan often gives nearly as great tensile strength in 3 to 1 mixtures as neat.

Puzzolan permanently assimilates but little water compared with Portland, its lime being already hydrated. It should be used in comparatively dry mixtures well rammed, but while requiring little water for chemical reactions, it requires for permanency in the air constant or continuous moisture.

Puzzolanic material has been suggested by Dr. Michaelis, of Germany, and Mr. R. Feret, of France (see Chapter XVI), as a valuable addition to Portland cement designed for use in sea water.

*Professional Papers No. 28, p. 28.

†Report of the Board of U. S. Army Engineers on Steel Portland Cement, 1900, p. 52.

HYDRAULIC LIME

The hydraulic properties of a lime, — its ability to harden under water, — are due to the presence of clay, or, more correctly, to the silica contained in the clay. Hydraulic lime is still used to quite an extent in Europe, especially in France, as a substitute for hydraulic cement. The celebrated lime of-Teil of France is a hydraulic lime.

Mr. Edwin C. Eckel states* that "theoretically the proper composition for a hydraulic limestone should be calcium carbonate 86.8%, silica 13.2%. The hydraulic limestones in actual use, however, usually carry a much higher silica percentage, reaching at times to 25%, while alumina and iron are commonly present in quantities which may be as high as 6%. The lime content of the limestones commonly used varies from 55% to 65%."

Although the chemical composition of hydraulic lime is similar to Portland cement, its specific gravity is much lower, lying between 2.5 and 2.8.†

In the manufacture of hydraulic lime the limestone of the required composition is burned, generally in continuous kilns, and then sufficient water is added to slake the free lime produced so as to form a powder without crushing.

COMMON LIME

The commercial lime of the United States is "quicklime," which is chiefly calcium oxide (CaO).

Lime is now manufactured by a continuous process. Limestone of a rather soft texture, so as to be as free as possible from silica, iron and alumina, is charged into the top of a kiln which may be, say, 40 ft. high by 10 ft. in diameter. The fuel is introduced into combustion chambers near the foot of the shaft, and the finished product is drawn out from time to time through another opening in the bottom of the shaft. The temperature of calcination may range from 1400° Fahr. (760° Cent.) to, at times, 2,000° Fahr. (1,090° Cent.). The product (see analysis, p. 47), in ordinary lime of the best quality, is nearly pure calcium oxide (CaO). Upon the addition of water the lime slakes, forming calcium hydrate (CaH_2O_2), and, with the continued addition of water, increases in bulk to twice to three times the original loose and dry volume of the lump lime as measured in the cask. In this plastic condition it is termed by plasterers "putty" or "paste."

The setting of lime mortar is the result of three distinct processes which, however, may all go on more or less simultaneously. First, it

*American Geologist, March, 1902, p. 152.

†Candlot's Ciments et Chaux Hydrauliques, 1898, p. 26.

dries out and becomes firm. Second, during this operation, the calcic hydrate, which is in solution in the water of which the mortar is made, crystallizes and binds the mass together. Hydrate of lime is soluble in 831 parts of water at 78° Fahr; in 759 parts at 32° and in 1136 parts at 140°. Third, as the per cent. of water in the mortar is reduced and reaches five per cent., carbonic acid begins to be absorbed from the atmosphere. If the mortar contains more than five per cent. this absorption does not go on. While the mortar contains as much as 0.7 per cent. the absorption continues. The resulting carbonate probably unites with the hydrate of lime to form a sub-carbonate, which causes the mortar to attain a harder set, and this may finally be converted to carbonate. The mere drying out of mortar, our tests have shown, is sufficient to enable it to resist the pressure of masonry, while the further hardening furnishes the necessary bond.*

Magnesian Limes evolve less heat when slaking, expand less, and set more rapidly than pure lime. A typical analysis is given on page 47.

Hydrated Lime is the powdered product formed by slaking quick lime with the requisite amount of water. The material as it comes in commerce is a very finely divided white powder, and if properly prepared contains no unhydrated particles of lime.

SUB-CLASSIFICATION OF PORTLAND CEMENTS

In addition to the gray-colored cements for ordinary uses, Portland cements are made from raw materials low in iron so as to produce a light colored cement, and also from raw materials low in aluminum and high in iron to produce a cement which better resists the action of sea water. This leads to a sub classification suggested by Mr. Eckel. The distinction is somewhat arbitrary, since the classes grade into each other, while normal Portlands vary in the relative proportions of iron and alumina.

(1) Normal Portlands. Containing, with the silica and lime, both alumina and iron oxide in appreciable quantity; usually from 4 to 10 per cent alumina with 1.5 to 5 per cent iron oxide. Product: the ordinary gray-colored commercial cement.

(2) Low Iron Portlands. Containing relatively high alumina, with only 1 per cent or less of iron oxide. Product, white or very light colored, quick setting, usually low in tensile strength.

(3) High Iron Portlands. Containing relatively high iron, with less than 2 per cent of alumina. Product: slow setting, high tensile strength in long time tests, resistant to sea and alkaline waters.

* The authors are indebted to Mr. Clifford Richardson for this paragraph.

CHAPTER VI

CHEMISTRY OF HYDRAULIC CEMENTS*

BY SPENCER B. NEWBERRY

INTRODUCTION

Hydraulic cements are compounds consisting chiefly of lime, silica, and alumina, which have the property, when mixed to a paste with water, of hardening to a stone-like mass. They may be classified as follows:

1. **Portland cement,** made by calcining at high heat an artificial mixture of carbonate of lime and clay or slag, in exactly correct proportions, and grinding the resulting clinker to powder.
2. **Natural cement,** made by burning at low heat limestone containing excess of clay and usually much magnesia, and grinding the product to powder.
3. **Hydraulic lime,** obtained by burning limestone containing a small amount of clay, slaking by sprinkling with water, and bolting the product.
4. **Puzzolan or slag cement,** consisting of a mixture of certain kinds of volcanic scoria, or of blast furnace slag, and slaked lime, ground together.

Each of these classes of cement shows peculiar qualities, and each may have advantages for certain purposes. Puzzolan cement is that used by the Romans, and many striking examples of its durability are seen in ancient structures. Slag cement, a mechanical mixture of slag and slaked lime, is made to a considerable extent in this country, and finds extended use for mortar and in work in which the greatest strength and hardness are not required. Hydraulic lime is made chiefly in France, and is but little known in the United States. Natural cement is manufactured on a very large scale from limestones containing a large proportion of clay. It is usually quick-setting, and the better qualities gain very good strength at long periods. Owing to its cheapness it is extensively used, chiefly as mortar for brickwork and masonry. All these earlier hydraulic materials, however, have gradually given way before the advance of Portland cement, as this product has been improved in quality and manufactured on a constantly increasing scale.

Portland cement was first made in England in 1827, and named from the

*The authors are indebted to Mr. Newberry for this chapter, which has been especially prepared by him for this Treatise.

resemblance in color of the hardened cement to the building stone quarried at the Island of Portland.

MATERIALS*

As above stated, hydraulic lime and natural cements are made by burning natural limestones containing suitable amounts of clay. Portland cement, on the other hand, is made from an artificial mixture of materials, of exactly correct composition. Limestones containing clay are of frequent occurrence. If a deposit of stone containing exactly the right amount of clay, and of exactly uniform composition, could be found, Portland cement could be made from it simply by burning and grinding. For good results, however, the composition of the raw material must be *exact*, and the proportion of carbonate of lime in it must not vary even by one per cent. No natural deposit of rock of exactly this correct and unvarying composition is known or likely ever to be found; therefore Portland cement is always made from an artificial mixture, usually, if free from organic matter, containing about 75% carbonate of lime and 25% clay.

For the manufacture of Portland cement the materials chiefly used are limestone, chalk or marl, and clay. In southeastern Pennsylvania and western New Jersey occurs an unlimited deposit of *cement rock*, which consists of a slate-like limestone containing usually rather more clay than is required for a correct mixture. This is largely used for Portland cement manufacture, and is generally ground with a small amount of purer limestone to bring it to correct composition. At some of the factories in that section a correct mixture is obtained by grinding together, in suitable proportions, the upper and lower layers of the quarry. In the Central States, pure limestone, or marl (a soft and finely divided form of carbonate of lime) and clay, are the materials employed. Whatever the materials used, the first stage of the process is the preparation of an intimate and finely ground mixture of carbonate of lime and clay, of a certain definite composition, and if this is accomplished the resulting cement will be the same, whatever the original materials may have been. Success in Portland cement manufacture depends, more than upon all other features of the process, in extremely fine grinding of the raw materials. Most of the faults found in inferior Portland cement are due to neglect in this regard. Either the wet or dry process may be used in preparing the mixture. The material is then dried and calcined at white heat, generally in revolving cylindrical kilns, from which it issues in the form of small, black, rounded fragments of clinker. By grinding this clinker to fine powder the finished Portland cement is obtained.

*The materials for cement and the manufacture of cement are also treated in Chapter XXX.

Magnesia in Portland cement, beyond a small percentage, has generally been considered objectionable. But little positive evidence on this point is, however, available. A committee of the German Portland Cement Manufacturers Association, many years ago, reported that magnesia up to 8 per cent. is harmless. Dyckerhoff, a member of the committee, presented a minority report stating that he had found more than 4 per cent. injurious. The subject was referred to another committee, in 1896, but this committee laid out a program of work which proved impracticable to complete, and nothing further has been accomplished. Van Blaese, in the *Thonindustriezeitung*, 1899, page 213, published a long series of tests of cements containing variable proportions of magnesia, which show that cement containing 8 per cent. is faultless, while that containing 15 per cent. is defective. The writer has made a similar series of experiments and has found that properly prepared cement with 9 per cent. magnesia passes the boiling test perfectly, while that with 15 per cent. magnesia shows expansion cracks after several hours boiling. Comparative tests of tensile strength and expansion of bars of these cements, over long periods, are now in progress. From the evidence now available it appears that the presence of magnesia up to 8 per cent., in a properly prepared Portland cement, is no disadvantage.

Sulphate of lime, in quantities exceeding about 2 per cent., is objectionable in the raw material, owing to liability of reduction to sulphide, causing the cement to turn dark blue in hardening and to give poor tests, especially with sand. This fault is more frequent with cement burned in vertical kilns than in those of the rotary type, since the former are more liable to imperfect draft and consequent reducing action.

Clay for Portland cement manufacture should be highly siliceous and practically free from coarse sand. Siliceous clays, in which the silica is from 2.5 to 3.0 times the sum of alumina and iron oxide, give mixtures which stand the high heat of the kiln without fusing, produce a clinker which is comparatively easy to grind, and yield slow-setting cements which show steady gain in strength over long periods. More aluminous clays give hard, fusible clinker and quick-setting cement, and are in many respects troublesome to use. Highly aluminous cements are believed to be especially severely attacked by sea water.

Alkalies (potash and soda) appear to exert very little influence, in the small amounts present in ordinary clays, on the character of burning or quality of the resulting cement. Excess of alkalies is believed to make cement unsound.

PROPORTION OF INGREDIENTS

Although Portland cement has been manufactured since 1827, definite rules for proportioning the ingredients have only lately been established, and are yet by no means generally accepted. In Germany it has been customary to adjust the ingredients, as recommended by Michaelis, so that the "hydraulic modulus," the ratio by weight of lime to silica, alumina and iron oxide, shall be from 1.8 to 2.2. It has, however, become generally recognized by cement chemists that much more lime combines with silica than with alumina or iron oxide. The "hydraulic modulus" is therefore a variable, and must be much higher in the case of siliceous materials than with those high in alumina and iron.

A clear explanation of the composition of Portland cement clinker was first given by Le Chatelier in 1887. From microscopic examination of clinker and hardened cement he came to the conclusion that the chief constituent of Portland cement is tri-calcium silicate, $3CaO.SiO_2$, which is the active element in the hardening. This tri-silicate is produced by chemical precipitation from a mass of a multiple silico-aluminate which serves as a vehicle for the silica and lime and permits their combination, but remains inert during the hardening. Le Chatelier stated that the lime and magnesia in Portland cement should not exceed a maximum.

$$\frac{CaO+MgO}{SiO_2+Al_2O_3} \leqq 3 \qquad (1)$$

nor be less than a minimum,

$$\frac{CaO+MgO}{SiO_2-Al_2O_3-Fe_2O_3} \geqq 3 \qquad (2)$$

These formulas represent chemical equivalents and not weights.

The best brands of modern Portland cement approach pretty closely to the above maximum formula, while one corresponding to the minimum formula would be so greatly over-clayed as to be practically useless.

The hardening of cement, according to Le Chatelier, consists in the decomposition of the tri-silicate by water, with the formation of crystalline calcium hydrate and hydrated mono-silicate.

Since the publication of the above researches the constitution of clinker and hardened cement have been investigated by numerous experimenters, and a great number of new theories have been propounded. It cannot be said, however, that any of Le Chatelier's important statements have been disproved, nor that any material advance has been made upon the theory which he proposed. At the present time Portland cement clinker is re-

garded by nearly all cement chemists as a crystalline mass of tri-calcium silicate, imbedded in a non-crystalline magma consisting of a fusible compound of silica and lime with practically all the alumina and iron oxide.

Le Chatelier's formulas are inconvenient in form and incomprehensible except to those familiar with chemical formulas. The writer published in 1897 (*Journal of the Society of Chemical Industry*, Nov. 30, 1897) a paper on the constitution of hydraulic cements, containing an account of a series of experiments based on the work of Le Chatelier. It was found that the maximum of lime which could be brought into combination to produce a sound cement is three molecules for each molecule of silica present, and two molecules for each molecule of alumina. The composition of cement containing the maximum of lime would therefore be expressed by the formula

$$X(3CaO.SiO_2) + Y(2CaO.Al_2O_3) \qquad (3)$$

It is understood that this formula is merely empirical, representing the relative proportions present, since the aluminate remains for the most part in the magma in combination with part of the silica and with other substances.

Substituting weights for equivalents, the above formula may be expressed as follows:

$$\text{Lime} = \text{silica} \times 2.8 + \text{alumina} \times 1.1.$$

It should be remembered that this formula represents the *maximum* of lime which a Portland cement, burned in the usual manner, may contain without showing unsoundness. This maximum can be reached only by extremely fine grinding of the raw material. This formula, also, by no means represents the composition of finished cement, since the ash of the fuel lowers the lime and raises the silica and alumina, above that calculated from the raw material, by at least 2 per cent.

In the laboratory, using gas as fuel, it will be found practicable to prepare sound cements corresponding to the above formula. In actual manufacture it is safer to reduce the lime slightly, to counterbalance possible defective grinding of raw material or unavoidable variations in composition It will be found that the raw material at factories where the best Portland cements are made rarely falls below the composition,

$$\text{Lime} = \text{silica} \times 2.7 + \text{alumina} \times 1.0. \qquad (4)$$

This may be taken as a safe practical formula for commercial use. With fine grinding of the raw material it will invariably yield sound cements,

while the use of a lower proportion of lime will be likely to produce quick-setting cement, low in tensile strength. As already explained, commercial cements are considerably lower in lime, owing to change in composition produced by the fuel-ash.

The writer's experiments have shown that magnesia forms with clay no products having hydraulic properties It should therefore be disregarded in calculating cement mixtures, the composition of which should be calculated on the basis of the silica, alumina and lime only, without regard to the magnesia present. Iron oxide, also, in the quantities usually met with in ordinary clays, plays an insignificant part so far as the proportions of the constituents are concerned, and may be disregarded in the calculation.

As a practical example of the use of the above formula, let us suppose that we wish to make cement from limestone and clay of the following composition:

	Limestone	Clay
Lime	52.6	2.2
Magnesia	0.7	1.9
Silica	3.2	65.4
Alumina	1.0	16.5
Iron Oxide	0.3	6.1
Loss on ignition, etc.	42.2	7.9
	100.0	100.0

The silica and alumina in the limestone will require

$3.2 \times 2.7 + 1.0 = 9.6\%$ lime, leaving $52.6 - 9.6 = 43.0\%$ lime available for combination with clay.

The silica and alumina in 100 parts clay will require

$65.4 \times 2.7 + 16.5 \times 1.0 = 193.1$ parts lime. Subtracting the lime contained in the clay we have

$193.1 - 2.2 = 190.9$ parts lime required for 100 parts clay.

As the 100 parts stone contain 43 parts available lime, that amount of stone will require

$$\frac{43 \times 100}{190.9} = 22.5 \text{ parts clay.}$$

The composition of the charge and of the resulting cement may be tabulated as follows:

	100 STONE	22.5 CLAY	122.5 MIX	78.52 CEMENT	100 CEMENT
Lime	52.60	0.50	53.10	53.10	67.63
Magnesia	0.70	.43	1.13	1.13	1.44
Silica	3.20	14.71	17.91	17.91	22.81
Alumina	1.00	3.71	4.71	4.71	6.00
Iron Oxide	0.30	1.37	1.67	1.67	2.12
Loss, etc.	42.20	1.78	43.98	....	....
	100.00	22.50	122.50	78.52	100.00

As stated above, the ash of the fuel will change the composition of the resulting cement materially; analysis of the product, burned with coal, will probably show about 65 per cent. lime and perhaps 24 per cent. silica. This fuel-ash is, however, not uniformly distributed through the product, but attaches itself chiefly to the surfaces of the clinker. It is not, therefore, found practicable to materially raise the proportion of lime to counterbalance the silica and alumina of the ash.

It will be noted that in the above calculated analysis of raw mixture and cement the

$$\frac{\text{Lime—alumina}}{\text{silica}} = 2.7$$

The writer proposes to call this figure the *lime factor* of the mixture. Adoption of this factor will give cements of practically maximum quality with any materials, whether siliceous or aluminous, provided the mix is finely ground and properly burned. Owing to the influence of the ash of the fuel, as above explained, the factor of finished cements will be found about 0.2 lower than that of the raw material. The best commercial cements generally show a factor of 2.5 to 2.6, though made from mixtures with a factor of 2.7 to 2.8.

The following analyses, taken from a paper by the writer in *Cement and Engineering News*, November, 1901, show the influence of the fuel-ash on the composition of the clinker. The samples of clinker were taken one

hour later than those of raw material, since the passage through the kiln required about one hour.

Lehigh Portland Cement Co., Allentown, Pa.

	Mix	Clinker, calculated from mix	Clinker found
SiO_2	14.33	22.18	22.96
Al_2O_3	4.32	6.68	6.78
Fe_2O_3	1.46	2.26	2.54
CaO	42.69	66.08	63.95
MgO and SO_3	1.81	2.80	2.94
Loss	35.14	....	....
	99.75	100.00	99.17
Factor $\frac{CaO - Al_2O_3}{SiO_2}$	....	2.68	2.40

Sandusky Portland Cement Co., Syracuse, Ind.

	Mix	Clinker, calculated from mix	Clinker found
SiO_2	13.50	22.02	22.33
Al_2O_3	3.43	5.60	5.53
Fe_2O_2	1.27	2.07	3.28
CaO	40.76	66.49	64.40
MgO and SO_3	3.27	3.82	3.61
Loss	38.30	....	....
	100.53	100.00	99.15
Factor $\frac{CaO - Al_2O_3}{SiO_2}$	....	2.76	2.63

Comparison of the above analyses of mix and clinker shows how greatly the ash of the fuel affects the composition. In commercial cement a still further reduction in the proportion of lime is caused by the addition of gypsum and the absorption of moisture and carbonic acid from the air. It will be readily seen, therefore, that analysis of finished cement gives but little indication of the true proportion of ingredients or of the quality of the product.

EFFECT OF COMPOSITION ON QUALITY

Too high proportion of lime (lime factor of mix above 2.8) will give a slow-setting cement which will fail in the boiling test. If the excess of lime is great, pats of cement kept in cold water will show radial expansion cracks at the edges after a certain time, perhaps even within a few days. The same defects result from *imperfect grinding of the raw material,* and are far more often due to this cause than to excess of lime. Cement which is unsound and shows expansion from either cause may be improved and perhaps made sound by storage or by exposure to air. It is not, however, safe to rely greatly on this remedy. Lack of soundness is in all cases due to faulty manufacture, since well-burned cement made from suitably prepared raw material will invariably pass all soundness tests when fresh from the grinding mills. Consumers are advised to accept no cement which fails to pass a reasonable boiling test, as they will thus err, if at all, on the safe side, and will influence careless manufacturers to improve their process.

Too low proportion of lime, giving an over-clayed mixture, produces a fusible clinker, liable to overburning. This is especially the case with aluminous materials. If hard-burned, such mixtures give a fused clinker liable to fall to dust on cooling, hard to grind, and yielding slow-setting cement of poor hardening properties. If light-burned, an over-clayed mixture yields soft brownish clinker, grinding to a brownish, quick-setting cement of inferior strength.

Overburning rarely occurs except with over-clayed mixtures or in consequence of the fluxing action of the fuel-ash or the brick lining of the kiln Properly proportioned mixtures stand a very high heat without injury.

Underburning, as stated above, in the case of an over-clayed mixture, yields quick-setting and weak cement. Normal mixtures, when underburned, usually give cement which fails in soundness tests. Light burning is generally indicated by heating of the cement on mixing with water. This behavior generally accompanies quick setting, and may be so marked as to be quite apparent to the touch of the fingers. Some cements, though slow-setting when first made, become very quick-setting on storage. Cases are on record in which this change has taken place within a few days. After longer periods the original slow-setting quality may return. The cause of this phenomenon has not been determined; it may be said, however, that troubles of this class, including quick setting and heating with water, are especially characteristic of cements made from aluminous materials.

CHAPTER VII

STANDARD CEMENT TESTS

The tests which are regarded as most suitable for the selection and acceptance of cement for important concrete construction are as follows:

Chemical analysis.
Specific gravity.
Fineness.
Activity, or time of setting.
Tensile strength of neat cement and sand mortars.
Soundness or constancy of volume.

The French Commission* in 1893, in addition to these tests, gave standard rules for testing weight, homogeneity (with the microscope), compressive strength, bending strength, yield of paste and mortar (*rendement*), porosity, permeability, decomposition, and adhesion, one or more of which tests may be desirable under certain conditions. As these are usually of minor importance, however, special mention of them is reserved for the following chapter.

In unimportant construction it is often safe to use a first-class American Portland cement without testing, and in other cases the test for soundness is the only one which need be actually made. Under almost all circumstances, however, when purchasing cement, full specifications (see Chapter III, p. 28) are advisable, so that if the cement does not work satisfactorily it may be more carefully examined and unused portions rejected.

In this chapter are presented, in addition to the description of the methods of making cement tests, complete lists of apparatus for a large and a small laboratory (p. 80), formulas and tables for determining the quantity of water in cement mortars (p. 85), comparisons of American and European practice in cement testing, a discussion of the causes of unsoundness and the results of soundness tests (p. 101), curves showing the growth in strength of typical cements and cement mortars (p. 99), and other information with reference to the qualities and testing of Portland cement.

STANDARD METHODS OF CEMENT TESTING

The following recommendations for testing are reprinted, with comments by the authors, from the preliminary or Progress Report of Special Com-

*Commission des Méthodes d'Essai des Matériaux de Construction, 1894, Vol. 1, p. 235.

mittee on Uniform Tests of Cement of the American Society of Civil Engineers,* as presented in 1903 and amended in 1904 and 1912. The methods are designed particularly for the testing of Portland cement, but certain paragraphs are applicable to Natural (and also to Puzzolan).

The standards which should be attained by first-class Portland and Natural cements are presented in the Standard Specifications in Chapter III, page 28.

Sampling. 1. *Selection of Sample.* The selection of samples for testing should be left to the engineer. The number of packages sampled and the quantity taken from each package will depend on the importance of the work and the facilities for making the tests.

2. The samples should fairly represent the material. When the amount to be tested is small it is recommended that one barrel in ten be sampled; when the amount is large it may be impracticable to take samples from more than one barrel in thirty or fifty. When the samples are taken from bins at the mill one for each fifty to two hundred barrels will suffice.

3. Samples should be passed through a sieve having twenty meshes per linear inch, in order to break up lumps and remove foreign material; the use of this sieve is also effective to obtain a thorough mixing of the samples when this is desired. To determine the acceptance or rejection of cement it is preferable, when time permits, to test the samples separately. Tests to determine the general characteristics of a cement, extending over a long period, may be made with mixed samples.

4. *Method of Sampling.* Cement in barrels should be sampled through a hole made in the head, or in one of the staves midway between the heads, by means of an auger or a sampling iron similar to that used by sugar inspectors; if in bags, the sample should be taken from surface to center; cement in bins should be sampled in such a manner as to represent fairly the contents of the bin. Sampling from bins is not recommended if the method of manufacture is such that ingredients of any kind are added to the cement subsequently.

Fig. 8. Sampling Iron. (See p. 64.)

A sampling iron is illustrated in Fig. 8.

With the usual packing of Portland cement, four bags to the barrel, one bag in forty is equivalent to one barrel in ten. There is no necessity because of the smaller size of the packages for testing a larger proportion of the total weight.

The practice of mixing samples taken from a number of packages is by many authorities not considered advisable except for the purpose, suggested above, "of determining the characteristics of a shipment." A mixture of samples will not reveal irregularities in quality.

*Proceedings, American Society of Civil Engineers, February, 1912.

Chemical Analysis. 5. *Significance.* Chemical analysis may serve to detect adulteration of cement with inert material, such as slag or ground limestone, if in considerable amount. It is useful in determining whether certain constituents, such as magnesia and sulphuric anhydride are present in inadmissible proportions.

6. The determination of the principal constituents of cement, silica, alumina, iron oxide, and lime, is not conclusive as an indication of quality. Faulty cement results more frequently from imperfect preparation of the raw material or defective burning than from incorrect proportions. Cement made from material ground very finely and thoroughly burned may contain much more lime than the amount usually present, and still be perfectly sound. On the other hand, cements low in lime may, on account of careless preparation of the raw material, be of dangerous character. Furthermore, the composition of the product may be so greatly modified by the ash of the fuel used in burning as to affect in a great degree the significances of the results of analysis.

7. *Method.* The method to be followed should be that proposed by the Committee on Uniformity in the Analysis of Materials for the Portland Cement Industry, reported in the *Journal* of the Society for Chemical Industry, Vol. 21, page 12, 1902; and published in *Engineering News*, Vol. 50, p. 60, 1903; and in *Engineering Record*, Vol. 48, p. 49, 1903, and in addition thereto, the following:

The insoluble residue may be determined as follows: To a 1-gramme sample of the cement are added 30 cu. cm. of water and 10 cu. cm. of concentrated hydrochloric acid, and then warmed until the effervescence ceases, and digested on a steam bath until dissolved. The residue is filtered, washed with hot water, and the filter paper and contents digested on the steam bath in a 5% solution of sodium carbonate. This residue is filtered, washed with hot water, then with hot hydrochloric acid, and finally with hot water, and then ignited at a red heat and weighed. The quantity so obtained is the insoluble residue.

An exceedingly simple test for determining adulteration with raw or partially burned rock, is the treatment of the cement with muriatic acid as described in the purity test on page 4. It does not furnish the percentage of foreign ingredients, but the black precipitation of the adulterant darkens the color of the yellow jelly to a degree depending upon the quantity of adulteration.

Specific Gravity. 8. *Significance.* The specific gravity of cement is lowered by adulteration and hydration, but the adulteration must be considerable to be detected by tests of specific gravity.

9. Inasmuch as the differences in specific gravity are usually very small, great care must be exercised in making the determination.

10. *Apparatus.* The determination of specific gravity should be made with a standardized Le Chatelier apparatus. This consists of a flask (*D*), Fig. 9, of about 120 cu. cm. capacity, the neck of which is about 20 cm. long; in the middle of this neck is a bulb (*C*), above and below which

are two marks (*F*) and (*E*); the volume between these two marks is 20 cu. cm. The neck has a diameter of about 9 mm., and is graduated into tenths of cubic centimeters above the mark (*F*).

11. Benzine (62° Beaumé naphtha) or kerosene free from water should be used in making the determination.

12. *Method.* The flask is filled with either of these liquids to the lower mark (*E*), and 64 grammes of cement, cooled to the temperature of the liquid, is slowly introduced through the funnel (*B*), (the stem of which

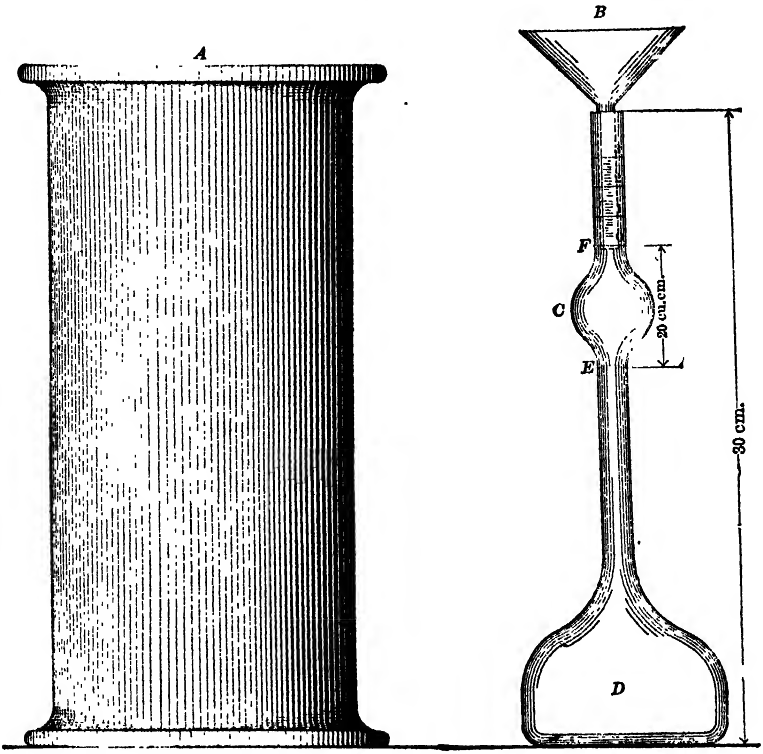


FIG. 9.—Le Chatelier's Specific Gravity Apparatus. (*See p. 65*)

should be long enough to extend into the flask to the top of the bulb (*C*)), taking care that the cement does not adhere to the sides of the flask, and that the funnel does not touch the liquid. After all the cement is introduced, the level of the liquid will rise to some division of the graduated neck; this reading, plus 20 cu. cm., is the volume displaced by 64 grammes of the cement.

13. The specific gravity is then obtained from the formula

$$\text{Specific Gravity} = \frac{\text{Weight of cement, in grammes,}}{\text{Displaced volume, in cubic centimeters.}}$$

14. The flask, during the operation, is kept immersed in water in a jar (*A*), in order to avoid variations in the temperature of the liquid in the flask, which should not exceed $\frac{1}{2}°$ Cent. The results of repeated tests should agree within 0.01. The determination of specific gravity should be made on the cement as received; if it should fall below 3.10, a second determination should be made after igniting the sample at a low red heat in the following manner: One-half gramme of cement is heated in a weighed platinum crucible, with cover, for 5 minutes with a Bunsen burner (starting with a low flame and gradually increasing to its full height) and then heating for 15 minutes with a blast lamp; the difference between the weight after cooling and the original weight is the loss on ignition. The temperature should not exceed 900° Cent., and the ignition should preferably be made in a muffle.

15. The apparatus may be cleaned in the following manner: The flask is inverted and shaken vertically until the liquid flows freely and then held in a vertical position until empty; any traces of cement remaining can be removed by pouring into the flask a small quantity of clean liquid benzine or kerosene and repeating the operation.

The usual specific gravities of different classes of cement are given on page 81.

Le Chatelier's apparatus, suggested above as most convenient, was also recommended by Mr. E. Candlot after experiments for the French Commission,* in which he employed for comparison the Mann, Keate, Schumann, and Candlot, as well as the Le Chatelier apparatus.

Mr. Daniel D. Jackson† has more recently devised an apparatus with which temperature corrections can be made more readily than with the older types.

Fineness. 16. *Significance.* It is generally accepted that the coarser particles in cement are practically inert, and it is only the extremely fine powder that possesses cementing qualities. The more finely cement is pulverized, other conditions being the same, the more sand it will carry and produce a mortar of a given strength.

17. *Apparatus.* The fineness of a sample of cement is determined by weighing the residue retained on certain sieves. Those known as No. 100 and No. 200, having approximately 100 and 200 wires per linear inch, respectively, should be used. They should be at least 8 in. in diameter. The wire cloth should be of brass wire, and should conform to the following requirements:

No. of sieve	Diameter of wire	Meshes per linear inch	
		Warp	Woof
	in.		
100	0.0042 to 0.0048	95 to 101	93 to 103
200	0.0021 to 0.0023	192 to 203	190 to 205

* Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 15.
† See *Engineering Record*, July 16, 1904, p. 82.

The meshes in any smaller space, down to 0.25 in., should be proportional in number.

18. *Method.* The test should be made with 50 grammes of cement, dried at a temperature of 100° Cent. (212° Fahr.)

19. The cement is placed on the No. 200 sieve, which, with pan and cover attached, is held in one hand in a slightly inclined position and moved forward and backward about 200 times per minute, at the same time striking the side gently, on the up stroke, against the palm of the other hand. The operation is continued until not more than 0.05 gramme will pass through in one minute. The residue is weighed, then placed on the No. 100 sieve, and the operation repeated. The work may be expedited by placing in the sieve a few large steel shot, which should be removed before the final one minute of sieving. The sieves should be thoroughly dry and clean.

Laboratory scales for weighing the samples and the residue are illustrated in Fig. 10.

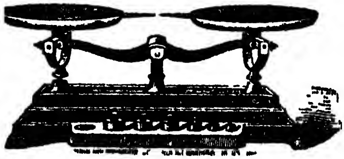


FIG. 10.—Delicate Laboratory Scales. (See p. 68)

A table is given on page 84 for comparing American and European sieves, and the effect of the fineness of cement upon its strength is discussed on page 82.

It is impracticable to sift cement through a sieve finer than 200 meshes per linear inch. The particles which will just pass a No. 200 sieve are about 0.10 millimeter (0.004 in.) in diameter.* For still further separating the cement, some method based on the principle of suspension in liquid is employed as described on page 85.

Normal Consistency. 20. *Significance.* The use of a proper percentage of water in making pastes† and mortars for the various tests is exceedingly important and affects vitally the results obtained.

21. The amount of water, expressed in percentage by weight of the dry cement, required to produce a paste of plasticity desired, termed "normal consistency," should be determined with the Vicat apparatus in the following manner:

*Allen Hazen in Report Massachusetts State Board of Health, 1892

†The term 'paste' is used in this report to designate a mixture of cement and water, and the word "mortar" to designate a mixture of cement, sand and mortar.

22. *Apparatus.* This consists of a frame (*A*), Fig. 11, bearing a movable rod (*B*), weighing 300 grammes, one end (*C*) being 1 cm. in diameter for a distance of 6 cm., the other having a removable needle (*D*), 1 mm. in diameter, 6 cm. long. The rod is reversible, and can be held in any desired position by a screw (*E*), and has midway between the ends a mark (*F*) which moves under a scale (graduated to millimeters) attached to the frame (*A*). The paste is held by a conical, hard-rubber ring (*G*), 7 cm. in diameter at the base, 4 cm. high, resting on a glass plate (*H*) about 10 cm. square.

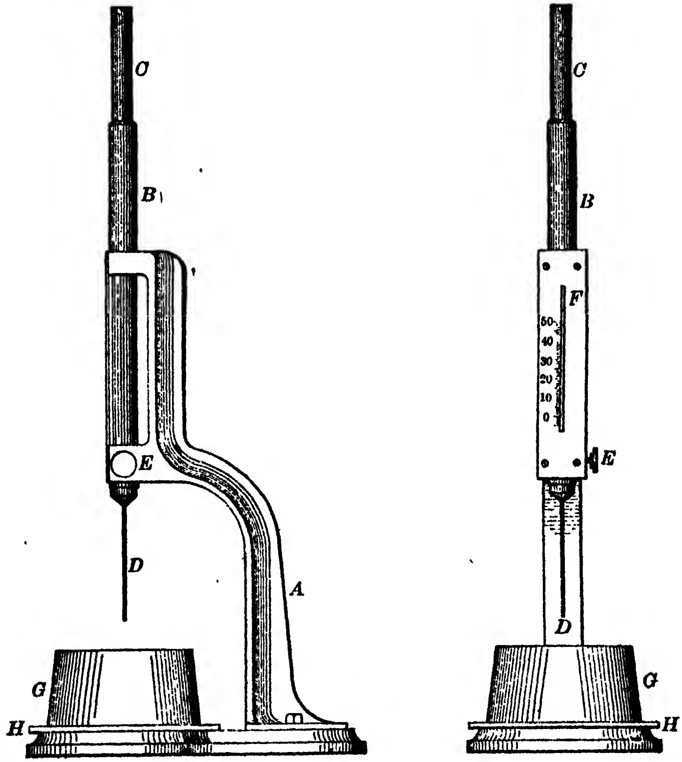


FIG. 11.—Vicat Apparatus. (*See p. 69*)

23. *Method.* In making the determination, the same quantity of cement as will be used subsequently for each batch in making the test pieces, but not less than 500 grammes, with a measured quantity of water, is kneaded into a paste, as described in Paragraph 45, and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained about 6 in. apart;

the ball resting in the palm of one hand is pressed into the larger end of the rubber ring held in the other hand, completely filling the ring with paste; the excess at the larger end is then removed by a single movement of the palm of the hand; the ring is then placed on its larger end on a glass plate and the excess paste at the smaller end is sliced off at the top of the ring by a single oblique stroke of a trowel held at a slight angle with the top of the ring. During these operations care must be taken not to compress the paste. The paste confined in the ring, resting on the plate, is placed under the rod, the larger end of which is brought in contact with the surface of the paste; the scale is then read, and the rod quickly released.

24. The paste is of normal consistency when the cylinder settles to a point 10 mm. below the original surface in one-half minute after being released. The apparatus must be free from all vibrations during the test.

25. Trial pastes are made with varying percentages of water until the normal consistency is obtained.

26. Having determined the percentage of water required to produce a paste of normal consistency, the percentage required for a mortar containing by weight one part of cement to three parts of standard Ottawa sand, is obtained from the following table, the amount being a percentage of the combined weight of the cement and sand.

Percentage of Water for Standard Mortars.

eat	One cement, three standard Ottawa sand	Neat	One cement, three standard Ottawa sand	Neat	One cement, three standard Ottawa sand
15	8.0	23	9.3	31	10.7
16	8.2	24	9.5	32	10.8
17	8.3	25	9.7	33	11.0
18	8.5	26	9.8	34	11.2
19	8.7	27	10.0	35	11.3
20	8.8	28	10.2	36	11.5
21	9.0	29	10.3	37	11.7
22	9.2	30	10.5	38	11.8

Formulas of Mr. R. Feret for determining the percentage of water for sand mortars, and a table formally used, are presented on pages 86 and 88.

The Boulogne Method for determining the proper consistency of neat paste was formerly in general use in France, and is still the best guide for determining the correct consistency of paste when the Vicat apparatus is not available. The Vicat apparatus, however, should be included in every well equipped cement laboratory, experiments by Messrs. P. Alexandre and R. Feret for the French Commission* showing that it gives much more uniform results than the Boulogne method.

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 49.

The Boulogne method requires that the paste shall be firm but well bonded, shining and plastic, and shall satisfy the following conditions:

1. The consistency shall not change if it is worked 3 minutes longer than the original 5 minutes.*

2. If dropped 50 centimeters (20 in.) from a trowel, it should leave the trowel clean, and fall without losing its shape or cracking.

3. Light pressure in the hand should bring water to the surface, and the paste should not stick to the hand. If a ball thus formed falls from a height of about 50 centimeters (20 in.) it should retain its rounded form without showing cracks.

4. The proportion of water should be such that more or less will produce opposite effects from those just described for the proper consistency.

Time of Setting. 27. *Significance.* The object of this test is to determine the time which elapses from the moment water is added until the paste ceases to be plastic (called the "initial set"), and also the time until it acquires a certain degree of hardness (called the "final set" or "hard set"). The former is the more important, since, with the commencement of setting, the process of crystallization begins. As a disturbance of this process may produce a loss of strength, it is desirable to complete the operation of mixing or moulding or incorporating the mortar into the work before the cement begins to set.

28. *Apparatus.* The initial and final set should be determined with the Vicat apparatus described in Paragraph 22.

29. *Method.* A paste of normal consistency is moulded in the hard-rubber ring, as described in Paragraph 23, and placed under the rod (*B*), the smaller end of which is then carefully brought in contact with the surface of the paste, and the rod quickly released.

30. The initial set is said to have occurred when the needle ceases to pass a point 5 mm. above the glass plate; and the final set, when the needle does not sink visibly into the paste.

31. The test pieces should be kept in moist air during the test; this may be accomplished by placing them on a rack over water contained in a pan and covered by a damp cloth; the cloth to be kept from contact with them by means of a wire screen; or they may be stored in a moist box or closet.

32. Care should be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point may increase the penetration.

33. The time of setting is affected not only by the percentage and temperature of the water used and the amount of kneading the paste receives, but by the temperature and humidity of the air, and its determination is, therefore, only approximate.

*The original working for the U. S. Standard tests is one minute (see paragraph 45).

For practical purposes in ordinary construction where laboratory apparatus is unavailable, the setting qualities of a cement or mortar may often be examined by making up pats from a number of the packages and trying their hardening by pressure of the thumb. When the thumb nail fails to indent the surface, the paste or mortar may be considered to have reached its final set.

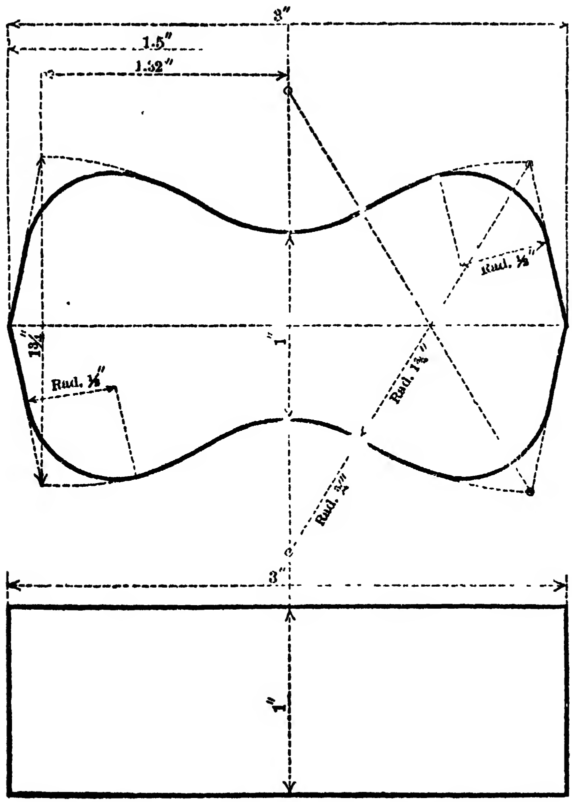


FIG. 12.—Details for Briquette. (*See p. 73.*)

The Gilmore needles, described on page 89 and there compared with the Vicat apparatus, were formerly the U. S. standard.

Standard Sand. 34. The sand to be used should be natural sand from Ottawa, Ill., screened to pass a No. 20 sieve, and retained on a No. 30 sieve. The sieves should be at least 8 in. in diameter; the wire cloth should be of brass wire and should conform to the following requirements:

No. of sieve	Diameter of wire	Meshes per linear inch	
		Warp	Woof
	in.		
20	0.016 to 0.017	19.5 to 20.5	19 to 21
30	0.011 to 0.012	29.5 to 30.5	28.5 to 31.5

Sand which has passed the No. 20 sieve is standard when not more than 5 grammes passes the No. 30 sieve in one minute of continuous sifting of a 500-gramme sample.*

Photographs of the grains of Ottawa and of crushed quartz sand are shown on page 175.

Form of Test Pieces. 35. For tensile tests the form of test piece shown in Fig. 12 should be used.

36. For compressive tests, 2-in. cubes should be used.

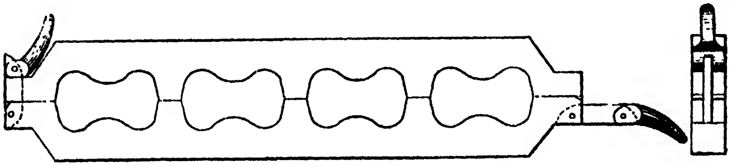


FIG. 13.—Details for Gang Mould. (*See p. 73*)

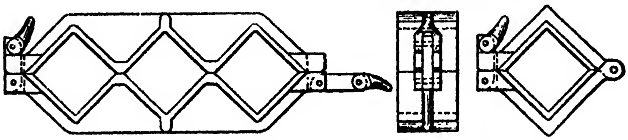


FIG. 13a.—Mould for Compression Test Pieces. (*See p. 73*)

European is compared with U. S. standard sand on page 90.

The German standard briquette is sketched on page 92.

Moulds. 37. The moulds should be of brass, bronze, or other non-corrodible material, and should have sufficient metal in the sides to prevent spreading during moulding.

38. Moulds may be either single or gang moulds. The latter are preferred by many. If used, the types shown in Figs. 13 and 13a are recommended.

39. The moulds should be wiped with an oily cloth before using.

Mixing. 40. The proportions of sand and cement should be stated by weight; the quantity of water should be stated as a percentage by weight of the dry material.

*This sand may now (1912) be obtained from the Ottawa Silica Co., at a cost of two cents per pound f. o. b. cars, Ottawa, Ill.

41. The metric system is recommended because of the convenient relation of the gramme and the cubic centimeter.

42. The temperature of the room and of the mixing water should be maintained as nearly as practicable at 21° Cent. (70° Fahr.).

43. The quantity of material to be mixed at one time depends on the number of test pieces to be made; 1 000 grammes is a convenient quantity to mix by hand methods.

44. The Committee has investigated the various mechanical mixing machines thus far devised, but cannot recommend any of them, for the following reasons: (1) the tendency of most cement is to "ball up" in the machine, thereby preventing working it into a homogeneous paste; (2) there are no means of ascertaining when the mixing is complete without stopping the machine; and (3) it is difficult to keep the machine clean.

45. *Method.* The material is weighed, placed on a non-absorbent surface (preferably plate glass), thoroughly mixed dry if sand be used, and a crater formed in the center, into which the proper percentage of clean water is poured; the material on the outer edge is turned into the center by the aid of a trowel. As soon as the water has been absorbed, which should not require more than one minute, the operation is completed by vigorously kneading with the hands for one minute. During the operation the hands should be protected by rubber gloves.

The apparatus required for mixing briquettes consists of a piece of 1-inch plate glass at least 24 inches square, counter scales (preferably metric system), recording from $\frac{1}{10}$ gram to $1\frac{1}{2}$ kilograms, a 250 cubic centimeter graduated measuring glass, rubber gloves, one 8-inch mason's trowel, one 4-inch pointing trowel, Fig. 14, and a thermometer.

FIG. 14.
(*See p. 73*)

European standards specify mixing five minutes instead of one minute. This difference in time is due to the methods of manipulation, in Europe the materials being mixed with a trowel or spoon. Experiments by the authors tend to show that a denser mixture can be obtained by kneading one minute than by mixing five minutes with a trowel, so that the American method is both quicker and better.

Moulding. 46. The Committee has not been able to secure satisfactory results with existing moulding machines; the operation of machine moulding is very slow; and is not practicable with pastes or mortars containing as large percentages of water as herein recommended.

47. *Method.* Immediately after mixing, the paste or mortar is placed in the moulds with the hands, pressed in firmly with the fingers, and smoothed off with a trowel without ramming. The material should be heaped above the mould, and, in smoothing off, the trowel should be drawn over the mould in such a manner as to exert a moderate pres-

sure on the material. The mould should then be turned over and the operation of heaping and smoothing off repeated.

48. A check on the uniformity of mixing and moulding may be afforded by weighing the test pieces on removal from the moist closet; test pieces from any sample which vary in weight more than 3% from the average should not be considered.

The method of introducing the paste or mortar into the moulds exercises considerable effect upon the strength of the specimen. If a comparatively dry mixture is employed and it is packed in thin layers into the mould, a denser mass results and the strength is higher, especially on short-time tests, than with specimens of a wet or plastic consistency. Results from plastic cements and mortars, however, show greater uniformity.

Although the French Commission in 1893 specified the method of using dry mortar, they recommended that after an international agreement standard plastic mortars be employed for all tests.

Experiments by Mr. R. Feret, made for the French Commission,* which are summarized in an article by the author† on *Variation in Strength of Mortars*, give the comparative strengths of specimens beaten with a spatule (the German method), pressed with a hand rammer, rammed in the Tetmajer apparatus, and rammed with the Bohme rammer (an alternate German method).

Storage of the Test Pieces. 49. During the first 24 hours after moulding, the test pieces should be kept in moist air to prevent drying.

50. Two methods are in common use to prevent drying: (1) covering the test pieces with a damp cloth, and (2) placing them in a moist closet. The use of the damp cloth, as usually carried out, is objectionable, because the cloth may dry out unequally and in consequence the test pieces will not all be subjected to the same degree of moisture. This defect may be remedied to some extent by immersing the edges of the cloth in water; contact between the cloth and the test pieces should be prevented by means of a wire screen, or some similar arrangement. A moist closet is so much more effective in securing uniformly moist air, and is so easily devised and so inexpensive, that the use of the damp cloth should be abandoned.

51. A moist closet consists of a soapstone or slate box, or a wooden box lined with metal, the interior surface being covered with felt or broad wicking kept wet, the bottom of the box being kept covered with water. The interior of the box is provided with glass shelves on which to place the test pieces, the shelves being so arranged that they may be withdrawn readily.

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol IV, p 73.
†*Cement*, July, 1903, p 165.

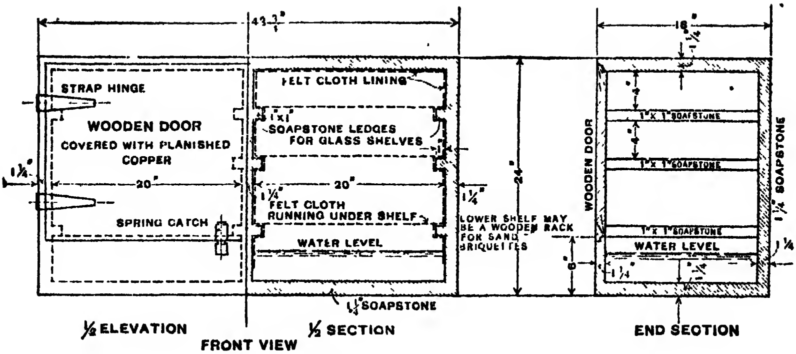


FIG. 15.—Moist Closet. (See p. 76)

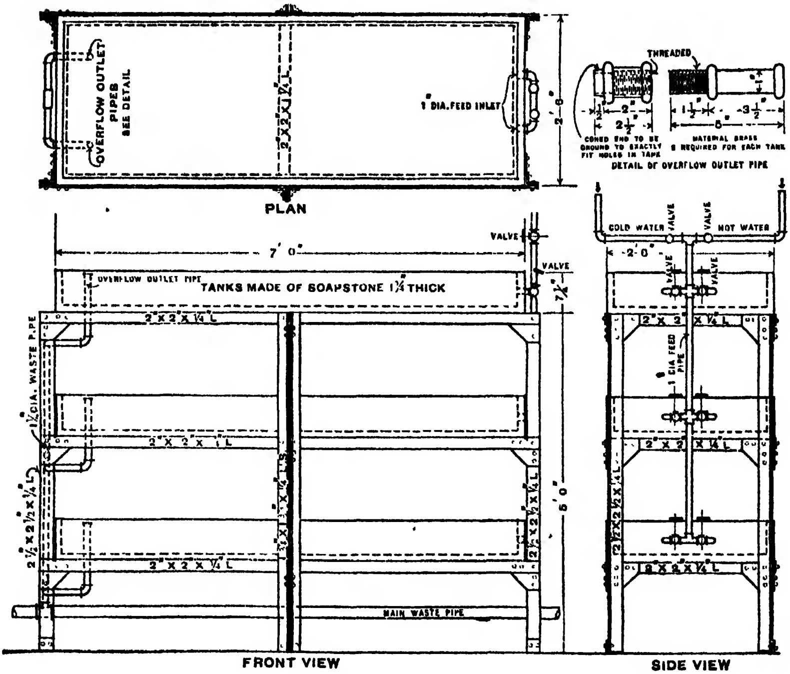


FIG. 16.—Immersion Tanks. (See p. 76)

52. After 24 hours in moist air, the pieces to be tested after longer periods should be immersed in water in storage tanks or pans made of non-corrodible material.

53. The air and water in the moist closet and the water in the storage tanks should be maintained as nearly as practicable at 21° Cent. (70° Fahr.).

A moist closet and storage pans designed by Mr. Richard L. Humphrey are shown in Figs. 15 and 16, page 76.

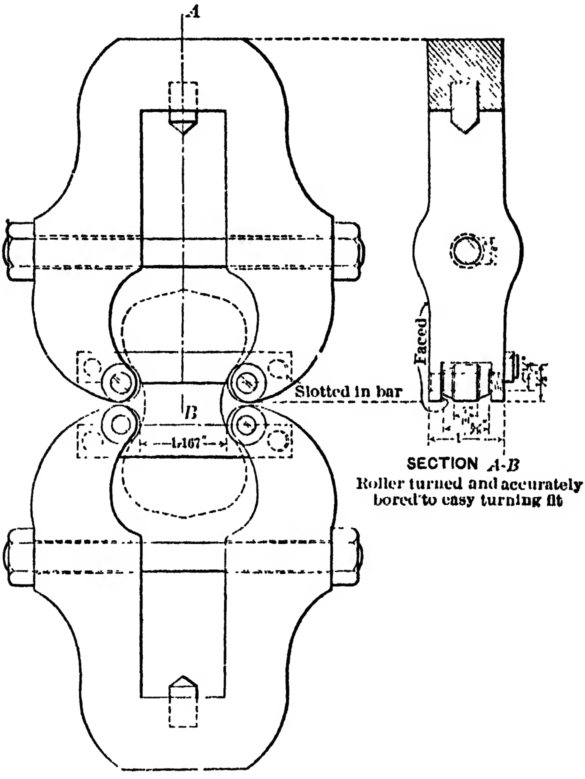


FIG. 17.—Form of Clip. (*See p. 77*)

Tensile Strength. 54. The tests may be made with any standard machine.

55. The clip is shown in Fig. 17. It must be made accurately, the pins and rollers turned, and the rollers bored slightly larger than the pins so as to turn easily. There should be a slight clearance at each end of the roller, and the pins should be kept properly lubricated and

free from grit. The clips should be used without cushioning at the points of contact.

56. Test pieces should be broken as soon as they are removed from the water. Care should be observed in centering the test pieces in the testing machine, as cross strains, produced by imperfect centering, tend to lower the breaking strength. The load should not be applied too suddenly, as it may produce vibration, the shock from which often causes the test piece to break before the ultimate strength is reached. The bearing surfaces of the clips and test pieces must be kept free from grains

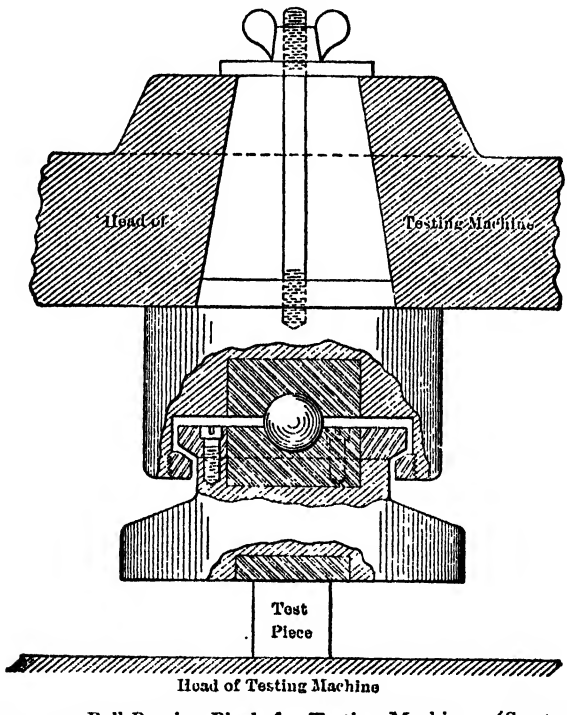
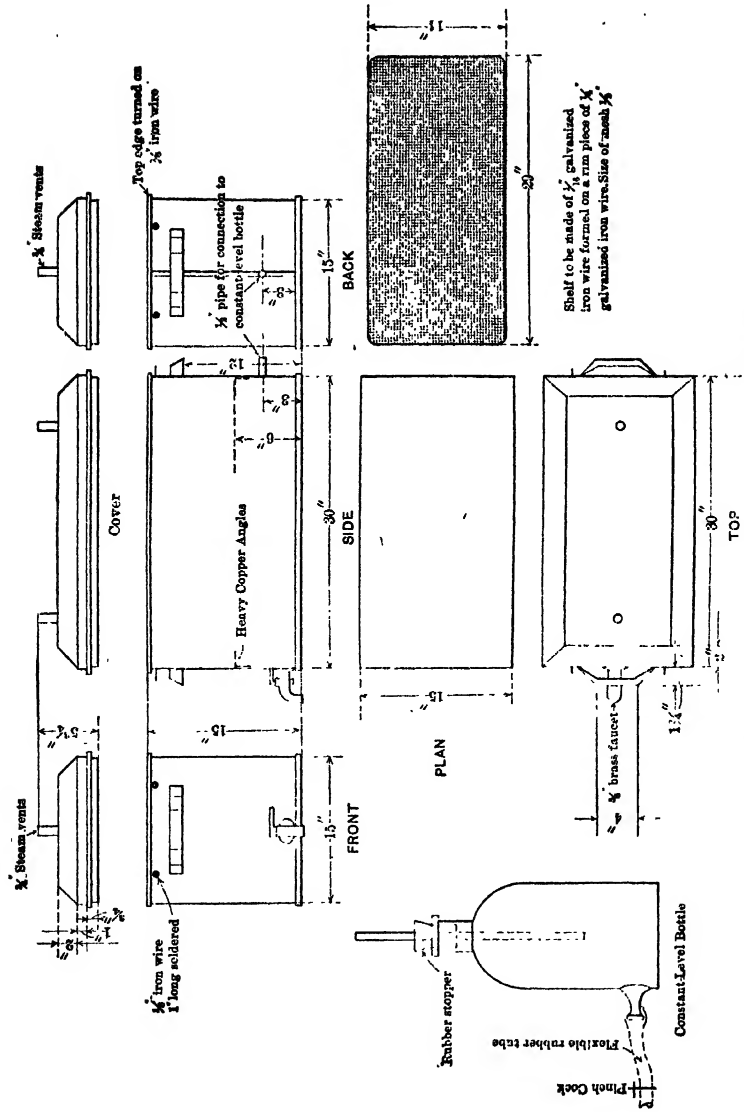


FIG. 17a.—Ball-Bearing Block for Testing Machine. (See p. 78)

of sand or dirt, which would prevent a good bearing. The load should be applied at the rate of 600 lb. per min. The average of the results of the test pieces from each sample should be taken as the test of the sample. Test pieces which do not break within ¼ in. of the center, or are otherwise manifestly faulty, should be excluded in determining average results.

Compressive Strength. 57. The tests may be made with any machine provided with means for so applying the load that the line of pressure is along the axis of the test piece. A ball-bearing block for this purpose is shown in Fig. 17a. Some appliance should be provided to



To be made of sheet copper weighing 22 oz. per sq. ft., tinned inside. All seams to be lapped where possible. Hard solder only to be used.

FIG. 18.—Apparatus for Making Accelerated Test for Soundness of Cement. (*See p. 78b*)

facilitate placing the axis of the test piece exactly in line with the center of the ball-bearing.

58. The test piece should be placed in the testing machine, with a piece of heavy blotting paper on each of the crushing faces, which should be those that were in contact with the mould.

Constancy of Volume. 59. *Significance.* The object is to detect those qualities which tend to destroy the strength and durability of a cement. Under normal conditions these defects will in some cases develop quickly, and in other cases may not develop for a considerable time. Since the detection of these destructive qualities before using the cement in construction is essential, tests are made not only under normal conditions but under artificial conditions created to hasten the development of these defects. Tests may, therefore, be divided into two classes: (1) Normal tests, made in either air or water maintained, as nearly as practicable, at 21° Cent. (70° Fahr.); and (2) Accelerated tests, made in air, steam or water, at temperature of 45° Cent. (113° Fahr.) and upward. The Committee recommends that these tests be made in the following manner:

60. *Methods.* Pats, about 3 in. in diameter, ½ in. thick at the center, and tapering to a thin edge, should be made on clean glass plates (about 4 in. square) from cement paste of normal consistency, and stored in a moist closet for 24 hours.

61. *Normal Tests.* After 24 hours in the moist closet, a pat is immersed in water for 28 days and observed at intervals. A similar pat, after 24 hours in the moist closet, is exposed to the air for 28 days or more and observed at intervals.

62. *Accelerated Test.* After 24 hours in the moist closet, a pat is placed in an atmosphere of steam, upon a wire screen 1 in. above boiling water, for 5 hours. The apparatus should be so constructed that the steam will escape freely and atmospheric pressure be maintained. Since the type of apparatus used has a great influence on the results, the arrangement shown in Fig. 18, page 78a, is recommended.

63. Pats which remain firm and hard and show no signs of cracking, distortion, or disintegration are said to be "of constant volume" or "sound."

64. Should the pat leave the plate, distortion may be detected best with a straight edge applied to the surface which was in contact with the plate.

65. In the present state of our knowledge it cannot be said that a cement which fails to pass the accelerated test will prove defective in the work; nor can a cement be considered entirely safe simply because it has passed these tests.

GEORGE S. WEBSTER, *Chairman.*
RICHARD L. HUMPHREY, *Secretary.*
W. B. W. HOWE,
F. H. LEWIS,
S. B. NEWBERRY,
ALFRED NOBLE,
CLIFFORD RICHARDSON,
L. S. SABIN,
GEORGE F. SWAIN.

ELEMENTARY DIRECTIONS FOR TESTING SOUNDNESS

Soundness tests, which are of greater importance than any other one test, may be made by those unskilled in laboratory practice, with no apparatus except a piece of plate glass at least ⅓ inch thick and 12 by 18 inches square, pieces of window glass 4 inches square, and a small trowel. Take samples at random from several barrels or bags, as described on page 64. From each sample make three pats of neat cement, requiring for the three about 8 ounces (250 grams) or one cupful of dry cement.

Cements of different classes and degrees of fineness require different percentages of water. The consistency must be such that the cement can be readily kneaded without crumbling and formed into a smooth pat with a thin edge, when pressed upon the piece of glass provided for it, without running or losing its shape.* Approximate amounts may be taken for the first trial of any cement, as, —

Portland Cement	....	20% of water by weight
Natural "	....	30% " "
Puzzolan "	....	18% " "

If these quantities after kneading give too wet or too dry a mixture, the paste should be thrown away and the trial repeated with less or more water until the desired consistency is attained. The percentage thus determined may generally be used in the remaining tests of the same shipment of cement.

Place a sample of the dry cement upon the plate glass in the form of a mound, and with the small trowel make a depression in the center. Weigh, or measure, a quantity of water which has been found by trial to give the proper consistency, and pour it into the depression, allowing it to soak into the cement, and then turn the material on the edges into the water with a trowel. As soon as the water is absorbed, the paste is kneaded for 1½ minutes with the hands, which should be protected with rubber gloves.

A piece of window glass about 4 inches square is required for each pat. A portion of the paste is made into a ball and pressed upon one of these pieces of glass so as to form a circular pat about 3 inches in diameter and ½ inch thick in the center, tapering to a thin edge. For the first 24 hours, to prevent the surface from drying too quickly, the pats must be kept under a cloth moistened and suspended above the pats, with its ends immersed in water to keep it wet. The temperature of the air while mixing, and of the water for mixing and storage, should be maintained as near as possible to 70° Fahr. (21° Cent.). At the end of 24 hours one pat should

*See also Boulogne method, p. 70.

be placed in water and another in air, to be observed at intervals for a period of 28 days, and the third pat placed upon some sort of a frame in a loosely covered vessel over boiling water, and kept there, with the water boiling, for 5 hours. The possible defects which are mentioned above in paragraphs 74 and 75 are described at length on page 103.

APPARATUS FOR A CEMENT TESTING LABORATORY†

(The apparatus is designed for one experimenter. Where the number of pieces is not stated, their number depends upon the quantity of cement to be tested.)

*One piece plate glass, one inch thick, 24 by 24 inches square;
*Two or more gangs of 4 or 5 molds each — A. S. C. E. standard (see Fig. 13, p. 73);
*One metric counter scale recording from 10 grams to 1½ kilograms.
*One No. 100 sieve (96 to 100 meshes to the linear inch) about 20 centimeters (7.87 ins.) in diameter and 6 centimeters (2.36 in.) high, made of woven brass wire cloth, with wires 0.0045 inches diameter;
*One No. 200 sieve (188 to 200 meshes to the linear inch) of similar size to the No. 100 sieve, and made of woven brass wire cloth, with wires 0.0024 inches diameter;
*One measuring glass graduated to 250 cubic centimeters;
*One 8-inch mason's trowel;
*One 4-inch pointing trowel (see Fig. 14, p. 74);
*One-half dozen pairs rubber gloves;
*Pieces of window glass 4 inches square for soundness tests;
*One tensile testing machine (see Figs. 22 to 27, pp. 94 to 98);
*Air thermometer;
*Standard sand;
Two or more gangs of 4 molds each for 2-inch cubes (see Fig. 43, p. 119);
Two or more molds for transverse specimens 1 by 1 by 6 inches (see Fig. 44, p. 121);
10-pound tin cans with covers for holding samples;
One special scale for weighing cement in ascertaining fineness (see Fig. 10, p. 68);
One pan of same diameter as the sieves and 5 centimeters (1.97 in.) deep, with cover, for holding sieve when shaking it;
One measuring glass graduated to 100 cubic centimeters;

*An asterisk designates the apparatus required for a temporary laboratory on construction work.

†This list has been criticised and approved by Mr. Richard L. Humphrey.

One cement sampler 24 inches long (see Fig. 8, p. 64)
One and one-half minute sand glass;
One moist closet (see Fig. 15, p. 75);
Galvanized iron waste cans;
Apparatus for steaming and boiling specimens (see Fig. 18, p. 78);
Tanks for immersing specimens (see Fig. 16, p. 76);
Vicat needle apparatus (see Fig. 11, p. 69);
One compression testing machine (adapted also to transverse tests), capacity 50,000 lb. (see Figs. 41 and 42, pp. 117 and 118);
Chemical thermometer;
Specific gravity apparatus (see Fig. 9, p. 66);
Microscope with 1½ inch objective;
Set of sieves, about 8-inch diameter, for analyzing sands, sizes No. 4, 8, 20, 50–100 (the number corresponds to the number of meshes to the linear inch) (see p. 159a);
Mechanical shaker for sifting sand (see Fig. 68, p. 195).

SPECIFIC GRAVITY OF DIFFERENT CEMENTS

The specific gravity test, by determining whether a cement is thoroughly burned, supplements the chemical analysis, since the latter does not indicate the degree of calcination. A Puzzolan cement may be distinguished from a true Portland because its specific gravity is usually between 2.7 and 2.9, while that of Portland ranges from 3.05 to 3.15. The adulteration of Portland cement lowers its specific gravity, because the substances used, — powdered sand, limestone, trass or slag, — are lighter than particles of pure cement. The test will not detect a small adulteration nor adulteration with a material of high specific gravity.

Natural cement usually has a specific gravity above 2.75, ranging from this sometimes as high as 3.1,* thus overlapping the inferior limit given for Portland cement.

The specific gravity of cement is lowered by exposure, because of the absorption of water and carbonic acid, hence the necessity of drying it at 100° Cent. (212° Fahr.) before determining. Even this temperature may not always be sufficient to restore old cements to their original condition.†

A neat little device for dropping fine material into a specific gravity apparatus so as to prevent the entraining of air has been devised by Mr. Thomas H. Wiggin. A thin wooden board with a circular hole in it is

*Tests of Metals, U. S. A., 1901, p. 476.

†See experiments in Tests of Metals, U. S. A., 1901, p. 476, and Dr. H. Kupfender in *Thonindustriezeitung*, translated in *Cement*, March, 1903, p. 23.

placed above the apparatus and a paper funnel fitted into the hole and filled with dry cement. An electro-magnet, such as is used with an ordinary electric door-bell, is connected with its storage battery and arranged so that the clapper, instead of striking a bell, strikes a metal plate attached to the corner of the board. The constant tapping jars the funnel so that the grains fall slowly into the apparatus without requiring the attention of the operator.

ADVANTAGES OF FINE GRINDING

The effects of fineness of grinding upon cements are to make them,—

Stronger when tested with sand;
Weaker when tested neat;
Quicker setting;
Capable of producing a larger volume of paste;
Less affected by free lime.

Fineness is expressed by the percentage of the total weight of the cement retained on each sieve.

A recognition of the value of extreme fineness has led to the adoption of higher standards than formerly, and manufacturers have accordingly improved the quality of their product in this respect. As an illustration of this, in 1875 it was a common requirement for Portland cement that 85% should pass, or not more than 15% be retained on, a sieve having 50 meshes per linear inch; in 1893 Max Gary gave the German standard as 90% to pass, or not more than 10% to be retained on, a sieve having 76 meshes per linear inch, while in 1904 specifications for first-class work required not more than from 6% to 10% to be retained on a sieve having 100 meshes per linear inch, and not more than 20% to 35% on a sieve having 200 meshes per linear inch. Some American factories are equipped to grind even finer than this, shipping cement of which less than 10% is retained on a No. 200 sieve. Standard requirements for different cements are given in the specifications on pages 30 and 31.

Strength affected by Fineness. With the same proportions of sand higher tensile and compressive strength is obtained from finely ground than coarsely ground cements. Conversely, a larger proportion of sand can be used with fine ground than with coarse ground cement, with the same resulting strength.

The chief cementing value of a cement lies in the grains which are fine enough to pass a sieve having 200 meshes per linear inch. Photographs of thin sections of sand briquettes several years old made by

Mr. E. W. Lazell show very clearly the coarser grains of cement which have never been penetrated and chemically changed by the water.

Tested neat, a coarse cement may give higher strength than the same cement after regrinding. This is chiefly due, in the opinion of the authors, to the fact that the fine cement requires more water in gaging to produce the same consistency of paste, so that the same weight of cement produces a larger volume of paste, which therefore has less density and consequently lower strength. When sand is added, on the other hand, less influence is exerted by the water, because in any case a smaller volume of it is required in proportion to the dry materials, and besides this the very fine grains, which also have higher cementing qualities, fit more readily into the voids in the sand. The relation of the density of a mortar to its strength is discussed in Chapter IX, page 132.

The effect of the fineness of cement upon its strength was brought to general notice by Mr. John Grant* in 1880, who quotes experiments made in Germany by Dyckerhoff. In 1883 Mr. I. J. Mann† illustrated the small cementing value of the coarse particles by substituting for them grains of sand of the same size, with but little reduction in the resulting strength.

The following table from tests reported in 1885 by Mr. Eliot C. Clarke‡ illustrates the effect of the fineness of cement on paste and mortars. All of these cements would be reckoned as coarse in modern practice, but the relative results are still of interest.

Tensile Strength of Mortar Affected by Fineness of Cement.

By Eliot C. Clarke.

Portland Cement			Rosendale Cement		
Proportions of cement to sand	Strength in pounds per square inch		Proportions of cement to sand	Strength in pounds per square inch	
	Ordinary cement 35% retained on No. 120 sieve	Finely ground cement 12% retained on No. 120 sieve		Coarse cement 17% retained on No. 50 sieve	Fine cement 6% retained on No. 50 sieve
1:0	403	304	1:0	98	92
1:3	105	180	1:1½	29	41
1:5	68	96	1:2	16	25

*Proceedings Institution of Civil Engineers, Vol. LXII, p. 149.

†Proceedings Institution of Civil Engineers, Vol. LXXI, p. 254.

‡Transactions American Society of Civil Engineers, Vol. XIV, p. 158.

Mr. D. B. Butler* in England has made extended tests to determine the value of coarse particles in cement and the effect of regrinding. A summary of one of his tables, illustrating also the effect of fineness upon the time of set, gives the average of his results from four brands of Portland cement.

Effect of Regrinding Coarse Particles and of Substituting Sand.
BY DAVID B. BUTLER.

CEMENT, HOW TREATED	Fineness residue per cent on sieves of meshes per linear inch			Setting Properties		TENSILE STRENGTH IN POUNDS PER SQUARE INCH									
						Neat cement					1 part cement to 3 parts sand				
	180	76	50	Initial set min.	Final set min.	7 days	28 days	3 mo.	6 mo.	12 mo.	7 days	28 days	3 mo.	6 mo.	12 mo.
As received	33.7	15.5	4.6	13	90	504	580	641	702	717	194	262	354	404	421
Reground	1.3	0.0	0.0	2	20	497	478	518	489	504	326	411	531	591	618
Sand substituted for coarse particles†						414	480	606	660	702	164	217	290	354	387

The fine grinding of commercial cements, by accelerating the setting, has been one of the causes for the necessity of adding gypsum or plaster during manufacture.

American vs. European Sieves. Standard sieves recommended by the American Society of Civil Engineers‡ and the French Commission§ are tabulated below with English and Metric equivalents.

American Sieves.

U. S. STANDARD					METRIC EQUIVALENTS			
No. of sieve	Meshes per linear inch	Meshes per square inch	Diam. of wire in.	Width of openings in.	Meshes per cm.	Meshes per sq. cm.	Diam. of wire mm.	Width of openings mm.
100	100	10 000	0.0045	0.0045	39	1 550	0.114	0.140
200	200	40 000	0.0024	0.0026	79	6 200	0.061	0.066

*Proceedings Institution of Civil Engineers, Vol. CXXXII, p. 343, and Butler's Portland Cement, 1899, p. 169.

†All particles not passing No. 180 sieve (averaging 33.7% by weight) were removed from the original cement as received, and sand having grains of similar size substituted for them.

‡See p. 67.

§Commission des Méthodes d'Essai des Matériaux de Construction, 1894, Vol. I, p. 248.

French Sieves.

FRENCH STANDARD				ENGLISH EQUIVALENTS			
Meshes per cm.	Meshes per sq. cm.	Diam. of wire mm.	Width of openings mm.	Meshes per linear inch	Meshes per square inch	Diam. of wire in.	Width of openings in.
18	324	0.20	0.36	46	2 120	0.0078	0.0124
30	900	0.15	0.18	76	5 780	0.0059	0.0071
70	4 900	0.05	0.09	178	31 680	0.0020	0.0035

Separating Material Passing No. 200 Mesh Sieve. The high cementing value of the grains of cement passing a No. 200 sieve leads in elaborate tests to still finer separations. In studies for soil analysis chiefly, the various methods of separating the different sized grains have been developed. They are fully described in Wiley's *Principles and Practice of Agricultural Analysis*, Vol. I, pages 171 to 281. The same principles are applicable to cement determinations, except that some liquid other than water must be employed.

Separation may be made by a winnowing device* in which a blast of air is directed against falling grains of cement; by settlement through water at rest, which in its simplest form may be accomplished by allowing the material to settle in a beaker, for a certain length of time and then decanting†; and by means of a liquid in motion, as illustrated in the Schöne apparatus, and, with still greater exactness, by Hilgard's churn elutriator.‡ The Schöne apparatus has been adapted by Dr. W. Michaelis to cement, and has also been employed by Mr. J. B. Johnson.§

QUANTITY OF WATER FOR NEAT PASTE AND MORTAR

The quantity of water used in gaging affects the results of tests, especially in the determination of the time of setting and of the strength. (See p. 151.) Different cements even of the same class require different proportions of water to produce the same consistency, chiefly because of differing degrees of fineness, the cement containing the largest proportion of fine particles requiring the largest percentage of water by weight.

For chemical combinations alone about 8 per cent of water to the weight of the cement is customarily assumed to be required, but in practice the percentage must be much greater.

*Tests of Metals, U. S. A., 1901, p. 474.

†Allen Hazen in Report Massachusetts State Board of Health, 1892.

‡Wiley's Principles and Practice of Agricultural Analyses, 1894, Vol. I, p. 226.

§Johnson's Materials of Construction, 1903, p. 412.

Percentage of Water for Mortar of Normal Consistency. The following table, based on the formula of Mr. Feret given on page 88, which is strictly applicable only to French sands and French methods, has been suggested provisionally by the Committee of the American Society for Testing Materials (1904), for the percentage of water for mortars of consistency corresponding to that of normal neat paste. To use the table select from the first column the percentage of water required for the neat paste of the selected cement and read in column of the desired proportions the percentage of water required for the mortar in terms of the sum of the weights of the cement and sand.

*Percentage of Water for Cement Mortars of Normal Consistency.**

Percentage of water for neat cement	Percentage of water to cement plus sand. Proportions cement to sand by weight 1:1	1:2	1:3	1:4	1:5	Percentage of water for neat cement	Percentage of water to cement plus sand. Proportions cement to sand by weight 1:1	1:2	1:3	1:4	1:5
18	12.0	10.0	9.0	8.4	8.0	33	17.0	13.3	11.5	10.4	9.6
19	12.3	10.2	9.2	8.5	8.1	34	17.3	13.6	11.7	10.5	9.7
20	12.7	10.4	9.3	8.7	8.2	35	17.7	13.8	11.8	10.7	9.9
21	13.0	10.7	9.5	8.8	8.3	36	18.0	14.0	12.0	10.8	10.0
22	13.3	10.9	9.7	8.9	8.4	37	18.3	14.2	12.2	10.9	10.1
23	13.7	11.1	9.8	9.1	8.5	38	18.7	14.4	12.3	11.1	10.2
24	14.0	11.3	10.0	9.2	8.6	39	19.0	14.7	12.5	11.2	10.3
25	14.3	11.6	10.2	9.3	8.8	40	19.3	14.9	12.7	11.3	10.4
26	14.7	11.8	10.3	9.5	8.9	41	19.7	15.1	12.8	11.5	10.5
27	15.0	12.0	10.5	9.6	9.0	42	20.0	15.3	13.0	11.6	10.6
28	15.3	12.2	10.7	9.7	9.1	43	20.3	15.6	13.2	11.7	10.7
29	15.7	12.5	10.8	9.9	9.2	44	20.7	15.8	13.3	11.9	10.8
30	16.0	12.7	11.0	10.0	9.3	45	21.0	16.0	13.5	12.0	11.0
31	16.3	12.9	11.2	10.1	9.4	46	21.3	16.1	13.7	12.1	11.1
32	16.7	13.1	11.3	10.3	9.5						

Weights of Cement and Sand for Different Proportions.

	1:1	1:2	1:3	1:4	1:5
Cement	500	333	250	200	167
Sand	500	666	750	800	833

The Engineers of the U. S. Army† advocate a dryer mixture than most

*In the final report of the Committee on Cement Tests of the American Society of Civil Engineers, dated 1912, the percentages of water to cement plus sand for normal consistency have been reduced in each case 0.5 below the values in the above table.

† Professional Papers, No 28.

authorities, and the following percentages suggested by them may therefore be taken as representing minimum quantities.

Portland Cement.

Neat..................................	20% of water by weight.
1 cement: 3 sand......................	12½% " "

Natural Cement.

Neat..................................	30% of water by weight.
1 cement: 1 sand......................	17% " "

Puzzolan Cement.

Neat..................................	18% of water by weight.
1 cement: 3 sand......................	10% " "

French Determination of Consistency of Neat Paste. The Vicat needle apparatus has been selected in America as well as in France as the standard appliance for determining normal consistency. The apparatus is shown in Fig. 11 on page 69, and the U. S. standard method of applying the test is there described.

A plastic paste is preferred to one of dryer consistency. The French Commission* advised a softer consistency than the American standard, the French requiring for normal consistency the penetration of a needle one centimeter (0.39 in.) in diameter and weighing 300 grams (10.58 oz.) through a disc of cement 40 millimeters (1.57 in.) thick to within 6 millimeters (0.23 in.) of the bottom, making a total depth of penetration of 34 millimeters (1.33 in.), while the American Society recommend the penetration of a similar needle into a like mass to a depth of 10 millimeters (0.39 in.) below the surface.

Feret's Formula† for percentage of water for mortar of normal consistency was evolved from a very interesting series of experiments.‡ He found that it was impracticable to determine with the Vicat needle the proper consistency of a mortar of cement and sand, and therefore based his determination upon the average judgment of several operators, plotting the consistencies designated by them upon cross-section paper.

*Commission des Méthodes d'Essai des Matériaux de Construction, 1894, Vol. I, p. 270.

†Commission des Méthodes d'Essai des Matériaux, 1895, Vol. IV, p. 103.

‡Methods of Mr. Feret's investigations are described and illustrated in an article by the authors on "Quantity of Water to Use in Gaging Mortars" in *Cement and Engineering News* (Chicago), November, 1903.

His formula is:

For mortars of plastic consistency,*

$$W = \frac{2}{3} \cdot \frac{P}{S+1} + 6.0 \qquad (1)$$

For mortars of dry consistency,*

$$W' = \frac{2}{3} \cdot \frac{P}{S+1} + 4.5 \qquad (2)$$

Where

W = percentage of water for mortar in terms of weight of the mixture of dry materials;

P = percentage of water required for neat cement of normal consistency;

S = parts of sand by weight to one part cement.

Mr. Richard L. Humphrey† states that from formula (2) he has obtained very uniform results with U. S. standard sand, although slight modifications are necessary for a mortar containing more or less than three parts of sand.

ARBITRARY PERIODS OF SETTING

The methods employed in mixing and depositing the mortar or concrete and the character of the construction form a guide to the necessary requirements for the time of setting of the cement.

The setting of cement is due to chemical reaction, as described by Mr. Spencer B. Newberry on page 57. The process is a gradual one, but may be arbitrarily divided into three periods:

Initial set.
Final set.
Hardening.

The dividing line between these periods is arbitrary, but the division is based upon the fact that after water is added the paste remains plastic for a certain period, and then commences to "stiffen" or crystallize. This is called the time of initial set. The setting process continues rapidly, and when a point is reached that the paste will withstand a certain pressure, arbitrarily fixed in practice, it is said to have reached its final set. The

*The original formula of Mr. Feret corresponding to formula (2) is $E = \frac{2}{3} NA + 60$, and to formula (3) is $E = \frac{2}{3} NA + 45$, in which E = weight of water in grams required for one kilogram of dry mixture of cement and sand, N = weight of water in grams required for one kilogram of neat cement, and A = weight in kilograms of cement in one kilogram of the dry mixture. The change in the form of the formula permits the direct use of percentages.

†Journal Franklin Institute, 1901–2.

process of hardening now continues more slowly, and proceeds with increasing slowness for an indefinite period.

Those unfamiliar with cement construction must bear in mind that a cement which has reached its final "set" is not hard nor is it capable of bearing a load. Natural cement, for example, usually reaches its initial and its final set much earlier than Portland cement, but it hardens more slowly, and Natural cement masonry will not bear loading nearly so quickly as Portland cement masonry.

EUROPEAN METHODS FOR DETERMINING SET

The French and German requirements are similar to the American (p. 70) except that in them the commencement of the set is taken as the time when the needle can no longer penetrate entirely to the bottom of the box instead of limiting it to a penetration to a depth of 5 millimeters above the bottom surface.

For sand mortars the French Commission designate the final set as the moment when the surface of the mortar can support pressure of the thumb without indentation. As an alternate method, they use the Vicat apparatus with a needle one centimeter (0.39 in.) in diameter and weighing 5 kilograms (11.02 lb.). The preliminary reports of Mr. R. Feret and Mr. P. Alexander in Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, pp. 111 and 139, describe experiments with different apparatus.

Comparison of Vicat and Gillmore Needles. The Gillmore needles, the former American standard, were first used by General Totten in 1830.*

By these needles the initial set of Neat cement is the time at which a wire one-twelfth-inch diameter, loaded to a ¼ pound, is just supported by the mass without appreciable indentation. The final set is taken as the time when a wire one-twenty fourth-inch diameter, loaded to weigh one pound, is supported without appreciable indentation.

The diagram in Fig. 19, page 90, from experiments made at the Watertown Arsenal† upon various cements (designated by letters) shows the difference in the nominal time of setting when measured by the Gillmore needle and the Vicat needle, employing with the latter the German method. (See above.) The diagram also shows the variation in time of set of Portland cement occasioned by varying the proportion of water, and the effect of leaving out the usual "restrainer" of plaster of Paris or gypsum.

*Gillmore's Treatise on Limes, Hydraulic Cements and Mortars, p. 80.

†Tests of metals, U. S. A., 1901, p. 492.

THE RATE OF SETTING

The rate of setting of cement, that is, the process of hardening, has been studied by the French Commission* in France and by Prof. Edgar B. Kay in the United States. The diagram, Fig. 20, page 91, shows curves of setting made with a machine of Prof. Kay's design and the corresponding tensile strength of briquettes of the same cement. Prof. Kay calls attention to the positive change from the plastic to the granular or crystalline structure which in the cement shown occurred between the periods of 35 and 40 minutes. The elongation of the briquette when being broken gradually changed from ⅜ inch at the 5-minutes period to 0.15 inch at 40 minutes, while at 200 minutes, or one hour before the initial set was completed, the elongation was not measurable.

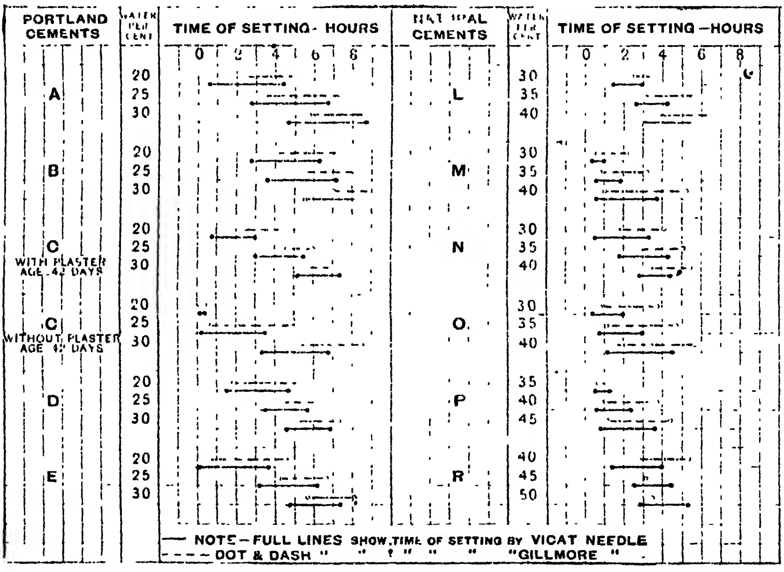


FIG. 19.—Time of Setting of Typical Cements,—and comparison of Vicat and Gillmore's Needles. (Tests of Metals, U. S. A., 1901.) (*See p.* 89.)

AMERICAN AND EUROPEAN STANDARD SANDS COMPARED

The character of the sand has so great an effect upon the strength of a mortar that for comparing different brands of cement or specifying requirements of strength a sand of standard size and quality is essential.

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV., p. 111.

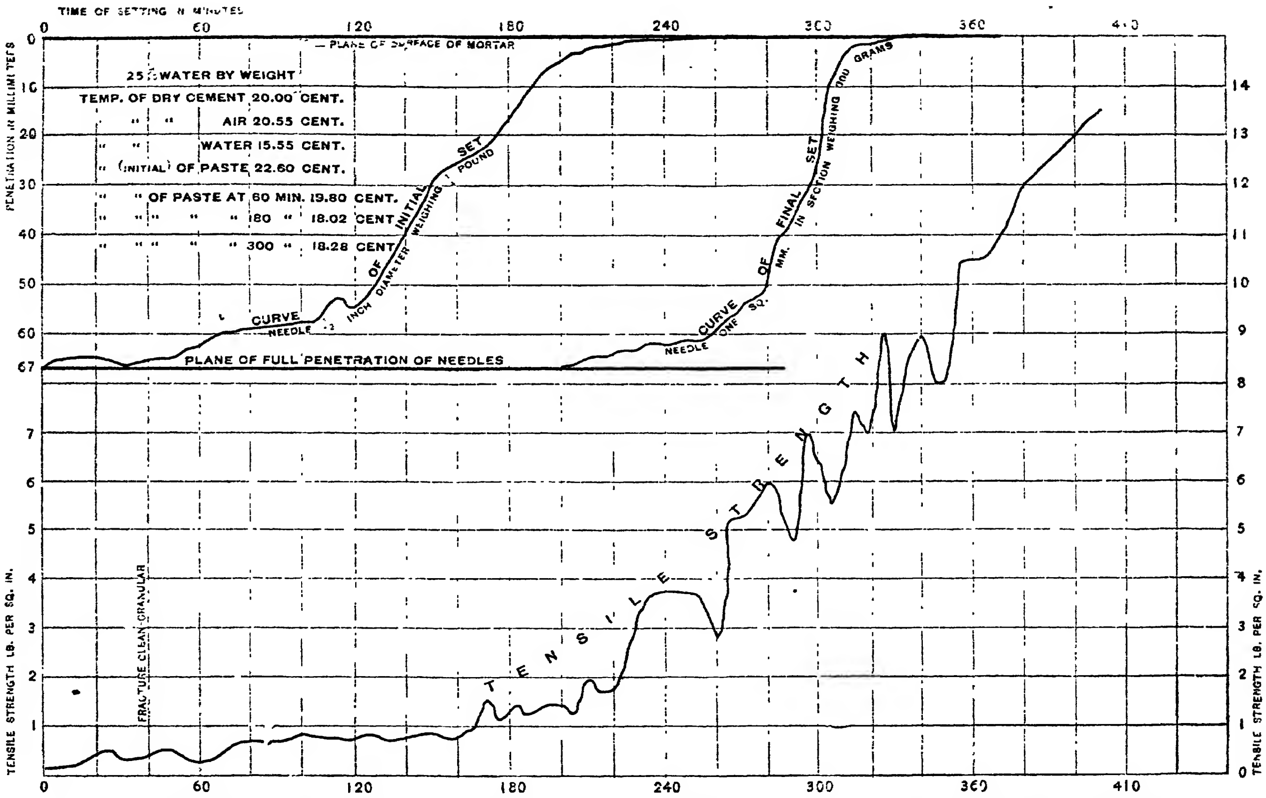


FIG. 20.—Rate of Setting and Corresponding Tensile Strength of Portland Cement Paste. (See p. 89.)
(Especially prepared by Prof. Edward R. Kay for this treatise.)

The U. S. Standard Sand recommended by the Committee of the American Society of Civil Engineers, as specified on page 71, is a natural sand from Ottawa, Ill., screened to pass a sieve having 20 meshes per linear inch, and retained on a sieve having 30 meshes per linear inch.

The change in America from artificial to natural sand is in accord with recent practice abroad.

The English Standard Sand is obtained from a pit at Leighton Buzzard,* and the screens are the same as in the United States.

The German Standard Sand is a natural quartz retained between sieves having respectively 20 and 28 meshes per linear inch.

The French Standard Sand, a natural sand from Leucate, France, is simple or compound. Simple standard sand must pass a screen having holes 1.5 millimeters (0.059 in.) in diameter, and be retained on a screen having holes one millimeter (0.039 in.) in diameter. Compound standard sand is made by forming a mixture of equal weights of the following:

(1) Grains passing holes of 2 mm. (0.079 in.) and retained by 1.5 mm. (0.059 in.).

(2) Grains passing holes of 1.5 mm. (0.059 in.) and retained by 1 mm. (0.039 in.).

(3) Grains passing holes of 1 mm. (0.039 in.) and retained by 0.5 mm. (0.020 in.).

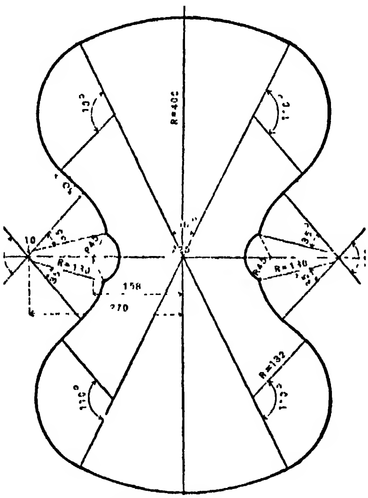


FIG. 21.—The German Standard Briquette (dimensions are in millimeters). (*See page 92.*)

THE FORM OF BRIQUETTE FOR TENSILE TESTS

Mr. John Grant in 1871† presented results of a series of experiments with different forms of briquettes and sizes of section. Ten years later‡ he adopted the form now used in England which is substantially the same as that recommended by the American Society of Civil Engineers in 1884, and, with a very slight alteration, in 1903. (See Fig. 12, p. 72.)

The German Standard Briquette, also adopted by the French Commission

*Butler's Portland Cement, 1899, p. 200.

†Proceedings Institution of Civil Engineers, Vol. XXXII, p. 282.

‡Proceedings Institution of Civil Engineers, Vol. LXII, p. 137.

in 1893, is shown in Fig. 21. The section is 5 square centimeters (0.78 sq. in.). Results with this form of briquette are lower per unit of area than those of the American pattern. Prof. Jerome Sondericker* in studying the quality of strength and uniformity of breaking of different forms, found that a groove in the sides of the specimen lowered the unit strength about 13%.

M. Feret† found that briquettes of 5 square centimeter section gave 46% higher strength per unit of area than briquettes of 16 square centimeter, and attributed this difference to lack of homogeneity throughout the section.

TO CONVERT METRIC UNITS OF STRENGTH TO ENGLISH UNITS

To convert values of kilograms per square centimeter (kg. per sq. cm.) to pounds per square inch (lb. per sq. in.), multiply the former by 14.2.‡ To convert values of pounds per square inch (lb. per sq. in.) to kilograms per square centimeter (kg. per sq. cm.), multiply the former by 0.07.§

MACHINES FOR TESTING TENSILE STRENGTH

A testing machine should be so designed that the strain can be applied to the briquette at a definite rate without irregularity or jar. The clips should be suspended from pivoted bearings to avoid friction, and should be stiff, so that they will not spread. The contact surfaces should hold the briquette firmly without crushing it.

Effect of Eccentricity in Placing Briquettes. One of the causes of irregularity in tests of similar briquettes is careless adjustment of the briquette in the clips of the machine, that is, placing it so that it is not exactly central. Prof. J. B. Johnson‖ has discussed this theoretically, and concludes that

if h = width of specimen,

and a = eccentricity of loading,

then $\frac{6a}{h}$ represents the percentage of increase in stress due to eccentricity.

"Thus if a cement briquette one inch thick be placed in the clips 0.01 inch out of center, its strength will be reduced by 6%. This assumes perfect freedom of motion of the clips at the surface of contact, which they do not

*Journal Association of Engineering Societies, January, 1899, p. 1.

†See p. 136.

‡More exactly, 14.2234.

§More exactly 0.07031.

‖1903 Edition, p. 446.

have. Experiments made at the Massachusetts Institute of Technology have shown that a displacement of one-sixteenth inch decreased the tensile strength by from 15% to 20%."

Rate of Applying Strain. The selections of the standard rate of 600 lb. per minute by the committee of the American Society of Civil En-

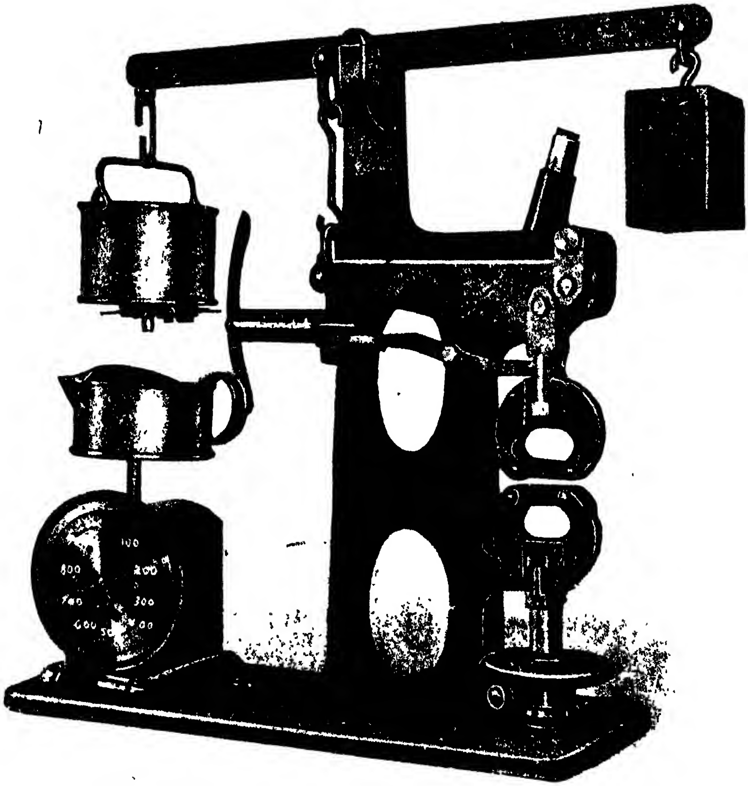


FIG. 22.—Shot Testing Machine. (*See p.* 95.)

gineers (see p. 76) is based on an extensive series of tests from which it was found that the breaking load increases with the speed up to a rate of at least 800 lb. per min., but that between the rates of 400 and 600 lb. the variation is slight. Mr. E. S. Wheeler's* experiments tend to confirm this conclusion.

*Report Chief of Engineers, U. S. A., 1895, p. 2916.

Types of Testing Machines. There are three most common types of tensile testing machines.

(*a*) The shot machine, originally designed by Dr. Michaelis and shown in its American patterns in Figs. 22 and 23, applies the load by the discharging of a stream of shot whose flow is automatically shut off when the

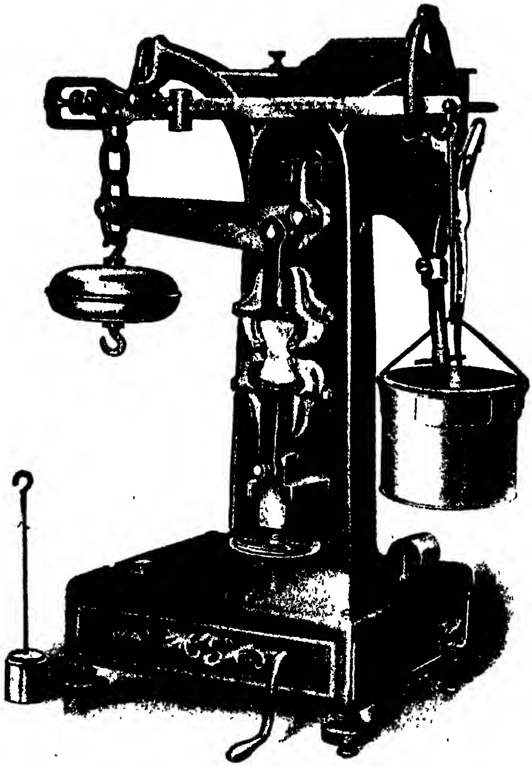


FIG. 23. Shot Testing Machine. (*See p.* 95.)

break occurs. The breaking load is determined from the weight of the shot.

(*b*) The simple or compound lever machines apply their load by a sliding weight operated by hand or by power. A compound lever power machine is illustrated in Fig. 24, page 96.

(c) The spring balance machine, which was originally designed and used by Mr. Henry Faija in England, transmits the strain from the crank to the briquette through a spring balance which records the load upon the dial. (See Fig. 25, p. 97.)

Johnson's Ring Testing Machine. A machine devised by Mr. A. N. Johnson for testing the tensile strength of cement and mortars is based on an entirely different principle from the clip machines just described

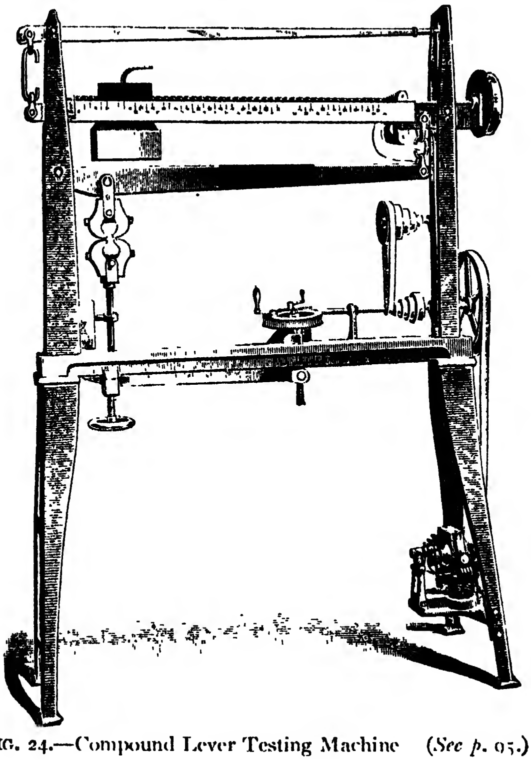


FIG. 24.—Compound Lever Testing Machine (*See p.* 95.)

The cement or mortar instead of being formed into standard briquettes is molded in the shape of rings. The apparatus is shown in Figs. 26 and 27, page 98. A cylinder A filled with water or other liquid contains a piston operated by a handwheel F. The pressure exerted by lowering the piston is transmitted by the liquid to the closed cylinder B, a section of which consists of rubber tubing which is expanded by the pressure from within until it bursts the ring of cement which encircles it. The pressure is also

transmitted to the gage whose reading for a certain diameter and thickness of ring of cement or mortar bears a definite ratio to the circumferential tensile stress upon the ring. Brass molds of special design for forming the rings are constructed either single or in gangs of five.

TENSILE TESTS OF NEAT CEMENT AND MORTAR

Tests of tensile strength are made primarily to determine whether the ingredients of the cement and the process of its manufacture are such that a continued and uniform hardening may be expected in the work, and whether its actual strength in mortar or concrete is so high that it can be depended upon to withstand the strain placed upon it. Tensile tests must be combined with other tests, most particularly the test for soundness, to arrive at correct conclusions on these points.

FIG. 25.—Spring Balance Testing Machine. (*See p.* 96.)

The dates which have been universally selected for making tensile tests to determine the quality of the cement are 7 days and 28 days after molding. In each case the briquettes remain for the first 24 hours in moist air, and the balance of the time in water at the standard temperature of 21° Cent. (70° Fahr.). For arriving at a quicker knowledge of the quality, standard specifications require one-day tests, the briquettes being broken after 24 hours in moist air. Longer periods than 28 days are useful for determining the rate of permanent hardening, although the rate of growth is different in neat cements, mortars and concretes. The growth in tensile strength is not strictly comparable with its growth in compressive strength.

A cement giving an extremely high test at a very short period may be regarded with suspicion, although if future tests show a good increase, no fault can be found. Specifications occasionally limit the strength of the one-day or the 7-days test. Others require a definite increase in strength between periods. The engineers of the New York Rapid Transit Com-

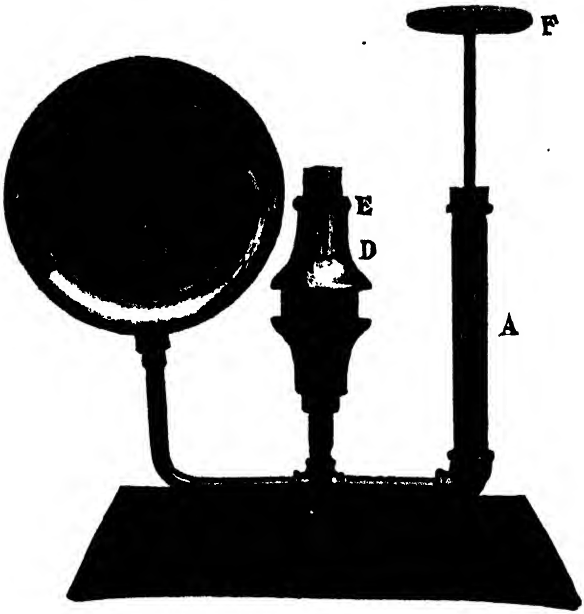


FIG. 26.—Machine with Cement Ring in Position ready for a Test. (See p. 96.)

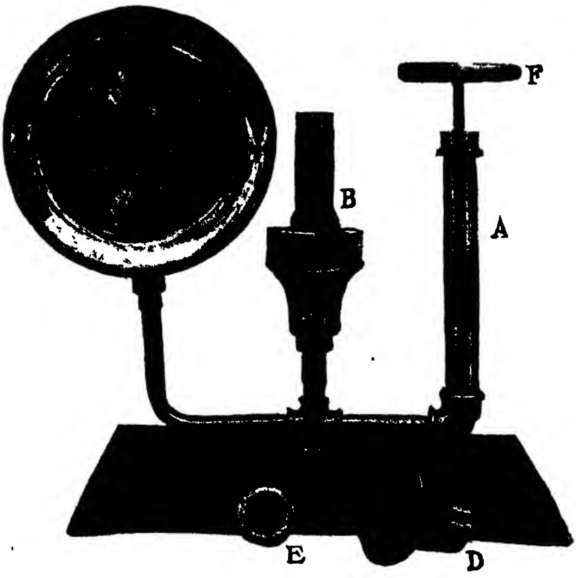


FIG. 27.—Machine after a Test with the Top Cap Removed, Showing the Broken Cement Ring and Distended Rubber Cylinder or Tube. (See p. 96.)

mission require, for example, "a specific ratio of increase," 15% in tensile strength "from 7 to 28 days, and furthermore that a cement showing as high as 750 lb. at the earlier stage should be generally refused as unlikely to give good results in long-time tests."* Manufacturers consider this a very severe requirement for Portland cement tested neat.

Specifications for tensile strength are given on pages 30 and 31. A comparison of these with the actual strengths of different cements as furnished by manufacturers will show that on the average the tensile strength of Portland cement as now manufactured is largely in excess of the specifications. In comparing these figures, however, it must be recognized that specifications are not for average strength, but are intended to cover the lowest limit which can be allowed on the work, and to provide for lack of uniformity in testing as well as in real quality.

GROWTH IN STRENGTH OF PORTLAND AND NATURAL CEMENTS AND CEMENT MORTARS

The curves in Fig. 28, for which we are indebted to Mr. W. Purves

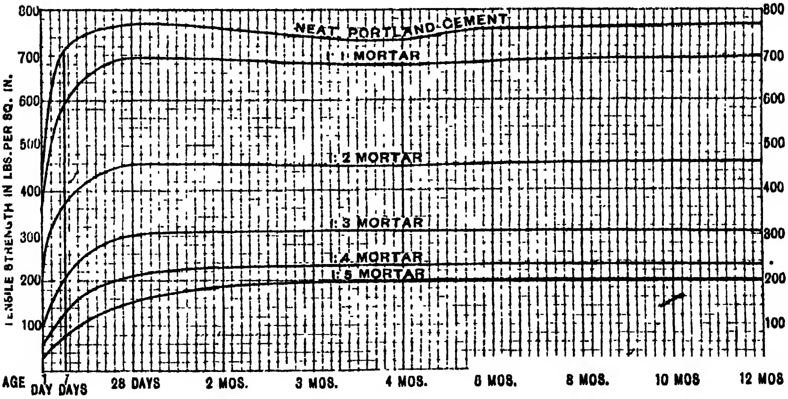


FIG. 28.—Growth in Tensile Strength of Neat Portland Cement and Portland Cement Mortars with Different Proportions of Standard Sand. (*See p.* 99.)
(Compiled for this treatise by W. Purves Taylor.)

Taylor, illustrate the growth in strength of neat Portland cement and Portland cement mortars. The tests from which the curves are drawn were made under his direction at the Philadelphia Municipal Laboratories.

*Report of New York Board of Rapid Transit Commissioners, 1900-01, p. 258.

The neat and 1 : 3 (*i. e.*, one part cement to 3 parts sand by weight) curves are averaged from over 100,000 briquettes, while the other curves are each based on tests of 300 to 500 briquettes.

The cements included a number of brands, American brands largely predominating. The sand was crushed quartz, the former U. S. standard. The Philadelphia records include tests of much longer time than one year, and there is a noticeable falling off in the observed tensile strength after the one-year period. This is most noticeable with neat cement of rotary kiln brands, but also occurs to a less degree with sand mortars. With cements from stationary kilns it is less marked. The falling off in tensile tests is generally attributed to the brittleness of the small sized specimens, which tends to irregularity of results with the ordinary testing machine, and to the unequal hardening of the surface and interior of the specimen, rather than to actual deterioration in the cement.

The average growth in strength of neat Natural cement and Natural cement mortars is illustrated in Fig. 29 from data kindly prepared by

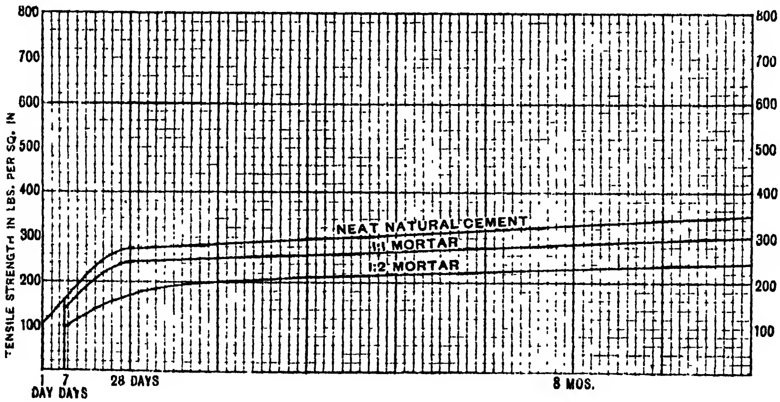


FIG. 29.—Growth in Tensile Strength of Neat Natural Cement and Natural Cement Mortars with Different Proportions of Standard Sand. (*See p.* 100.)
(From data by Richard L. Humphrey and A. W. Munsell.)

Mr. Richard L. Humphrey from Philadelphia tests, and by Mr. A. W. Munsell from tests made for the Baltimore & Ohio R. R. Cements from seven different sections of the United States are included in the averages from which the curves are drawn, representing the Akron, Cumberland, James River, Lehigh Valley, Louisville, Milwaukee, Rosendale and Utica districts.

SOUNDNESS OR CONSTANCY OF VOLUME

The term "soundness" is more commonly used in America and England than the expression "constancy of volume" suggested by the Committee of the American Society of Civil Engineers, or "deformation" as employed in France. The purpose of the test is to determine in advance whether a cement is in danger of disintegrating, that is, crumbling, or of expanding or contracting so as to cause distortion or cracking in the masonry.

If a cement satisfactorily passes the tests for soundness, it will in all probability withstand the effect of the elements without swelling or disintegration, and will continue to harden for an indefinite period. Failure, on the other hand, to pass the tests for soundness, especially the hot test, is not positive proof of inferiority, for a cement which fails to pass may possibly, through subsequent exposure to the air before being used, or because of mixing with sand or other aggregate, produce durable masonry. We may, however, with safety adopt the following conclusion:

If a Portland cement passes the hot test it may be used immediately with reasonable certainty of its ultimate soundness. If it fails to pass, it should be regarded with suspicion and thoroughly tested.

Causes of Unsoundness. Disintegration, or crumbling, of work in Portland cement properly mixed and laid, is usually due to an excess of lime in a form which can be attacked by the elements. This may come about in two entirely distinct ways, either (1) by the use of too high a proportion of lime in the raw materials from which the cement is made, (2) by under-burning the cement, or (3) by too coarse grinding.

The presence of magnesia in excess in a thoroughly burned cement may produce a gradual expansion which will disintegrate the mortar or concrete after several years. This action, brought to notice by tests of Mr. H. Le Chatelier,* is generally recognized, but opinions differ as to the limit to the percentage of magnesia which may occur in Portland cement without deleterious effect. Le Chatelier's experiments led him to consider 5% as injurious. The Association of German Cement Manufacturers first placed the limit at 3½%, and later raised it to 5%. Mr. Spencer B. Newberry states (page 56) that recent experiments made by himself and by Van Blaese show that cements containing 8% or 9% of magnesia will pass the boiling test, while those with 15% magnesia will expand. The limit of 4% recommended by the Committee of the American Society for Testing Materials in 1904 (see p. 30) is undoubtedly conservative. Natural cement, which is burned at a lower temperature,

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 229.

may contain a much larger quantity of free lime and of magnesia without injury.

The expansion caused by an excess of free lime is due to the hydration or slaking of the calcium oxide (CaO). This is readily understood from the expansion of common lime, which in slaking with water will produce a bulk of paste from 2 to 3 times greater than the volume of the loose powder. The presence of lime in a free or loosely combined state must not be confounded with other compounds of calcium. A thoroughly slaked lime paste, or powder, that is, one which is completely hydrated, may in fact be added to a Portland cement mortar without injurious results, to lengthen its time of setting or to produce a more water-tight mixture.

The small amount of free lime which frequently occurs and sometimes produces unsoundness in first-class Portland cement, tested when fresh, may be hydrated and rendered harmless by air-slaking after, say, two or three weeks' storage, or after spreading the cement out in the air.

Adulteration with slag may cause a cement containing an excess of free lime to pass the boiling test.

Tests for Soundness. The presence of ingredients which will render a cement unsound, that is, which will cause it to expand or disintegrate, is determined by the eye, or by measuring appliances in specimens which have been exposed under conditions which as nearly as possible produce the same effect as the practical effects of time and the elements.

There is apparently no reliable method for determining the presence of free lime by chemical analysis. Mr. E. Candlot* says that "there is in fact no method for finding the percentage of free lime in the cement," and Dr. Schuman* concurs in this view in the following statement·

> I do not know a method for finding out the percentage of free lime in Portland cement. I do not think there exists such a method, and I am myself of the opinion that chemists will never find out one; the solutions capable of taking away the free lime from the cement will always work in a more or less strong degree on the cement itself.

This inability to detect free lime by chemical analysis necessitates a resort to physical tests. Specimens for testing soundness are generally circular pats tapering toward the edges, so that the expansion of the mass will tend to enlarge the circumference and thus produce cracks at the edges.

*Quoted by W. W. Maclay in Transactions American Society of Civil Engineers, Vol XXVII, p. 448.

Egg-shaped specimens and also briquettes are sometimes used. Both of these show deterioration by the appearance of the surface.

Appearance of Soundness Specimens. Cracks which appear on pats are not always caused by unsoundness. Expansion cracks, which reveal an unsound cement, are distinguished from shrinkage cracks, which usually appear during setting instead of after the cement is set, in Figs. 30 to 37. Hair cracks also sometimes appear upon specimens, and in practice upon neat cement or very rich mortar, where so large an excess of water has been employed in mixing that it does not dry off until the cement has set, and causes the deposition of a very thin coating of partially decomposed cement which had remained in suspension in the water. An unsound cement in air or in water at the ordinary temperature will generally show defect within 28 days, although in very exceptional cases several months or even years have been known to elapse before signs of deterioration appear in specimens which have not been subjected to heat.

Photographs of pats illustrating the appearance of defective specimens which have been subject to the boiling test are shown in Figs. 38 and 39, pages 108 and 109. Figs. 30 to 37, pages 104 and 105, are sketches* employed in the Philadelphia Municipal Laboratories for distinguishing harmless appearances in neat pats from evidences of unsoundness. Mr. Taylor describes the pats as follows:

Fig. 30 represents a normal pat in good condition.

Fig. 31 represents shrinkage cracks. These cracks are ordinarily due to the use of a too wet mixture or to too quick a drying out. If the pats are left exposed to dry air during setting these cracks are often developed. Shrinkage cracks ordinarily, therefore, indicate only a lack of care in manipulation, and not dangerous properties in the cement.

Fig. 32 shows cracks caused by the expansion of the cement and the curling of the edges of the pat from the glass while the pat still adheres, which is often coincident with the expansion. In the air pats these cracks are developed in nine-tenths of the pats adhering to the glass, and unless very decidedly marked are not dangerous. They should not exist in the water pats. If they do exist, however, to an appreciable extent, it denotes the presence of a too great proportion of expansives, which ordinarily is sufficient to condemn the sample.

Fig. 33 shows blotching, a pat which is usually indicative of either adulteration or under-burning. This condition in itself should not necessarily mean rejection, but should always induce an investigation of the causes producing it, which may or may not be sufficient to warrant rejection.

Fig. 34 shows pats which have left the glass (*A*) by mere lack of adhesion, (*B*) by contraction, and (*C*) by expansion. (*A*) is never dangerous

*Presented to the authors by Mr. W. Purves Taylor.

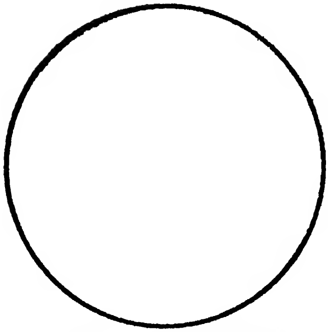


Fig. 30. Normal Pat.*

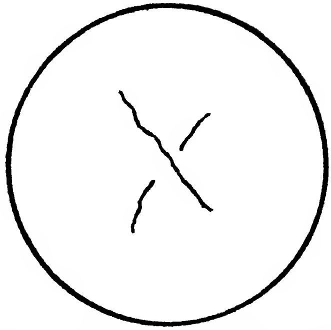


Fig. 31.—Harmless Shrinkage Cracks *

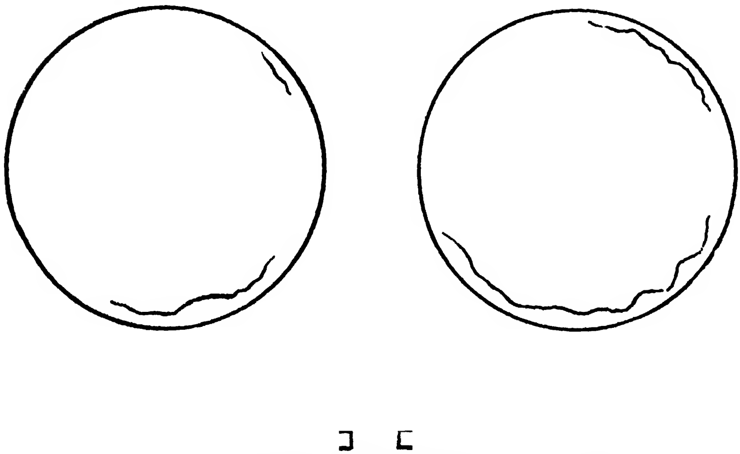


Fig. 32.—Expansion Cracks, Harmless in Air Pats.*

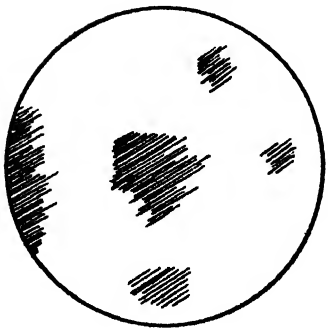


Fig. 33.—Blotches Requiring Investigation.*

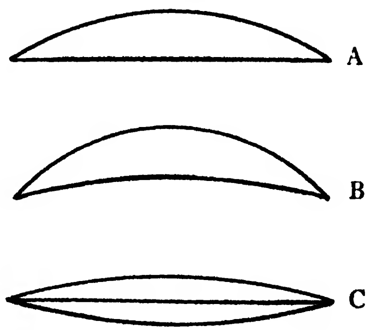


Fig. 34.—Pats which have left Glass.*

*See pp. 103 and 106.

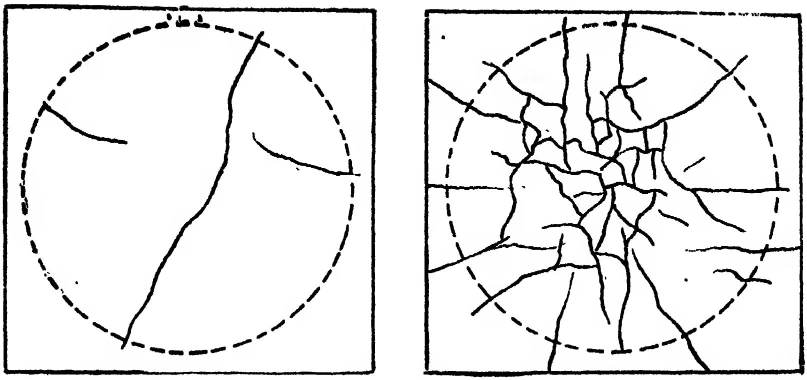


FIG. 35. —Cracked Glass (pat removed.)*

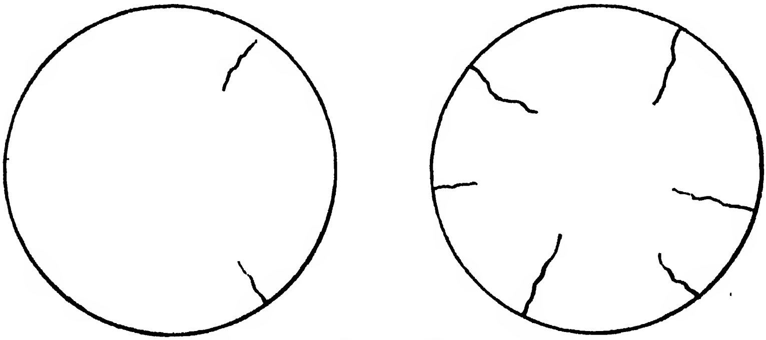


FIG. 36.—Incipient Disintegration.*

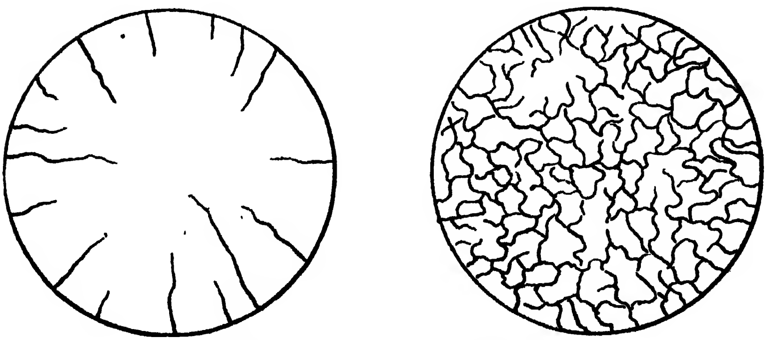


FIG. 37.—Complete Disintegration.*

*See pp. 103 and 106.

in either air or water. (*B*) and (*C*) are dangerous only when existing in a marked form. A curvature of about a quarter of an inch can be considered about the limit of safety in a 3-inch pat. Case (*C*) rarely, if ever, occurs in water.

Fig. 35 shows a peculiar condition in which the pat is perfectly sound and hard, but the glass on which it is made is badly cracked.* This has often been laid to chemical action, but this conclusion is doubtless erroneous. It is probably due entirely to expansion of the pat, when the adhesive strength of the cement to the glass exceeds the strength of the glass itself. It is only found in the water pats, and is not usually indicative of dangerous qualities of the cement.

Fig. 36 shows the radial cracks of incipient disintegration. These are the danger marks to be looked for in the normal pat tests, and are always sufficient to warrant rejection.

Fig. 37 shows cases of complete disintegration, which almost invariably begins merely by showing radial cracks, as in Fig. 36.

Accelerated or Hot Tests. The object of all forms of hot tests is to produce in a few hours the results which at a normal temperature require several days or perhaps months. Engineers are by no means agreed as to the value of accelerated tests, the chief objection to their use being that some cements which fail in these tests prove satisfactory in construction.

An argument for the hot test lies in the fact that Portland cement manufacturers are coming to recognize it as the very best test for them to use in determining whether their own cement will fulfil the requirements of permanent construction. In a recent letter to the authors the superintendent of one of the largest factories in the United States writes, "So far as we are concerned, we consider the hot test of the greatest importance. If this shows up well, we feel quite satisfied that all other tests will show up properly." Those desiring to investigate the various opinions upon the subject are referred to References, Chapter XXXI.

Mr. W. Purves Taylor, in a paper read before the Cement Section of the American Society for Testing Materials, at the Sixth Annual Meeting, 1903,† gives the results of a large number of accelerated tests made at the Philadelphia Testing Laboratory by boiling balls or pats (after 24 hours in moist air) for three or four hours, and the results of some of the conclusions there given are quoted as follows:

"The condition in a cement most affecting the result of an accelerated test is its age or the amount of seasoning it has undergone. Every cement,

*Similar causes may produce one or two cracks in the glass.

†Proceedings American Society for Testing Materials, 1903, Vol. III, p. 374, also printed in *Engineering News*, July 23, 1903, p. 81.

no matter how well proportioned and burned, will contain at least a small amount of free or loosely combined lime, which will usually cause unsoundness if used or tested at once. This lime, however, will hydrate in a very short time on exposure to air, thus rendering it inert and preventing any expansive action. It will, therefore, be found in a large majority of cases that if a cement failing in the accelerated tests be stored for two or three weeks, this unsoundness will disappear, and the cement pass the test with ease."

This is illustrated in the following table and in Fig. 38, page 108, the first three photographs also showing various conditions which may be expected in specimens which fail to pass accelerated tests.

Effect of Age of Cement on Results of Boiling Test.

BY W. PURVES TAYLOR. (*See p.* 107.)

Age of cement when tested	TENSILE STRENGTH					NORMAL PAT TESTS		BOILING TEST
	Neat			1:3 sand				
	1 day	7 days	28 days	7 days	28 days	28 days in air	28 days in water	
1 week	550	765	762	171	225	Curled and soft.	Slightly checked.	Partly disintegrated.
2 weeks	548	67	771	17[illegible]	246	Slightly curled.	Slightly curled.	Checked and cracked.
3 "	492	718	763	182	244	"O. K."	"O. K."	Slightly checked.
5 "	427	692	747	183	249	"O. K."	"O. K."	Sound.

"Coarseness of grinding is also a frequent cause of unsoundness for the reason that the larger particles are not readily susceptible to hydration, and contain for a long period of time expansive elements which very rapidly develop a disintegrating action when treated in the accelerated tests."

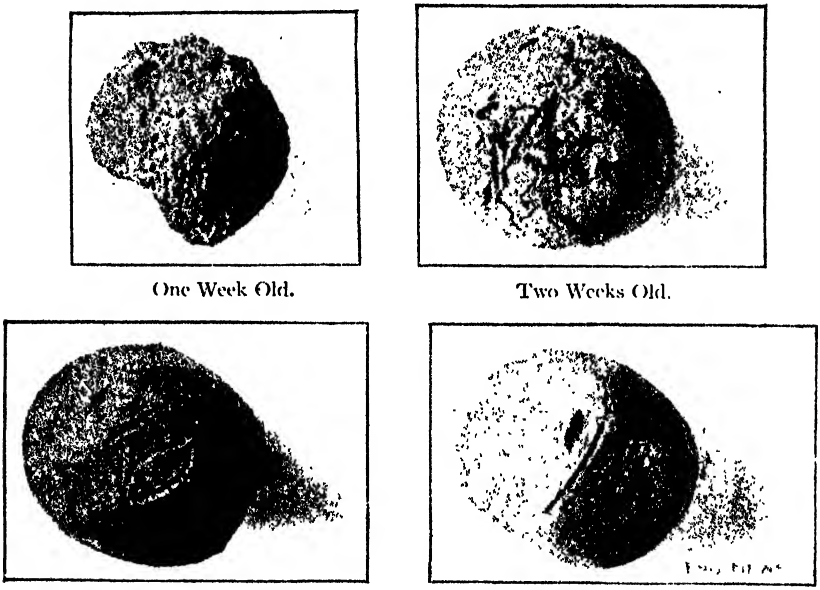
"A large number of tests on different cements were made and the time at which failure occurred was observed. In these tests it was found that of those samples which did not pass the test, 22% failed in the first half hour, 57% failed in the first hour, 85% failed in two hours, 96% in three hours, and 99% in four hours," "thus showing generally that a test piece of cement standing three or four hours of boiling will almost invariably stand a much greater length of time, and also that at least three or four hours should always be allowed for the test."

"Pats of cement allowed more than about twelve hours to harden will, if unsound, fail when tested by boiling at almost any time in the future."

"We now come to the very important question of the relation of the boiling tests to the other tests for soundness and strength as made in the

laboratory. No one who has had much experience with the boiling test questions that, although it is by no means infallible, the results obtained from it are generally corroborated by either the tensile tests or the normal tests for soundness. The writer has recently compiled some data in regard to this point, covering over a thousand tests on many varieties of cement, with the following results:

"Of all samples failing to pass the boiling test, 34% of them developed checking or curvature in the normal pats — or a loss of strength in less than twenty-eight days. Of those samples that failed in the boiling test but re-



One Week Old. Two Weeks Old.

Three Weeks Old. Five Weeks Old.

FIG. 38.—Specimens showing the Effect of the Age of the Cement upon its Soundness. (See p. 107.)

mained sound at twenty-eight days, 3% of the normal pats showed checking or abnormal curvature in two months, 7% in three months, 10% in four months, 26% in six months, and 48% in one year; and of these same samples, 37% showed a falling off in tensile strength in two months, 39% in three months, 52% in four months, 63% in six months, and 71% in one year. Or, taking all these together, of all the samples that failed in the boiling test, 86% of them gave evidence in less than a year's time of possessing some injurious quality.

"On the other hand, of those cements passing the boiling test, but one-half of 1% gave signs of failure in the normal pat tests, and but 13% showed a falling off in strength in a year's time.

"This certainly makes a very strong showing in favor of the boiling test, at least considered from a laboratory standpoint.

"In order to show the great value sometimes obtained from the results of the boiling test, several examples are given in the table on page 110 of tests of cements occurring in the regular routine work of the laboratory."

The air and water pats of sample 2 of this table are shown in Fig. 39 at the age of four months. These pats were sound at twenty-eight days.

In conclusion Mr. Taylor lays special emphasis upon the fact that many cements which do not pass the boiling test will give excellent results in

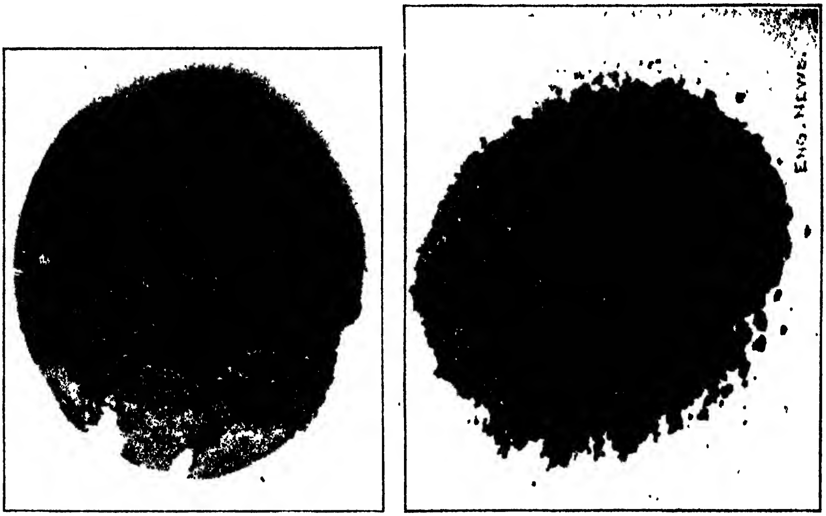


Fig. 39.—Examples of Unsound Pats at 4 months which were sound at 28 days. (*See p.* 109.)

practice. He gives as the probable reason for this that the test for soundness is generally made immediately upon the receipt of a shipment, while the cement used in construction has opportunity to season, and upon the fact "that the disintegrating action of a cement is always far greater when mixed neat than when mixed with an aggregate, and the greater the amount of the aggregate the less the tendency to unsoundness." It is often good policy before rejecting a cement which fails to pass the hot test to hold it for a week or two so that it may further season and then retest it.

Methods of Making Accelerated Tests. The methods of conducting accelerated tests are numerous, the object of all of them being to hasten

Evidences of Failure in Cement Indicated by the Boiling Test.
By W. Purves Taylor. (*See p.* 109.)

Tensile strength							Normal pat tests				Boiling test
Neat				1:3 sand			Air		Water		
1 day	7 days	28 days	4 months	7 days	28 days	4 months	28 days	4 months	28 days	4 months	
522	793	797	Disintegrated.	204	257	52	Very slightly curled: left glass.	Badly curled; soft and crumbly.	Left glass.	Disintegrated.	Disintegrated.
503	872	586	Disintegrated.	184	239	47	Very slightly curled: left glass.	Badly curled; soft and crumbly.	Left glass.	Disintegrated.	Disintegrated.
498	762	700	Disintegrated.	176	231	119	Very slightly curled: left glass.	Badly curled; soft and crumbly.	"O. K."	Disintegrated.	Disintegrated.
427	751	603	223	183	227	94	Very slightly curled; left glass.	Disintegrated.	"O. K."	Disintegrated.	Disintegrated.
503	827	717	177	220	252	132	Left glass.	Badly curled; soft and crumbly.	"O. K."	Disintegrated.	Disintegrated.
492	883	620	202	195	217	147	Left glass.	Badly curled; soft, and crumbly.	"O. K."	Disintegrated.	Badly checked.
535	864	743	94	197	241	77	Very slightly curled; left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked.	Checked and cracked.
502	829	722	320	203	247	65	Very slightly curled, left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked	Checked and cracked.
Neat tests not made.				172	219	93	Very slightly curled; left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked.	Checked.
Neat tests not made.				198	231	101	Left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked.	Checked.

Note.—All of these cements were normal in specific gravity, time of setting, and fineness.

the hardening of the cement so as to produce in a few hours results which under ordinary conditions require weeks or months. Boiling the specimens, instead of steaming them as recommended by the Committee of the American Society of Civil Engineers, while more common, is more severe. Other methods are employed in Europe.

The Steam Test, recommended by the Committee of the American Society of Civil Engineers, requires, as already described (p. 77), that the pat after twenty-four hours in moist air shall be placed in an atmosphere of steam above boiling water.

The Boiling Test was originated by Prof. Tetmajer in Germany. After twenty-four hours in moist air, or until it is thoroughly set, the specimen is placed in cold water, which is raised to and then maintained at the boiling point for several hours. Three or four hours is the time specified by Mr. W. Purves Taylor, and often used in the United States, although some cement factories boil for twenty-four hours. Dr. Michaelis advocates six hours' boiling, and this period is specified by the French Commission.

Combined Boiling and Tensile Test. A regular test at many Portland cement factories consists in testing the tensile strength of briquettes which have been subjected to the hot test. A briquette of neat cement after twenty-four hours under a damp cloth is placed in an atmosphere of steam over boiling water for an hour or two, and then immersed in water at about the boiling point and boiled for about twenty-four hours, when it must show a certain tensile strength.

The Hot Water Test, as adopted by Mr. Henry Faija in England, and advocated there by Mr. David B. Butler, consists in subjecting a newly mixed pat to a moist heat of 100° to 105° Fahr. (38° to 40° Cent.) for six or seven hours, or until thoroughly set, and then placing it in warm water at a temperature of 115° to 120° Fahr. (46° to 49° Cent.) for the remainder of the twenty-four hours. Mr. Deval in France employed a temperature of 176° Fahr. (80° Cent.) for a period of six days.

Other Accelerated Tests which have been employed in Europe are oven tests, where the specimen is heated in an oven; glow tests, where a ball is heated over a gas flame, and Prussing disc tests, where discs are formed under heavy pressure and then exposed to hot water.

Measurement of Expansion. Appliances have been devised for testing the soundness of cement by measuring the amount of expansion or deformation which it undergoes in different periods of time. The principal of these are the long bar apparatus, devised by Messrs. Durand-Claye and

Debray, which was recommended by the French Commission, Bauschinger's caliper apparatus, and Le Chatelier's tongs.*

The Chimney Expansion Test, in which a small quantity of neat cement is solidly pressed into a straight lamp chimney with the idea that an unsound cement will break the glass, is worthless, as all first-class cements expand to a greater or less degree.

*Described in Spalding's Hydraulic Cement, 1903, p. 166.

CHAPTER VIII

SPECIAL TESTS OF CEMENT AND MORTAR

The most important tests for comparing the qualities of different cements and for determining their practical value have been described in the preceding chapter. Certain other tests are often made to investigate special qualities of a cement or mortar, or for scientific research.

Such special tests may be enumerated as follows:

Color.
Weight.
Microscopical.
Compressive.
Transverse.
Adhesive.
Shearing.
Abrasive.
Porosity.
Permeability.
Yield of mortar.
Rise in temperature.

COLOR

The color of a cement bears but slight relation to its quality, but a variation of color in the same brand is sometimes an indication of inferiority. Natural cements made in different localities may often be distinguished from each other and from Portland cements by their color.

Portland Cement. The chemical composition of Portland cements made by different processes is so uniform that the color of different brands varies less than that of Natural cements.

The color of Portland cement is described as a cold blue gray. In England the term "foxy" is applied to a Portland cement of a brownish color. According to Mr. David B. Butler* this denotes "insufficient calcination or the use of unsuitable clay or possibly excess of clay." He further states that if a Portland cement contains a large quantity of underburned particles, on account of their lower specific gravity they tend to rise to the surface on troweling, thus forming a yellowish brown film which is noticeable in the section of the briquette after fracture.

*Butler's Portland Cement, 1899, p. 255.

The dark color of the coarser particles of a Portland cement left as residue on a screen is due simply to the fact that cement clinker is black, and pieces which are not finely ground retain the color of the clinker.

Natural Cement. The color of Natural cement varies with the character of the rock and consequently with the locality in which it is produced. It ranges from the light écru of the Utica (Ill.) cement to the dark grayish brown of the Rosendale (N. Y.). Samples received by the authors from various manufactories show the James River cement to be a light yellowish brown, the Akron (N. Y.) cement, écru, the Milwaukee (Wis.) cement, drab, and the Louisville (Ky.) cement, a brownish gray. Certain other brands are similar in color to Portland.

Puzzolan Cement. Puzzolan cement made from slag is of a light lilac shade, much lighter than Portland. After being kept under water it assumes, when freshly fractured, a bluish green tint. This green tint, which according to Candlot* is due to sulphide of calcium present in the cement, is especially noticeable in a sample kept in sea water, and fades on exposure to dry air.

WEIGHT OF CEMENT

Weight is no indication of quality. Formerly, nearly all specifications required that a cement should reach a certain standard of weight per struck bushel or per cubic foot, on the principle that, other things being equal, a thoroughly burned cement is heavier than one which is underburned. But when, on the other hand, the degree of fineness was found to affect the weight much more than any difference in calcination, the worthlessness of this requirement became apparent, and the test for specific gravity was substituted.

The following table by Eliot C. Clarke† illustrates the difference in weight between cements of the same manufacture which contain different percentages of coarse particles.

Weights of Cements Containing Varying Percentages of Coarse Particles. (*See p* 114.)

BY ELIOT C. CLARKE.

Percentage of cement retained on No. 120 sieve	Weight per cu. ft.
0	75 lb.
10	79 "
20	82 "
30	86 "
40	90 "

*Candlot's Ciments et Chaux Hydrauliques, 1898, p. 159.

†Transactions American Society of Civil Engineers, Vol. XIV, p. 144.

Mr. Henry Faija's experiments* arranged in the following table prove that the weight of a cement decreases with age. His explanation for this is that the lower specific gravity of the moisture and carbonic acid absorbed from the air tends to increase the bulk of the cement without correspondingly increasing its weight.

Decrease of Weight of Cement with Age. (*See p.* 115.)

BY H. FAIJA.

	Weight per cu. ft. lb.	Percentage of loss in weight per cent.
When received	88	..
After one month	85½	2.7
" three months	79½	9.9
" six "	78	11.4
" nine "	75½	14.2
" one year	74	15.9

Method of Weighing Cement. The apparatus finally recommended by the French Commission, after a series of tests by Mr. P. Alexandre,† was a circular funnel with screen, as shown in Fig. 40. The cement placed upon the screen is stirred with a wooden spatula 4 cm. (1⅝ in.) wide, and 25 cm. (10 in.) long, and falls through the screen into the cylindrical measure of one liter capacity (61 cu. in.).

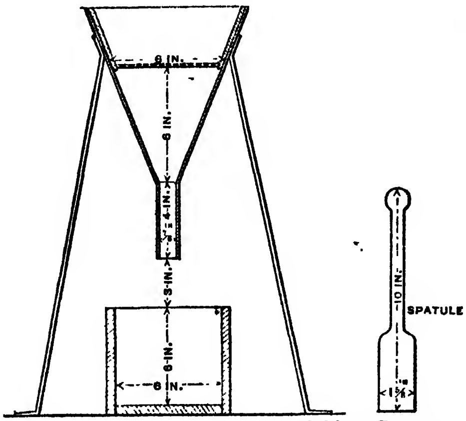


FIG. 40. Funnel Used in Weighing Cement. (*See p.* 115.)

MICROSCOPICAL EXAMINATION OF PORTLAND CEMENT CLINKER

The structure of Portland Cement clinker can be clearly discerned with the aid of the microscope and polarized light by preparing thin sections of it in the same way as those of rocks made by petrographers.

Le Chatelier, a French engineer, and Törnebohn,

*Butler's Portland Cement, 1899, p. 240.

†Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 21.

a Swedish petrographer, some years ago identified two essential mineral entities, and three others of less importance, as constituents of Portland cement clinker. Törnebohn denominated the two essential constituents alite and celite.

Mr. Clifford Richardson has within the last few years taken the subject up very elaborately in this country, and his results go to show that optical methods of examining clinker will eventually prove of great interest, not only in determining the character of clinker, but also in pointing out means of improving the methods of production.

COMPRESSIVE TESTS OF CEMENT

Compression testing machines are coming into general use in America. For merely determining the quality of a cement, tensile tests are more convenient because they can be made more quickly and require less powerful machines, but for comparing different sand aggregates and for its adaptability to testing concrete by compression or by transverse, *i.e.*, beam, tests, the compression machine possesses great advantage. The French Commission recommend compression tests in addition to tension, and many engineers in the United States advise them in well equipped laboratories.*

Types of Compression Testing Machines. Machines especially adapted for compressive tests are built with capacities ranging from 30 000 to 400 000 lb., or even larger. The Emery Machine at the Watertown Arsenal, U. S. Army, is of 800 000 lb. capacity while the machine designed in 1908 for the structural materials laboratory of the U. S. Geological Survey at St. Louis has a capacity of 10 000 000 lb. A machine with a capacity of not less than 40 000 lb. is required for 2-inch cubes of neat cement or mortar, while for 6-inch cubes of mortar or concrete a machine should run to at least 150 000 lb.

A testing machine for general laboratory work driven by power is illustrated in Fig. 41, in which the pressure is continuously applied by means of a screw pump. It may be operated either by hand or by power and is built for maximum capacities of 200 000 lb. and upwards.

An American machine of about 40 000 lb. capacity of the same type as the German Amsler-Laffon compression testing machine is illustrated in Fig. 42, page 118. The hydraulic power is applied by turning the hand wheel and the load is read directly from the pressure gage.

* Proceedings American Society of Civil Engineers, April, 1900, p. 125.

Form of Compression Specimens. Extended tests were made for the French Commission by Mr. P. Siméon,* in which he employed specimens of various shapes and sizes, and compared the results with those obtained

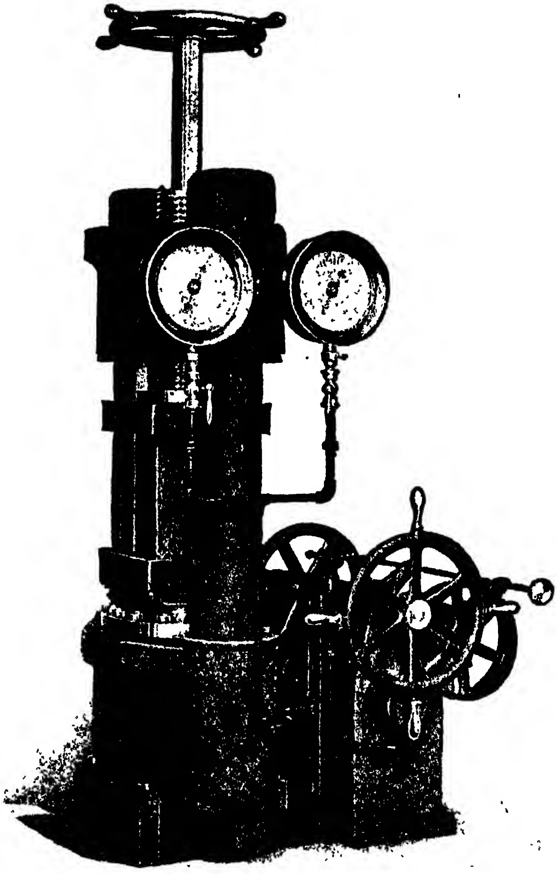


FIG. 41.—Compression Testing Machine. (See page 116.)

from crushing the halves of briquettes which had been broken in tension. Quoting from a discussion of Mr. Thompson† upon the Report of the Cement Committee of the American Society of Civil Engineers:

*Commission des Méthodes d'Essai des Matériaux de Construction, Vol. IV, 1895, p. 187.
†Sanford E. Thompson in Proceedings American Society of Civil Engineers, August, 1903, p. 646.

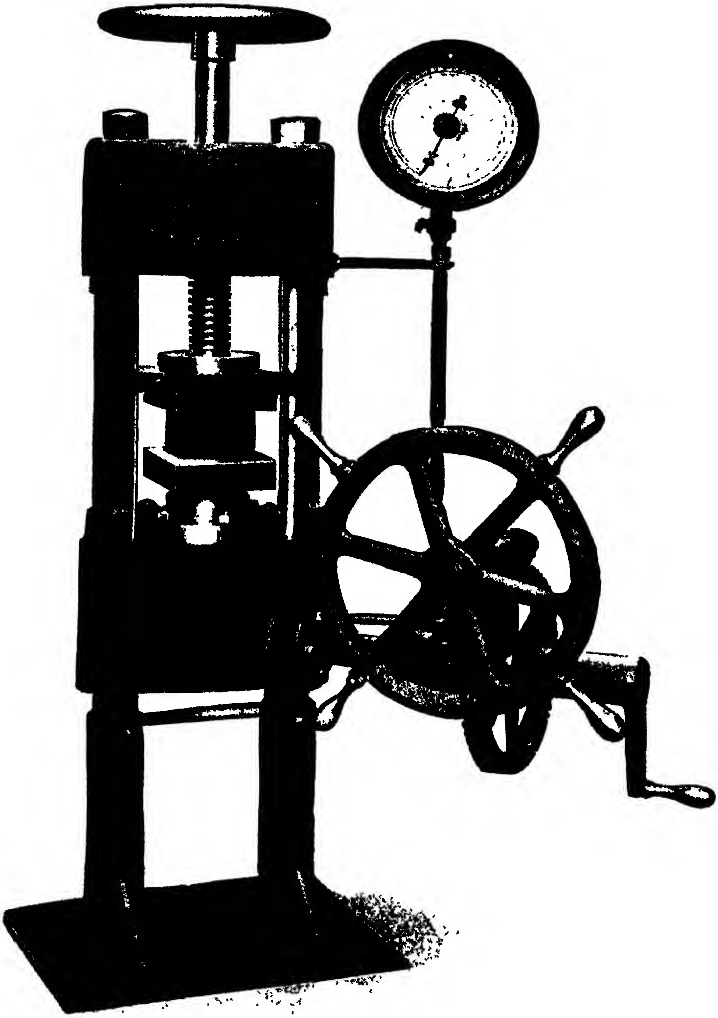


FIG. 42.—Hydraulic Compression Testing Machine. (See page 116)

The Commission reached the conclusion that the briquettes which had been broken in halves by tension should be used for the compressive tests. The two halves of each briquette are crushed separately and the sum of the two results divided by the total area of the briquette, thus obtaining the compressive strength per unit of surface. The surface area of the United States standard briquette recommended by our Committee is almost exactly 4 sq. in. Instead of the halves of a briquette, a single cylinder having the same thickness and the same area of surface as a whole briquette may be used with substantially equivalent results.

Specimens which are rough or uneven may be smoothed by gentle rubbing on a stationary grindstone.

In breaking, the pressure should increase uniformly, and at such speed that it will require between one and two minutes to crush each specimen.

For comparing the strength of cement paste or mortar, with that of other materials which cannot readily be molded in cement molds, the Commission recommends cubes having an area of 50 sq. cm. (7.75 sq. in.) on each face. For a United States standard, cubes 2 in. on an edge, that is, with all faces having an area of 4 sq. in., conform to most common usage, and are therefore best for this class of comparative tests.

A mold for cubes is shown in Fig. 43.

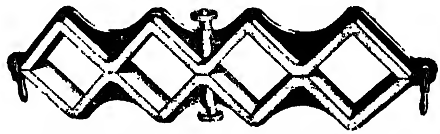


FIG. 43.—Gang Mold for Compression Cubes. (*See p.* 119.)

Relation of Compressive to Tensile Strength. Mr. R. Feret* concludes, after an extended series of tests, that there is no constant relation between resistances to compression and tension. He also concludes that the rate of increase in strength varies with the different cements, so that "two different mortars having the same resistance to compression do not necessarily break under the same tension." He claims that compression tests give better results than tension and furnish "the real measure" of the cohesion of mortars. These opinions are generally corroborated by cement experts.

The ratio of compression to tension also varies with the character of the sand or other aggregate. With a larger proportion of cement the compressive strength increases faster than the tensile strength, thus giving a higher ratio. This law continues to hold with concrete of different proportions, that containing the largest proportion of cement showing the highest compressive strength in comparison to its tensile strength.

* Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Series 5, Vol. II.

A comparison of the compressive and tensile strength of 1:3 mortars based upon tests at the U. S. Government Structural Materials Laboratory at St. Louis, in 1908, gives a formula

$$\frac{\text{Compressive strength}}{\text{Tensile strength}} = 6.6 + 2.3 \log. A,$$

where

A = age of the cement mortar in months.

By this formula it will be seen that the ratio varies from 6.8 on a one-month test up to 10.3 on a 12-months test. The formula is in the same form, but the ratios are somewhat greater than those obtained by Prof. J. B. Johnson * from Prof. Tetmajer's tests at Zurich.

TRANSVERSE TESTS OF CEMENT

Transverse, or flexion, tests of beams or prisms while very convenient for concrete are now seldom used for testing the quality of cement, although Gillmore and other of the older experimenters largely employed this form of test. Transverse tests are of value in comparing the relation between fiber stress and tension, and with proper care may give as uniform results as tension tests. As is stated below, the fiber stress bears a definite relation to the tensile strength, but since there is no fixed relation between tension and compression, there can be no fixed relation between transverse strength and compressive strength. Compression testing machines (see Figs. 41 and 42, pages 117 and 118) may be adapted for transverse tests by a suitable arrangement of supports and knife edges.

Size of Specimen. Mr. Durand-Claye† made for the French Commission an extended series of tests by flexion or bending. As a result of his report, the Commission adopted for this form of test square prisms 12 cm. (4.72 in.) long by 2 cm. (0.79 in.) on a side.

In breaking, a prism is placed on its side — that is, on a face which has been in contact with the mold — upon two knife-edges, 10 cm. (3.94 in.) apart, and the load is applied at the center through a slightly rounded knife-edge. The load should be applied continuously at the rate of 1 kgr. (2.2 lb.) per second. The same number of specimens should be broken as in tensile tests, and the results averaged.

*Johnson's Materials of Construction, 1903, p. 419.

†Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 211.

English measure will naturally change the dimensions of the specimen to 1 by 1 by 6 in., to be broken upon knife-edges 5 in. apart.*

A prism 2 by 2 by 8 in. was employed by General Gillmore in experiments described in his famous "Treatise on Limes, Hydraulic Cements and Mortars," and has been adopted by other American engineers, but with apparatus of sufficient delicacy there is no reason why the specimens need be larger in section than tensile specimens, and the dimensions of 1 by 1 by 6 inches suggested above are recommended for comparative tests of neat cements and mortars. A form of mold is shown in Fig. 44.

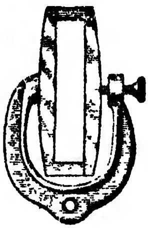


FIG. 44. —Mold for Prism. (*See p. 121.*)

Relation of Tensile to Fiber Stress. In the experiments mentioned above Mr. Durand-Claye compared all of his tests for flexion with tensile tests of briquettes made and tested at the same time. As a result, he obtained as the ratio between the ultimate fiber stress in flexion and the tensile strength, 1.92 at 7 days and 1.86 at 28 days; or in round numbers, 1.9 for both. That is, tensile fiber stress is 1.9 times the simple tensile stress of the same material. In this connection he calls attention to the fact that a briquette tested in tension gives a result less than the real resistance, while a prism tested in flexion gives a greater result. He judges that the real resistance may be approximated by taking the mean of the two results.

Mr. Durand Claye also found the mean error by the two methods of testing to be very similar, with tensile briquettes the variation being about 2.02 % as compared with 2.52% variation in the flexion tests. In tests with mortar there was less variation with prisms than with briquettes.

Prof. Edgar B. Kay states that in recent experiments he has obtained more uniform results with tranverse than with tensile tests.

Comparative tests of Mr. R. Feret in tension, flexion, and compression are shown in the table on page 136.

ADHESION TESTS OF CEMENT

Mr. E. Candlot† made a large number of tests of adhesion for the French Commission, and designed a mold adopted as the French Standard. With reference to such tests he says that since the adhesion of mortar to a stone depends upon the state of the surface and the nature of the cement,

*Sanford E. Thompson in Proceedings American Society Civil Engineers, August, 1903, p. 646.
†Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 281.

absolute tests are of little value, but comparative tests, if made under identical conditions, are of real interest to the builder.

Thus, to cite several examples, the tests of adhesion prove that a mortar regaged after having set possesses a strength of adhesion much smaller than the same mortar gaged and put in place before its set, the resistance to tension and compression of these two mortars remaining, however, almost the same; that mortars gaged dry have a more feeble adhesion than mortars gaged slightly liquid; that mortars gaged with an excess of water have in tension a resistance less than their adhesive strength, etc.

Method of Making Adhesion Tests. In the same report Mr. Candlot describes the forms of specimens suggested by Dr. Michaelis and others, and then presents a form which he considers to best meet the requirements. On account of the difference in section of the French standard briquette, the mold he describes is not suitable for making specimens to fit the clips

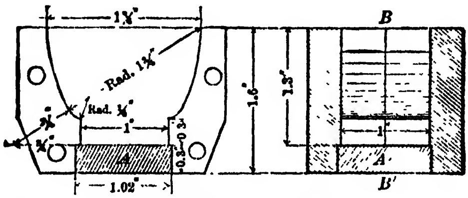


Fig. 45.—Mold for Adhesion Block. (*See p.* 122.)

on American testing machines. To adapt his mold to American standards, the authors have designed the mold shown in Fig. 45. The method of making tests is described by Mr. Thompson* as follows:

Adhesion is considered by Mr. Candlot in two ways: First, with reference to the relative adhesive qualities of different cements; and, second, with reference to the adhesion of the same cement mortar to other materials of different natures. The same general method is advocated in both cases.

Briquettes are formed, as described below, of a shape which can be broken in an ordinary tensile testing machine. The European tensile briquette is of small section, 5 sq. cm. (0.775 sq. in.), and of an inconvenient shape for molding in halves. The area of the breaking section is therefore doubled by the Commission, while the curves where the clips take hold remain the same, so that the distance between the two points of each clip is unchanged. The shape of the United States standard briquette is such that fewer changes have to be made in its outline, and the regular section of 1 sq. in. need not be altered.

*Proceedings American Society of Civil Engineers, August, 1903, p. 647.

The Commission found that adhesion briquettes could not be molded satisfactorily in the manner used for tension briquettes. They advised finally a mold in which a half briquette could be made, and then when this had set, the same mold could be used for completing the other half. In Fig. 45 is shown the style of mold selected, but with the dimensions changed to adapt the briquette to the United States standard form of clip. It consists of a bottomless box, which divides vertically in the center on the line *BB*, so that the half briquette can be removed readily. The bottom is formed of a movable bronze plate, shown at *A*.

For the first class of tests, to determine the relative adhesion of different cements, a normal adhesion block is formed of a mortar composed, by weight, of 1 part of highest quality Portland cement, which has passed a No. 75 sieve, and 2 parts of fine sand, gaged 9% of water. As soon as it is rammed into the mold, the mold is removed, and after remaining in moist air for 24 hours the half briquette is placed in water until it is required. It must set for at least 28 days. When required for use, the block is dried and the surface polished with emery paper. The block is then placed on a table with the large end down, the half mold, with the disc *A* removed, set on top of it and filled with plastic mortar consisting of the cement which it is desired to test mixed with sand in the required proportions, thus completing the briquette. This briquette is treated and tested as an ordinary tension specimen.

For the second class of tests, if the material can be molded, it is formed as a half briquette, and the specimen completed with the mortar to be tested. If solid, a plate of the material, several millimeters thick, having one smooth face, is prepared, and placed at the bottom of the mold, on top of the bronze plate, and the first half of the specimen is formed by filling the mold with neat cement. After setting, the half of the briquette is completed with the mortar which it is desired to test.

Adhesive Strength of Mortar. The following table from tests of Mr. Candlot, presented to the French Commission,* shows the results of adhesive tests upon Portland cement mortars cemented to the normal adhesion block by the method described in the preceding paragraphs. It is noticeable that, in the same column, the values, each of which represents a single specimen, are fairly regular, but that there is a very great variation in the adhesive strength of mortars made from different cements, and no uniform relation between the strength of mortars of different proportions.

Adhesion of Mortar to Various Materials. The results of tests made by Professor Tetmajer in Germany, quoted by Mr. E. Candlot, are briefly as follows: 1:2 Portland cement mortars cemented to sandstone gave an adhesive strength after 28 days of from 5.5 to 8.8 kg. per sq. cm. (78 to 125 lb. per sq. in.). To rough glass the adhesion was about 3.5 kg. per sq. cm

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 285.

(50 lb. per sq. in.). Tests made at Boulogne-sur-Mer using blocks of marble showed, after 28 days, variations of 3.1 to 8.3 kg. per sq. cm. (44 to 118 lb. per sq. in.). Regaged mortar showed about half the strength in adhesion of fresh mortar.

*Adhesive Strength of Portland Cement Mortars in Pounds per Square Inch.**

BY E. CANDLOT.

Cement	A				B		C		D	
Proportions of mortar.	1:3	1:3	1:2	1:2	1:3	1:2	1:3	1:2	1:3	1:2
Per cent. of water.	12	13.8	9.5	15	12	13	15	17	12	13
	lb	lb.	lb.	lb.	lb.	lb	lb.	lb.	lb.	lb.
7 day tests.	107 195 131 156	135 131 156 152	142 145 135 135	140 152 128	36 36 28 28 36	36 43 36 50	36 38 38	43 50 36 36	60 60 57 67 57	65 65 71 82 107
Average. ...	117	143	130	143	33	41	37	41	60	78
28 day tests.	164 178 178 109	192 206 220 220	188 204 124 185	178 152 192 156	92 81 85 60 67	142 114 114 85 102	78 74 71 81	95 78 117 100 88	152 128 114 107	95 88 74
Average. ...	180	209	198	160	77	111	76	96	125	86

Mr. E. S. Wheeler† has made several series of tests, inserting thin discs of different materials in the center of briquettes. Although the irregularity in the results cast considerable doubt upon his method of testing, the experiments tended to show that the adhesive strength to sawn limestone of Portland cement mortar in proportions 1:0 to 1:2 is about one-third the cohesive strength of the mortar alone. Mr. Wheeler concluded that grooving the surface of the stone has no appreciable effect on the adhesive strength. For the maximum adhesive strength more water is required than for the maximum cohesive strength even if the surface of the stone be saturated. The substitution of a small portion of lime for a part of the cement apparently increases the adhesive strength.

*Molded upon normal adhesion blocks, see pp. 122 and 123.

†Report Chief of Engineers, U. S. A., 1895, p. 3019 and 1896, pp. 2799 and 2834.

Mr. R. Feret* states that adhesion to stone increases as the stone becomes more porous. He found, as did Mr. Wheeler, that irregularities of surface of the stone do not seem to affect the adhesive strength. With iron, however, roughening the surface increases the adhesion of the mortar. A dirty surface or insufficient moistening of the surface lowers the adhesion.

The method adopted by various experimenters of crossing two bricks and cementing them together, then determining the loads required to separate them, is obviously inaccurate because of the difficulty of distributing the pull uniformly over the entire surface.

The adhesion of mortar to iron or steel is of such practical importance in the use of iron or steel for reinforcement, and the setting of bolts in mortar and concrete, that the subject is discussed in connection with reinforced concrete in Chapter XXI.

SHEARING TESTS OF CEMENT AND MORTAR

Mr. R. Feret made a series of shearing tests upon different mortars which are quoted in column (20) of the table on page 136. He employed for the shearing test the halves of small prisms which had been broken to determine the transverse strength, placing the specimens and loading them as is shown in Fig. 46.

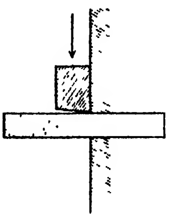


FIG. 46.—Shearing Test. (*See p. 125.*)

ABRASION

Abrasion or wearing tests have been made by pressing the specimen against a grindstone, an emery wheel, or a cast-iron disc, the last requiring sand in definite proportions to be poured upon it to increase the friction.

Tests by Mr. Eliot C. Clarke† tend to indicate that for Portland cement mortar the best proportions to resist abrasive forces are 1 : 2 and for Natural cement mortar 1 : 1, the resistance of Portland cement mortar mixed with two parts of sand being nearly double that of both the richer 1 : 1 mixture and the leaner 1 : 2½ mixture.

POROSITY TESTS

The determination of the porosity of a specimen is often necessary in scientific research and for comparing the relative absorptive properties of

*Communication au Congrès de Budapest, 1901.

†Transactions American Society of Civil Engineers, Vol. XIV, p. 167.

building materials. Porosity is a passive quality referring to the actual voids, *i.e.*, air and uncombined water in a substance as distinguished from permeability or percolation, the quality of a substance which permits the flow of a liquid or gas through it.

Method of Testing Porosity. Messrs. P. Alexandre, P. Debray, and H. Le Chatelier* recommended a method for making the test for porosity which, with the units converted into English measure, is summarized by Mr. Thompson† in his "Discussion on the Report of the Committee of the American Society of Civil Engineers on Uniform Tests of Cement." This method is suitable for testing the porosity of concrete as well as of mortar.

The porosity of a mortar is expressed as the ratio or percentage of voids to the total volume. In measuring the voids all water in the mortar is included except that of crystallization.

If

V = total apparent volume of mortar;
v = volume of solid portion of mortar;
v' = volume of voids in mortar;

then

$$\text{Porosity} = \frac{v'}{V} = \frac{V-v}{V}.$$

The size of specimen recommended is that having a volume of between 0.3 and 0.5 liter (18 to 30 cu. in.).

The solid volume, v, is found by the application of the principle of Archimedes, that the difference between the weight of a body in air and its weight when suspended in a liquid is equal to the weight of the liquid displaced. From the weight of the displaced liquid, its volume, which is manifestly the volume, v, of the mortar, can be readily calculated.

In English measure, if

P = weight of specimen after drying;
p = weight suspended in water after saturation;
W = weight of 1 cu. ft. of water;
v = volume of solid portion of mortar;

then

$$v \text{ (in cubic feet)} = \frac{P-p}{W}.$$

In order that the specimen may be thoroughly impregnated with water and all air driven from the pores when determining p, its weight in water, the specimen is first exposed for $\frac{1}{4}$ hour in rarefied air at a pressure not greater than 25 mm. of mercury. Water is made to cover it, and then the atmospheric pressure is restored. It must now remain in the water 24 hours before being weighed. If apparatus for rarefying the air is not at

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 247.

†Proceedings American Society of Civil Engineers, Aug. 1903, p. 648.

hand, and if the specimen will stand exposure to heat, an alternate method may be used. The specimen, after 48 hours in water, is placed in cold water, raised to boiling, and boiled for 2 hours, then allowed to cool for 24 hours. The weight, P, used in this determination, is taken after exposing it to a heat of between 40° and 60° Cent. (104° and 140° Fahr.), until there is no loss in weight, care being taken to prevent any access of carbonic acid gas from the heating apparatus.

The apparent volume, V, of the specimen, can be found by direct measurement, or by calculation from its loss of weight in water, using again the principle of Archimedes. To prevent saturation in the later proceeding, it can be covered with a thin coating of grease spread with the fingers. The weight in water must be taken before that in air.

The standard test of porosity is made with 1:3 mortars of normal plastic consistency, 28 days old. Other proportions and ages suggested are 1:2, and 1:5, at 7 days, 28 days, 6 months and 1 year.

The Porosity of Different Mortars. Porosity includes the voids or pores occupied by both air and water, the relative volumes of the two classes of voids varying with the freshness of the mortar.

In different fresh mortars there is much less variation in the volume of air voids than is generally supposed. If we leave out of consideration mortars that are mixed to such a dry consistency that voids are apparent to the eye, we notice from column 10 of the table on page 136 that in mortars proportioned richer than 1:5 the air voids rarely exceed 12%, and for most mixtures are in the neighborhood of 4% to 8%, that is, 4% to 8% by volume of air is entrained when gaging. Although experiments of Messrs. Candlot* and Alexandre show similar results, the authors, by mixing the materials with gloves, as recommended by the American Society of Civil Engineers, and using more water than required for standard consistency, — about, in fact, the consistency used by stone masons, — have obtained mortars in proportions of cement to either standard sand or bank sand of 1:0, 1:1 and 1:2 with no appreciable entrained air, and leaner mixtures with 1% to 2% air. A few experiments carefully made tend to show that in larger batches thoroughly mixed to soft consistency these low percentages may be obtained. Such mortars require no ramming, in fact the volume cannot be reduced after it is carefully introduced into the measure, except that if a very wet mixture is used it will expel a portion of its surplus water when setting so that the volume set is less than the volume green. One would naturally expect a greater variation with different materials and different proportions of water, but as a matter of fact, in a fresh mor-

*Candlot gives a range of from 2 or 3% for mortars of coarse sand, up to 10% with fine sand.

tar slightly softer than standard consistency, the spaces between the particles of sand and cement are not occupied by air but by water.

As the mortar dries after setting, the variation between different mortars is more appreciable, since the additional amount of water which is required with mortars of fine sand partially evaporates and leaves air voids. It is evident from experiments by Mr. Alexandre that the percentage of air voids due to evaporation of water ranges from 7% with a very coarse sand to 18% with a very fine sand. Assuming a small allowance for entrained air in the fresh mortar, due to imperfect mixing, we may estimate a range of from 7% to 25% total air voids in mortar after setting and drying. An average mortar of Portland cement and fairly coarse bank sand, in proportions 1 : 2 by weight or 1 : 2½ by volume, from experiments of the authors, may be expected to contain about 10% of air voids after setting and hardening, and to have a total porosity of about 25%. The porosity of well proportioned concrete is much lower (see p. 339). The porosity is but slightly affected by the percentage of water used in gaging, because an excess of water rises to the surface. (See p. 338.)

PERMEABILITY OR PERCOLATION TESTS

The permeability of mortar and concrete is discussed and the laws which govern it formulated in Chapter XIX, page 338 Permeability is distinguished from porosity on page 126.

Method of Testing Permeability. When preparing its final report, the French Commission* first experimented with cylindrical blocks having in the center a truncated well into which a vertical tube was introduced for a short distance to convey the water under pressure. They finally recommended instead of this form a cube of cement or mortar with a pipe cemented to its upper surface. Quoting again from Mr. Thompson's Discussion† on Uniform Tests of Cement:

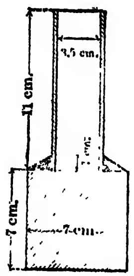


Fig. 47.—French Test for Permeability. (See p. 128.)

The permeability of neat cement and mortars is expressed by the number of liters of water passed in one hour through a cubical block, 50 sq. cm. (7.75 sq. in.) on a face, the size used for compressive tests. The block is placed on its side, that is, with a face which has been against the mold uppermost; this surface is carefully cleaned and a glass tube 3.5 cm. (1.38 in.) in diameter, and 11

*Commission des Méthodes d'Essai des Matériaux de Construction, 1894, Vol. I, p. 313.

†Proceedings American Society of Civil Engineers, August, 1903, p. 649.

cm. (4.33 in.) high is sealed vertically to it by means of neat cement, as shown in Fig. 47. For varying the pressure, a rubber pipe is attached to this tube, and its upper end connected with a reservoir. The height of pressure, according to the permeability of the mortar, may be 10 cm. (4 in.), 1 m. (3 ft. 3 in.) or 10 m. (33 ft.).

Before taking the test, the block is immersed in water for 48 hours, and remains in water during the test. The periods recommended are: 24 hours, 7 days, 28 days, 3 months, etc. The standard test is made at 28 days. Tests are made on three blocks, and an average taken of the two which most nearly agree.

Logically, we should suggest for the block to be used for testing permeability in this country, the size mentioned for compression, that is, a 2-inch cube. Further investigation is considered necessary, however, before adopting either the size or shape as a standard.

Since the publication of the above discussion, the authors have performed a series of tests on the relative permeability of concretes, as described on page 348, obtaining satisfactory relative results by cementing a short length of pipe to the surface of a solid block of concrete in a manner similar to that adopted by the French Commission.

YIELD TESTS OF PASTE AND MORTAR

The French Commission* recommend the testing of cement paste and mortar to determine the volume occupied. The yield or *rendement* is the volume of mortar obtained by gaging to any given consistency a unit of weight of cement or of a mixture of cement and sand in the selected proportions. One kilogram of cement, or of the required mixture of cement and sand, gaged to the given consistency, is introduced into a graduated cylindrical glass test tube about 6 cm. (2.37 in.) in diameter, with care to avoid imprisonment of air, and its volume is noted.

Another method, which they consider more accurate, is to allow the paste or mortar to harden and then determine the difference in weight in air and in water.

Mr. R. Feret in his report to the French Commission† on the production and density of mortars considers that sands should be submitted to a thorough test. He advises determining their granulometric composition, as described on page 142, the proportion of gravel (that is, of particles remaining on a sieve with holes of 50 mm. (0.19 in.) diameter, the mineralogical nature, and the form of the grains. Disregarding the yield test he would study the absolute volumes of the cement, the sand, the water,

*Commission des Méthodes d'Essai des Matériaux de Construction, 1894, Vol. I, p. 307.

†Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 243.

and the voids in a unit volume of fresh mortar, and would estimate the cost per cubic meter of mortar made with different sands, and its strength under various conditions, as is discussed at length in the following chapter.

TEST OF RISE IN TEMPERATURE WHILE SETTING

The determination of the rise in temperature which takes place in a cement while setting has often been suggested as an indication of its quality, but the increase in temperature is due to so many causes that it is of slight value as a test of the cement.

Mr. Le Commandant Ribaucour* found that the temperature commenced to rise at the commencement of the setting, and the rise was generally higher with quick-setting cements.

Mr. J. E. Howard at the Watertown Arsenal† discovered that the

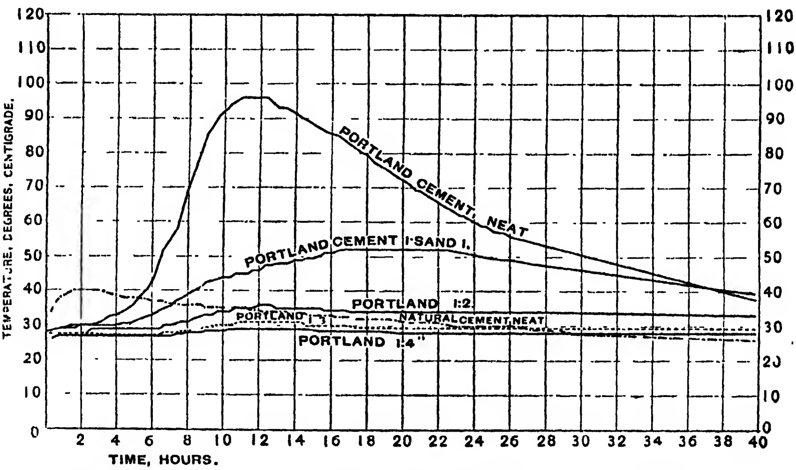


Fig. 48.—Rise in Temperature in 12-inch Cubes of Cement and Mortar. (Tests of Metals, U. S. A., 1901.) (See p. 130.)

temperature was largely dependent upon the size of the specimen, small cubes showing very slight increase. He therefore made a series of tests upon 12-inch cubes to determine the temperature acquired by different brands of cement and mortars during setting, and plotted his volumes in a series of curves. The curves for a first-class brand of American Port-

*Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 133.
†Tests of Metals, U. S A., 1901, p. 493.

land cement with and without sand, and for a typical Natural (Rosendale) cement, are shown in Fig. 48.

Mr. Howard found that while first-class American brands of neat Portland cement often reached a maximum temperature of 100° Cent. (212° Fahr.); the maximum temperature of the various brands of American Natural cement was generally from 35° to 40° Cent. (95° to 104° Fahr.), and was reached at a shorter time than the Portland cements. The rise in temperature of the German brands of Portland cements was in general less than that of the American Portlands.

The rise in temperature in Portland cement concrete is less than in neat Portland cement, but in the interior of a large mass like a dam may reach nearly 100° Fahrenheit.

TESTS OF SAND FOR MORTAR

Tests of sand for mortar and concrete are as important as tests of cement Methods of making tests are given on page 159.

CHAPTER IX

STRENGTH AND COMPOSITION OF CEMENT MORTARS

The following are the important conclusions in this chapter:

(1) The strength of a mortar depends primarily upon (*a*) percentage of cement in a unit volume, and (*b*) density. (See p. 133.)

(2) The strongest mortar for any given proportions, by weight, of cement to dry sand, is obtained from sand which with the given cement produces the smallest volume of plastic mortar. (See p. 148.)

(3) The best sand is in general that which will produce the smallest volume of mortar of standard consistency when mixed with the given cement in the required proportions. (See pp. 133 and 149.)

(4) The density of a mortar is determined by calculating the absolute volumes of its ingredients. (See p. 138.)

(5) The qualities of different sands may be studied by screening each into three sizes and comparing their granulometric compositions with Feret's curves. (See p. 142.)

(6) Sharpness of the sand grains is of slight importance. (See p. 154a.)

(7) Coarse sand produces stronger mortar than fine sand. (See p. 146.)

(8) Fine sand requires more water than coarse sand to produce a mortar of like consistency, and consequently its mortar is less dense. (See p. 145.)

(9) Mixed sand, *i. e.*, sand containing fine and coarse grains, in mortars leaner than 1: 2, usually produces stronger and more impervious mortars than coarse sand. (See p. 146.)

(10) Screenings from broken stone usually produce stronger mortars than sand because of their greater density. The relative value of screenings and sand may often be determined by comparing their densities or the densities of mortar made from them. (See pp. 150 and 153.)

(11) Mixtures of fine and coarse sand or of sand and screenings often produce better mortar than either material alone. (See p. 149.)

(12) The variation of the sand in different portions of the same bank may be utilized by requiring the contractor to mix two sizes without exact measurement, so that the material as delivered shall contain not less than a definite percentage of sand coarse enough to be retained on a certain sieve. (See p. 149.)

(13) Mineral impurities in sand, such as clay, in small quantities, may strengthen a lean mortar, and weaken a rich mortar. (See p. 154b.)

(13a) Organic impurities in sand, such as vegetable loam, even in minute quantities may destroy the strength of the mortar or concrete. (See p. 154b.)

(14) Gaging with sea water does not affect the ultimate strength of mortars. (See p. 159b.)

(15) The unit fiber stress in a cement or mortar beam is about the same for a prism 4 cm. (1.6 in.) on edge as for one 2 cm. (0.8 in.) on edge. (See p. 134.)

(16) The unit fiber stress in bending is about 1.89 times the unit tensile strength of briquettes of 5 sq. cm. (See p. 134.)

(17) The unit tensile strength of specimens decreases as the breaking area is enlarged. (See p. 134.)

(18) The unit compressive strength of similar specimens of cement or mortar is not greatly affected by their size. (See p. 134.)

Laws of Strength. There are two fundamental laws of strength which apply to mortars composed of the same cement with different proportions and sizes of sand.

(1) With the same aggregate,* the strongest and most impermeable mortar is that containing the largest percentage of cement in a given volume of the mortar.

(2) With the same percentage of cement in a given volume of mortar, the strongest, and usually the most impermeable, mortar is that which has the greatest density,† that is, which in a unit volume has the largest percentage of solid materials.

The first of these rules is understood by ordinary users of cement, but the second rule states a fact which is appreciated only by experts.

The value of a first-class cement is universally recognized, the effects of impurities have been studied in various ways, and the variations in strength of mortars made from different sands or broken stone screenings have been recorded, but the fundamental law of the relation of the density of a mortar to its strength, — a function nearly as important as the quality of the cement itself and explaining many of the seemingly paradoxical results of tests with different aggregates and different proportions of water, — is but vaguely comprehended by the majority of experimenters and most of the users of cement.

The importance of this subject claims for it a full investigation, and its study is taken up on page 134. The application of these laws to concrete is discussed in Chapter XX.

*The word *aggregate* is defined on page 1.

†The meaning of *density* may be understood by referring to the figures on pp. 172 and 173.

STRENGTH OF SIMILAR MORTARS SUBJECTED TO DIFFERENT TESTS*

Mr. René Feret, Chief of the Laboratory of Bridges and Roads at Boulogne-sur-Mer, France, has made very extended tests of strength of mortars, studying his results scientifically, and in many cases formulating laws and formulas applicable to different conditions. The tests of one series in particular are of so wide a range in character and in proportions used that the authors have converted the values into English units, and reproduce the table in full on pages 136 and 137.

After plotting the strengths in various ways, Mr. Feret reaches conclusions which may be summed up as follows:

(*a*) The unit fiber stress for prisms 4 centimeters (1.6 in.) on an edge is about the same as for prisms 2 centimeters (0.8 in.) on edge.

(*b*) The tensile strength per square centimeter of prisms having a breaking area of 16 square centimeters (the strength of which he found to be similar to that of briquettes of the same section) is about two-thirds the strength per square centimeter of the normal briquettes which have an area of 5 square centimeters. This difference is attributed partly to the lack of homogeneity of the specimens, especially on their surfaces, but principally to the unequal distribution of the stress on the area of the section.

(*c*) Resistance to flexion, that is, the unit fiber stress in bending, is about 1.89 times the tensile strength per unit of area of briquettes of 5 square centimeters.

(*d*) The form and dimensions of the specimen do not greatly influence the strength per unit of area in compression when the height and width of the block are approximately equal.

(*e*) Resistances to flexion and tension are proportional to each other, and resistances to compression, shearing, and punching are proportional to one another, but there is no constant relation between the resistance to compression and the resistance to tension or flexion.

THE RELATION OF DENSITY TO STRENGTH

In the same paper from which we have quoted, Mr. Feret treats of the density and elementary volumetric composition of mortars, using in his studies the results given in the table just described. He calls particular attention to the fact that the properties of hydraulic mortar, such as durability, permeability, porosity, and ability to resist the decomposing action of sea water, depend not only upon the quality of the cement, but "in a measure greater than is generally believed, upon the granular physical

* A valuable series of tests has also been made by Messrs. Humphrey and Jordan at the U. S. Government Testing Laboratory at St. Louis, see Bulletin No. 331 U. S. Geological Survey, 1908.

composition of the mortars, that is to say, upon the dimensions and relative positions of the different elements entering into their composition."

The density (*compacité*) of a mortar is represented by the total volume of the solid particles, — exclusive of the water and the voids, — entering into a unit volume of mortar.*

The "elementary volumes" in a unit volume of fresh mortar consist of the absolute volumes of the cement, sand, water, and voids, each expressed in the form of a decimal. To illustrate, the "elementary volumetric composition" of the mortar in Item 8 of the table on page 136, which is mixed in proportions by weight of one part cement to 1½ parts of natural sand, is:

Cement	$(c) = 0.226$
Sand	$(s) = 0.499$
Water	$(w) = 0.234$
Air voids	$(v) = 0.041$
Total volume	$= 1.000$

Expressing this in more familiar terms, 22.6% of the unit volume of the given mortar consists of solid particles of cement, 49.9% of particles of sand, 23.4% of water, and the remaining 4.1% of air voids.

The porosity, represented by the sum of the water and air voids, is 27.5%. The term *voids* is often employed to represent the porosity, that is, the sum of the air and water.

It is obvious that

$$c + s + w + v = 1;$$

also that

$$v = 1 - (c + s + w),$$

which is equivalent to the statement that the entrained air in any volume of fresh mortar is equal to the measured volume of the mortar minus the space occupied by the cement, sand, and water.

The density of the mortar considered above is $c + s$, or, $0.226 + 0.499 = 0.725$ as given in column (11) of the table on pages 136 and 137.

A thorough understanding of the use of these symbols is essential to the study of strength of concrete and mortar, for, as will be shown further on, practical tests of strength are of small value unless the density and exact mechanical composition of the specimens are clearly defined.

*If the word density is applied to sand alone, it means the proportion of the measured volume of the sand, which is occupied by the solid sand grains; a sand, for example, having under certain conditions 40% voids, would have a density of $1.00 - 0.40 = 0.60$.

Strength and Composition of Portland Cement Mortars.

BY R. FERET. (*See p.* 134.)

(Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897. Vol. II, p. 1593.)

SAND.*	ITEM	Approximate Proportions by Weight: Cement (1)	Sand (2)	Composition of Mortar by Weight: Cement in 1,000 grams dry mixture ($c+s$) (3) gr.	Water per 1000 grams dry mixture (4) gr.	Weight of a cu. yd. of fresh mortar (5) lb.	Price of a cu. yd. of mortar (6) $	Elementary Volumetric Composition: Cement c (7)	Sand s (8)	Water w (9)	Air voids v (10)	Density $c+s$ (11)	$\left(\frac{c}{1-s}\right)^2$ (12)	Strength per sq. in. after 5 months in fresh water: Flexion F_{16} (13) lb.	Flexion F_4 (14) lb.	Tension T_{16} (15) lb.	Tension T_5 (16) lb.	Compression P_{50} (17) lb.	Compression P_{16} (18) lb.	Compression P_4 (19) lb.	Average Strength: Shearing C_4 (20) lb.	Tension and flexion T (21) lb.	Compression P (22) lb.
G	(1)	1	18.6	51	62.5	3296	2.37	0.030	0.670	0.115	0.185	0.700	0.0083	134	134		65	210	340	160	170	69	240
G	(2)	1	9.9	92	70	3426	2.95	0.055	0.663	0.132	0.150	0.718	0.0266	287	256	58	152	840	980	800	570	146	870
G	(3)	1	6.9	126	76	3539	3.46	0.078	0.655	0.148	0.119	0.733	0.0511	418	370	129	220	1340	1690	1580	1070	212	1540
G	(4)	1	5.2	160	81.5	3654	4.00	0.102	0.648	0.163	0.087	0.750	0.0841	509	461	144	260	2060	2620	2360	1440	258	2350
G	(5)	1	4.1	196	88	3750	4.56	0.127	0.631	0.180	0.062	0.758	0.1190	597	568	206	326	2840	3680	3440	2000	314	3320
G	(6)	1	3.2	237	95	3814	5.17	0.155	0.605	0.196	0.044	0.760	0.1537	693	687	245	371	3750	4510	4250	2560	367	4170
G	(7)	1	2.5	287	104.5	3810	5.82	0.186	0.559	0.214	0.041	0.745	0.1772	804	807	277	407	4690	5430	5500	2790	421	5210
G	(8)	1	1.8	355	116	3792	6.64	0.226	0.499	0.234	0.041	0.725	0.2034	935	939	299	451	5720	5870	6320	3580	480	5970
G	(9)	1	1.2	449	133	3755	7.74	0.280	0.415	0.262	0.043	0.695	0.2304	1040	1060	356	502	6400	6500	7110	3930	537	6670
G	(10)	1	0.7	602	163	3694	9.35	0.359	0.287	0.306	0.048	0.646	0.2530	1070	1120	395	532	7050	6270	>7110	3640	563	>6810
S	(11)	1	12.9	72	90	3111	1.96	0.039	0.592	0.152	0.217	0.631	0.0092	141	158		85	270	360	310	256	81	310
S	(12)	1	7.0	125	96.5	3290	2.70	0.070	0.587	0.171	0.172	0.657	0.0289	330	341	54	192	970	970	940	669	182	950
S	(13)	1	5.0	167	101.5	3409	3.29	0.097	0.576	0.186	0.141	0.673	0.0524	451	438	141	249	1380	1640	1490	1040	240	1510
S	(14)	1	4.1	196	105	3485	3.74	0.116	0.569	0.196	0.119	0.685	0.0724	516	526	152	282	1780	2120	2080	1350	278	1990
S	(15)	1	3.1	241	111	3622	4.45	0.148	0.555	0.215	0.082	0.703	0.1109	612	572	212	333	2460	2870	2820	1810	320	2720
S	(16)	1	2.5	283	117.5	3645	5.02	0.173	0.525	0.227	0.075	0.698	0.1325	694	684	222	374	3130	3560	3600	2250	368	3430
S	(17)	1	2.0	333	125	3674	5.71	0.204	0.486	0.242	0.068	0.690	0.1576	821	768	246	405	4010	4440	4680	2650	415	4380
S	(18)	1	1.4	412	138	3701	6.76	0.252	0.428	0.265	0.055	0.680	0.1945	994	953	326	458	5230	5350	5730	2750	521	5440
S	(19)	1	0.9	518	157	3659	7.99	0.309	0.340	0.294	0.057	0.649	0.2190	1080	1070	307	489	5830	5920	6560	3580	541	6100
S	(20)	1	0.5	648	180	3630	9.44	0.376	0.241	0.329	0.054	0.617	0.2460	1180	1200	378	543	6600	6510	7050	3540	602	6720

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
D	(21)	1	12.3	75	167	3178	1.49	0.038	0.564	0.269	0.129	0.602	0.0076	114	141		65	160	200	130	156	67	160
	(22)	1	5.8	148	168.5	3230	2.38	0.077	0.528	0.276	0.119	0.505	0.0269	247	238	60	122	600	550	480	370	126	540
	(23)	1	3.5	225	170	3262	3.32	0.118	0.485	0.281	0.116	0.603	0.0529	408	415	119	208	1080	1290	1320	768	214	1230
	(24)	1	2.4	299	172	3318	4.26	0.159	0.444	0.289	0.108	0.603	0.0818	576	576	196	296	1790	1910	2120	1410	302	1940
	(25)	1	1.8	363	175	3367	5.09	0.195	0.409	0.298	0.098	0.604	0.1082	703	693	233	354	2380	2970	3170	2130	364	2840
	(26)	1	1.3	429	179	3412	5.98	0.234	0.370	0.307	0.089	0.604	0.1376	814	828	233	439	3470	3570	4100	2570	436	37.0
	(27)	1	1.0	500	183.5	3458	6.92	0.274	0.331	0.318	0.077	0.605	0.1681	980	963	302	502	4750	4410	5830	2750	510	5000
	(28)	1	0.7	573	190	3510	7.90	0.320	0.282	0.333	0.065	0.602	0.1908	1100	1120	373	549	5750	5450	6070	3070	574	5760
	(29)	1	0.5	659	197	3495	8.92	0.361	0.223	0.341	0.075	0.584	0.2171	1210	1280	371	623	6630	5970	6900	3570	647	6500
	(30)	1	0.3	771	208	3545	10.39	0.426	0.151	0.361	0.062	0.577	0.2530	1280	1370	401	671	7400	6830	7110	(4120)	691	>7110
M′	(31)	1	5.0	167	123	3650		0.102	0.607	0.237	0.054	0.709	0.0676	613	623	239	330	2120	2350	2570	1720	328	2350
	(32)	1	3.0	250	136	3642		0.150	0.539	0.259	0.052	0.689	0.1056	842	808	304	475	3640	3870	4510	3100	450	4010
	(33)	1	2.0	333	148	3645		0.199	0.474	0.278	0.049	0.673	0.1429	990	981	336	513	4640	4860	4920	3070	518	4810
N′	(34)	1	3.0	250	91	3620		0.156	0.557	0.179	0.108	0.713	0.1246	862		293	455	3600	3670			456	3640
C	(35)	1	0	1000	245	3552		0.534	0.000	0.414	0.052	0.534	0.2851	1280	1510	385	624	8720	7350	>7110	3680	698	8040

NOTE.—All are plastic mortars except N′.

*Description of Sands.

SAND	Nature of sand	Form of grains	Granulometric Composition† Coarse grains G	Medium grains M	Fine grains F	Approximate weights per cu. yd. lb.	Approximate price per cu. yd. $
G Sand from Gattemarre near Cherbourg	Granitic	Large and rounded	0.73	0.25	0.02	2815	1.18
S Sand from Saint-Malo	Very shelly	Varied	0.17	0.70	0.13	2537	0.59
D Sand from the dunes	Strongly siliceous	Fine and rounded	0.00	0.01	0.99	2461	0.15
M′ Ground quartzite sifted and remixed in equal parts	Quartz	Angular	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		

N′ Ground quartzite passing a sieve of 64 meshes, and retained on one of 144 meshes per sq. cm (20 and 31 meshes per linear inch).

C Neat Portland Cement.

†Granulometric composition is defined on p. 142.

EXPLANATION OF COLUMNS.

Col. (6) based on price and weight of given sand, on cement at 50 francs per "tonne" ($1.66 per bbl.). and on labor at 3 francs per cubic meter (44 cts. per cu. yd.) of mortar.

Cols. (7) to (12) are discussed on page 135.

Col.	Number of specimens of each mortar.	Size of specimens, centimeters.	Remarks.
(13)	15 prisms	4 x 4 x 16	Supports 10 cm. apart
(14)	15 "	2 x 2 x 13	" 10 " "
(15)	15 "	4 x 4 x 8±	Halves of broken prisms
(16)	25 briquettes	5 sq. cm. section	French standard briquettes
(17)	5 cubes	50 sq. cm. face	
(18)	15 prisms	4 x 4 x 8±	Halves of broken prisms
(19)	30 "	2 x 2 x 3±	Quarters of " "
(20)	15 "	2 x 2 x 6±	Halves " " "

(21) Average of cols. (13), (14) and (16) by formula

$$T = \frac{\left(\frac{F_{13} + F_{14}}{2}\right) + T_{5}}{1.89 + 1}$$

(22) Average of cols (17), (18), (19).

In practice density of volumetric tests are of great value for comparing the relative values of different aggregates, and for determining the proportions for the most economical concrete. They are also useful for studying the effect of varying quantities of water. As is shown in the following pages, the density of mortars or concretes made from similar materials bears a definite relation to the strength, so that it is frequently possible to determine the best mixture as soon as the density tests are completed, instead of waiting for the tests of tensile or compressive strength. The test has been used by the authors in a practical way for comparing sands and for grading sands in special work, and also for concrete to fix on the best proportions when using merely one fine and one coarse aggregate, and in other cases to determine the proper proportions for a scientifically graded mix.*

Density of Mortars and Concrete. The density of fresh mortars of ordinary proportions, as shown by tests of the authors, averages about 0.70 (corresponding to 30% air plus water voids). Mortars of fine sands may run as low as 0.60 (40% air plus water voids), while by special grading or the use of an exceptionally good coarse sand the density may be as high as 0.75 (25% voids). The density of neat cement usually ranges between 0.50 and 0.55. The density of concrete ranges† from 0.76 to 0.88, depending upon the grading of the aggregates and the cement.

The values apply to the materials freshly mixed before setting. The chemical combination of the cement and water reduces the porosity further.

Density or Volumetric Tests of Mortar.‡ To obtain accurate results, considerable care is necessary in making the experiments. An approximate method suited to rough comparisons will be given first and this will be followed by more accurate methods advised for laboratory work.

The rough volumetric test may be made in almost any vessel or mold so long as the capacity is readily computed and its dimensions such that the depth of mortar or concrete can be measured exactly. A deep mold is more accurate than a shallow one. The volume

* See Chapter XI, p. 183.

† From the "Laws of Proportioning Concrete," by Wm. B. Fuller and Sanford E. Thompson, Transactions American Society Civil Engineers, Vol. LIX, 1907. p. 67.

‡ The French Commission determine the "yield" of a mortar (see p. 129) by measuring its volume green, that is, just after introduction into the molds, when an excess of water may affect the volume, and thus give misleading results with very wet mixtures.

In his Report to the French Commission, 1895, Vol. IV, p. 243, Mr. Feret also measures the mortar wet, but he employs a vessel of known capacity,—a cylindrical measure whose height and interior diameter are each about 8 centimeters,—and uses only a portion of the mortar which he mixes, calculating his percentages by ratio of the weight of mortar made to the weight of mortar introduced into the measure to fill it exactly. This method eliminates inaccuracies in measuring the level of the surface.

of mortar and concrete of dry consistency will measure the same after setting as when green, but wet mixtures must be measured before setting, and again after they have become sufficiently hard to expel the surplus water. The measurement before setting is necessary in order to calculate the volume of air bubbles entrained in the wet mortar or concrete. The volume after setting, or partially setting, however, is the only one of real importance for studying the characteristics of strength, permeability, and cost. The sand is dried, or its moisture is determined by weighing and drying a sample of it. If stone of a porous nature is used the pores of its particles should be filled with water, but there should be no perceptible moisture on their surfaces. The quantities of dry materials for a single tube or mold are weighed in the required proportions, mixed with a known weight of water, and placed compactly in the mold, whose lateral dimensions have been exactly measured so that the volume of mortar in it may be obtained by measuring down from the top. The exact space occupied by the particles of each of the solid materials and by the water is calculated, if the metric system is employed, by dividing their total weight by the specific gravity of each, or, if English units are used, by dividing the weight times 1728 (the number of cubic inches in a cubic foot) by the specific gravity multiplied by the weight of a cubic foot of water. After partially setting, the exact depth of the mortar in the mold is measured and its volume calculated. The percentage of each of the dry materials, which really determines the density,—which is represented by the sum of the absolute volumes of the dry material,—is found by dividing the absolute volume of each material by the total volume of the set mortar or concrete.

The specific gravity of cement which has been stored for a short time may be taken at 3.10 and the specific gravity of dry sand at 2.65.

The following example from the authors' note book illustrates the method of finding the density when the measurements are in English weights and measures:

Example:—Find density of a mortar composed of Newburyport sand and Portland cement in proportions 1 : 2 by weight.

Solution:—For the mold used, it was estimated that 8 lb. cement and 16 lb. dry sand would be required. Gaging these with 3 lb., 12.6 oz. (3.79 lb.) of water, the quantity necessary for the desired consistency, the volume of the mortar was found by measurement to be 348 cu. in. when green, and 336 cu. in. after setting and pouring off the surplus

water. The absolute volumes are expressed below, first in cubic inches and finally in terms of the density $(c + s)$, of the set mortar.

$$\text{Cement} = \frac{8 \times 1728}{3.1 \times 62.3} = 71.6 \text{ cu. in.}$$

$$\text{Sand} = \frac{16 \times 1728}{2.65 \times 62.3} = 167.4 \text{ cu. in.}$$

$$\text{Water} = \frac{3.79 \times 1728}{62.3} = 105.1 \text{ cu. in.}$$

Absolute volume cement, sand and water,	344 cu. in.
Measured volume green mortar,	348 cu. in.
Volume of entrained air,	4 cu. in.
Percentage of entrained air,	1.2%

$$\text{Density of set mortar, } c + s = \frac{71.6}{336} + \frac{167.4}{336} = 0.213 + 0.498 = 0.711$$

Volumetric Tests of Mortar at Jerome Park Reservoir. The methods used by Messrs. Fuller and Thompson at Jerome Park Reservoir in tests for the New York Aqueduct Commission in 1906* have since been adopted, with slight variations, in the authors' laboratory. The procedure is indicated in the blank form used in the tests, a copy of which filled out is here reproduced on page 139. While somewhat lengthy in appearance, it is arranged to correct almost automatically for the unavoidable losses due to free water and mortar sticking to the tools. The chief object of the test is to find the density of a fresh mortar, that is, the ratio of solid material in it to the total volume, and also to determine the elementary volumes of each ingredient. In the test illustrated, for example, the density is 0.696 and the air plus water voids are therefore 30.4%.

The apparatus used for density tests of mortar are a shallow pan about 9 inches diameter, a small pointing trowel, scales to weigh to one-tenth gram, measuring glass or graduate about 1½ inches diameter and 250 cubic centimeters capacity, one or two beakers, and a stick for tamping the mortar in the glass. 300 or 400 grams of mixed cement and aggregate may be used in the tests.

It has been found that the material which sticks to the tools is either cement or similarly fine aggregate, so that the weight of the aggregate which passes a No. 100 sieve should be recorded for use in the computations.

* See paper by Messrs. Fuller and Thompson, Transactions American Society Civil Engineers, Vol. LIX, p. 67.

Volumetric test for *Reservoir* ....File W. R.
Cement *B* Aggregates *Clean Sand*............................Date 4-26 06.
Computed by *Brown* ....Checked by *T*.

(1)	Experiment No.	152
(2)	Nominal proportions by volume	1 : 2
(3)	Proportions by weight	1 : 1.78
(4)	Description of aggregate	Sand
(5)	Wt. of cement	150.0
(6)	Total weight of aggregate	267.0
(7)	Wt. of the aggregate passing a No. 100 sieve	53.4
(8)	Wt. of vessel and water (before using)	287.7
(9)	" " " " " (after using)	228.7
(10)	" " water used = (8) — (9)	59.0
(11)	Percentage of water = $\frac{(10)}{(5)+(6)}$	14.2
(12)	Consistency	Soft
(13)	Temperature water	65°F.
(14)	Total weight mixed = (5) + (6) + (10)	476.0
(15)	Weight tray and tools (after using)	325.8
(16)	" " " " (before using)	322.2
(17)	Weight mix adhering = (15) − (16)	3.6
(18)	Weight measuring glass or graduate	295.4
(19)	Weight glass + mix	767.9
(20)	Weight glass + mix − free water	767.9
(21)	" free water = (19 − 20)	0.0
(22)	" mix set = (14) − (17) − (21)	472.4
(23)	" " " = (20) − (18)	472.5
(24)	Discrepancy = (23) − (22)	.1
(25)	Time mixing completed	10.15 a.m.
(26)	Volume of mix, in cu. cm	210.0
(27)	Time settling	2 hrs.
(28)	Final volume of mix in cu. cm	209.5
(29)	Water left on tray = (10) × $\frac{(17)}{(5)+(7)+(10)}$	0.8
(30)	Cement left on tray = (5) × $\frac{(17)}{(5)+(7)+(10)}$	2.1
(31)	Aggregate left on tray = (7) × $\frac{(17)}{(5)+(7)+(10)}$	0.7
(32)	Wt. water in set mortar = (10) − (21) − (29)	58.2
(33)	Wt. cement in set mortar = (5) − (30)	147.9
(34)	Wt. aggregate in set mortar = (6) − (31)	266.3
(35)	Specific gravity cement	3.11
(36)	" " aggregate	2.71
(37)	Absolute volume water = $\frac{(32)}{(28)}$	.278
(38)	" " cement = $\frac{(33)}{(28) \times (35)}$	.227
(39)	" " aggregate = $\frac{(34)}{(28) \times (36)}$	.469
(40)	Total absolute volume = (37) + (38) + (39)	.974
(41)	Density = (38) + (39)	.696
	Remarks: Fine Material on Surface	3 cc.

Note: Weights are in grams; volumes in cubic centimeters.

The materials are carefully weighed, and enough water added,—the quantity varying with the fineness of the sand,—to produce a mortar softer than standard consistency which will scarcely hold its shape in the mixing pan. An examination of the various items in the table will show the purpose of each, the object being to correct for all losses and obtain a resulting volume corresponding to that of the mortar after setting. The figures following many of the items refer to the numbers of the other items, the fraction following item (29), for example, representing the water of the mix which adheres to the tray and tools. The weight of the water in this mortar which adheres is found from the proportion,—Mix adhering: total fine mortar = water in mix adhering : total water. Expressed in item numbers this becomes

$$\text{Item (29)} = \frac{\text{Item (17)}}{\text{Items (5)} + (7) + (10)} \times \text{Item (10)}.$$

The cement and aggregate left on tray, items (30) and (31), are similarly computed, and from these the weight of each of the materials in the set mortar is found. The absolute volumes, items (37) to (39), are then readily computed and the density determined.

Volumetric Tests of Concrete. For volumetric or density tests of concrete, molds at least 8 inches in diameter are necessary, but the process throughout is similar to that already described for the volumetric tests of mortar and a similar blank form may be readily made for records.

The density tests as made at Jerome Park Reservoir are fully described in the paper by Messrs. Fuller and Thompson already referred to† and results of the tests are there given.

Feret's Formula for Strength. For studying the relation of absolute volumes to strength, let

P = compressive strength of the mortar.
K = a constant which differs for different cements and at different ages of the same mortar.
c = absolute volume of cement.
s = absolute volume of sand.
w = absolute volume of water voids.
a = absolute volume of air voids.

The value of determining the density of mortars is made evident by the following law of Mr. Feret:*

"For any series of plastic mortars made with the same binding material

*Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II, p. 1604.
†See also Chapter XI of this Treatise.

and inert sands, the resistance to compression after the same length of set, under identical conditions, is solely a function of the ratio $\frac{c}{w+v}$ or $\frac{c}{1-(c+s)}$, whatever be the nature and size of the sand and the proportions of the elements, — cement, inert sand and water, — of which each is composed."

It follows from this law, as Mr. Feret says, that the strength of any

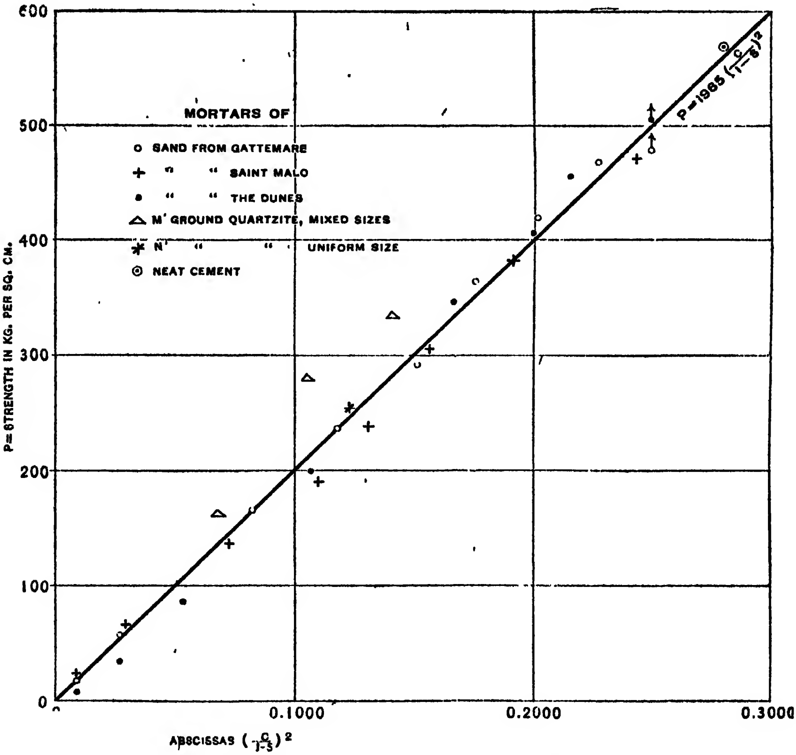


FIG. 49. — Derivation of Feret's Formula for Strength. (*See p.* 142)
(Bulletin de la Société d'Encouragement pour l'Industrie Nationale — 1897.)

mortar increases with the absolute volume of the cement (c) in a unit volume of fresh mortar, and also with the density ($c + s$), whatever may be the relative volumes filled with water and air.

From very numerous experiments such as those tabulated on page 136 Mr. Feret evolves the approximate formula

$$P = K\left(\frac{c}{1 - s}\right)^2 \qquad (1)$$

By suitably changing the value of K the formula may be adapted to either the English or the metric system of measurement.

As a proof of this formula Mr. Feret plots on a diagram, shown in Fig. 49, values of $\left(\frac{c}{1 - s}\right)^2$ from column (12) in the table on pages 136 and 137 for abscissas, and the average compressive strengths of the various mortars, from column (22), for ordinates. Since, in formula (1), K is equal to P divided by the square of the quantity in brackets, the value of K is the tangent of the straight line passing through the points. In Fig. 49

$K = 1965$, if the strength is in kg. per sq. cm.

or

$K = 28\ 000$, if the strength is in lb. per sq. in.

This particular value is applicable only to the cement used by Mr. Feret in his experiments and to specimens at the age of five months, but the principles involved are of general application.

The most practical application of this formula is in the determination of the relative compressive strengths of various mortars made from the same cement, with sand in differing proportions and of different compositions. Mr. Feret calls attention also to its possible use in laboratory experiments and specifications. A cement, for example, may be required to furnish, when mixed with any sand, a definite value of K, since the value of K is independent of the choice of the sand and of the composition of the mortar.

Experiments by the authors tend to show that the formula does not apply strictly to specimens of different consistency, but that the general law of the increase of strength with the density is applicable except in extreme cases. The formula is inapplicable to tensile tests, although here, too, the general principle appears to hold good.

This subject as related to concrete is discussed on pages 355 to 362

GRANULOMETRIC COMPOSITION OF SAND

Feret's Three-Screen Method of Analyzing Sand.

The determination of the physical characteristics of the sand, which, mixed with a cement, will produce the densest mortar, has been the object

of a large number of experiments by Mr. Feret, which are recorded in Annales des Ponts et Chaussées, 1892. In America Messrs. William B. Fuller and Sanford E. Thompson have extended the researches, by a different method, to the investigation of the properties of concrete. The mechanical analysis of sand and stone is discussed in Chapter XI, and the results of earlier experiments are tabulated on page 376.

Mr. Feret, in studying any sand, separates it by screening into three sizes. He then recombines these three sizes in varying proportions, so as to obtain results which are applicable to any natural or artificially mixed sand. He distinguishes sand from gravel as consisting of grains which will pass through a screen having circular holes of 5 millimeters diameter (0.20 in.). The three sizes of sand he then calls G, M, and F, representing, respectively, the large (*gros*), medium (*moyens*), and fine (*fins*) particles as defined by sifting through metallic sieves with circular holes, or wire cloth of definite mesh, as follows:

Large grains, G, passing circular holes	5 mm.	(0.20 in.)	diameter.
Retained by circular holes	2 mm.	(0.079 in.)	"
Medium grains, M, passing circular holes	2 mm.	(0.079 in.)	"
Retained by circular holes	0.5 mm.	(0.020 in.)	"
Fine grains, F, passing circular holes	0.5 mm.	(0.020 in.)	"

These sizes, Mr. Feret states, are nearly equivalent to sand screened through sieves of wire cloth as follows:

Large grains, G, passing screen of	4 meshes per sq. cm.	(5 meshes per linear inch.)
Retained on "	36 " "	(15 " " ")
Medium grains, M, passing "	36 " "	(15 " " ")
Retained on a "	324 " "	(46 " " ")
Fine grains, F, passing "	324 " "	(46 " " ")

Sometimes, for experimental purposes, he divides each of the sands, G, M, and F, into three intermediate sizes.

The granulometric composition of any sand is represented by its relative proportions, expressed either in weights or absolute volumes, of G, M, and F. For example, a sand containing by weight 50% of the largest grains, 30% of the medium, and 20% of the fine grains, has a granulometric composition of $g = 0.50$, $m = 0.30$, $f = 0.20$.

The granulometric composition of a sand which has been mechanically analyzed, and plotted on a diagram similar to that shown on page 194, may be ascertained readily by drawing three ordinates corresponding respectively to screens of 5, 15, and 46 meshes per linear inch, and determining by the length or the difference in length of these ordinates the proportions which pass and which are retained by the screens of these three meshes. These three proportions or percentages represent the granulometric com-

position. An illustration of this method of transforming mechanical analysis to granulometric composition is shown in Fig. 57 on page 151.

Feret's Triangles. To simplify the tabulation of results, and arrange them so that they may be understood at a glance, Mr. Feret has used a graphical arrangement which is exceedingly ingenious. In nearly all his writings we find little triangles with the apexes labeled G, M, and F. Curves or contours in these triangles, representing the various properties of the sands or mortars, are based on a system of three instead of two

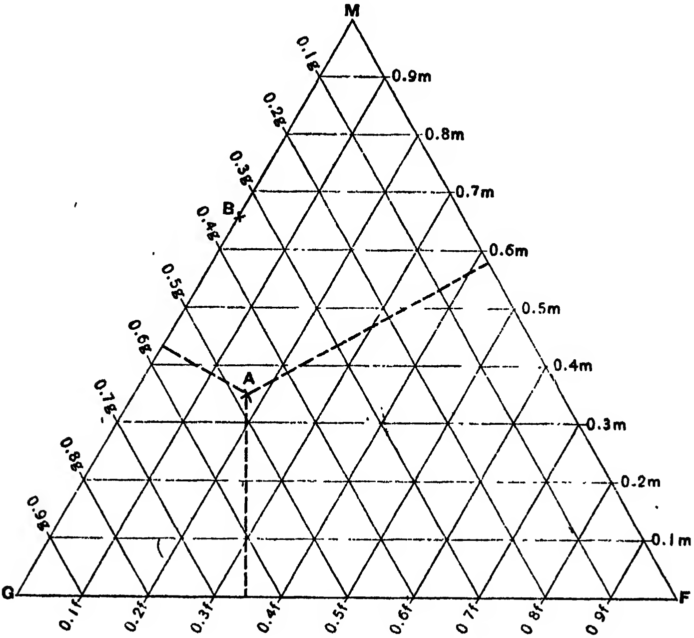


FIG. 50.—Feret's Three-Screen Method of Analyzing Sand. (*See p.* 144.)

co-ordinates, that is, each curve is the loci of points measured from 3 axes placed at angles of 60° with each other. A full discussion of the theory of this is given in his paper "Sur la Compacité des Mortiers Hydrauliques" in Annales des Ponts et Chaussées, 1892, II, but the principles may be understood by reference to Fig. 50. The apexes of the triangle are labeled G, M, and F, corresponding to the three sizes of sand described on page 143. The granulometric composition of any sand is plotted as a single point in this triangle. The proportion of each of the three sizes in the sand is represented by its perpendicular distance from the side opposite each apex.

For example, exactly at the apex G, the granulometric composition is g = 1.00, m = 0, f = 0. A sand represented by the point "*A*" in the triangle has for its granulometric composition, g = 0.48, m = 0.35, f = 0.17. Sand, *B*, whose point is on the line G M is a mixture of G and M with no fine particles. It can be readily proved by geometry that if the altitude of the triangle is 1.00, the sum of the three perpendicular distances from any given point in the triangle to the three sides equals 1.00. Also, that any combination of G, M, and F is contained in the triangle or else on one of its sides. To use Mr. Feret's language, "any sand will be represented by a point in the triangle and by one alone, and, reciprocally, one granulometric composition of sand, and only one, will correspond to a given point on the interior or sides of the triangle." If the altitude of the triangle

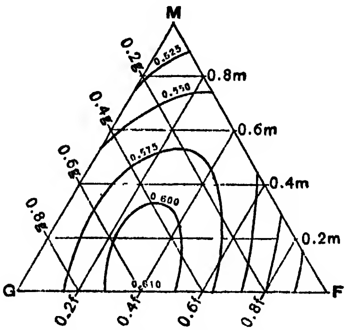


FIG. 51.—Absolute Volumes of Sand per Unit Volume of Sand not Shaken. (*See p.* 147.)

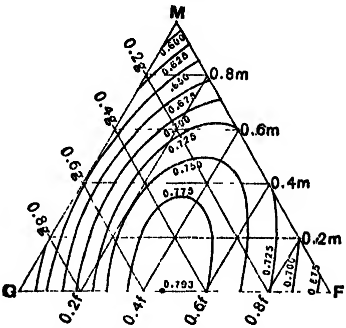


FIG. 52.—Absolute Volumes of Sand per Unit Volume of Sand Shaken to Refusal. (*See p.* 147.)

is considered 1.00, any point, *A*, in the triangle is readily plotted by locating it at perpendicular distances from each of the three sides corresponding to each component of its granulometric composition. For example, suppose that the granulometric composition of a sand, *A*, is g = 0.48, m = 0.35, f = 0.17. As the apex G represents a sand containing only coarse grains, and the line opposite to it, M F, all sands containing no coarse grains, the locus of a sand containing coarse grains (g = 0.48) will lie somewhere upon a line parallel to M F and at a distance 0.48 from M F. By similar reasoning it will also lie on a line parallel to G F and at a distance 0.35 from it. The intersection of these two lines is the locus of the sand *A*, and it will be seen that this intersection is at a perpendicular distance of 0.17 from the line M G (the side opposite F), which checks the plotting, since f = 0.17.

For comparing a special property of different sands, or of mortars com-

posed of different sands, each sand employed in the tests is plotted and labeled with its value, — which may be in units of strength, weight, or volume, — and "contour lines" are sketched in by the eye, as one would draw contours from elevations on a topographical drawing.

Any point on the same contour line represents a sand made up of the

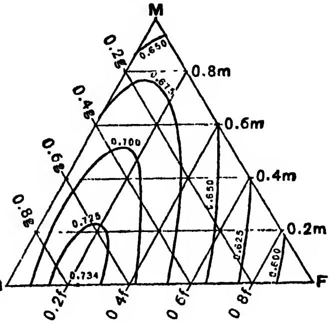


FIG. 53.—Absolute Volumes of Solid Materials (c+s) per Unit Volume of Fresh Mortar in Proportions 1:3 (by Weight). (*See p.* 147.)

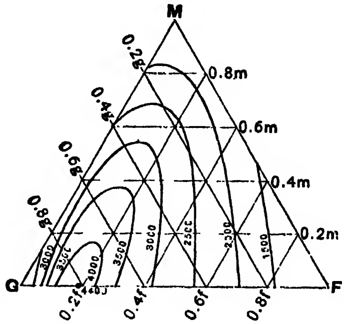


FIG. 54.—Compressive Strength in Pounds per Square Inch of 1:3 (by Weight) Mortars with Different Mixtures of Sand, after 9 Months in Air and 3 Months in Sea Water. (*See p.* 148.)

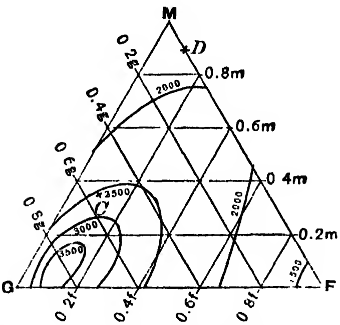


FIG. 55.—Compressive Strength in Pounds per Square Inch of Mortars with Various Mixtures of Sand, after One Year in Fresh Water. Proportions 100 lb. Portland Cement to 3.2 cu. ft. Mixed Sand. (*See p.* 148.)

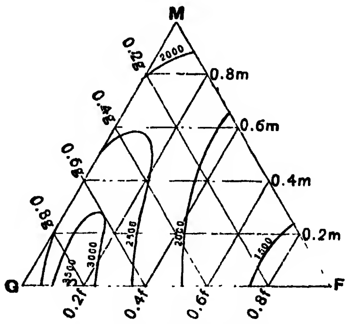


FIG. 56.—Compressive Strength in Pounds per Square Inch of Mortars with Various Mixtures of Sand, after One Year in Air. Proportions 100 lb. Portland Cement to 3.2 cu. ft. Mixed Sand. (*See p.* 148.)

different sizes, G, M, and F, in proportions corresponding to its perpendicular distances from the sides opposite each apex, but having the same strength, weight, volume, humidity, or whatever special function may be represented, as every other point on the same line.

Figs. 51 and 52, page 145, illustrate the use of the triangle for showing the volumes of sands composed of different sizes of grains. Any sand, for example, whose granulometric composition is represented by any point on the contour line labeled 0.575, in Fig. 51, has, when measured loose, 0.575 of its volume, or 57½%, of absolutely solid matter, or, taking the complement, 42½% of voids. In Fig. 51 it will be seen that the greatest solid volume of loose sand is obtained by mixing G and F in proportions 60% G and 40% F by weight. The amount of solid matter in this mixture of maximum density is 0.61 of the unit volume; in other words, the sand contains 39% voids. By interpolating between the contour lines we may see that a sand consisting of equal parts of the three sizes, which would be represented by a point at the geometrical center of the triangle, has about 0.597 solid matter, or 40.3% voids. In sands shaken to refusal, Fig. 52, the mixture of maximum density consists of sands G and F alone, in proportions about 55% G and 45% F, and the total solid matter, that is, the absolute volume of sand, in a unit volume of the shaken sand of maximum density, is 0.798, corresponding to 20.2% voids.

EFFECT OF SIZE OF SAND UPON THE STRENGTH OF MORTAR

As a matter of fact, the actual size of a sand, that is, the size of its grains, is subordinate, in its influence upon the strength and other qualities of a mortar, to the density of the mortar produced from it. One naturally would suppose that the densest sand, that is, the sand which contains, when dry, the fewest voids, when mixed with a given proportion of cement, would make, inevitably, the densest and therefore the strongest mortar. Such, however, is not necessarily the case, for the addition of both the cement and water change the mechanical composition. A mixture of fine sand and cement, for example, requires a larger percentage of water in gaging than a mixture of coarse sand and the same cement. The total volume of a mortar of plastic consistency is affected by the quantity of water used, as well as by the volumes of the dry materials. Hence, a mortar consisting of fine sand and cement will be less dense than one of coarse sand and the same cement, even though the fine and coarse sands, when weighed or measured dry, each contain the same proportions of solid matter and voids.

Fine sand has more grains in a unit measure and therefore a greater number of points of contact of the grains. The water forms a film (see Fig. 63, p. 175,) and separates the grains by surface tension.

The fact is graphically illustrated in Feret's triangle, Fig. 53, page 146,

in which the contour lines show the combined absolute volumes of the cement and sand in 1:3 mortar (proportioned by weight) made from sand of various compositions. It will be noticed that the point of maximum absolute volume, which is labeled 0.734, is much farther to the left than in Figs. 51 and 52, showing that for a mortar of maximum density, a sand is required containing more large particles, G, in proportion to the fine particles, F, than for maximum density with the same sand in its dry state.

From such experiments Mr. Feret* derives the law that:

> The plastic mortars, which, per unit of volume, contain the greatest absolute volume of solid materials (c + s), are those in which there are no medium grains, and in which coarse grains are found in a proportion double to that of fine grains, cement included.

Figs. 54, 55, and 56, page 146, show the strength in compression, converted to pounds per square inch, of mortars made from various mixtures of the three sizes of sand.

Comparing these with Fig. 53 it will be seen that the curves of strength follow the same general direction as the curves of density. This is in conformity with the general laws stated at the commencement of the chapter and with the principles upon which Feret's formula (page 142) is based.

There is one point which must be noticed when studying these and other similar triangles of Feret, namely, that his results, as shown by the curves on his triangles, apply exactly only to sands and cements, and not to mixtures of sand and coarse stone. In all the triangles, sands for maximum density are composed of a mixture of fine and coarse grains with no medium grains. It is shown on page 172 that a denser mixture can be obtained with stone and sand and cement, that is, with three sizes of materials, than with sand and cement, and it is consequently probable that Feret could have obtained greater densities by making the size of G larger (that is, employing for G gravel or broken stone) and the size of F smaller, and that with this arrangement a portion of the medium grains would have been absolutely necessary to obtain the maximum density. In this connection, however, it must be remembered that Feret's experiments were intended to cover, as far as possible, practical combinations of sizes of sand for mortar. It is noticeable, even with the sizes of sand which he uses, that the curves in Fig. 53 run sharply upward, and that mortars from mixtures of three sizes of sand are therefore very nearly as dense and strong as those made from two sizes. Furthermore, when the three sizes

*Annales des Ponts et Chaussées, 1896, II, p. 182.

G, M, and F are mixed together, a graded mixture is formed in which there are particles ranging from 0.2 inch down to fine dust.

Experiments indicate, as stated on page 206, that sand for concrete requires for best results more fine material than mortar sand.

TESTS OF DENSITY AND STRENGTH OF MORTARS OF COARSE VS. FINE SAND

The application of Mr. Feret's tests is shown in the table on pages 136 and 137, and to illustrate its practical use in comparing the quality of different sands the following table is presented, giving the density and strength of three natural bank sands as tested in the laboratory of one of the authors.*

Compressive Strength and Elementary Volumetric Composition of 2-inch Cubes of Portland Cement and Bank Sand

By Sanford E. Thompson

Sand	Proportions by Weight	Proportions by Volume (nominal)	Percentages Passing Sand Sieves: 1" Sieve	No. 8 Sieve	No. 20 Sieve	No. 50 Sieve	No. 200 Sieve	Elementary Volumes: Cement c	Sand s	Density $c+s$	$\left(\frac{c}{1-s}\right)^2$	Actual Average Compressive Strength, Age 7 days	Estimated Compressive Strength at 6 months, K=28,000
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Coarse	1 : 2.6	1 : 3	100	84	62	28	3	0.171	0.518	0.689	0.126	715	3530
Fine	1 : 2.6	1 : 3	100	84	100	77	6	0.154	0.466	0.620	0.083	405	2320
Very Fine .	1 : 2.6	1 : 3	100	84	100	92	27	0.149	0.451	0.600	0.074	330	2070

PRACTICAL APPLICATIONS OF THE LAWS OF DENSITY

It is probable that many who read this chapter will question the practical use of it all. Sand from the same bank usually varies largely in different places, and even when sands of a uniform character are to be obtained, it is considered impracticable to mix two or more sizes on account of the expense involved. In other cases, only one quality of sand is obtainable, and consequently there is no opportunity for choice.

In answer to such critics, we outline below several conditions under which the investigation of the physical properties of the sand is not only interesting but essential from the standpoint either of quality or of maximum economy.

(*a*) The variation of the sand in different portions of the same bank may be utilized by requiring the contractor to mix two sizes without exact

* From paper by Sanford E. Thompson on "Sand for Mortar and Concrete," Bulletin No. 3, Association American Portland Cement Manufacturers, 1906.

measurement, so that the material as delivered shall contain not less than a certain percentage of sand coarse enough to be retained on a certain sieve.

(*b*) If two sands are available, a study of their physical characteristics will determine which is better suited to the work in hand as *the sand which produces the smallest volume of plastic mortar, when mixed with cement in the required proportions by dry weight, furnishes the strongest and least permeable mortar.*

(*c*) A good sand brought from a distance at a high price may be more economical than a poor sand from a neighboring bank.

(*d*) The relative value of crusher dust or of sand in a given locality may be determined by comparing their densities or the densities of mortars made from them.

(*e*) Frequently, a mixture of a fine and coarse sand, or of sand and crusher dust, proportioned according to their relative granulometric compositions or analyses, may be shown to produce a better mortar than either material alone.

(*f*) To produce impermeable mortar or concrete, it may be economical to screen a mixed gravelly sand into different sizes, and remix these in proportions which will produce a mortar of greater density.

(*g*) The value of "sand cements" for use in mortar and concrete under certain conditions may be made evident.

The use of mixed sand, as described in (*a*), was adopted by Mr. Thomas F. Richardson, Engineer, for the 1 : 2 Natural cement mortar employed in the stone masonry of the Wachusett dam of the Massachusetts Metropolitan Water Works, after an exhaustive study of the comparative tensile strength and permeability of mortars made with different sands. He required the contractors to furnish sand so coarse that at least 50% would be retained on a sieve having 30 meshes per linear inch. The sand was excavated by scrapers, and the condition was readily complied with, whenever the sand in one section was shown by samples to be running too fine, by taking alternate scraper loads of coarse sand from another place in the bank.

Mixed or graded sands are specially advantageous when concrete is made at a central plant such as a block manufactory. By using graded screenings, instead of the fine stone as it came from the crusher, and by slightly increasing the size of the coarse aggregate, Mr. Thompson obtained a strength two and one-half times as great with the same proportions of cement and, on the other hand, maintained equal strength with 40% less cement.

Comparative Tests of Different Sands. One of the most important applications of the laws of density is in the comparison of different sands. Void determinations of sand are valueless because of variations in moisture and compactness, but if equal dry weights of each of the sands to be compared are mixed with the same cement in the proportions required on the work, and then gaged to plastic consistency as described on page 138a, the best sand, provided it does not contain vegetable loam or other impurities to affect it chemically, is that which produces the smallest volume of mortar.

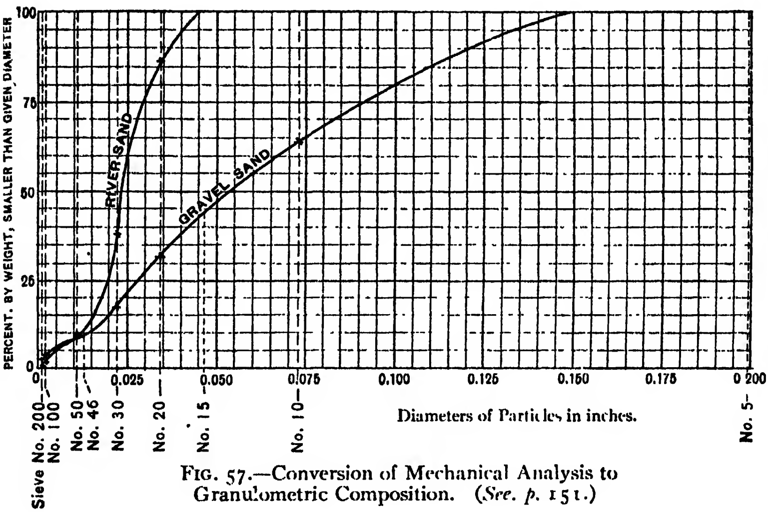


FIG. 57.—Conversion of Mechanical Analysis to Granulometric Composition. (*See. p.* 151.)

CONVERSION OF MECHANICAL ANALYSIS TO GRANULOMETRIC COMPOSITION

As an illustration of methods of contrasting two different sands and of making practical use of Feret's researches, we may compare tests made by Mr. R. L. Humphrey* in connection with the construction of the Pennsylvania Avenue Subway, Philadelphia. He found the tensile strength at the age of one year, of 1:3 mortar made with sand screened from gravel, to be about 50% stronger than that made with sand dredged from the Delaware River. The mechanical analyses† of the two sands are plotted by

*Transactions American Society of Civil Engineers, Vol. XLVIII, p. 558.

†Mechanical Analysis Curves are described in Chapter XI, page 190.

the authors in Fig. 57, page 151, from tables presented by Mr. Humphrey.

To transform these mechanical analysis curves to Feret's granulometric composition, we may draw on the diagram, ordinates corresponding to the sizes of sieves used by him, namely, No. 5, No. 15, and No. 46. (See p. 143.) From inspection of the curve it is evident that the granulometric composition of the gravel sand is g = 0.56, m = 0.35, f = 0.09, and of the river sand is g = 0.00, m = 0.89, f = 0.11. Plotting these granulometric compositions as *C* and *D* on Feret's triangle, Fig. 55, and interpolating between contours. we find the relative compressive strengths of mortars made from the two sands to be, after one year in fresh water, about as 1775 is to 2550, or as 1: 1.44, while Mr. Humphrey's ratio of tensile strength for the two mortars at the age of one year is as 304 is to 470, or as 1: 1.53. These ratios are remarkably similar when the differences in conditions are considered.

Numerous tests have been made in America* in proof of the general law that coarse sands are stronger than fine. Many experimenters have seemed to reach the result that coarse sand is stronger than mixed sand. In certain cases this is undoubtedly true, because of mixing the different sizes in wrong proportions, or because the mortar of coarse sand contains so large a proportion of cement that the voids are completely filled and the addition of fine sand decreases, instead of increasing, the density. Mortar, for example, as rich as 1 : 2 (*i.e.*, one part cement to two parts sand) of coarse sand is as strong as, and often stronger than, mortar of similar proportions made of almost any mixed sands, but with leaner mortars, a small admixture of from 10% to 25% of fine sand improves it. Natural sand, which in appearance is very coarse, almost invariably has a small percentage of very fine particles which, with the fine grains of cement, may assist, in the leaner mixture, in producing a dense mortar. The mechanical analysis curves of sand shown in Fig. 72, on page 200, are an illustration of the fine matter contained in all bank sands.

EFFECT OF QUANTITY OF WATER UPON THE STRENGTH OF MORTARS

Fine sands require in gaging a larger percentage of water than coarse sands, in order to produce a mortar of the same consistency. This, as discussed on page 147, exerts an indirect influence upon the strength.

The influence of different percentages of water upon the same cement and aggregate is largely physical, although a deficiency may affect the

*E. S. Wheeler in Report Chief of Engineers, U. S. A., 1895, p. 3013, A. S. Cooper in Journal Franklin Institute, Vol. CXL, p. 326, Ira O. Baker in Journal Western Society of Engineers, Vol. I, p. 73

permanent strength of a mortar, while an excess may for reasons given on page 271 injure the cement by dissolving a portion of it.

The effect of different proportions of water upon the ultimate strength (as suggested on p. 142) depends chiefly upon the density of the resulting mortar; the consistency which produces with a given weight of the same materials, the smallest volume, after setting, of Portland cement paste or mortar, gives the highest strength. Dry mixed mortars usually test higher than wet, — especially at short periods, as they set and harden more rapidly, — because they can be more densely compacted, but more uniform results in practice as well as in experiment, can be attained with plastic mixtures.

Tests by Mr. E. S. Larned,* a portion of which are shown in the table on page 154, illustrate the practical effect of different proportions of water upon the strength of neat cement pastes at various periods. It is noticeable that although the Natural cement mixed very wet finally attains a high strength, its very low strength up to 28 days shows the inadvisability of mixing Natural cement with an excess of water.

SAND VS. BROKEN STONE SCREENINGS

The relative strength of mortars made from sand and from screenings of broken stone or crusher dust has occasioned much discussion and dispute. It is probably dependent chiefly upon the relative density of the different mortars. Usually, a mortar from screenings will show higher tests, while occasionally mortar from sand will be superior, because of the difference in size or of the relative sizes of the particles or grains composing the two materials.

In some cases the form of the grain† and the mineralogic composition‡ may exert a certain influence, although tests show that these are usually of inferior importance to the mechanical or granulometric composition of the sand or screenings. It is possible that the fine dust or impalpable powder in certain stone may chemically react upon the cement.

On the other hand, screenings from a soft stone like slate, shale or soft limestone, may contain so much dust as to produce a poor mortar or concrete, for the same reason that a very fine sand results in a weak mortar.

*Proceedings American Society for Testing Materials, Vol. III, 1903, p. 401.

†Baumaterialienkunde, V Jahrgang (1900), p. 21, and Annales des Ponts et Chaussées, 1892, II, p. 124.

‡Mr. P. Alexandre found calcareous sands to give relatively high strength, and Mr. Feret obtained similar high results with marble.

Table Showing Strength of Cements Mixed Neat with Different Proportions of Water.

BY EDWARD S. LARNED. (*See p.* 153.)

Cement brand	Water per cent	Sieve test residue on			Wire minutes		Tensile strength					
		No. 50	No. 100	No. 180	Light	Heavy	24 hours	7 days	28 days	3 months	6 months	12 months
Portland A	13											
	14											
	15	0.15	5.4	21.2	12	207	371	655	875	941	720	787
	16	..	..	..	29	297	303	750	973	1008	735	816
	18	..	..	..	80	355	260	649	773	831	645	748
	20	..	..	..	142	402	233	500	693	716	621	676
	22	..	..	..	268	473	184	546	635	658	601	589
	24	..	..	..	327	912	167	539	649	644	629	755
Portland B	13	0.1	7.0	18.0	13	270	366	775	859	1067	892	832
	14	..	..	..	18	303	404	780	891	972	852	781
	15											
	16	..	..	..	22	327	363	602	725	844	806	723
	18	..	..	..	15	383	308	570	723	785	728	724
	20	..	..	..	56	703	225	590	718	760	674	636
	22	..	..	..	52	833	166	554	649	731	643	604
	24	..	..	..	188	918	42	510	691	695	632	574
Natural (Lehigh Valley)	23	0.1	4.6	10.2	13	32	212	251	252	311	275	356
	24											
	25	..	..	..	18	39	185	218	215	289	300	341
	27	..	..	..	21	42	150	188	220	257	272	314
	29	..	..	..	20	52	128	178	202	246	248	256
	31	..	..	..	21	57	112	173	199	224	259	309
	33	..	..	..	27	85	104	172	182	267	246	290
	35	..	..	..	38	137	93	121	178	260	286	319
	37	..	..	..	34	160	85	108	168	262	306	326
	39	..	..	..	67	233	85	119	202	252	371	400
Natural (Rosendale)	23	2.3	12.4	21.9	22	59	138	177	271	332	284	264
	24	..	..	..		78	125	141	264	342	309	310
	25	..	..	..	35	120	150	164	216	308	318	321
	27	..	..	..	49	143	117	116	194	305	345	272
	29	..	..	..	76	166	96	105	164	272	320	267
	31	..	..	..	117	212	72	72	159	270	371	225
	33	..	..	..	115	235	62	71	147	277	379	244
	35	..	..	..	127	400	50	64	112	245	318	315
	37	..	..	..	198	828	59	62	96	..	284	351
	39	..	..	..	260	1057	54	56	85	..	355	364

NOTE. — Results shown are the averages of six briquettes made.

Such dusty screenings are also especially bad for granolithic surfacing for sidewalks, and must not be used.

SHARPNESS OF SAND

In the past all specifications have called for clean, "sharp" sand in spite of the fact that in many parts of the country where sharp sand is not obtainable, sand with rounded grains is furnished and used with perfect satisfaction.

Comparative laboratory tests under conditions as nearly as possible identical uphold the practice of using sand with rounded grains. They indicate, as may be inferred from the previous discussion in this chapter, that the chief difference in natural sands is due to the size of the grains, and while the sharpness of grain may exert a certain influence it is of so much less importance than the size of the grain that *the requirement of sharpness for sand should be omitted from concrete specifications.*

Referring to columns (11) and (22) in the table on page 136, and to Fig. 49, page 141, it is evident that the difference in strength of nearly all the mortars made with the various sands is explained by the differing percentages of cement and densities without reference to the character of the grains. The only noticeable exception is with the artificial sand, M′, which consists of mixed sizes of crushed quartz. Mr. Feret‡ believes that this exception may be due to chemical action produced by the large quantity (⅑ its weight) of impalpable quartz. Sand N′, also crushed quartz, but containing none of this fine powder, produces a mortar similar in strength to like mortars of natural sand having rounded grains.

Other tests of Mr. Feret§ and comparative tests, in the United States, of mortar with crushed quartz and natural sands generally confirm the above conclusion. The variation in the shape of the grains of natural sands and crushed quartz is illustrated in Figs. 62, 64, and 65, page 175.

EFFECT OF NATURAL IMPURITIES IN THE SAND UPON THE STRENGTH OF MORTAR

A clause to the effect that a sand for mortar or concrete shall be "clean" is almost universally found in masonry specifications. The necessity for this requirement is often questioned by cement experimenters, because the results of tests of mortar to which percentages of loam or clay have been added, often give higher results than those of mortar made with cement and pure sand.

‡Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II.

§Annales des Ponts et Chaussées, 1892, II, p. 124.

As a matter of fact, it is impossible to make a general statement either to the effect that natural impurities in sand are beneficial or that they are detrimental. In some cases fine material may be of actual benefit, while in others the contrary is true.

The case is covered by three conditions: (1) the character of the impurities; (2) the coarseness of the sand; (3) the richness of the mortar.

Character of Impurities. If the fine material is of ordinary mineral composition, such as clay, the mortar is affected only mechanically, and the results depend upon the coarseness of the sand of which the fine material is a part and the richness of the mortar, as indicated in paragraphs which follow. One exception to this general rule is when the clay is in such condition as to "ball up" and stick together so as to remain in lumps in the finished concrete. On the other hand, a small percentage of clay well distributed may be valuable for making the concrete or mortar work smooth, and especially for increasing its water-tightness (see p. 343).

Vegetable or Organic Impurities. When the impurities are of an organic nature, like vegetable loam, they frequently have been found to prevent the mortar or concrete from hardening or to retard the hardening for so long a period as to make the sands entirely unfit for use. A very minute quantity of vegetable matter may produce injury, so small a percentage in fact that frequently a sand which has passed careful inspection fails in practice to set properly with any brand of cement; therefore a test is absolutely necessary for any sand which has a suspicion of organic matter.

The following tests of 1 : 3 mortar made with sand satisfactory in appearance, but which nevertheless caused the fall of a concrete building, are given

Effect of Vegetable Impurities in Sand

By Sanford E. Thompson, 1908 See p. 154b.

Sand.	Tensile strength of 1:3 mortar at 7 days. Lb. per sq. inch	Tensile strength of 1:3 mortar at 28 days. Lb. per sq. inch.
A*	4	93
B†	43	114
B washed	129	201
W ‡	165	
Standard Ottawa	200	300

* Poorest portion of bank; reddish and dark in appearance.

† Average sand from bank which passed inspection.

‡ A medium good sand from another bank similar to B in appearance, mechanical analysis, and chemical composition except nearly free from vegetable impurity.

in the following table. They are averaged from different series and for convenience in comparison the results are all converted to the basis of standard sand mortar, considered as 200 pounds in 7 days and 300 pounds in 28 days. The mortars were stored in air to conform to the actual conditions. Comparative tests on mortars from the same sands stored in moist air and in water corroborated the results.

The cause of the failure was traced in the expert investigation, to vegetable impurities in the sand which had washed down into the bank from the soil above. The poorest sand, *A*, showed by mechanical analysis only 4% by weight of fine material passing a No. 100 sieve and 1.61% silt by washing, but this silt was found to contain nearly 30% of vegetable matter corresponding however to only 0.5% in the total sand. The vegetable matter appeared to coat the grains of sand so as to prevent adhesion of the cement and also retarded the setting.

Effect of Fine Material in Filling Voids. Lean mortars may be improved by small admixtures of pure clay or by substituting dirty for clean sand. provided it is free from vegetable matter, because the fine material increases the density. Rich mortars, on the other hand, do not require the addition of fine material, and it may be positively detrimental, because the cement furnishes all the fine material required for maximum density. This is illustrated in experiments by Mr. Griesenauer* in which an admixture of even 2 per cent of clay (based on the weight of the sand) slightly reduced the strength of 1 : 2 mortar, while 20% of clay, added to the 2 parts of sand, reduced the strength about 30%. In 1 : 3 mortar, on the other hand, the addition of 2% slightly increased the strength, and there was no appreciable injury up to 20% addition.

In experiments by Mr. E. S. Wheeler† clay reduced the strength of neat and 1 : 1 mortars, but improved leaner mixtures.

In this connection, of course, it must be borne in mind that if the sand is composed largely of fine material, the strength of the mortar is comparatively low, as indicated in preceding pages.

EFFECT OF MICA IN THE SAND UPON THE STRENGTH OF MORTAR

The effect of mica in screenings from stone of a micaceous nature has been the subject of considerable controversy. Tests by Mr. Feret‡ in France indicated that the presence of 2% of mica has but slight influence upon the tensile strength of mortar, but a greater one upon its compressive

* *Engineering News*, April 28, 1904, p. 413.
† Report Chief of Engineers, U. S. A., 1895, p. 3004, and 1896, p. 2827.
‡ Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II.

strength. More recent tests by Mr. W. N. Willis* in 1907 on mortars made with standard Ottawa sand into which mica was introduced are illustrated in Fig. 57a. He found that the presence of mica increased the voids and decreased the strength. The sand used in tests, loosely shaken, contained 37% voids, but as mica was added, the voids increased rapidly until with 20% mica the voids were 67% with a corresponding decrease in weight, and three times the amount of water was required for mixing.

It is thus evident that the reduction in strength was largely due to the decrease in density and not entirely caused by the slippery character of the grains. In crushed stone screenings it is probable that the effect of the same percentage of mica in the natural state would be less marked.

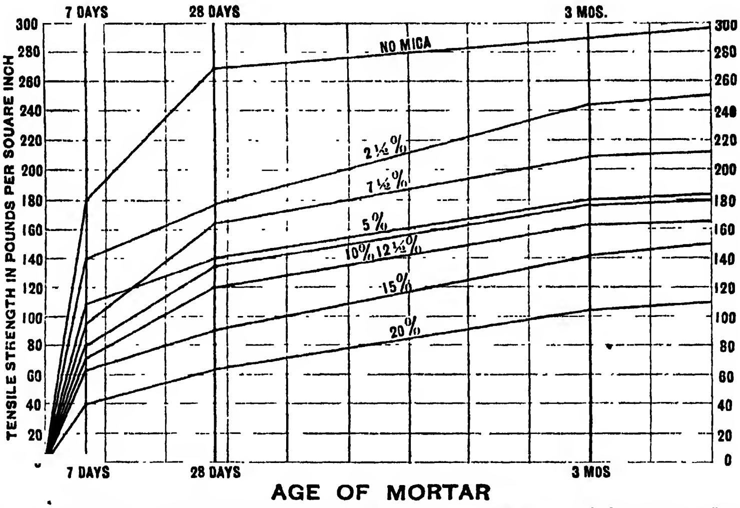


Fig. 57a.—Effect of the Addition of Mica upon 1 : 3 Mortar of Standard Sand. By W. N. Willis. (*See p.* 154d.)

Black mica, which has a different crystalline form, is not injurious to mortar.

EFFECT OF LIME UPON THE STRENGTH OF MORTAR

As a principal constituent of mortar in masonry construction, lime is inferior to cement in durability and strength. However, not only because of its relative cheapness, but also because a small addition of slaked or hydrated lime may increase the density of the mortar and cause it to work easier under the trowel, a limited quantity often can be used to advantage in mortar which is to be subjected to high loading.

* *Cement Age*, Mar. 1907, p. 172.

For concrete, lime has been suggested, as mentioned in Chapter XIX, on Water-tightness, as a suitable ingredient to fill the voids and thus render it more impermeable.

Although lime mixed with neat cement is apt to decrease its strength, in combination with sand for cement mortars, a small admixture of lime may add to the strength of the mortar. The questions as to whether lime is beneficial, and as to the amount which can be used, are determined by the character of the cement, the coarseness of the sand, and the proportions in which the two are mixed. The effect of lime in cement mortar or concrete is chiefly mechanical. In a porous mortar or concrete a small quantity of it assists in filling the voids, and if it is thoroughly slaked so as to contain no quicklime, its expansion need not be feared.

Since even a neat cement paste has 35% to 45% water plus air voids, the inference might be drawn that the addition of lime would increase its density, and thus that the lime would be valuable even in very rich mortars. However, it seems to be practically impossible, except under high pressure, to replace the water which occupies the voids in neat cement paste with lime or any other fine powder. But it is evident that a lean mortar, such as a 1:4, or even a 1:3, should be improved by the addition of lime, and that this is true is illustrated in the following tests by Mr. E. S. Wheeler.* In these experiments the addition of 10% of lime — based on the weight of the cement — increases the strength of 1:3 mortar, and as shown by item (3) in the table, a 1:3⅓ mortar with 10% of lime is stronger than a 1:3 mortar with no lime. Items (4) and (5) illustrate the reduction in

Effect of Lime Paste upon the Strength of Portland Cement Mortar.
BY E. S. WHEELER. (*See* *p.* 155.)

Item	Proportions cement plus lime to sand by weight	Proportions cement to sand by weight	Cement	Lime†	Sand	Average Tensile Strength.	
						at 28 dys.	at 3 mos
	parts	parts	grams	grams	grams	lb. per sq. in.	lb. per sq. in.
(1)	1:3	1:3	200	0	600	201	236
(2)	1:2¾	1:3	200	20	600	242	265
(3)	1:3	1:3⅓	180	20	600	238	264
(4)	1:3	1:4	150	50	600	168	171
(5)	1:3	1:6	100	100	600	57	70

*Report Chief of Engineers, U. S. A., 1896, p. 2823.

†The weight of the lime paste was 2.7 times the weights in this column.

strength when the lime becomes more nearly a principal ingredient. Each value is an average of five briquettes**

With another brand of cement and sand of different coarseness the relative quantity of lime to produce similar results will differ, but the general principle will still hold. In determining the amount of lime to add without decreasing the strength of a certain mortar, tests should be made with the materials to be employed.

In scientific experiments by Mr. Feret* the maximum strength of 1:4 mortar of Portland cement and sand from Saint Malo† was reached with an addition of 4% or 5% by weight of hydrated lime powder. As the mortar became richer, the lime had less effect, until at proportions 1:2, the addition of lime reduced the density, and at proportions 1:1½ the strength was also lowered.

A larger number of bricks can be laid in a given time with mortar containing lime than with a lean cement mortar because the lime fills the pores in the mortar so that it spreads more readily without crumbling and adheres better to the bricks in "buttering" them.

Unslaked Lime. Unslaked lime mixed with cement either for mortar or concrete is liable to produce expansion in the masonry and it is therefore never permissible to use it under any circumstances. Builders recognize that lime, putty, or paste is much improved by standing for several days, or, better, for months, before being used, because all the small lumps are thus slaked. This thorough slaking is especially necessary when lime is to be used, even as a very small ingredient, in important concrete and masonry construction; an admixture of even 2% of ground quicklime may seriously reduce the strength of the mortar.‡

Weight and Volume of Lime. In proportioning lime to cement, the method of measurement must be clearly stated. The volume of common lime or quicklime increases in slaking to about 2½ times its volume measured loose in the lime cask, the exact increase varying with the chemical composition and the purity of the lime. The weight of lime paste is about 2½ times the weight of the same lime before slaking. Hydrated lime powder also occupies more volume than quicklime from which it is made.

GROUND TERRA-COTTA OR BRICK AS A SUBSTITUTE FOR SAND

Experiments by Mr. E. S. Wheeler§ indicated that for a mortar of light weight terra-cotta may be ground and used instead of sand. Tests with

*Chimie Appliquée, 1897, p. 481.

†See p. 137.

‡Report Chief of Engineers, U. S. A., 1895, p. 2999.

§Report Chief of Engineers, U. S. A., 1896, p. 2866.

**See tests by Dr. E. W. Lazell, Transactions American Society for Testing Materials, Vol. VIII, 1908, p. 418.

both Portland and Natural cement mixed with the ground terra-cotta in various proportions gave at the end of three months tensile strengths which are not appreciably different from the strengths obtained with standard crushed quartz. Red brick pulverized* may also be used for the same purpose with good results.

EFFECT OF REGAGING MORTAR AND CONCRETE

Engineers have frequently specified and insisted that concrete or mortar be used immediately, that is, within one hour or one-half hour after it is gaged. As opposed to this requirement, tests by various experimenters indicate with singular unanimity that, at least for Portland cements, it is unnecessary, and that Portland cement concrete or mortar may remain for at least two hours in the mortar bed without deterioration. In fact, the ultimate tensile and compressive strength appears to be thus increased.

The results of such tests lead to the following conclusions:

(1) The tensile or compressive strength of Portland cement mortars or concretes is not lowered by standing two hours after mixing.
(2) Continuous gaging increases the ultimate strength.
(3) Regaging makes the cement slower setting.

Because of the Slow Setting and Hardening it is Scarcely ever Advisable in Practice to Permit the Regaging of Mortar or Concrete.

With Natural cements, however, the results of experiments are somewhat contradictory. It is probable that some Natural cements are injured, and, therefore, if circumstances require delay in placing Natural cement mortar, the effect of such delay should be determined by tests upon the brand to be used.

Mr. E. Candlot (see page 124) states that the adhesive quality of cement mortar is reduced by regaging.

Extended tests to determine the effect of regaging neat cements and mortars have been made by Mr. P. Alexandre† and Mr. E. Candlot‡ in France, by Mr. Henry Faija§ in England, by Mr. James E. Howard¶ at the Watertown Arsenal, U. S. A., and by Mr. Thomas F. Richardson at the Wachusett Dam, Massachusetts.

Mr. Richardson in the course of his experiments made a batch of 1:2 mortar from each cement, cut it into two portions and, leaving half of it in

*Report Chief of Engineers, U, S. A., 1896, p. 2830.

†Annales des Ponts et Chaussées, 1890, II, p. 340.

‡Candlot's Ciments et Chaux Hydrauliques, 1898, p. 355.

§Butler's Portland Cement, 1899, p. 307.

¶Tests of Metals, U. S. A., 1901, p. 497.

the mortar box, had the other half worked continuously. At various periods ranging from seven minutes to two hours, samples were taken from each portion, and made into tensile briquettes. Several brands of American and English Portland cements, both slow and quick-setting, and several brands of Natural cement having different periods of set, were tested. Referring to the results Mr. Richardson states:*

For the quicker setting cements there was a considerable falling off in strength in the briquettes broken seven days after being mixed, and a somewhat less falling off for those broken twenty-eight days after mixing; but at the age of six months all the mortars which had been allowed to stand, or which were worked continuously for one and one-half and two hours, showed a considerable gain in tensile strength.

A typical series of tests with Rosendale cement, which attained its initial set in forty minutes and its final set in ninety minutes, and coarse sand (passing a No. 8 and retained on a No. 30 sieve) is presented in the following table:

Effect of Regaging upon the Tensile Strength of 1:2 Natural (Rosendale) Cement Mortar. (See p. 158.)

BY THOMAS F. RICHARDSON.

Age	Periods of Sampling.				
	Immediately	After one hour		After two hours	
		Worked	Not Worked	Worked	Not Worked
	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.
7 days..........	27	23	21	19	15
28 days..........	22	34	27	32	29
3 months........	120	155	141	192	150
6 months........	163	223	191	225	213

As a result of his tests, Mr. Richardson allowed the contractor, when necessary, to use the mortar on the dam up to two hours after being mixed. This was often a great convenience because of the distance of the mortar-mixing machine from the dam.

Mr. Howard at the Watertown Arsenal took samples of neat Portland

*Personal correspondence.

cement after longer periods of setting, in some cases up to one hundred and two hours. In general, his specimens showed at the age of one month no appreciable difference, whether they were taken when first gaged or at four, or in some cases eight, hours after gaging. The strength of specimens taken after longer periods of standing was found at the age of one month to be lower. Natural cements showed an immediate falling off, due to regaging, on the thirty days' tests, but the tests were not extended beyond this age.

The Setting of Regaged Mortars. The experiments of Mr. Candlot were made chiefly upon mortars which had attained their final set, as determined by the pressure of the thumb. These mortars, after regaging, set much more slowly than normally gaged mortars, and he states that the set occurred at approximately the same time with all cements. "Thus, whether a mortar originally sets in ten minutes or three hours, when regaged it requires, in either case, about eight to ten hours." He concludes from this action that, in Portland cements, aluminate of lime, which plays an important part in the setting, has no action on the hardening.

Consequently regaging should have little influence upon siliceous products, while it would be expected to seriously affect aluminous cements. This is the effect in practice, for limes and Portland cements can be regaged without bad results, while the strength of Natural Vassy cement is considerably lowered by regaging.*

Effect of Regaging upon Adhesion. Mr. Candlot* found that mortars which had set several hours before molding, although usually showing as great compressive or tensile strength as normal mortars, gave much lower strength in adhesion, the reduction in strength being often 50%. (See p. 124.)

TESTS OF SAND FOR MORTAR AND CONCRETE

Since it is frequently impossible even for the most expert engineer to determine positively whether or not sand is fit to use for mortar and concrete,† it should always be tested for important structures. The experience of one of the authors during the last few years in the investigation of failures of concrete structures leads to the conclusion that unless the sand is from a bank of known quality **it is even more necessary to test the sand than to test the cement.**

The test recommended by the Joint Committee on Concrete and Reinforced Concrete in 1909 is as follows:

Mortars composed of one part Portland cement and three parts fine

* Candlot's Ciments et Chaux Hydrauliques, 1898, pp. 358 and 360.
†See p. 154b.

aggregate, by weight, when made into briquets should show a tensile strength of at least 70 per cent of the strength of 1 : 3 mortar of the same consistency made with the same cement and standard Ottawa sand. To avoid the removal of any coating on the grains which may affect the strength, bank sands should not be dried before being made into mortar but should contain natural moisture. The percentage of moisture may be determined upon a separate sample for correcting weight. From 10 to 40 per cent more water may be required in mixing bank or artificial sands than for standard Ottawa sand to produce the same consistency.

Sieves for Testing Sand. Since the relative strength of sand mortars, which are free from organic or other impurities is governed by the sizes and relative sizes of the grains, mechanical analysis tests are recommended by the Reinforced Concrete Committee of the National Association of Cement Users, 1909, as frequently of great value in selecting a sand.

The relative strength of mortars from different sands is largely affected by the size of the grains. A coarse sand gives a stronger mortar than a fine one, and generally a gradation of grains from fine to coarse is advantageous. If a sand is so fine that more than 10 per cent of the total dry weight passes a No. 100 sieve, that is, a sieve having 100 meshes to the linear inch, or if more than 35 per cent of the total dry weight passes a sieve having 50 meshes per linear inch, it should be rejected or used with a large excess of cement.

For the purpose of comparing the quality of different sands a test of the mechanical analysis or granulometric composition is recommended, although this should not be substituted for the strength test. The percentages of the total weight passing each sieve should be recorded. For this test the following sieves are recommended:*

0.250 inch diameter holes.†

No.	8 mesh holes		0.0955 inch width			No. 23 wire	
No.	20	"	"	0.0335	"	"	No. 28 "
No.	50	"	"	0.0110	"	"	No. 35 "
No.	100	"	"	0.0055	"	"	No. 40 "

The effect of mechanical analysis or granulometric composition upon the strength of mortar is illustrated in table, page 159b. By this table the relative strength of different sands may be approximately estimated.

Washing Test for Organic Impurities. To determine the percentage of organic impurities, the silt can be removed from the sand by placing it in a large bottle and washing it with several waters. The wash water is evaporated, and the residue is screened through a No. 100 mesh sieve to remove coarse particles which do not affect the strength. The silt passing

* Sheet brass perforated with round holes passes the material more quickly than square holes. Round holes corresponding to sieves No. 8, 20 and 50 respectively are approximately 0.125, 0.050, 0.020 inch diameter.

† A No. 4 sieve, having 4 meshes per linear inch, passes approximately the same size grains as a sieve with 0.25 diameter holes.

this sieve is weighed to obtain the percentage in the original sand, and then ignited in a platinum crucible to determine, after driving off the water, the percentage of combustible organic matter.

Although data on the subject is incomplete, tests by Mr. Thompson tend to indicate that if the silt in a sand has more than 10% organic matter, and at the same time if the organic matter amounts to over 0.1% of the total sand, the use of the sand may be dangerous.*

Microscopical Examination of Sand. An examination of grains of dirty sand with a microscope will frequently show a crust of organic matter on the grains which is not readily brushed off.

Chemical Composition of Sand. A sand found by chemical test to contain a large per cent, say, 95 per cent, of silica is apt to be of excellent quality for mortar. However, this is by no means a sure test or a necessary test, since sands are frequently found with as low as 75% of silica which make first-class mortar or concrete.

Tests by New York Board of Water Supply of 1 : 3 Mortar Made With Sands of Different Mechanical Analysis. (*See p.* 159a)

Percentages Passing Sieves.				Tensile Test. Lb. per sq. in.		Compression Test. Lb. per sq. in.	
No. 4.	No. 8.	No. 50.	No. 100.	7 days.	90 days.	7 days.	90 days.
100	70	12	5	213	613	2690	5640
100	86	21	6	263	412	1915	4660
100	99	26	2	177	325	905	2170
100	97	28	6	178	282	1075	1500
100	94	44	12	139	228	905	1130
100	100	52	14	122	170	275	810
100	100	94	48	80	149	330	490

EFFECT OF GAGING WITH SEA WATER

Mr. Alexandre† concludes from his own and other experiments which extend to a three-year period, that there is no essential difference in strength of mortars gaged with fresh and with sea water. Briquettes gaged with sea water, however, usually set very much slower than those gaged with fresh water.‡

Crushing tests made by the authors in 1909 on six 3-inch cubes of 1 : 2 : 4 concrete 14 months old, three of which were gaged with sea-water and three with fresh water, gave a result which indicated no appreciable difference between the two; the specimens gaged with sea-water averaging 4070 lb. per sq. in. and the fresh water cubes 3870 lb. per sq. in.

* See "Impurities in Sand for Concrete" by Sanford E. Thompson, Transactions American Society of Civil Engineers, 1909.

†Annales des Ponts et Chaussées, 1890, II, p. 332.

‡Alexandre and Feret in Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol IV, p. 111.

CHAPTER X

VOIDS AND OTHER CHARACTERISTICS OF CONCRETE AGGREGATES

In this chapter are given tables of the specific gravities and voids of different materials, and the method of determining them, also laws relating to the voids in concrete aggregates, and the effect of compacting such materials.

Laws of Volumes and Voids. The most important of these general laws relating to volumes of different materials, and to their voids, may be stated as follows:

(1) A mass of equal spheres, if symmetrically piled in the theoretically most compact manner, would have 26% voids whatever the size of the spheres, but by experiment it is found that it is practically impossible to get below 44% voids. (See p. 168.)

(2) If a dry material having grains of uniform shape be separated by screens into grains of uniform dimensions, the separated sizes (except when finer than will pass a No. 74 screen) will contain approximately equal percentages of voids; in other words, a dry substance consisting of large particles, all of similar size and shape, will contain practically the same percentage of voids as a substance having grains of the same shape but of uniformly smaller size. (See p. 170.)

(3) In any material the largest percentage of voids occurs with grains of uniform size, and the smallest percentage of voids with a mixture of sizes so graded that the voids of each size are filled with the largest particles that will enter them. (See p. 171.)

(4) An aggregate consisting of a mixture of coarse stones and sand has greater density — that is, contains a smaller percentage of voids — than the sand alone. (See p. 172.)

(5) By Fuller and Thompson's experiments, perfect gradation of sizes of the aggregate appears to occur when the percentages of the mixed aggregate passing different sizes of sieves are defined by a curve which approaches a combination of an ellipse and straight line. (See Chap. XI, p. 201.)

(6) Materials with round grains, such as gravel, contain fewer voids than materials with angular grains, such as broken stone, even though

the particles in both may have passed through and been caught by the same screens. (See p. 174.)

(7) The mixture of a small amount of water with dry sand increases its bulk. In the case of most bank sands the maximum volume — and hence the smallest amount of solid matter per unit of volume, that is, the largest percentage of absolute voids — being reached with from 5% to 8% of water. (See p. 176.)

CLASSIFICATION OF BROKEN STONE.*

Rocks which are commonly employed for concrete or for road making are commercially classified as (*a*) traps, (*b*) granites, (*c*) limestones, (*d*) conglomerates, and (*e*) sandstones.

The trade term "trap" includes dark green to black, heavy, close textured, tough rocks of igneous origin, thus covering a variety of rock whose mineralogical names are diabase, norite, gabbro, etc. As shown in the table below, the traps usually range in specific gravity from 2.80 to 3.05.

Granites, commercially so called, include the lighter colored, less dense rock, such as not only true granite, but syenite, diorite, gneiss, mica schist, and several other groups. Their specific gravities range from about 2.65 to 2.85, averaging close to 2.70. Although, as road metal, the traps are usually far superior to granites, for concrete there appears to be no great difference in the value of the two classes. The distinction, however, is worth keeping because a concrete stone is often purchased from road metal quarries.

Limestones of normal type range in specific gravity from 2.47 to 2.76, averaging about 2.60, although the very soft stones, which are not suitable for high class concrete, may fall below 2.0.

Conglomerate, or pudding stone as it is often termed, is essentially a very coarse grained sandstone, ranging in specific gravity from 2.50 to 2.80. It makes a good concrete aggregate.

Sandstones of compact texture, such as the Potsdam and Medina sandstones, and the Hudson River bluestone, may run as high in specific gravity as 2.75, while the looser textured, more porous sandstones may fall as low as 2.10, a fair average being about 2.40.

Shale and slate make poor concrete aggregates, because their crushing and shearing strength is low.

*The authors are indebted to Mr. Edwin C. Eckel for the material under this heading, which has been especially prepared by him for this Treatise.

Specific Gravity of Stone from Different Localities.

COMPILED BY EDWIN C. ECKEL.

TRAP.

Locality.	Specific Gravity.
MASSACHUSETTS	
Boston	2.78
MINNESOTA	
Duluth	3.00
Duluth	2.80
Taylors Falls	3.00
NEW JERSEY	
Jersey City Heights	3.03
Little Falls	2.99
NEW YORK	
Staten Island	2.86

GRANITE.

Locality.	Specific Gravity
CALIFORNIA	
Penrhyn	2.77
Rocklin	2.68
CONNECTICUT	
Greenwich	2.84
New London	2.66
GEORGIA	
Stone Mt.	2.69
MAINE	
Hallowell	2.66
MARYLAND	
Port Deposit	2.72
MASSACHUSETTS	
Quincy	2.70
NEW HAMPSHIRE	
Keene	2.66
NEW YORK	
Ausable Forks	2.76
RHODE ISLAND	
Westerly	2.67
VERMONT	
Barre	2.65
WISCONSIN	
Amberg	2.71
Montello	2.64

LIMESTONE.

Locality.	Specific Gravity.
ILLINOIS	
Joliet	2.56
Lemont	2.51
Quincy	2.57
INDIANA	
Bedford	2.48
Salem	2.51
MINNESOTA	
Frontenac	2.63
Winona	2.67
NEW YORK	
Canajoharie	2.68
Glens Falls	2.70
Kingston	2.69
Prospect	2.72
Sandy Hill	2.76
Williamsville	2.71
Soft Limestone	
FRANCE	
Caen	1.84

SANDSTONE.

Locality.	Specific Gravity.
COLORADO	
Ft. Collins	2.43
Trinidad	2.34
CONNECTICUT	
Portland[1]	2.64
MASSACHUSETTS	
Longmeadow[1]	2.48
MINNESOTA	
Fond du Lac	2.24
NEW JERSEY	
Belleville[1]	2.26
NEW YORK	
Albion[2]	2.60
Medina[2]	2.41
Potsdam[3]	2.60
Oxford[4]	2.71
Malden[5]	2.75
Oswego	2.42
OHIO	
Berea[6]	2.14
Cleveland	2.21
Massillon	2.11

[1]Brownstone.
[2]Medina sandstone.
[3]Potsdam sandstone.
[4]Bluestone.
[5]Hudson River Bluestone.
[6]Berea grit.

AVERAGE SPECIFIC GRAVITY OF SAND AND STONE

The specific gravity of a substance is the ratio of the weight of a given volume to the weight of the same volume of distilled water at a temperature of 4° Cent. (39° Fahr.). For ordinary tests of stone and sand, the water need not be distilled and may be at ordinary temperature.

A knowledge of the specific gravity of the particles of the sand and stone is important to the engineer as a ready means of determining the percentages of voids.

The uniformity in the specific gravity of different sands is very convenient for calculation. Different authorities who have tested large quantities of sand have reached almost identical conclusions as to the average specific gravity, and all state that it is practically a constant. Mr. Allen Hazen gives 2.65, Mr. William B. Fuller, 2.64, Mr. R. Feret in France states that "one may without appreciable error adopt an average specific gravity of 2.65 for siliceous sands,"* while Mr. E. Candlot gives limits of 2.60 to 2.68 for sands which are not porous.† The specific gravity of calcareous sands averages about 2.69 by absolute determination, or about 2.55 if measured by the total volume of the particles having their pores filled with air.

Gravels also have quite uniform specific gravity. According to Mr. A. E. Schutté, who has tested gravel from more than forty localities in the United States and Canada, an average value is 2.66.

The following table gives average values of various concrete aggregates. In every case, the specific gravity is the ratio of the weight of an absolutely solid unit volume of each material to the weight of a unit volume of water. Specific gravities of stone from various localities are given on page 162.

Average Specific Gravity of Various Aggregates. (*See p.* 163.)

Material.	Specific Gravity.	Weight of a solid cu. ft. of rock. lb.	Authority.
Sand	2.65	165	Allen Hazen
Gravel	2.66	165	A. E. Schutté
Conglomerate	2.6	162	Robert Spurr Weston
Granite..................	2.7	168	Edwin C. Eckel
Limestone	2.6	162	Edwin C. Eckel
Trap	2.9	180	Edwin C. Eckel
Slate	2.7	168	Tod's Tables‡
Sandstone	2.4	150	Edwin C. Eckel
Cinders (bituminous)	1.5	95	The authors

*Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II, p. 1591.

†Ciments et Chaux Hydrauliques, 1898, p. 246.

‡Encyclopedia Britannica.

METHOD OF DETERMINING SPECIFIC GRAVITY

The specific gravity of a sample of material is determined by dividing its weight by the weight of water which it displaces when immersed.

The size of sample necessary for the accurate determination of a sand or stone of fairly uniform texture depends chiefly upon the delicacy of the apparatus employed. If scales reading to grams, and measures reading to cubic centimeters, are employed, a sample of 250 grams should give accurate results to two decimal places. With scales reading to ¼ ounce, a sample of 4 lb. is necessary for similar accuracy. The water must be maintained at 68° Fahr. (20° Cent.).

The sample should be taken by the method of quartering described on page 398.

Before finding the specific gravity of siliceous sand, the sample should be dried in an oven at a temperature as high as 212° Fahr. (100° Cent.) until there is no further loss in weight. A porous stone, on the other hand, may be first moistened sufficiently to fill its pores, and then the surfaces of the particles dried by means of blotting paper. If this method is followed, the material should be in a similar condition when its voids are determined by the method given on page 165. The absolute specific gravity of the porous stone may be afterward found by drying in an oven and correcting for the moisture lost.

The apparent specific gravity of sand or stone may be determined with an apparatus consisting of scales reading to ¼ ounce or to 5 grams, and a tall glass vessel with a reference mark, such as a cylinder or a pharmacist's graduate. The method is as follows:

Make a mark at any convenient place on the neck of the vessel;
Fill the vessel with water at a temperature of 68° Fahr. (20° Cent.) up to this mark;
Take a known weight in grams or ounces of the material;
Pour material into vessel carefully, a few grains at a time, so that no bubbles of air are carried in with it;
Pour out the clear water displaced by the material (leaving water in the vessel up to the level of the mark), and weigh the water poured out.

Let

S = Weight of material placed in vessel.
W = Weight of water displaced.

Then

$$\text{Specific gravity of material} = \frac{S}{W} \qquad (1)$$

It is essential that the weight of water displaced be weighed to within $\pm 2\%$. If the scales are not sufficiently sensitive, more material must be taken and a larger vessel used. With balances sensitive to 1 gr. or $\frac{1}{16}$ oz. the displacement of more than 3 ounces of water is necessary.

METHOD OF DETERMINING VOIDS

The voids in sand, gravel, and broken stone may be obtained directly from the tables on pages 166 and 167. Special determinations may be made as described below.

The percentage of voids in sand or fine broken stone cannot be accurately obtained by the ordinary method of placing in a measure and pouring in water, because it is physically impossible to drive out all the air. There may be enough of this held to amount to 10% of the volume of the sand, and thus cause a corresponding error in the percentage of voids.

The voids in coarse stone containing no particles under ½-inch diameter may be determined by placing in a box or pail of known volume and pouring in water, but if the specific gravity is known, the method described below is simpler and more accurate.

The only apparatus required are scales of fair accuracy and an exact measure which contains not less than ½ cu. ft. If a cubic foot measure is not available a 10-quart pail will answer the purpose, although compactness of the sand is less easily adjusted because of the small diameter. Such a pail holds slightly over ½ cu. ft. and the exact measure is determined by weighing the pail, pouring in 31 lb. 2 oz. of water, and marking the level of the surface. The pail up to this mark contains ½ cu. ft. of any material.

The method of determining the voids is as follows:

Weigh the measure;

Fill the measure to the required level with the material in the state in which the percentage of voids is required, that is, loose, shaken, or packed;

Weigh, and deduct the weight of the measure, calling the net weight of a cubic foot of the material, S;

If the material consists of, or contains, sand or fine stone, correct for moisture by taking an exact weight, — about 10 lb., — drying in an oven at a temperature of at least 212° Fahr. (100° Cent.) until there is no further loss in weight, and after calculating the percentage of moisture in terms of the weight of the original moist sand or stone, express the percentage as a decimal, p.

Select the weight of a cubic foot of absolutely solid rock* from the table on page 163, and call it R.

$$\text{Per cent of absolute voids} = \left(1 - \frac{S - Sp}{R}\right)100 \qquad (3)$$

The air voids are determined, if desired, by deducting the volume of moisture (its weight divided by the weight of one cubic foot of water)

Percentages of Voids Corresponding to Different Weights per Cubic Foot of Sand, Gravel, and Broken Stone Containing Various Percentages of Moisture. (*See p.* 168.)

Weight of one cu. ft. of sand or gravel.†	Percentages of absolute voids in material containing moistures by weight.‡ 0%	2%	4%	6%	8%	Moisture by volume corresponding to 1% by weight.‡	Weight of one cu. ft. of sand or gravel.†	Percentages of absolute voids in material containing moistures by weight.‡ 0%	2%	4%	6%	8%	Moisture by volume corresponding to 1% by weight.‡
	%	%	%	%	%	%		%	%	%	%	%	%
70	57.6	58.4	59.3	60.1	61.0	1.1	98	40.6	41.8	43.0	44.2	45.3	1.6
75	54.5	55.4	56.4	57.3	58.2	1.2	99	40.0	41.2	42.4	43.6	44.8	1.6
80	51.5	52.5	53.4	54.4	55.4	1.3	100	39.4	40.6	41.8	43.0	44.2	1.6
81	50.9	51.9	52.9	53.9	54.8	1.3	101	38.8	40.0	41.2	42.5	43.7	1.6
82	50.3	51.3	52.3	53.3	54.3	1.3	102	38.2	39.4	40.7	41.9	43.1	1.6
83	49.7	50.7	51.7	52.7	53.7	1.3	103	37.6	38.8	40.1	41.3	42.5	1.6
84	49.1	50.1	51.1	52.2	53.2	1.4	104	37.0	38.2	39.5	40.8	42.0	1.7
85	48.5	49.5	50.6	51.6	52.6	1.4	105	36.4	37.6	38.9	40.2	41.4	1.7
86	47.9	48.9	50.0	51.0	52.0	1.4	106	35.8	37.0	38.3	39.6	40.9	1.7
87	47.3	48.3	49.4	50.4	51.5	1.4	107	35.2	36.4	37.7	39.0	40.3	1.7
88	46.7	47.7	48.8	49.9	50.9	1.4	108	34.6	35.9	37.2	38.5	39.7	1.7
89	46.1	47.1	48.2	49.3	50.4	1.4	109	33.9	35.3	36.6	37.9	39.2	1.7
90	45.5	46.5	47.6	48.7	49.8	1.4	110	33.3	34.7	36.0	37.3	38.7	1.8
91	44.8	45.9	47.0	48.2	49.2	1.5	115	30.3	31.7	33.1	34.5	35.9	1.8
92	44.2	45.4	46.5	47.6	48.7	1.5	120	27.3	28.7	30.2	31.6	33.1	1.9
93	43.6	44.8	45.9	47.0	48.1	1.5	125	24.2	25.8	27.3	28.8	30.3	2.0
94	43.0	44.2	45.3	46.5	47.6	1.5	130	21.2	22.8	24.4	25.9	27.5	2.1
95	42.4	43.6	44.7	45.9	47.0	1.5	135	18.2	19.8	21.4	23.1	24.7	2.2
96	41.8	43.0	44.1	45.3	46.4	1.5	140	15.2	16.8	18.5	20.2	21.9	2.2
97	41.2	42.4	43.6	44.7	45.9	1.6							

*The weight per cubic foot of a solid is the specific gravity of the rock multiplied by the weight of a cubic foot of water.

†Also applicable to broken stones such as granite, conglomerate, and limestone, whose specific gravity averages from 2.6 to 2.7. Table is based on specific gravity of 2.65.

‡The per cent. of absolute voids given in the columns include the space occupied by both the air and the moisture. To determine the per cent. of air space, multiply the figure in the last column, opposite the weight of sand under consideration, by the per cent. of moisture by weight, and deduct result from the per cent. already found.

in a unit volume of the sand or stone, from the total voids. Expressed in percentages with notation same as above,

$$\text{Per cent. of air voids} = \text{Per cent. of absolute voids} - \frac{Sp}{62.3}\,100 \qquad (4)$$

Example. — Given a sand whose loose weight per cubic foot is found to be 92 lb. and its moisture 3% by weight. Find the percentage of voids in the loose sand.

Solution by formula. — Since from the example $S = 92$ and $p = 0.03$, and, from table on page 163, $R = 165$,

$$\text{Percentage of absolute voids} = \left(1 - \frac{92 - 0.03\,(92)}{165}\right)100$$

$$= 45.9\%$$

This percentage includes the space occupied by the moisture. The net percentage of voids occupied by air alone is the difference between the absolute voids and the percentage of moisture by volume. Moisture is $92 \times 0.03 = 2.76$ lb., or $\frac{2.76}{62.3} = 0.044$ cu. ft., corresponding to 4.4% voids by volume, hence air voids are $45.9\% - 4.4\% = 41.5\%$.

Percentages of Voids Corresponding to Different Weights per Cubic Foot of Dry Broken Stone of Various Specific Gravities. (*See p. 168.*)

Weight of one cu. ft. of dry broken stone.	PERCENTAGES OF ABSOLUTE VOIDS CORRESPONDING TO SPECIFIC GRAVITIES OF STONE OF					
	2.4*	2.5	2.6†	2.7‡	2.8	2.9§
	%	%	%	%	%	%
70	53.2	55.0	56.8	58.4	59.9	61.3
75	49.8	51.8	53.7	55.4	57.0	58.5
80	46.5	48.6	50.6	52.4	54.1	55.7
85	43.2	45.4	47.5	49.5	51.3	53.0
90	39.8	42.2	44.5	46.5	48.4	50.2
95	36.5	39.0	41.4	43.5	45.5	47.4
100	33.1	35.8	38.3	40.6	42.7	44.7
105	29.8	32.6	35.2	37.6	39.8	41.9
110	26.4	29.4	32.1	34.6	36.9	39.1
115	23.1	26.2	29.0	31.6	34.1	36.4
120	19.8	23.0	25.9	28.7	31.2	33.6
125	16.4	19.8	22.8	25.7	28.3	30.8
130	13.1	16.6	19.8	22.7	25.5	28.1
135	9.7	13.3	16.7	19.7	22.6	25.3
140	6.4	10.1	13.6	16.8	19.7	22.5

NOTE.—Average specific gravity of bituminous coal cinders may be taken as 1.5.

*Sandstone. ‡Granite and slates.

†Limestone and conglomerates. §Trap.

Solution by table (p. 166.) — Opposite 92 lb. per cu. ft., interpolating between 2% and 4% moisture, is 46.0% of absolute voids. From last column 3% by weight corresponds to 3% x 1.5 = 4.5% by volume. 46.0% — 4.5% = 41.5% air voids.

Tables of Voids. From the tables on pages 166 and 167, the voids in sand, gravel, and broken stone may thus be determined simply by weighing the material and finding the percentage of moisture contained in it, as above described. Since the percentage of moisture by volume is always greater than its percentage by weight, and the two are not proportional to each other, the final column is inserted in the first table for convenience in calculating the moisture by volume.

VOIDS AND DENSITY OF MIXTURES OF DIFFERENT SIZED MATERIALS

The term *density* as applied to mortar is defined on page 135. Similarly, in a dry material, such as a concrete aggregate, it is represented by the total volume of the solid particles entering into a unit volume of the aggregate. In dry materials the density is the complement of the voids, since a material which has, say, 40% voids will have a density of 0.60; but density is a more correct term to use than voids because it is applicable to concretes and mortars in which connection the term voids is somewhat ambiguous. The example on page **138a** illustrates the method of determining the density of a concrete or mortar.

The densities of dry aggregates of uniform specific gravity, or of mixtures in uniform proportions of materials with different specific gravities, are in direct proportion to their weights. For example, the densities of different dry sands may be compared by weight; or the densities of different mixtures of sand and broken trap in proportions, say, 2 parts sand to 4 parts trap may be compared by weight; but the density of sand and the density of trap screenings cannot be compared by weights unless the differing specific gravities are taken into account.

In the following discussion of the laws formulated on page 160, both the terms *density* and *voids* are used in relation to the dry materials.

Voids in Masses of Similar Sized Particles. (1) The fact that the percentage of voids in a mass of equal spheres symmetrically piled in the theoretically most compact manner is independent of the actual diameter is simply a geometrical proposition, evident without demonstration by inspection of Fig. 58.

In actual experiment it has been found that while the percentage of voids is uniform regardless of the size of the spheres, it is impossible to

pour spheres into a measure so that they will arrange themselves symmetrically, and the rather astonishing result has been reached by Mr. Fuller (see p. 185) that 44% is the smallest percentage of voids which can be obtained with equal perfect spheres, no matter what may be their actual diameters or the size of the receptacle.

The following simple demonstration,* which is of theoretical interest, proves that the percentage of voids in a mass of equal spheres symmetrically piled in the most compact manner is 26%, and that the radii (and consequently the diameters) of the two next smaller spheres which can

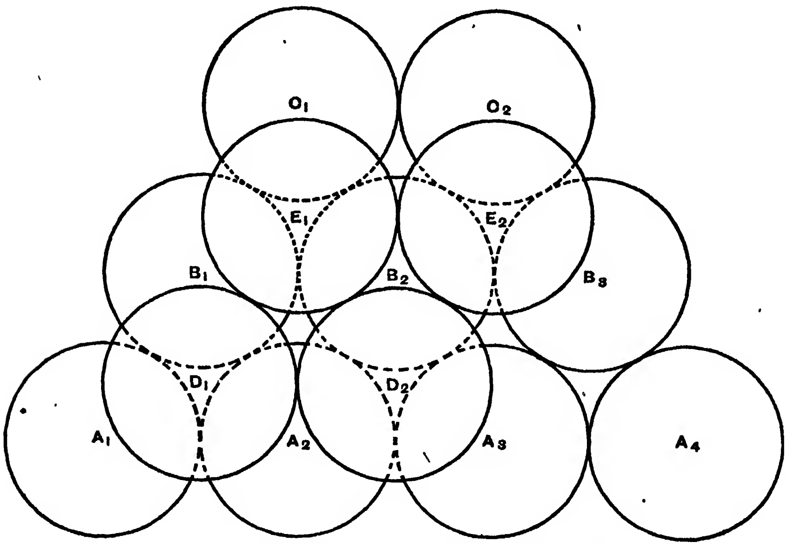


FIG. 58.—Spheres of Equal Size. (*See p.* 168.)

be inscribed between the larger ones are respectively 0.41 and 0.22 of the radius of the large spheres.

The circles in Fig. 58 represent a horizontal plan of two layers of spheres.

The centers A_1 A_2 B_1 D_1 form a regular tetrahedron.

Let edge be 2.

Altitude = difference between level of centers A, B, C, and level of centers D, E is $\frac{2}{3}\sqrt{6}$

Let number of spheres in a layer be *m*, number of layers *n*.

*For which the authors are indebted to Dr. Harry W. Tyler.

Volume of one sphere is $\frac{4\pi}{3}$

Volume of spheres in a layer, $\frac{4m\pi}{3}$

Volume of all spheres, $\frac{4mn\pi}{3}$ (approx.) $= V_1$

Cross-section of including space is $2\sqrt{3}\,m$ (approx.)

Volume of including space is $2\sqrt{3}\,m \times \frac{2}{3}\sqrt{6}\;n$ (approx.)

$= 4\sqrt{2}\,m\,n$ (approx.) $= V_2$

Ratio $\frac{V_1}{V_2} = \frac{4\,m\,n\,\pi}{3 \times 4\,m\,n\sqrt{2}} = \frac{\pi}{3\sqrt{2}} = 0.74$ (approx.) corresponding to about 26% voids.

Inscribed Spheres.

1. Sphere inscribed between spheres A_1 A_2 B_1 and D_1:
Distance from any vertex A_1 of tetrahedron to center is $\frac{1}{2}\sqrt{6}$
Radius of small sphere $= \frac{1}{2}\sqrt{6} - 1 = 0.22$ (approx.) or about $\frac{22}{100}$ of the radius of the large spheres.

2. Sphere inscribed between A_2 B_1 B_2 and D_1 D_2 E_1:
Distance from A_2 to E_1 is $2\sqrt{2}$
Radius of small sphere $= \sqrt{2} - 1 = 0.41$ (approx.) or about $\frac{41}{100}$ of the radius of the large spheres.

(2) The proposition that if a dry material such as sand, pebbles, or irregular broken stone, having grains of fairly uniform shapes, be separated by screens into grains of uniform dimensions, the separated sizes will contain approximately equal percentages of voids, is not so self-evident, but experiment proves that in portions of the same material screened to uniform sizes the percentages of voids will be substantially alike until very fine sizes are reached, such as will pass a No. 74 sieve; below this degree of fineness the particles are entangled by air. The authors have found by experiments given in the following table, that different lots of broken stone from the same quarry, each screened to uniform size, will contain substantially the same percentages of voids, but that lots of stone from different quarries screened to the same size may differ because of the structure of the rock. Published records usually show slight variations in the weight per cubic foot of different sized broken stone, but it is noticeable that some authorities give the heaviest weight,

which corresponds to the smallest percentage of voids, for the larger sizes, while others give the reverse. For example, Patton's Civil Engineering gives the smallest percentage of voids in the coarsest broken stone, while Butler's Portland Cement gives the smallest percentage in the finest stone. The variation in results is undoubtedly due to differences in methods of compacting and to the variations in the sizes of the stones of each lot.

Experiments by Mr. Feret in France, and Mr. Thomas F. Richardson in the United States, show that the percentages of voids in absolutely dry sand which has been screened to uniform size are almost identical. Mr. Feret, experimenting by shoveling dry sand loosely into a 50 liter (1.8 cu. ft.) box, — a measure large enough to eliminate errors of placing, — found that fine (F) medium (M) and coarse (G) sands each contained about 50%

Voids and Compression of Broken Trap and Gravel. (*See p.* 170.)

Size of Stone	Class of Stone	Crusher	Size of Particles	Voids in loose stone %	Compression by light ramming or shaking %	Voids in lightly rammed or shaken stone %	Compression by heavy ramming %	Voids in heavily rammed stone %	
No. 2	Hard Trap	Rotary	2½" to 1"	54.5			19.2	43.7	
No. 3		"	1" to ½"	54.5	14.3	46.0	20.5	42.8	
Nos. 2, 3, 4		"	2½" to dust*	43.0	14.5	35.7	30.8	30.6	
No. 2	Soft Trap	Jaw	2" to ¾"	51.2	11.0	44.6	17.8	40.0	Variation is due to trap breaking under rammer
No. 3		"	¾" to ⅜"	51.2	14.3	43.1	23.9	35.0	
	Gravel		2½" to ½"	36.5	12.5†	27.1	11.5†	28.2	

Loose stone is as thrown by a laborer into a measuring box or barrel.
Material rammed in 6-inch layers.

voids, while mixing the sizes, which are defined on page 142, in the best proportions reduced the voids to 34%.

Densest Mixture of Sand and Stone. (3) The fact that the densest mixture occurs with particles of different sizes is so evident as to require no proof, and this being recognized, it follows that the least density and hence the largest percentage of voids occurs when the grains are all of the same size. The converse of this proposition, that the smallest percentage of voids occurs in a mixture graded so that the voids of each size are filled with the largest particles which will enter them, is

*Mixed in proportions 44.4% No. 2, 33.3% No. 3, and 22.2% No. 4 (dust).

†Another gravel tested, compressed, 8.5% on shaking, and 11.2% on hard ramming.

illustrated in Figs. 59, 60, and 61, and is important in its application to the selection of materials for concrete.

(4) The fact that an aggregate consisting of a mixture of stones and sand has greater density, that is, contains fewer voids than the sand alone,

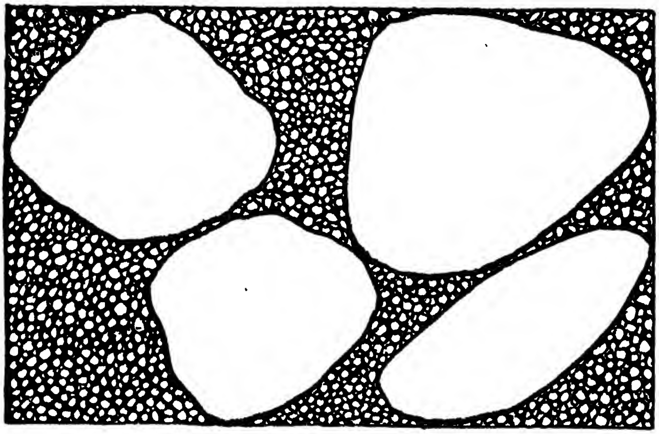


Fig. 59.—Large Stones with Voids filled with Sand. (*See p.* 172.)

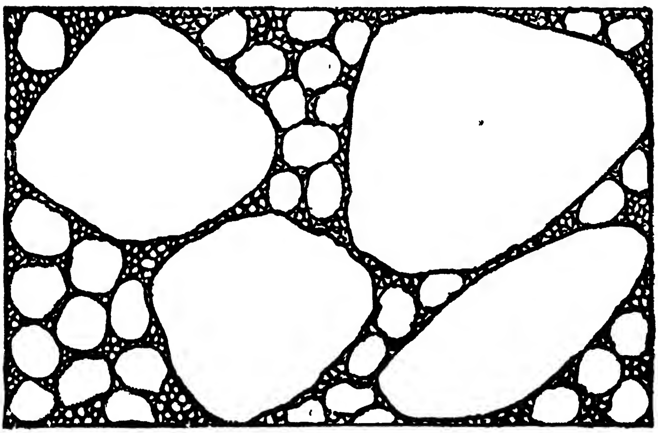


Fig. 60.—Large Stones with Voids filled with small Stones and Sand. (*See p.* 172.)

is illustrated by comparison of Figs. 59 and 61. The voids of the large stone in Fig. 59 are filled with sand, while the voids in the same large stone in Fig. 61 are filled with mixed sand and stone, and the mass of the mixture is evidently denser, that is, it contains more solid material. This

law relates directly to the difference between mortar and concrete. The substitution of stones for small masses of sand reduces the voids and consequently the quantity of cement required. Extending the principle to the fixing of proportions of sand and stone, it is evident that for maximum

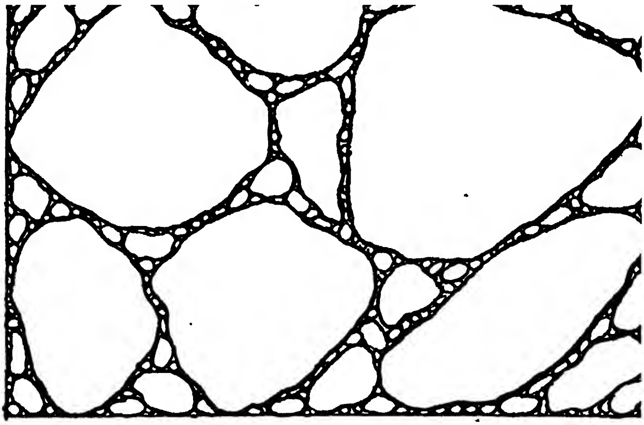


FIG. 61.—Large Stones, with Voids filled with medium sized Stones surrounded by smaller Stones and Sand so as to give Graded Mixture. (See p. 172.)

economy and equal strength there should be used the largest possible quantity of stone in proportion to the sand, the strength of concrete being often actually increased simply by substituting more stone for a portion of the sand. In the following table this is illustrated by tests selected from Mr. Fuller's 6-inch beam experiments, which are given in full on page 376.

Relation of Strength of Concrete to Relative Proportions of Sand and Stone. (See p. 173.)

Proportions by weight of cement to total aggregate.	Proportions by weight of cement to sand and broken stone.	Modulus of Rupture lb. per sq. in.
1:6	1:1:5	504
1:6	1:2:4	439
1:6	1:3:3	355
1:6	1:4:2	210
1:6	1:6:0	93

The total amount of aggregate in each case is the same, namely, one part cement to 6 parts sand and stone, but the strength varies with the relative proportions of each, from 93 lb. to 504 lb.

(5) The discussion of Fuller's experiments on the relation of the best

practical mixture of sizes to a parabolic curve is given in Chapter XI, page 201.

Effect of Shape of Grain. (6) The fact that round grains, such as gravel, contain fewer voids than material with angular grains, such as broken stone, even if the particles in both are the same size, is proved from experiments in America and France. Mr. Allen Hazen states* that round grained water-worn sands have from 2% to 5% less voids than corresponding sharp grains of sand. Mr. Feret† also has studied the effect of the shape of the grain upon the density of sand, using in each case an artificial mixture of three sizes, with the following results:

Effect of Character of Sand Grains upon the Volume of the Sand. (*See p.* 174.)
BY R. FERET.

Nature of Sand	Shape of Grains	Actual solid volume per liter of sand: Not shaken, liter	Actual solid volume per liter of sand: Shaken to refusal, liter
Quartzite crushed in jaw crusher..............	Laminated	0.525	0.654
Crushed shells..............................	Flat	0.557	0.682
Ground quartzite	Angular	0.579	0.726
Natural granitic sand	Rounded	0.651	0.744

The voids in each case are the complements of the figures given.

The conclusion to be drawn is that the real volume increases (and therefore the voids decrease) as the sand approaches the round form.

When experimenting upon gravels and broken stone Mr. Feret‡ separated each into three sizes which he called respectively:

G (coarse) passing holes of 6 cm. (2.36 in.) diameter and retained by holes of 4 cm. (1.57 in.) diameter;
M (medium) passing holes of 4 cm. (1.57 in.) diameter and retained by holes of 2 cm. (0.79 in.) diameter;
F (fine) passing holes of 2 cm. (0.79 in.) diameter and retained by holes of 1 cm. (0.39 in.) diameter.

Each size of broken stone loosely measured gave about 52% voids, and each size of gravel about 40% voids. The voids in the broken stone were reduced to 47%, the lowest result obtainable, by mixing G and F in about

*Twenty-fourth Annual Report, Massachusetts State Board of Health, 1892.
†Annales des Ponts et Chaussées, 1892, II, p. 32.
‡Annales des Ponts et Chaussées, 1892, II, p. 153.

equal parts with no M, and in the gravel to 34% with about 3½ parts of G to one part of F. These figures are of course directly applicable only

FIG. 62.—Standard Ottawa Sand, dry.* No. 20 to No. 30 Sieves. (*See p.* 175.)

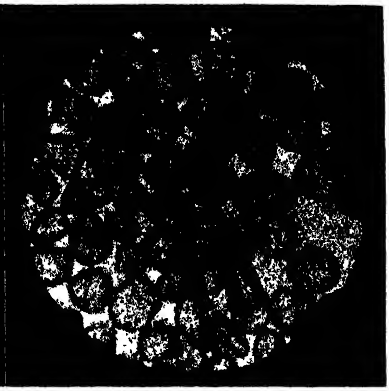


FIG. 63.—Standard Ottawa Sand with 6% moisture.* No. 20 to No. 30 Sieves. (*See p.* 175.)

FIG. 64.—Natural Bank Sand.* No. 20 to No. 30 Sieves. (*See p.* 175)

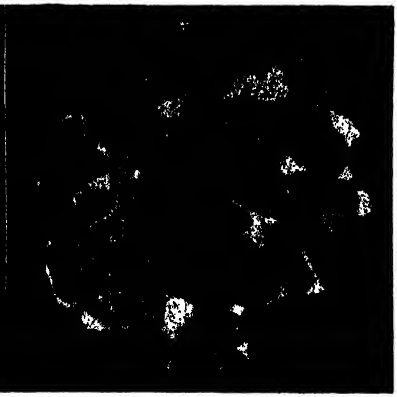


FIG. 65.—Crushed Quartz.* No. 20 to No. 30 Sieves. (*See p.* 175.)

to the special materials which he studied, and do not apply to gravel or stone containing sand or dust.

Photographs of Sand. Photographs of three types of sand are shown in Figs. 62 to 65. Figures 62 and 63 are photographs of the Ottawa,

*Each sand has passed a No. 20 and been retained on a No. 30 sieve. Magnified 10½ diameters.

Illinois, bank sand screened to the size selected for the standard sand by the Committee of the American Society of Civil Engineers. They illustrate the effect of moisture upon the arrangement of the sand grains, which is more fully described below. Fig. 64 is an ordinary bank sand from Eastern Massachusetts which has passed through and been retained by the same screens as the Ottawa sand. Fig. 65 is a sample of crushed quartz sand, formerly the standard in the United States. The sands are all reduced by the same number of diameters. The Ottawa sand, Figs. 62 and 63, is apparently of finer grain than either the bank sand or the crushed quartz, but close inspection will show that its grains, very uniform in size, are of about the same diameter as the smallest grains in the other sands. In other words, all the grains correspond very closely to a No. 30 sieve, the lot of sand from which it was screened containing no larger particles.

Effect of Moisture on Sand and Screenings. (7) Moist sand occupies more space and weighs less per cubic foot than dry sand. This is directly contrary to what one would naturally suppose. Indeed, it is almost incredible that the addition of water can reduce the weight of any material. The statement is readily proved, however, by shoveling a small quantity of natural sand as it comes from the bank with, say, 3% or 4% of moisture into a measure and drying it. The sand will settle, leaving the surface much below the level of the top of the measure. The explanation of this apparent anomaly lies in the fact that a film of water coats each particle of sand and separates it by surface tension from the grains surrounding it. This is illustrated in Figs. 62 and 63, page 175, the grains of the moist sand being separated from each other by the film of water. Fine sand, having a larger number of grains, and consequently more surface area, is more increased in bulk by the addition of water than coarse sand. The volume of coarse broken stone and gravel is but slightly, if at all, changed by moisture, while small broken stone composed largely of particles of less than ½-inch diameter is affected like sand.

If a small quantity of water is poured into a vessel containing dry sand, the bulk is not increased because of the inertia of the particles, but if the sand after moistening is dumped out and then turned back into the vessel with a shovel or trowel, its bulk will be increased. On the same principle, a sand bank does not swell in bulk during a shower, but the effect of the moisture is shown in the excavated material as soon as it is loosened with the shovel, and therefore its loose measurement for concrete or mortar is effected.

The diagram in Fig. 66, plotted by Mr. Fuller* from experiments upon a single sample of natural sand mixed by weight with varying percentages of water, illustrates the effects of moisture upon the actual percentages of voids in sands loose and tamped. The volumes produced by varying degrees of compacting are located between the two curves. It is noticeable that both the loose and tamped sand increase in volume with the addition of water and reach a maximum with about 6% of water, then decrease, and finally, when saturated, return to slightly less than their original dry bulk. The same sand, it is seen, may contain from 27% to 44% of absolute voids, according to the percentage of water and the degree of compacting.

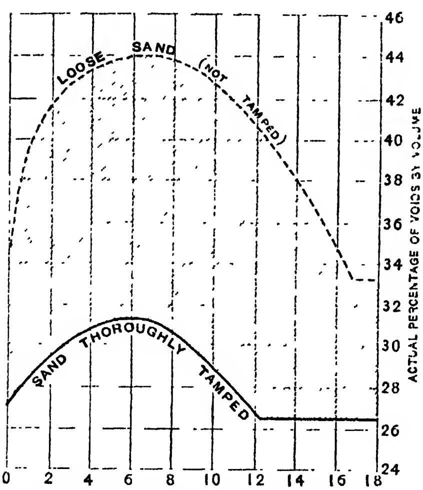


FIG. 66.—Percentage of Absolute Voids in a Natural Bank Sand containing Varying Percentages of Moisture. (*See p.* 177.)

The percentage of water by weight which will give the greatest bulk,—corresponding, of course, to the largest percentage of absolute voids,—varies with different sands from 5% to 8%.

The actual variation on different days in the percentage of moisture in a natural bank sand was found by the authors, in a series of experiments, to range from $1\frac{1}{2}$% to $5\frac{1}{4}$% of the total weight, or from $2\frac{1}{2}$% to $7\frac{1}{4}$% of the bulk of the moist sand. The sand, screened from a gravel bank in Eastern Massachusetts, ranged in coarseness from very fine to that which would pass a $\frac{3}{8}$-inch mesh screen. The moist sample was taken from the pile the day after a shower, and weighed $84\frac{1}{2}$ lb. per cubic foot, while the dryer sample, taken after a period of dry weather, weighed 107 lb. per cubic foot.

A sample of very fine sand which had been standing in a pile through the same shower contained $9\frac{1}{2}$% of moisture by weight, corresponding to 13% by volume. Ordinary gravel, on the other hand, from which the sand had been screened, was found after a heavy rain to contain only 1.8% of moisture by weight, this being apparently the maximum quantity which it would hold.

Engineering News, July 31, 1902, p. 81.

The maker of concrete is especially interested in the influence of moisture upon the bulk of sand and upon its voids (1) because of its effect upon the actual measurement of sand used in construction work, and (2) because of its effect upon his experimental determinations of proportions.

Rather incomplete experiments of the authors tend to show that the actual effect of moisture upon the volume of sand used in concrete and mortar may often be less than would naturally be inferred from the various experiments cited, and depends largely upon the processes of handling the sand. For example, fairly dry sand (3% moisture) shoveled by laborers from the pile into the regular sand-measuring box weighed 454 lb., while after a rain, the sand (with 5% moisture) shoveled from the pile into the same box weighed 464 lb., that is, the moist sand was slightly heavier than the dry. Further handling reversed these relations, for on weighing these two sands in a half cubic foot measure, the moist sand, as we should expect, was lighter than the dry.

The explanation of this apparent discrepancy is undoubtedly due to the fact that as the rain which affected the moisture occurred after the sand had been excavated and piled near the mixing platform, its bulk, as suggested on page 176, was not affected. The laborers handling the moist sand took large shovelfuls and the arrangement of the grains was not greatly disturbed. If the sand had been excavated after the rain, the handling with shovels and dumping from the cart probably would have rearranged the grains so that the moist sand would have weighed less than the dry in the large measure as well as in the small box.

Mr. Feret* calls attention to the fact that mortars of nominally the same proportions are richer in winter than in summer because of the greater amount of moisture in the sand, which, by increasing its bulk, reduces the absolute volume of the grains in a unit of measure. On the other hand, mortars are leaner in dry than in damp weather because the sand has greater density when dry.

In the experimental study of sand for determining the proportions of cement to be used, the effect of moisture is exceedingly important. The voids in absolutely dry sand are certainly no criterion of its qualities for mortar, while a moist sand will give entirely different results on different days. The best that can be done, if the study can be pursued no further than void determination, is to select conditions as near as possible to the average, and after determining the voids, considered as air alone and also as space occupied by the air and moisture, to use the results as a basis for judgment, bearing in mind that the volume of paste made from 100 lb.

*Annales des Ponts et Chaussées, 1892, II, p. 26.

of neat Portland cement, while varying largely with different brands, averages about 0.86 cubic feet, and that the volume of the additional water required for the sand (see pages 146 and 221) actually occupies space in the resulting mortar.

The most important conclusion to be drawn from the extreme variation in the same sand under different conditions is the impossibility of attaining results by the usual void experiments upon sand alone, which will be of accurate value in the consideration of mortar and concrete, and the practical necessity of employing methods such as are described by the authors in Chapter IX, page 138, or by Mr. Fuller in Chapter XI.

In the preceding paragraphs we have referred chiefly to the variation in the condition of the same sand. The importance of studying mortars rather than the sand alone is still further emphasized by the varying effect of moisture upon sands of different sizes. This is brought out very clearly in Mr. Feret's paper.* In studying the normal consistency of mortars he finds that not only every cement but also every sand has a definite percentage of water necessary to bring it to what may be called normal consistency. This he illustrates in the triangle shown in Fig. 67 (constructed as described on page 143), giving the "proportions of water (by weight) required for ground quartz sands of all granulometric composition." It is evident from the diagram that coarse sands,† G, require 3% by weight of water, medium sands, M, 9%, and fine sands, F, 23%, while mixtures of the three sizes require intermediate percentages.

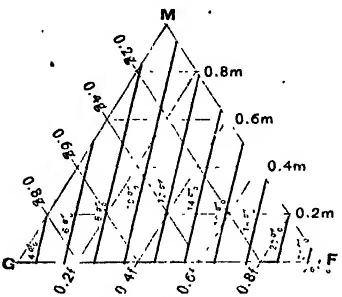


FIG. 67.—Percentages of Water Required to Gage Ground Quartz Sand of all Granulometric Compositions. (*See p.* 179.)

Compacting of Broken Stone and Gravel. Since concrete is usually compacted by ramming or lubrication of semi-liquid mortar, the density or the percentage of voids in compacted material is an important function. The statement has been made frequently that the aggregate compacts more when rammed in concrete than when rammed dry or merely moistened with water, because the mortar acts as a lubricant. Experiments by the authors indicate that broken stone under the same ram-

*Annales des Ponts et Chaussées, 1892, II.

†The sizes of screens defining coarse, medium, and fine sands are given on page 142.

ming will compress on the average 1% more when it is moistened than when dry, and that an amount of mortar sufficient to lubricate without filling the voids produces no further reduction in volume. For example, a volume of broken stone mixed with 20% of mortar and rammed in 6-inch layers produced a volume exactly equal to that of the rammed broken stone which had been merely moistened.

Further experiments, partially outlined in the table on page 171, upon gravel and also upon varying sizes and mixtures of trap rock from two quarries, the one producing a soft and the other an exceedingly hard stone, lead to the conclusion that with stones of the same general structure, the percentage of reduction in volume by similar ramming in 6-inch layers is quite uniform, irrespective of the actual sizes of the particles, their relative sizes, the percentage of voids, and, within certain limits, the degree of hardness. On the other hand, the method of ramming the same stone will very largely affect the amount of compacting. Broken stone of the nature of trap, whether hard or soft, was found to compact when spread in 6-inch layers about 14% either under light ramming or shaking the measure, and about 21% under heavy ramming. In actual concrete work this large reduction of volume is of course seldom reached, because imperfect mixing and the necessary coating of the particles require a larger percentage of mortar than will just fill the voids of the rammed stone, and the bulk of concrete is usually greater than that of the original stone.

Screened gravel spread in 6-inch layers and unconfined, compacted about 12% under either light or heavy ramming.

These percentages of compacting are based upon the loose measurement of the material as thrown by a laborer into a barrel or box measure. Rehandling a material like broken stone as it comes from the crusher tends to mix particles of unequal size and therefore to compact it very slightly. In one case a screened stone fresh from the crusher compacted 1% when rehandled once, and an additional 1% when rehandled the second time.

It is interesting to note that the method of shoveling broken stone into a measure has but slight effect upon its shrinkage; for example, a lot of stone thrown with force into an inclined barrel occupied a space scarcely appreciably less than when very carefully and lightly placed. On the other hand, dropping from a considerable height does affect the volume, for Mr. Desmond Fitzgerald* states that broken stone dropped 12 feet into a car shrank to a volume 7% less than when it was measured in a box.

*Transactions American Society of Civil Engineers, Vol. XXXI, p. 303.

Sand, unlike stone, is largely affected by the manner of shoveling and the size of the receptacle.

Compacting of Sand. The degree of compacting of sand is largely dependent upon the percentage of moisture which it contains. The dry sand shown in diagram in Fig. 66, page 177, when thoroughly tamped compacted from 34% to 27% voids or 9.6% in volume,* the sand with 6% moisture from 44% to 31% voids or 18.8% in volume, and the saturated sand from 33% to 26½% voids or 8.8% in volume.

Attention is called by Mr. Feret to the fact that the measurement of the weight of a given sand depends not only upon the quantity of moisture in it, but also upon the depth of the box which is used for the measure, the quantity of sand introduced at a time, — that is, the size of a shovelful, — the height from which it falls, the amount of shaking, if any, given to the box during filling, the amount of compacting given to the mass when leveling it off, and the smoothness of the surface left. As an illustration of the difference due to the method of placing in the measure, the authors found that a certain coarse sand shoveled into a pail about as a laborer would fill a measure weighed 88.0 lb. per cubic foot, while the same sand carefully poured into the pail weighed 83.3 lb. per cubic foot.

DEFINING COARSENESS OF SAND BY IT'S UNIFORMITY COEFFICIENT

The size of a sand may be indicated by what is termed its uniformity coefficient. This gives an idea of the actual variation in the size of the particles, and thus affords a means for comparing sands in different localities. A sand which is termed *coarse* in one section of the country is often considered *fine* in another.

To find the uniformity coefficient of a sand, screen it into at least five sizes, determine the percentage by weight of each size, and plot the mechanical analysis curve as described on page 196, and illustrated in Fig. 72, page 200. Then divide the diameter of the particles represented by the point at which the curve of the sand crosses the 60% horizontal line by the diameter of the particles where the curve crosses the 10% line. The quotient is the uniformity coefficient.

As an illustration of the value of the uniformity coefficient (u. c.) for different sands, reference may be made to the three mechanical analysis curves in Fig. 72, page 200. The curve of the coarse sand crosses the

*Ratio of compacting $= \frac{0.34-0.27}{1.00-0.27} = 0.096$

horizontal 60% line at the ordinate corresponding to a diameter of 0.117 inch, and the 10% horizontal line at ordinate 0.023 inch. Its uniformity coefficient and similarly the uniformity coefficients of the other sands are as follows:

			Uniformity Coefficient
Coarse sand	$\frac{0.117}{0.023}$	=	5.1
Medium sand	$\frac{0.038}{0.009}$	=	4.2
Fine sand	$\frac{0.018}{0.008}$	=	2.2

In general, it may be said that a sand with a uniformity coefficient above 4.5 is a good coarse sand for concrete work, and in comparing different natural sands the one having the highest uniformity coefficient may be considered the best.

As in ordinary bank sands the size of the particles at the 10% line (which is termed the effective size,* e. s.) does not greatly vary, the diameter at the 60% line alone is a very good indication of the coarseness of the sand. A knowledge of the effective size and the uniformity coefficient of any sand enables one accustomed to mechanical analysis diagrams to form a picture of its character.

Mr. Allen Hazen † who first used these terms in the examination of filter sand, states with reference to the percentage of voids or "open space" in compacted sand corresponding to different coefficients:

> A rough estimate of the open space can be made from the uniformity coefficient. Sharp-grained materials having uniformity coefficients below 2 have nearly 45 per cent. open space as ordinarily packed; and sands having coefficients below 3, as they occur in the banks or artificially settled in water, will usually have 40 per cent. open space. With more mixed materials the closeness of packing increases, until, with a uniformity coefficient of 6 to 8, only 30 per cent. open space is obtained, and with extremely high coefficients almost no open space is left.

For loose sand at least 10 should be added to these percentage values.

* The effective size itself is of considerable value for comparison of sand for filters, but not for concrete.

† Twenty-fourth Annual Report of State Board of Health of Massachusetts for 1892.

CHAPTER XI

PROPORTIONING CONCRETE

By William B. Fuller*

IMPORTANCE OF PROPER PROPORTIONING

The proper proportioning of concrete materials increases the strength obtainable from any given amount of cement, and also the water-tightness. Conversely, it permits, for a given requirement of strength and water-tightness, a reduction in the amount of cement, thereby reducing the cost.

Upon large or important structures it pays from an economic standpoint to make very thorough studies of the materials of the aggregates and their relative proportions. This fact has been seriously overlooked in the past, and thousands of dollars have sometimes been wasted on single jobs by neglecting laboratory studies or by errors in theory. Since cement is always the most expensive ingredient, the reduction of its quantity, which may very frequently be made by adjusting the proportions of the aggregate so as to use less cement and yet produce a concrete with the same density, strength and impermeability, is of the utmost importance.

As an example of such saving, the ordinary mixture for water-tight concrete is about 1 : 2 : 4, which requires 1.57 barrels of cement per cubic yard of concrete. By carefully grading the materials by methods of mechanical analysis the writer has obtained water-tight work with a mixture of about 1 : 3 : 7, thus using only 1.01 barrels of cement per cubic yard of concrete. This saving of 0.56 barrels is equivalent, with Portland cement at $1.60 per barrel, to $0.89 per cubic yard of concrete. The added cost of labor for proportioning and mixing the concrete because of the use of five grades of aggregate instead of two was about $0.15 per cubic yard, thus effecting a net saving of $0.74 per cubic yard. On a piece of work involving, say, 20 000 cubic yards of concrete such a saving would amount to $14 800.00, an amount well worth considerable study and effort on the part of those in responsible charge.

Proper proportioning is also important for reinforced concrete so as to give the uniformity and homogeneity which cannot be obtained without careful attention to the proportions and grading of the aggregates.

* The authors are indebted to Mr. Fuller for the material for this chapter.

METHODS OF PROPORTIONING

It is recognized generally that for maximum strength a concrete should be as dense as possible, that is, that it should have the smallest practicable percentage of voids. The various methods of aiming toward this result have been outlined as follows:*

(1) Arbitrary selection; one arbitrary rule being to use half as much sand as stone, as 1 : 2 : 4 or 1 : 3 : 6; another, to use a volume of stone equivalent to the cement plus twice the volume of the sand, such as 1 : 2 : 5 or 1 : 3 : 7.

(2) Determination of voids in the stone and in the sand, and proportioning of materials so that the volume of sand is equivalent to the volume of voids in the stone and the volume of cement slightly in excess of the voids in the sand.

(3) Determination of the voids in the stone, and, after selecting the proportions of cement to sand by test or judgment, proportioning the mortar to the stone so that the volume of mortar will be slightly in excess of the voids in the stone.

(4) Mixing the sand and stone and providing such a proportion of cement that the paste will slightly more than fill the voids in the mixed aggregate.

(5) Making trial mixtures of dry materials in different proportions to determine the mixture giving the smallest percentage of voids, and then adding an arbitrary percentage of cement, or else one based on the voids in the mixed aggregate.

(6) Mixing the aggregate and cement according to a given mechanical analysis curve.

(7) Making volumetric tests or trial mixtures of concrete with a given percentage of cement and different aggregates, and selecting the mixture producing the smallest volume of concrete; then varying the proportions thus found, by inspection of the concrete in the field.

The most practical method known to the writer for accurately determining the proportions of each material is by mechanical analysis of the aggregates, as described on page 211.

Volumetric synthesis, or proportioning by trial mixtures (p. 210) is another method which is sometimes useful, and produces fairly scientific results.

Since in many cases the proportions for a concrete must be selected more or less arbitrarily, after outlining the principles of proper proportioning, some of the less exact methods which are frequently used in practice will be

* From "Proportioning Concrete," by Sanford E. Thompson, Journal Association Engineering Societies, Vol. XXXVI, Apr. 1906, p. 185.

taken up before referring to the more scientific ones, and some of the causes for inaccuracies of these approximate methods discussed.

PRINCIPLES OF PROPER PROPORTIONING

The principles underlying the correct proportions of the materials of concrete are the same as those for mortar, namely, that the mass when compacted shall have the greatest possible density. In order, therefore, to obtain a knowledge of correct proportioning it will be best to first study the general conditions which are known to affect density.

Perfect spheres of equal size piled in the most compact manner theoretically possible leave but 26% voids. If the spaces between such a pile of equal-sized perfect spheres were filled with other perfect spheres of diameter just sufficient to touch the larger spheres, it would take spheres having relative diameters of 0.414 and 0.222 of the larger spheres, and the voids in the total included mass would be reduced to 20%. Using in this same manner smaller and smaller perfect spheres, it is conceivable that the voids could be reduced to so low a per cent of the total mass and to a size so small as to be only in a capillary form, and thus prevent the passage of water. This is assuming that every particle is placed exactly in its assigned place, but it is inconceivable that such an arrangement should take place under practical conditions, and in fact numerous trials by the writer with large masses of equal-sized marbles have demonstrated that they cannot be poured or tamped into a vessel so as to give less than 44% voids.

If equal quantities of spheres of, say, three sizes are mixed together, the per cent of voids in the total mass immediately increases, becoming about 65%, due probably to the smallest spheres getting between and forcing apart the largest. If, however, the containing vessel is continually shaken and the spheres stirred around, the smallest spheres will gradually all gravitate to the bottom and the largest to the top and the amount of voids in the total mass will again approach 44%. If a large number of different sized spheres are used, employing an increasingly large number of the smaller sizes so that each larger size may be said to be wholly surrounded by the next smaller size, the voids remain the same, no matter what the shaking, and will in some cases reach as low as 27%.

With ordinary stones and sands the same law holds as with perfect spheres except that they do not compact as closely, and the percentage of voids under comparable conditions is larger, varying with the degree of roughness and other features of the stones and sands used for the experiments.

When dry cement is added to a dry aggregate of stone and sand it acts

in the same manner as fine sand, and for obtaining the greatest density with dry cement, the cement must replace an equivalent amount of fine sand.

The theory of a concrete mixture is well stated by Mr. Feret* as follows:

> The problem of making the best concrete is thus reduced to the selection of a mixture of materials whose granulometric composition† corresponds to the maximum of density, since when this composition is known absolute volumes of cement may be substituted for equal absolute volumes of fine sand and vice versa, so as to vary the strength as desired while the density remains the same.

In other words, having mixed dry, inert materials in proportions necessary for greatest density, a portion of the grains of the very finest aggregate (that is, the finest particles of sand or dust) may be replaced by a corresponding quantity of cement to the extent required for the desired strength. This is not strictly true for concrete mixtures, because, when water is added to dry cement, the cement particles are separated from each other by the surface tension of the film of water, and it is no longer possible to obtain as dense a mixture as is theoretically possible with the dry mixture.

The density of concrete therefore has been found to depend upon the varying degree of roughness of the stone and sand, the relative sizes of the diameters of the stone, sand and cement, and the amount of water used.

The fineness of the cement particles and the amount of water to be used are determined by questions discussed elsewhere, and we have to deal here only with the proportioning of the sand and stone.

DETERMINATION OF THE PROPORTION OF CEMENT

The most difficult question to decide with accuracy in proportioning is the proportion of cement to use. This is to a considerable extent a matter of mature judgment, depending upon the nature of the construction, the degree of strength required within a certain limit of time, the required watertightness, the character of the aggregates, and many other matters which must be considered in direct connection with the work to be done and the available materials. An engineer experienced in concrete construction and tests can estimate approximately the strength of concrete made with certain materials, and select the proportions accordingly. The surest plan after selecting and grading the aggregates is to make up specimens of concrete and test its crushing strength, but this is usually impracticable for lack of time. The next best plan is to have the tensile strength determined of mortar made from the sand to be used and by comparing

*Chimie Appliquée 1897, p. 523.
†Proportioning of sizes.

this with the strength of the mortar of standard sand an idea can be formed of the proportion of cement to select. If a sand is fine, a richer mortar must be used, frequently instead of a 1 : 2 selecting a 1 : 1½ or even 1 : 1, and the amount of coarse aggregate also reduced to accord with this.

An experimental plan which has been followed to determine the minimum quantity of cement which will produce a concrete practically free from air voids is to mix the aggregates in the correct proportions as described in the pages which follow, compact them by ramming or hard shaking, and then determine their voids by weighing and correcting for specific gravity.* The sand should be in the natural state of moisture found in the interior of the bank, not because this is the condition in which it will be mixed in the concrete, but because it may be assumed in the natural state to contain a quantity of moisture varying with its fineness. If gravel is used it may be taken in the same way, while coarse broken stone should be dry, and dry broken stone screenings may be mixed with about 4% of water by weight. Correction must be made for this moisture after weighing the mixed material, so that the voids calculated will be simply air voids.

In determining the quantity of cement to fill these air voids it may be assumed without appreciable error that 100 lb. of cement will make 1.0 cu. ft. of neat paste. This is a larger volume than would result with ordinary plastic paste, but makes a slight allowance for the additional moisture required for the sand and stone. To the quantity of cement thus determined 10% may be added, *i. e.*, 10% of the cement, not of the total mixture, to provide for imperfect mixing.

PROPORTIONING BY ARBITRARY SELECTION OF VOLUMES

The common custom of specifying arbitrarily the proportions of cement, sand and stone in parts by volume, while convenient in construction, causes wide discrepancies in results because of different methods of measuring the materials. A concrete called a 1 : 2 : 4 mixture by one man may not contain any more cement than a concrete termed a 1 : 3 : 6 mixture by another.†

Notwithstanding this, if the units of measurement and the methods of measuring are stated definitely, arbitrary selection of proportions may give good results in practice, although necessitating a larger quantity of cement with consequently a greater net cost than more scientific proportioning would require.

The percentage of volume of sand required for ordinary gravel or broken

*See page 165.

†These variations are discussed more fully by the authors on page 218.

stone from which the finest material has been screened may be taken between the limits of 40% and 60% with an average, which is suitable under many conditions, of 50%. If the cement is taken as additional, which is not strictly correct, this ratio corresponds to proportions 1 : 1½ : 3, 1 : 2 : 4, 1 : 2½ : 5, and 1 : 3 : 6, which are suggested by the authors in Chapter II as standard mixtures for the use of those who are inexperienced in concrete work.

In cases where the coarse material contains a good many small particles, as does crusher run, broken stone or graded gravel, or the sand is so fine as to flow readily into the voids of the stone, the proportion of sand should be slightly less than half the volume of stone. Since the cement also increases the bulk of mortar and hence assists to fill the voids in the stone, it is suggested that with such aggregates the volume of the stone be made equal to the cement plus twice the volume of the sand. This would give proportions 1 : 1½ : 4, 1 : 2 : 5, 1 : 2½ : 6, and 1 : 3 : 7 for these special conditions.

Proportions adopted by various authorities and tabulated on page 212 may serve as a guide to arbitrary selection.

It is a good plan on work which will not warrant special tests and for which there is no choice of aggregates, to use at first twice as much stone or gravel as sand and then vary the relative proportions of the sand to the stone as the work progresses, governing this by the way the concrete works into place. Too much sand will be indicated by the harsh working of the concrete, while if there is too little sand, stone pockets are apt to occur on the surface of the concrete, and it will be difficult to fill the voids of the stone.

Screened vs. Unscreened Gravel or Broken Stone. Unscreened gravel is often used alone for the aggregate, but there is scarcely any case where the cost of screening and re-mixing the materials will not be less than the saving in the cement by using screened aggregates. The quantity of sand in different parts of the same gravel bank always varies greatly and the run of the bank rarely contains sufficient coarse stone to make a dense concrete. If, as is sometimes the case, the quantity of material coarser than ¼ inch is about the same as that which passes a ¼-inch sieve, then, if used without screening the same quantity of total aggregate must be used as would otherwise be specified for the coarse aggregate; that is, instead of 1 : 2 : 4 proportions, the unscreened gravel would require 1 : 4.

Broken stone as it runs from the crusher will contain considerable dust, and may sometimes be used economically by simply adding sand without screening. However, there is apt to be a separation of the coarse particles from the fine as they roll down the pile so that less homogeneous propor-

tions can be attained. Consequently the writer is in favor of separating the aggregate into as many parts as is consistent with economy for the work in hand. Even on small work he believes it preferable to screen out the sand or dust and re-mix it in the specified proportions.

PROPORTIONING BY VOID DETERMINATION

The determination of proportions by finding the volume of water which may be poured into the voids of a unit volume of stone and selecting a volume of sand equal to this volume of water is one which gives no better results in practice than arbitrary selection of the proportions, as described in the preceding paragraphs, and varying the relative proportions of sand to stone when placing. The determination of the proportion of cement to sand by void measurement is still more misleading; in fact, for reasons discussed below, it is so inaccurate that no consideration will here be given to it.

The theory of proportioning by voids is that if the stone or gravel contains, say, 40 per cent voids as measured by the contained volume of water, the required volume of sand is theoretically 40% of the volume of the stone, and supposing the ratio of cement to sand to be as 1 : 2, the relation of parts of sand to parts of the coarse aggregate would be as 2 : 5, thus making the proportions 1 : 2 : 5. Because of the inaccuracy of this method of procedure, as discussed below, it is necessary in most cases, even although the cement and water will still further increase the bulk, to take a volume of sand, say 5% to 10% in excess of the voids; that is, for gravel with 40% voids to use 45% to 50% of its volume of sand, thus making the proportions 1 : 2 : 4½. If the coarse material is screened broken stone of large size, say 1½ or 2-inch, the volume of sand may be taken equal to the volume of voids instead of in excess of them, because the particles of sand will all be small enough to fit into the voids of the stone without appreciably increasing its bulk. Such stone usually has about 45% to 50% voids, so that we should have proportions 1 : 2 : 4½ or 1 : 2 : 4, the same as for the gravel concrete.

The irregular distribution of the materials by imperfect mixing may usually be disregarded, because the volume of gaged mortar is always in excess of the volume of sand from which it is made.

Care must be exercised in any case to guard against a larger excess of sand than is absolutely necessary, because the voids in a concrete are lessened by using stone in place of sand. Take, for instance, sand having 45% voids and stone having 40% voids. With the sand just filling the voids of the stone it is easily calculated that the resultant mass has 18%

voids; but supposing an excess of 10% of sand, there would be 10% of the material having 45% voids, which means there would be 2.5% more voids in the resultant mass.*

Authorities differ as to whether the stone should be loose or shaken when determining the voids. Loose measurement is generally considered preferable because it corresponds more nearly to the final volume of the concrete, and more sand is always necessary than will just fill the voids of rammed stone, since the sand and cement separate the stones and prevent their lying close together in concrete. In determining, however, the quantity of cement required for the mixture of aggregates the materials should be compacted as described on page 211.

The chief inaccuracy of this method of basing the proportions of the finer materials of a concrete mixture upon the water contents of the voids in the larger is due to the difference in compactness of the materials under varied methods of handling, and to the fact that the actual volume of voids in a coarse material may not and usually does not correspond to the quantity of sand required to fill the voids, and that therefore the common method of proportioning by basing the volume of sand or of mortar upon the volume of water which can be poured into the broken stone leads to false conclusions. The reasons for this inaccuracy are chiefly because the grains of sand thrust apart the particles of stone, and because with most aggregates a portion of the particles of sand or fine screenings are too coarse to enter the voids of the coarsest material.

Even in a mass of stones of uniform size many of the separate voids are much smaller than the particles. If we have, then, a mass of gravel ranging from fine to coarse or a mass of crusher-run broken stone, even with the finest sand or the dust screened out of them, the individual voids are many of them so small that a large number of the particles of natural bank sand will not fit into them, but will get between the stones and increase the bulk of the mass. On account of this increase in bulk, even with thorough mixing more sand is required than the actual volume of the voids in the coarse material. The separation of the particles of stone by the sand is illustrated in the mixture shown in Fig. 2, page 15.

To illustrate this important principle, an extreme example may be cited. Suppose that we have a mixture in equal parts of 1-inch stone and $\frac{7}{8}$-inch stone. By the usual method of reasoning employed in proportioning concrete, if the 1-inch stone has 50% voids, we should require a volume of $\frac{7}{8}$-inch, equal to 50% of the volume of the 1-inch stone, in order to fill

* See discussion by the writer in Transactions American Society of Civil Engineers, Vol. XLII, p. 142.

the voids in the latter. The absurdity of this is apparent, because the two stones are so near a size that the smaller cannot fit into the voids of the latter, and the bulk of the mixture is inappreciably less than the sum of the separate volumes, that is, the mixture still has nearly 50% voids. The principle is just as true, although the total effect is less, if we consider it with reference to the finer particles of the gravel or the crusher-run broken stone and the sand or fine screenings which are to be introduced to fill the voids. The sizes of many of the particles of the latter are so nearly equal to the sizes of the smallest particles of the coarse material that they increase the total bulk instead of reducing the voids. They also get between the surfaces of the stone particles and prevent the stones touching each other.

We might conclude from the above that the best concrete can be made with a coarse stone of uniform size and a sand whose particles are all small enough to fit into its voids; in fact, this is the conclusion reached by the advocates of broken stone of uniform size in preference to crusher-run stone.

Our experiments indicate that while this may be true in theory, in practice in making concrete the graded materials give about the same density and work rather smoother in handling and placing.

The point, however, which is to be emphasized is the inaccuracy of determining the exact volume of sand or mortar by simply measuring the water contents of the voids in the coarse aggregate.

The selection of the proportion of cement by determination of the water contents of the voids in sand is even more inaccurate than the proportioning of sand to stone by void measurement. The varying effect of moisture on the sand so influences the volume of the voids that their determination is chiefly important as an aid to the judgment; and as a matter of fact, although in practice the quantity of cement is supposed to depend upon the volume of voids in the sand, it is customary to select a definite relation of cement to sand varying according to the character of the construction from 1 : 1 to 1 : 3, recognizing, however, that fine sand—and fine sands in an ordinary state of moisture will almost always have the distinguishing characteristic of a lighter weight per cubic foot than coarse sands and a consequently larger percentage of voids—requires more cement for equivalent strength.

As already stated, if the work is too small to warrant a thorough study of the materials by mechanical analysis or volumetric synthesis, or some other scientific method, it is evident from the above discussion that it is nearly as accurate to determine the proportions by arbitrary selection (see p. 186) as by a study of voids.

RAFTER'S METHOD OF PROPORTIONING

Mr. George W. Rafter* has called attention to the method of proportioning the mortar as a percentage of the volume of the stone slightly shaken, the relation of cement to sand having been determined by the required strength of concrete.

Quoting from specifications for the Genesee Dam, the concrete is proportioned as follows:

> In forming concrete such a proportion of mortar of the specified composition will be used as may be found necessary by trial to a little more than fill the voids in the aggregate. Tests of the voids will be made from time to time under the direction of the engineer, and instructions given as to the per cent of mortar of the specified composition to be used. For the information of the contractor, in the way of computing the cost of concrete of the quality herein required, it may be stated that ordinarily the per cent of mortar will be about 33 per cent of the measured volume of the aggregate. In case of the use of a certain proportion of gravel in the aggregate, the proportion of mortar may be reduced to somewhat less than 30 per cent.

This method of proportioning is more accurate than the usual procedure, because there is less apt to be an excess of mortar. It does not, however, take account of the fact that with a coarse aggregate of varying sized particles some of the grains of sand are too large to fit into the voids of the stone, and that therefore the coarse and fine aggregates must be studied together.

An examination of the analysis of the sand used by Mr. Rafter indicates that to its fineness was due the small proportion of mortar to stone which he was able to use. Ninety-two per cent of the sand passed a No. 30 sieve, so that the grains were small enough to enter the voids of the stone without appreciably increasing the bulk of the concrete.

FRENCH METHOD OF PROPORTIONING

In France, proportions are ordinarily stated in terms of the volume of mortar to the volume of stone, and the mortar is described by the number of kilograms of Portland cement to 1 cubic meter or liter of sand.

The following table gives the nominal proportions in English measure based on a volume of 3.8 cubic feet corresponding to similar French proportions based on kilograms of cement to a cubic meter of sand.

*"On the Theory of Concrete" Transactions American Society Civil Engineers, Vol. XLII, p. 104.

American Equivalents of French Proportions. (*See p.* 999.)

French measure, kilograms cement per cubic meter of sand.	American measure, cement to sand by volume.*	Pounds of cement per cubic foot of sand.	French measure, kilograms cement per cubic foot of sand.	American measure, cement to sand by volume.*	Pounds of cement per cubic foot of sand.
200	1 : 8.0	12.5	700	1 : 2.3	43.7
300	1 : 5.3	18.7	800	1 : 2.0	50.0
400	1 : 4.0	25.0	1000	1 : 1.6	62.5
500	1 : 3.2	31.3	1200	1 : 1.3	75.0
600	1 : 2.7	37.5	1600	1 : 1.0	100.0

*Proportions based on standard weight of cement, i. e., 100 pounds per cubic foot

Concrete in France is frequently designated with respect to the ratio of mortar to stone; for example, one volume of mortar to two volumes of stone, the mortar then being designated as indicated in the above table. To express the parts more definitely, the basis is sometimes a cubic meter of sand; for example, 650 kilograms cement to one cubic meter sand to 1.8 cubic meter stone, this corresponding substantially to proportions 1 : 2½ : 4½ by volume, as ordinarily used in America.

MECHANICAL ANALYSIS

Mechanical analysis consists in separating the particles or grains of a sample of any material, — such as broken stone, gravel, sand or cement, — into the various sizes of which it is composed, so that the material may be represented by a curve (see Fig. 70, p. 198) each of whose ordinates is the percentage of the weight of the total sample which passes a sieve having holes of a diameter represented by the distance of this ordinate from the origin in the diagram.

The objects of mechanical analysis curves as applied to concrete aggregates are (1) to show graphically the sizes and relative sizes of the particles; (2) to indicate what sized particles are needed to make the aggregate more nearly perfect and so enable the engineer to improve it by the addition or substitution of another material; and (3) to afford means for determining best proportions of different aggregates.

To determine the relative sizes of the particles or grains of which a given

*Chimie Appliquée, 1897, p. 523.

†Proportioning of sizes.

sample of stone or sand is composed, the different sizes are separated from each other by screening the material through successive sieves of increasing fineness. After sieving, the residue on each sieve is carefully weighed, and beginning with that which has passed the finest sieve, the weights are successively added, so that each sum will represent the total weight of the particles which have passed through a certain sieve. The sums thus obtained are expressed as percentages of the total weight of the sample and plotted upon a diagram with diameters of the particles as abscissas and percentages as ordinates.

The method of plotting and the uses of the curves thus obtained are more fully described in the pages which follow.

Sieves and Other Apparatus. Fig. 68 illustrates a convenient outfit for such a mechanical analysis as above described, consisting of a set of sieves, an apparatus for shaking the sieves, and scales for weighing. A standard size of sieve is 8 inches in diameter and 2½ inches high. Sieves with openings exceeding 0.10 inches are preferably made of spun hard brass with circular openings drilled to the exact dimensions required. Sieves with openings of 0.10 inch and less are preferably of woven brass wire set into a hard brass frame. Woven brass sieves are made for many purposes, and are sold by numbers which approximately coincide with the number of meshes to the linear inch. As the actual diameter of the hole varies with the gage of wire used by different manufacturers, every set of sieves must be separately calibrated.

An approximate idea of the diameters of holes which may be expected in commercial sizes of sieves is presented in the following table, which is sufficiently exact to serve as a guide to the purchase of the sieves:

Commercial No. of sieve.	Diameter of hole in inches.	Commercial No. of sieve	Diameter of hole in inches.
10	0.073	60	0.009
15	0.047	74	0.0078
16	0.042	100	0.0045
18	0.037	140	0.003625
20	0.034	150	0.00325
30	0.022	170	0.0031
35	0.017	180	0.00306
40	0.015	190	0.0028
50	0.011	200	0.00275

For separating particles smaller than those passing through a No. 200 sieve, recourse must be had to processes of elutrition which have been developed to great precision by soil analysis chemists.*

*See page 85.

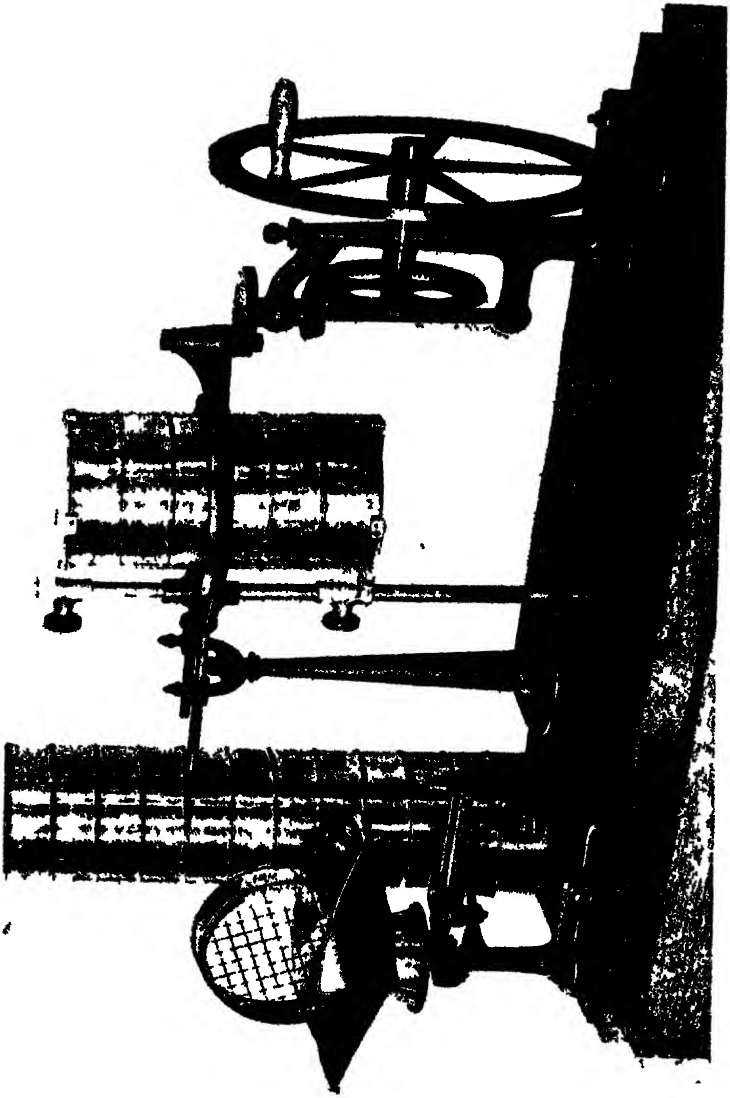


FIG 68. — Mechanical Analysis Sieves and Shaker (*See p.* 194)

In selecting the right series of sieves to purchase, first decide on the limiting diameters, say, from 3.00 inches to No. 200 = 0.00275 inches. Then decide on the total number of sieves, say, twenty. Look up the logarithm of 3.00 and of 0.00275 and by proportion find eighteen other logarithms between these having equal differences between each. Look for the number corresponding and take the nearest commercial sieve giving this diameter. The diameters of holes exceeding 0.10 inch can be made as required. A convenient set of twenty sieves, — ten for stone, which give the diameter of the holes in inches, and ten for sand, giving the commercial number (see p. 194), — is as follows:*

Stone Sieves inches	Sand Sieves Commercial No.
3.00	10
2.25	15
1.50	20
1.00	30
0.67	40
0.45	60
0.30	74
0.20	100
0.15	150
0.10	200

After the sieves are obtained it is necessary that they should be very carefully calibrated to ascertain the average diameter of the mesh. This should be done by averaging the diameters of the openings measured in two positions at right angles to each other, as the meshes of commercial sieving are not exactly square. Sieves having meshes exceeding 0.10 inch are most conveniently calibrated by ordinary outside calipers; those having meshes of less diameter, by a micrometer microscope.

When many analyses are to be made, it is convenient to have a printed cross section form, such as is shown in Fig. 69, p. 197, with appropriate spaces for filling in the number of the analysis, description of the material, location of the work, and other facts relating to the material.

Plotting Analysis Curves. For those who are unfamiliar with mechanical analysis a detailed explanation of the method of locating the curve is here given. The method can best be understood by referring to the diagrams of typical materials which are also of practical interest as illustrating the curves which may be expected in special cases.

Fig. 70, p. 198, represents a typical mechanical analysis of crusher-run micaceous quartz stone which has been run through a ¼-inch revolving screen so as to separate particles finer than ¼ inch, that is the dust, for use with sand.

For a sample of stone, which may be taken by the method of quartering

*A still smaller set for ordinary use is suggested on page 159a.

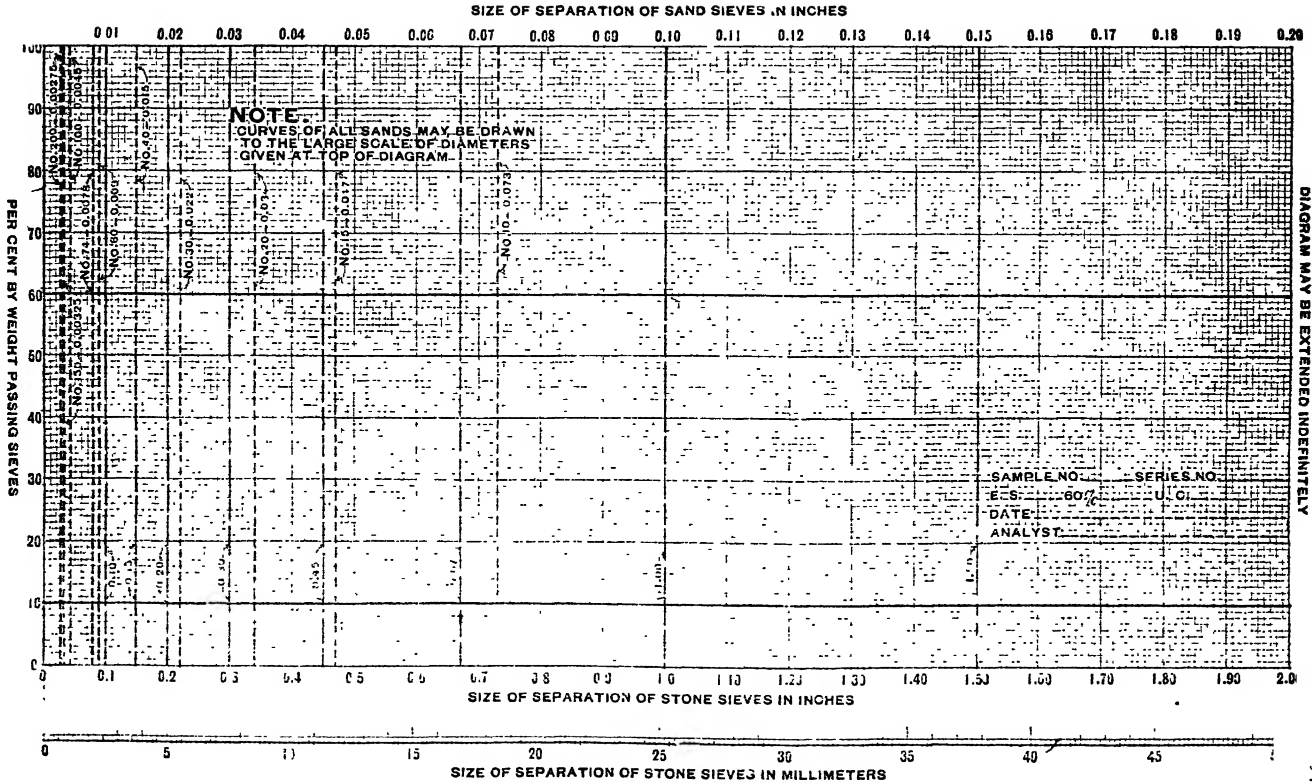


FIG. 69. — Blank Form for Mechanical Analysis Diagram. (*See p.* 106.)

described on page 280, 1 000 grams is a convenient quantity for 8-inch diameter sieves 2¼ inches in depth, and also permits of easy reduction from weights to percentages. To obtain the analysis shown in Fig. 70, the sample of stone is placed in the upper (coarsest) sieve of the nest of stone sieves given on page 190, and after 1 000 shakes the nest is taken apart, and the quantity caught on each sieve is weighed. The results obtained in the

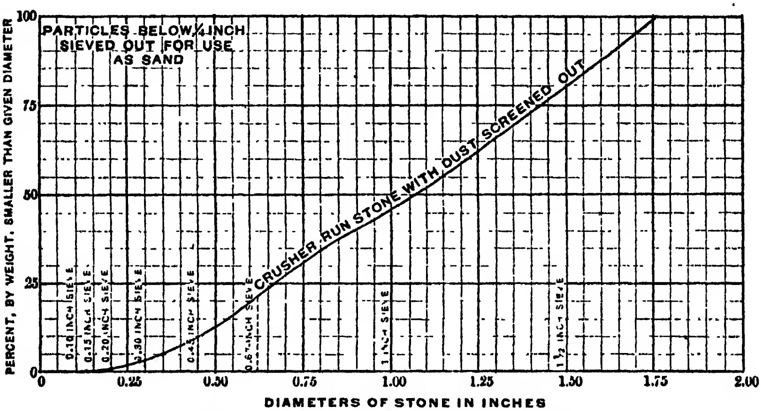


FIG. 70.—Typical Mechanical Analysis of Crusher-Run Micaceous Quartz Stone. (*See p.* 198.)

particular case under consideration are illustrated in the following table, which shows the method of finding the percentages:

Results of Screening Samples of Stone of Fig. 70.

Size sieve inches.	Retained in each sieve* grams.	Amount finer than each sieve grams.	Percentage finer than each sieve %
0.10	8	0	
0.15	11	8	1
0.20	8	19	2
0.30	72	27	3
0.45	123	99	10
0.67	235	222	22
1.00	344	457	46
1.50	199	801	80
Total,	1000		

*In practise this column is not required, the weights in the next column being obtained directly by placing each successive residue on the scale pan with that already weighed.

The various percentages are plotted on the diagram and the curve drawn through the points. The vertical distance from the bottom of the diagram

to the curve, that is, the ordinate at any point, represents the percentage of the material which passed through a single sieve having holes of the diameter represented by this particular ordinate. Since the percentage of material passing any sieve is always the complement of the percentage of grains coarser than that sieve, the vertical distances from the top of the diagram down to the curve represents the percentages which would be retained upon each sieve if employed alone. For example, taking 1.25, 62%, the distance from the bottom of the diagram, represents the percentage of material finer than 1¼ inch diameter, and 38%, the distance down from the top of diagram, represents the percentage coarser than 1¼ inch.

Fig. 71 represents a typical analysis of crushed trap rock which has been

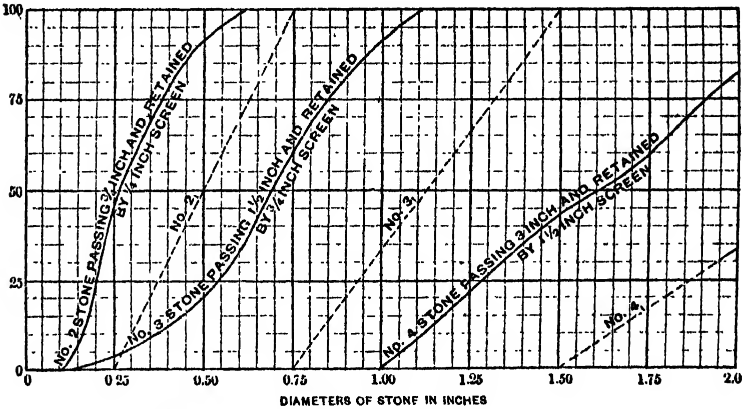


FIG. 71.—Typical Mechanical Analysis of Crushed Trap Rock Separated into Three Sizes by Revolving Screens having 3, 1½, ¾ and ¼ inch perforations. (*See p.* 199.)

separated into stone of three sizes and dust, by a revolving screen 2 feet 6 inches in diameter and 12 feet long set on a slope of 1 foot 9 inches. This was made up of four sections having respectively 3, 1½, ¾ and ¼ inch perforations. The curves not only show the sizes of trap rock which ordinarily pass through crusher screens of given diameter of hole, but also illustrate how inefficient the screening process may be. For example, if the sizes of the particles had corresponded exactly to the diameters of the holes and the screening had been more perfectly done, we should have had curves whose general direction and location is shown by the dotted lines No. 2_1, No. 3_1, and No. 4_1; that is, for example, No. 3, since it represents stone which passes a 1½ inch screen and which is retained on a ¾ inch screen, should occupy a position between the ordinates representing 1.50 and

0.75 diameters. If the stone had rumbled longer in the screen because of flatter slope or screen sections of greater length, the curves would have approached more nearly to these dotted lines.

Typical curves of a fine, a medium well graded, and a coarse sand are shown in Fig. 72. For convenience in plotting, the horizontal scale is ten

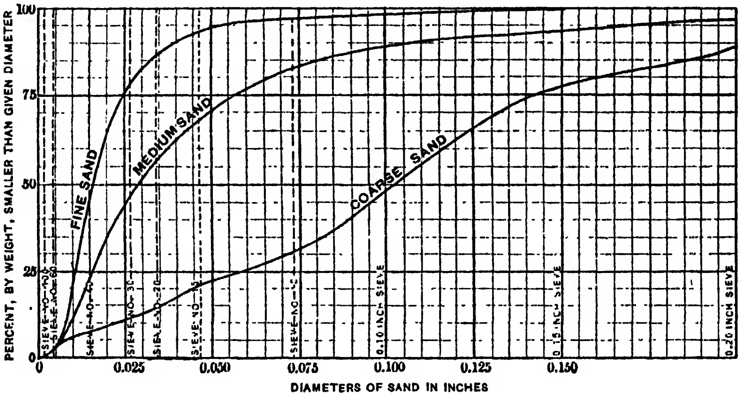


FIG. 72.—Typical Mechanical Analyses of Fine, Medium, Well Graded and Coarse Sands. (*See p.* 200.)

times greater than that of Figs. 70 and 71, the diagram showing diameters ranging from 0 to 0.200 inches diameter. The "granulometric composition" of these sands may be determined if desired by reference to page 149.

The mechanical analysis of crusher dust is apt to vary between the curves of fine sand and medium sand which are shown in Fig. 72.

STUDIES OF THE DENSITY OF CONCRETE

In the year 1901 the writer, through the permission and assistance of Mr. E. LeB. Gardiner, Vice-President, and Mr. J. Waldo Smith, Chief Engineer, of the East Jersey Water Company, was enabled to make an extended series of experiments on the comparative strengths of different proportions of concrete aggregate. Many mixtures of different proportions were made up into beams, their curves of mechanical analyses drawn as explained above, and the strength of the beams determined by breaking tests.*

These tests indicated that the strength of concrete varies with the percentage of cement contained in a unit volume of the set concrete, also with

* The results of these tests are presented in the table on pages 376 and 377.

the density of the specimen. With the same percentage of cement, the densest mixture, irrespective of the relative proportions of the sand and stone, was in general the strongest. These tests further indicated that for the materials used there was a certain mixture of sizes of grains of the aggregate which, with a given percentage by weight of cement to the total aggregate, gave the highest breaking strength. In practice also it was found that the concrete made with this mixture worked most smoothly in placing.

These tests led to a still more extended series by the writer and Mr. Sanford E. Thompson at Jerome Park Reservoir, New York, in 1903 and 1904, under the authorization of the Aqueduct Commission of the city of New York, Mr. J. Waldo Smith, Chief Engineer.

The method of procedure and the results of the tests are given in full in a paper on "The Laws of Proportioning Concrete," by William B. Fuller and Sanford E. Thompson, Transactions American Society Civil Engineers, Vol. LIX, p. 67, 1907. The experiments were begun with a series of tests on the density of different mixtures of aggregate and cement to determine the laws of proportioning for maximum density for different materials, and these density experiments were followed by the manufacture of concrete specimens in the attempt to determine the relation between the laws of strength and the laws of density.

The mechanical analysis diagram furnished a ready means of studying the effect of various sized particles on the density of concrete. For this purpose crusher-run stone and bank gravel were screened into twenty-one sizes ranging from 3 inches down to that passing a No. 100 sieve, having meshes 0.0027 inch in diameter. These sized materials were then re-combined in a predetermined mechanical analysis curve by weighing out the necessary quantities of each size.

This material was next thoroughly mixed with a given weight of cement and the whole amount wet and mixed and tamped into a strong cylinder in which its volume could be measured. This batch was then thrown away and another batch made up according to another mechanical analysis curve and its volume recorded. In this way over 400 different mechanical analysis curves were tested as to volume for the purpose of determining the ideal curve corresponding to the densest concrete mixture.

Both broken stone and gravel were used in the tests, and to reduce the number of variables, most of the experiments were made upon the same proportions, using 10 per cent by weight of cement to the total dry materials, corresponding to proportions 1 : 9 by weight.

In all of the tests instead of following the more usual plan of testing the

aggregate separately, every experiment was performed with a mixture of the aggregate and cement gaged with the water necessary to produce the proper consistency. The water was found necessary both in theory and practice. The cement and water actually occupy space in the mass, since many of the voids are too small for the grains of cement to fit into them without expanding the volume and the water also occupies actual bulk in the concrete. Besides this, a concrete mixed up with water is easier and smoother to handle than a mixture of dry materials alone which tend to separate when being placed.

Curve of Maximum Density. The Little Falls tests made by the writer indicated that the curve at greatest density was substantially a parabola. The Jerome Park tests based on a larger number of experiments define the curve still more accurately as a combination of an ellipse and a straight line.*

One of the most interesting developments was that a curve of substantially the same form would fit different materials whatever the maximum size of the stone. The ½-inch stone, for example, required but very slight change in curve equation from the 2¼-inch stone.

The maximum density curve then was found to consist of a combination of an ellipse† and a straight line, the ellipse being first constructed with its

* Mr. Fuller's method of proportioning the materials so that their mixture will form a smooth, clearly defined curve appears, on its face, to conflict with Mr. Feret's conclusion (see p. 147) that the best mixture of sand and cement for mortar is made up of coarse and fine grains only, with no intermediate grains. For sand mortars, Mr. Feret's methods are undoubtedly more exact than Mr. Fuller's, but for a concrete mixture the conditions are different, and, as we have stated on page 172, more than two sizes of materials are theoretically necessary for obtaining the densest mixture. In practice, too, all classes of materials are more or less varied, and experiments show that the particles will best fit into each other if the sizes are graded. The best proof of the practical efficiency of Mr. Fuller's method lies in the fact that he has employed it day after day for determining the proportions of the aggregate for concrete used in constructing thin, water-tight walls. The proportions used by him for such work are about 1 : 3 : 7, whereas for water-tight construction where the materials are not scientifically graded 1 : 2 : 4 mixtures are commonly used.

The method is exact and scientific and not "rule-of-thumb." The nature of the materials and their variation from hour to hour makes great refinement unnecessary, so that an accuracy of, say, 2% or 3% in the percentages are all that is necessary in practice. Although further tests may show that for other materials the form of the curve varies from that indicated by Mr. Fuller, the general method of analyzing materials and combining the curves is undoubtedly applicable whatever the form of the curve, so that Mr. Fuller's general principles and methods still hold.

† In practice ellipses may be most readily plotted graphically by the Trammelpoint method as follows:

Plot the major and minor axes on the diagram. The major or horizontal axis in all cases is on a line 7% above the base. The minor or vertical axis is at a distance, a, to the right of the vertical zero ordinate of the diagram. Lay a strip of paper or a thin straight-edge upon the major or horizontal axis, and mark upon it two points to represent the length of the semi-major axis, calling one of these points—the point on the zero ordinate—O, and the other point A. Mark off on the strip or straight-edge, in the same direction from O, the length of the semi-minor axis, calling this point B. Now, swing the strip of paper or straight-edge little by little so that the outline of the curve may be marked off by the point O, while the points A and B are kept at all times upon the axes b and a respectively. The straight lines to continue the curves are drawn as tangents to them, or may be readily plotted from the data on the following page.

major axis coinciding with 7 per cent line of percentages, and the equation of the ellipse, using the zero coördinates of the diagram, being $(y-7)^2 = \frac{b^2}{a^2}(2ax - x^2)$. One of the ideal curves is illustrated in Fig. 73, page 207, showing the general form which it takes.

In practice it was necessary to raise the curve somewhat higher, that is, to use more sand than the very careful laboratory tests would indicate as the ideal mix.

The values of *a* and *b* for the different materials, including the cement for the Ideal Mix, based on the Jerome Park stone and Cowe Bay sand and gravel, which, as already stated, were fairly representative materials, are as follows:

Data for Plotting Ellipses in Curves of Ideal Mix.

Materials.	Ideal Mix Axes of Ellipse.	
	a	*b*
Crushed stone and sand	0.04 + 0.16D	28.5 + 1.3D
Gravel and sand......	0.04 + 0.16D	26.4 + 1.3D
Crushed stone and screenings...........	0.035 + 0.14D	29.4 + 2.2D

In this table, D = the maximum diameter of the stone, in inches.

For the Practical Mix the values of *b* must be greater so as to give a higher curve with more of the finer material. A quick and sufficiently accurate method of drawing the curves for the practical mix is to draw a straight line from the point where the largest diameter stone reaches the 100% line to the point on the vertical ordinate at zero diameter given in Column (1) in the following table.

Data for Plotting Curves of Practical Mix.

Materials.	Intersection of tangent with vertical at zero diameter (1)	Height of tangent point (2)	Axes of Ellipse. *a* (3)	Axes of Ellipse. *b* + 7 (4)
Crushed stone and sand.	28.5	35.7	0.150D	37.4
Gravel and sand	26.0	33.4	0.164D	35.6
Crushed stone and screenings................	29.0	36.1	0.147D	37.8

Then mark the tangent point on this line where it is intersected by the vertical ordinate for one-tenth the maximum diameter stone. This mark should check with the values given in column (2) of above table. Then plot the location of minor axis of the ellipse from the values of a and b + 7, given in columns (3) and (4) in the above table. This point, together with the tangent point and the point at + 7 on the vertical ordinate at zero diameter where the curve begins, gives three points on the ellipse, which is usually sufficient for drawing the curve with the aid of an irregular curve. If more points are wanted, they may be plotted graphically by the trammel point method as given in the note on page 202.

RELATION OF DENSITY TO STRENGTH

Having determined the maximum density curve as just explained, it was important to know if the greatest strength coincided with the greatest density, and for this purpose a large number of beams, six inches square and six feet long, were made up and tested for transverse and crushing strength, for permeability and modulus of elasticity. Some beams were made using the proportions determined by the maximum density curve and other beams according to higher and lower curves to note if there were any decrease in these properties as the maximum density curve was departed from. The full results of the tests are given in the paper referred to,* but in general it may be said that a departure from the maximum density curve represented a reduction in all these properties except that when the curve was modified so as to use a uniform size of coarse stone instead of the graded stone it gave practically the same results as the graded. Any curving above the straight line in the coarse material decreased the density, and also the strength, indicating that the coarse aggregate should not have an excess of medium particles.

LAWS OF PROPORTIONING

From these experiments, laws of proportioning and also laws relating to strength and permeability which are outlined in full in the paper by Messrs. Fuller and Thompson* were evolved.

Those relating specifically to strength are given on page 390 and those relating definitely to permeability on page 349, and reference should be made to these for complete conclusions. The laws relating especially to the grading of the aggregates are as follows:

*See footnote p. 201

1.—Aggregates in which particles have been specially graded in sizes so as to give, when water and cement are added, an artificial mixture of greatest density, produce concrete of higher strength than mixtures of cement and natural materials in similar proportions. The average improvement in strength by artificial grading under the conditions of the tests was about 14 per cent. Comparing the tests of strength of concrete having different percentages of cement, it is found that for similar strength the best artificially graded aggregate would require about 12% less cement than like mixtures of natural materials.

2.—The strength and density of concrete is affected but slightly, if at all, by decreasing the quantity of the medium size stone of the aggregate and increasing the quantity of the coarsest stone. An excess of stone of medium size, on the other hand, appreciably decreases the density and strength of the concrete.

3.—The strength and density of concrete is affected by the variation in the diameter of the particles of sand more than by variation in the diameters of the stone particles.

4.—An excess of fine or of medium sand decreases the density and also the strength of the concrete, as will also a deficiency of fine grains of sand in a lean concrete.

5.—The substitution of cement for fine sand does not affect the density of the mixture, but increases the strength, although in a slightly smaller ratio than the increase in the ratio of cement.

6.—It follows from the foregoing conclusions that the correct proportioning of concrete for strength consists in finding, with any percentage of cement, a concrete mixture of maximum density, and increasing or decreasing the cement by substituting it for the fine particles in the sand or vice versa.*

7.—In ordinary proportioning with a given sand and stone and a given percentage of cement, the densest and strongest mixture is attained when the volume of the mixture of sand, cement and water is so small as just to fill the voids in the stone. In other words, in practical construction, use as small a proportion of sand and as large a proportion of stone as is possible without producing visible voids in the concrete.

8.—The best mixture of cement and aggregate has a mechanical analysis curve† resembling a parabola, which is a combination of a curve approaching an ellipse for the sand portion and a tangent straight line for the stone

* This very important law requires further tests for confirmation, outside of the limits of the present tests.

† For definition of mechanical analysis, see page 193.

portion. The ellipse runs to a diameter of one-tenth of the diameter of the maximum size of stone, and the stone from this point is uniformly graded.

9.—The ideal mechanical analysis curve, *i.e.*, the best curve, is slightly different for different materials. Cowe Bay sand and gravel, for example, pack closer than Jerome Park stone and screenings, and therefore require less of the size of grain which the authors designate as sand.

10.—The form of the best analysis curve for any given material is nearly the same for all sizes of stone, that is, the curve for ½ inch, 1-inch, and 2¼-inch maximum stone may be described by an equation with the maximum diameter as the only variable. In other words, suppose a diagram in which the left ordinate is zero, and the extreme right ordinate corresponds to 2¼-inch stone, with the best curve for this stone drawn upon it. If, now, on this diagram the vertical scale remains the same, but the horizontal scale is increased two and a quarter times, so that the diameter of 1-inch stone corresponds to the extreme right-hand ordinate, the best curve for the 1-inch stone will be very nearly the one already drawn for the 2¼-inch stone. The chief difference between the two is that the larger size stone requires a slightly higher curve in the fine sand portion.

11.—It follows from this last conclusion that from a scientific standpoint the term *sand* is a relative one. With 2¼-inch stone, the best sand would range in size from 0 to 0.22 inch diameter, while the best sand for ½-inch stone would range in size from 0 to 0.05 inch diameter.

APPLICATION OF MECHANICAL ANALYSIS DIAGRAMS TO PROPORTIONING

The mechanical analysis diagram offers a very exact method of determining the proper proportions of any materials for concrete by sieving each of the materials, plotting their analyses and combining these curves so that the result is as near as possible similar to the maximum density curve.

Plot on the diagram the maximum density curve for the given materials to be used; if the equation for this material is not known use the practical equation previously given. Make a mechanical analysis of all of the materials which it is desired to mix together in the right proportions and plot the result of each analysis on the diagram on which the maximum density curve has been plotted.

The aim is to find a new curve representing the mixture of the materials, but which will conform as nearly as possible to the curve of maximum density. The proportions of different materials required to produce this curve will show the relative quantity of each which must be used in proportioning. The theory of the combination and complete discussion of the

methods to be employed with different forms of curves are treated in Appendix IV.

A less exact method, but one which is convenient in practice, is by inspection and trial of different percentages. To illustrate this trial plan, the method of forming a curve of a mixture of several materials in stated proportions such as 1 : 2 : 4 will be given, then the curve for the mixture of the same materials which corresponds nearest to the curve of maximum density, and finally the application will be made to material like run of the bank gravel which may be separated into two or three parts.

In reading this discussion it must be borne in mind that the same principles will apply to mixtures of several aggregates, although for simplicity the principal part of the discussion refers to two aggregates. The same

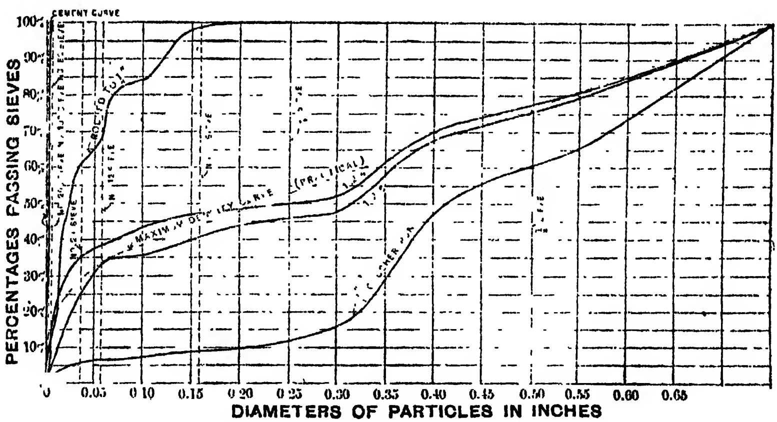


Fig. 73.—Curves of Fine and Coarse Crushed Stone and Mixtures. (p.207)

approximate plan may be used for the larger number of aggregates or the more exact method in the Appendix may be adopted.

Plotting Curve of Mix in Studying Proportions. In Fig. 73 we have $\frac{3}{4}$-inch Shawangunk grit as one aggregate and the same material rolled to $\frac{1}{8}$-inch maximum size as the other, giving the mechanical analysis curves shown in the diagram.*

In this diagram a curve of cement is also plotted so that the 1 : 2 : 4 curve represents the combination of the three materials. The curve marked 1 : 2 : 4 then represents the analysis of the mixture of cement, screenings

* This diagram and the ones which follow are made up from materials used in subsequent studies by the New York Board of Water Supply, and referred to in the Discussion by Mr. James L. Davis, Transactions American Society Civil Engineers, Vol. LIX, p. 144.

and stone in these proportions. This curve is made up by plotting various points and connecting these by a smooth curve. To find the point, for example, where the curve cuts the ordinate corresponding to the No. 20 sieve, the sums of the percentages of the individual materials at this same ordinate are taken in the proportion which they bear to the concrete mixture. All of the cement is finer than the No. 20 sieve, and since the cement is one part of the seven parts in the mixture, one-seventh of 100 per cent represents the percentage of cement in the mixture at the given ordinate. Similarly, since there are two parts of sand in the seven parts, the sand percentage at the No. 20 ordinate, 61 per cent, is multiplied by two-sevenths, and the stone percentage, 6 per cent, by four-sevenths, thus giving as the point on the No. 20 sieve ordinate in the combined curve:

$\frac{1}{7} \times 100$ per cent $= 14.3$ per cent for cement
$\frac{2}{7} \times 61$ per cent $= 17.4$ per cent for sand
$\frac{4}{7} \times 6$ per cent $= 3.4$ per cent for stone

Total......... 35.1 per cent for the point in the curve.

The other points in the curves are found in a similar manner.

Curve of Mix to Best Fit the Maximum Density Curve. Take the same two aggregates plotted in Fig. 73, but in this case disregard the cement or rather consider it a part of the sand. (Frequently the cement must be considered in the trial mixtures in order to study the part of the curve representing the fine material to see that the percentages of the finest particles are satisfactory). The slide rule is convenient for this proportioning.

Averaging the $\frac{3}{4}$-inch stone by a straight line, we see that it crosses the 0.15 line at about 9%; we note also that the $\frac{1}{8}$-inch sand crosses the same line at 98% and the maximum density curve crosses the line at 43%, that is, along this line it is 34% from the $\frac{3}{4}$-inch stone to the maximum density curve and 55% to the $\frac{1}{8}$-inch sand. The percentages to be used to obtain a 43% mixture would be an inverse ratio of these two numbers to their total, that is, $\frac{34}{89} = 38\%$ of fine material and $\frac{55}{89} = 62\%$ of the coarse material. With the slide rule take these percentages of each curve, add together and plot a new curve, and see if it conforms reasonably with the maximum density curve. If it does not, make another trial of percentages, the plot of the curve indicating by inspection the new percentages.

It must be remembered that the fine portion of the curve includes also the cement, so having decided on the amount of cement to use, say the equivalent of a 1 : 7 mix, which has $12\frac{1}{2}\%$ of cement, the actual proportions would be $12\frac{1}{2}$ parts cement to $38 - 12\frac{1}{2} = 25\frac{1}{2}$ parts fine aggregate to 62 parts coarse aggregate, or translated into the usual nomenclature, 1 : 2.04 : 4.95, or practically 1 : 2 : 5, showing that the ordinary mixture with this particu-

lar material is the best. Supposing, however, the equivalent of a richer mixture, say 1 : 2 : 4, is wanted. This would contain 1 : 6 = $14\frac{1}{2}$% cement and the proportions would be

$$14\tfrac{1}{2} : 23\tfrac{1}{2} : 62,$$

or

$$1 : 1.62 : 4.27,$$

or practically

$$1 : 1\tfrac{2}{3} : 4\tfrac{1}{4},$$

showing that for richer mixtures less fine materials is desirable.

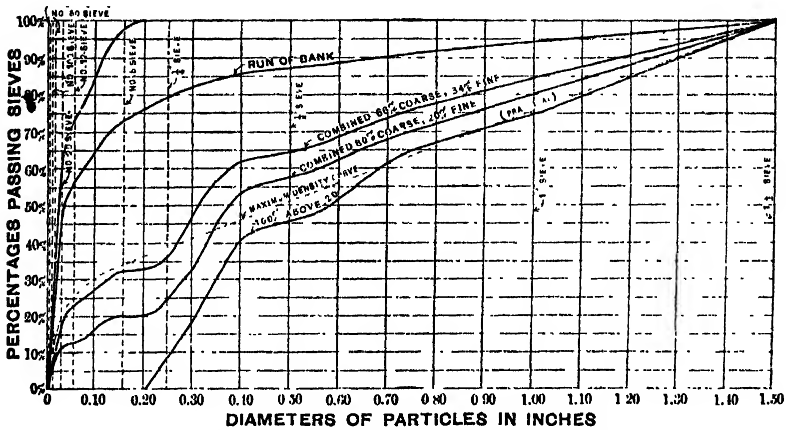


Fig. 74.—Cortland Gravel Screened to Two Sizes. (*see p.* 210.)

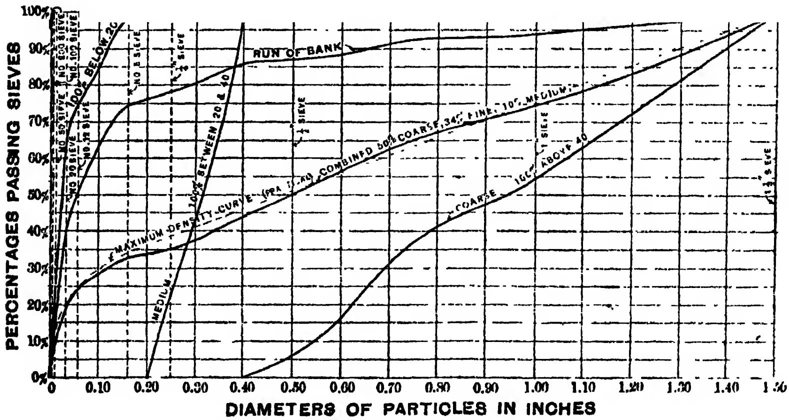


Fig. 75.—Cortland Gravel Screened tc Three Sizes. (*see p.* 210.)

Run of Bank Gravel. Gravel as it is found in the natural bank almost always contains too much fine material. In many cases screening this into two sizes produces a good curve which fits very closely to the curve of maximum density.*

Other gravels, especially where the sand is greatly in excess, require two screenings for the best result. Fig. 74 represents a common run of such gravel, showing that screening into two sizes will not permit a mixture fitting very near to the maximum density curve. The figure also shows how far away the original analysis of the run of the bank is from the ideal curve. In Fig. 75 the same sand is shown screened into three sizes, and illustrates the improvement that can be obtained in this case by the extra screening, the effect of which is to leave out some of the medium size particles which are too large to fill the voids of the coarse stones, and therefore decrease the density and the strength of the mixture.

VOLUMETRIC SYNTHESIS OR PROPORTIONING BY TRIAL MIXTURES

The density tests at Jerome Park and the relation there found of the strength to the density indicate a method of proportioning by trial mixtures, which in fact compared the density of the same materials mixed in different proportions or different materials mixed in similar proportions.

Having determined the particular sand and stone which are to be used on any piece of work, a simple and accurate way of determining proportions is by actual trial batches of fresh material. For this it is only necessary to have good scales and a strong and rigid cylinder, say, a piece of 10-inch wrought-iron pipe capped at one end. Carefully weigh out and mix together on a piece of sheet steel or other non-absorbent material all the ingredients, having the consistency the same as is intended to be used in the work. Place these in the pipe, carefully tamping all the time, and note the height to which the pipe is filled. Weigh the pipe before filling and after being filled, thus checking weight of material mixed. Throw this material away before it has time to set, and clean the pipe. Make up another batch, using the same weights of cement and water and the same total weight of sand and stone, but have the ratio of weights of the sand and stone slightly different from the first. Note whether, after placing, the height in the cylinder is less or more than was the height of the first batch, and this will be a guide to further similar mixes, until a proportion is found which gives the least height in the cylinder, and at the same time works

*An illustration of this is given by Mr. James L. Davis, in Transactions American Society of Civil Engineers, Vol. LIX, p. 145.

well while mixing and looks well in the cylinder, all the stones being covered with mortar. This method, if carefully followed, will give very accurate results, but of course does not indicate, as does mechanical analysis, what other changes can be made in the physical sizes of the sand and stones so as to get the best available composition.

Mr. A. E. Schutté, in studying the proportions of materials for bituminous macadam pavement for the Warren Brothers Company, has very effectively developed the method of volumetric synthesis with dry materials. His experiments included various classes and sizes of stone, sand, and screenings ranging from 3 inches diameter down to that which passes a No. 200 sieve. He found that the best method for compacting dry materials, such as sand, gravel or broken stone, is to place them in a vessel the shape of a truncated cone, with the largest diameter at the bottom. The cone is filled with the coarsest material and taken by a laborer, who compacts it by repeatedly striking the cone against the ground, keeping the measure full by adding new material of the same kind. When it ceases to settle, the contents is emptied and mixed with a portion of a finer material, replaced in the measure and compacted as before. By repeated trials the exact size and maximum volume of successive finer materials, which may be added without appreciably increasing the bulk of the coarsest after thoroughly compacting, are determined. Mr. Schutté has found that for different shapes of particles the proportions of each size must be varied, but having determined the required percentages for a certain stone, that is, for a stone from a certain quarry, the proportions of the sizes from day to day need be varied but little.

Practical Proportioning During Progress of the Work. The above methods of mechanical analysis and volumetric synthesis are methods to be used in the office or laboratory in determining the relative values of all the aggregates available for the work. When the work is begun, however, and the same general character of aggregate is used day by day, it is only necessary to see that the material does not change or, if it does, simply to readjust the relation between the fine and coarse aggregate. To do this by the mechanical analysis method, it is only necessary to have a nest of about six 8-inch sieves: say, stone sieves with 1 inch, ½-inch and ¼-inch diameter holes and sand sieves No. 8, 20, 50 and 100, together with a cover and pan. The shaking can be done by hand, and the sievings beginning with the finest emptied into a long glass tube. If a standard sample has been previously put in the tube in the same way and the points of division between the different sievings marked on a paper pasted on the outside of the tube, the difference between the standard and the sample under test can be quickly seen and modifications made in the mix accordingly.

Proportions in Actual Structures.

Compiled by TAYLOR AND THOMPSON.

Structure.	Nominal Proportions.	Portland Cement bbl of 380 lbs.	Loose Sand cu. ft.	Loose Stone cu ft.	Volumes based on nominal or actual measurement.	Authority.	Reference.
New Brooklyn Bridge Piers		1	8.5	19.5	nominal	Asst. Engineer	
Boston El. Ry. Column Foundations	1:2½:5*	1	9.5	19.1	nominal	G. A. Kimball	Jour. A. E. S. June '03, p. 353
N. Y C. & H R. R. R.							
Footings	1:4:7½†	1	13.9	26 2	actual	W. J. Wilgus	Assn. of Ry. Supts. 1900, p. 207
Abutments	1:3:6†	1	12 2	23.7	actual		
Facing Old Masonry	1:2:4	1	7.0	14.0	actual		
Coping and Bridge Seats	1:1:2	1	3.5	7.1	actual		
C. M. & S. P Ry.							
Piers and Abutments	1:2:5	1	7.8	2: 4	actual	W. A. Rogers	Assn. of Ry. Supts. 1900, p. 228
Culverts and Foundations	1:3:7½	1	10 5	28.5	actual		
Or R. R & Nav. Co.							
Abutments, Piers and Culverts	1:3:5	1	11.0	18.3	nominal	W. H. Kennedy	Assn of Ry. Supts. 1900, p. 182
Foundations and Light Buildings	1:3½:6	1	12.8	22.0	nominal		
	1:4:7	1	14.7	25.7	nominal		
C. & E. I. R. R.	1:2:5	1	9.3	26 7	actual	A. S. Markley	Assn. of Ry. Supts. 1900, p. 245
Northern Pacific Ry.							
Foundations	1:3:5‡	1	11.2	20.2	actual	E. H. McHenry	Assn. of Ry. Supts 1900, p. 235
Abutments and Piers	1:3:5	1	11.2	20.2	actual		
C., B. & Q. R. R.	1:3:6	1	12.5	22.5	actual	Fred Eilers	Assn. of Ry. Supts. 1900, p. 231
Mexican Central Ry.	1:3:6	1	13.5	27.0	nominal	Lewis Kingman	Assn. of Ry. Supts. 1900, p. 212
N. Y. Subway						N.Y.R.T Com.	Spec. 1900, p. 83
Roofs and Sidewalls not over 18 in. thick	1:2:4	1	7.2	14.4	nominal		
Sidewalls or Tunnel Arches	1:2½:5	1	9 0	18.0	nominal		
Wet Foundations not over 24 in. thick	1:2:4	1	7 2	14.4	nominal		
Wet Foundations exceeding 24 in. thick	1:2½:5	1	9 0	18.0	nominal		
Boston Subway	1:2½:4	1	8.3	13.2	nominal	H. A. Carson	
Harvard University Stadium	1:3:6						
Maine Fortifications							
Leveling for Foundations	1:5:10	1	18.2	36.5	nominal	S. W. Roessler	Report Chief of Engrs. U. S. A. 1901, p. 911
Walls and Masses not exposed to fire	1:4:8	1	14.6	29.2			
Walls and Masses exposed to fire	1:3:6	1	11.0	22.0	nominal		
Masses for greater imperviousness	1:3:5	1	11.0	18.3	nominal		
Little Falls						W. B. Fuller	
Mass Concrete	1:3:7	1	11.4	26 6	nominal		
Tanks, Buildings, etc.,	1:2:4	1	7.6	15.2			
Duluth Ship Canal Piers		1	11.8	23 8	nominal	C. Coleman	Cement, Sept., '00, p. 144
Boonton, N. J., Dam	1:2¾:6½§	1	10.5	23.8		W. B. Fuller	
Genesee Dam	33% mortar	1	11.4	36 8	nominal	Geo. W. Rafter	
Buffalo Breakwater		1	5	30‖	nominal	Emile Low	Trans. A. S. C. E. Vol. LII. p. 102
Pennsylvania Tunnel	1:2½:5	1	9.6±¶	19.3±¶	nominal	Specifications	Eng. News. Oct. 15, '03, p. 337
East Boston Tunnel	1:2½:4	1	7.7	12.4	nominal	H. A. Carson	Specifications, 1900

* Mixture varied with loading from 1:1:3 to 1:3:6. † 25% of the mass is rubble.
‡ Boulders added. § 55% of the mass is rubble.
‖ 15 cu. ft. gravel and 15 cu. ft. broken stone Actual volumes of aggregates, 25% higher.
¶ The specifications give proportions in volumes shaken, hence 10% has been added to convert them to loose measurement.

The test by volumetric synthesis is one easily made in a modified way in the field and with care gives good results. Procure a galvanized tin pail and a spring balance graduated to half pounds; take a representative sample of concrete, being careful that it contains no more stones or mortar than the regular concrete; tamp it into the pail until level full and weigh. Any variation from the standard weight will show a change in the character of material, and this change can usually be detected and corrected by observing the materials and mixing. If not, then mechanical analysis methods will have to be used.

PROPORTIONS OF CONCRETE IN PRACTICE

The proportion of cement to the aggregate depends upon the nature of the construction and the required degree of strength or water-tightness as well as upon the character of the inert materials. Strength and impermeability are discussed in Chapters XX and XIX respectively, but the table which follows, compiled by the authors, giving the proportions adopted upon important structures, may in some cases be useful as an arbitrary guide. Actual measurement, that is, measurement of proportions as actually used, almost invariably shows leaner mixtures than the nominal proportions called for. This is largely due to the heaping of the measuring boxes in practise.

In general, as both strength and imperviousness increase with the proportion of cement to aggregate, relatively rich mixtures are necessary for loaded columns and beams in building construction, for thin walls subjected to water pressure, and for foundations laid under water.

* Pages 214 and 215 are omitted in this Edition.

CHAPTER XII

TABLES OF QUANTITIES OF MATERIALS FOR CONCRETE AND MORTAR

This chapter presents tables, curves, and formulas (pp. 221 to 235), by which the volumes of materials required for a known volume of concrete may be estimated, and emphasizes the importance of distinctly stating the proportions (p. 217).

The volume of concrete, even when made from materials in the same proportions, varies largely with the character of the materials and the methods of placing it. A mixed aggregate like gravel contains fewer voids and with the same proportions by volume of the same cement and sand produces a larger quantity of concrete than a screened broken stone. The fineness of the sand also largely affects the volume of the concrete and mortar, a fine sand requiring more water, and therefore producing a larger volume of mortar than coarse sand in the same proportions by volume. If the sand is dry, a slightly larger bulk of mortar is produced than with the same sand when containing a larger percentage of moisture, because the latter is less compact (see p 176) Some cements require more water in gaging than others, and produce a larger amount of paste, which increases the volume of the concrete or mortar The method of mixing and placing the concrete also affects the resulting volume, since an imperfectly mixed or poorly compacted mass contains voids which increase the volume. An excess of water in mixing affects the resulting volume of the set concrete or mortar to a slight extent, although most of the surplus water is expelled during setting.

It is possible to provide for all these variations, except those relating to improper mixing and placing, in rational formulas from which the resulting volumes may be accurately estimated if the characteristics of all the materials are known. For most practical purposes, however, average values, such as are presented in the tables and curves, are sufficiently accurate for estimating quantities. These average values are based upon a large number of tests in the United States, France, and Germany.

The theory of a concrete mixture is discussed, and formulas for volumes and quantities are given on pages 220 to 227 preceding the tables.

EXPRESSING THE PROPORTIONS

In framing concrete specifications, the proportions of the constituents should be stated so distinctly that there can be no misunderstanding between the engineer and the contractor as to the quantities which will be required for the work. The quantity of cement should invariably be regulated by its weight; if the proportions are stated by volume a definite weight or number of packages of cement must be assumed to the unit volume. For reasons discussed in Chapter XI, it is also more accurate and scientific to measure the aggregates by weight than by volume, and since with a properly constructed plant using materials of several sizes, the cost need be no more than volume measure, the authors believe this will eventually become common practice in the case of important construction.

With our present system of weights and measures, it is advisable either to specify the number of cubic feet (or pounds) of sand and gravel, stone, or mixed material to a definite weight of cement, or else to stipulate a definite weight of cement to a cubic yard of concrete tamped in place, with an aggregate of clearly described material proportioned as the engineer may direct.

In stating the proportions for both mortar and concrete, it is now customary in the United States to separate the materials by colons, the first figure always representing the cement, followed by the aggregates in the order of the size of their grains. For example, 1: 3: 6 means 1 part cement (the unit of measurement should be stated), 3 parts sand, and 6 parts coarse material; or 1: 8 means 1 part cement (of defined weight) to 8 parts of graded aggregate. Mortar in proportion 1: 2 signifies one part cement to two parts sand by either weight or volume as specified.

In France, proportions are stated as one or more volumes of mortar to a definite number of volumes of stone, — "un volume de mortier pour deux volumes de cailloux."

Unit for Proportioning. If the proportions must be stated in parts, it is recommended that the weight of cement be assumed as 100 lb. per cubic foot, and the corresponding volume of a barrel as 3.8 cu. ft. By this system of units, proportions 1: 3: 6 would represent 100 lb. cement to 3 cu. ft. of sand to 6 cu. ft. of gravel or stone; or, 1 bbl. cement (*i.e.*, 4 bags or 376 lb.) to 11.4 cu. ft. sand to 22.8 cu. ft. gravel or stone.

The authors offer these recommendations after correspondence or personal interview with some fifty authorities* (members of the American

*See Preface.

Society of Civil Engineers) on concrete construction, representing all sections of the United States.

With reference to the unit which should be selected for the volume of a cement barrel (corresponding to 376 lb. Portland cement) the opinions were varied, but nearly every authority advocated specifying a definite weight of cement instead of measuring it loosely by volume. The units which met with the most favor were 3.5, 3.6, 3.8 and 4.0 cu. ft. The advocates of the first two values based their figures upon the measured volume of a cement barrel, while those selecting the last two did so on the presumption that the unit is an arbitrary one in any case, and 100 lb. per cubic foot, or 95 lb. per cubic foot (the latter equivalent to 1 cu. ft. to the bag), is convenient for calculation. An approximate average of all the figures suggested was 3.8 cu. ft. to the barrel, corresponding to 100 lb. per cubic foot, the advocates of this value being, among others, Messrs. Charles E. Fowler, William B. Fuller, Peter C. Hains, Allen Hazen, Rudolph Hering, George A. Kimball, Leonard Metcalf, J. Waldo Smith, and J. H. Wallace. Accordingly, in cases where it is advisable to specify the proportions by parts, the authors have adopted this unit as their standard.

When stating the proportions by volume, too much stress cannot be laid upon the necessity for the adoption of a standard unit, such as a barrel of 3.8 cu. ft. or the equivalent assumption that a cubic foot of cement weighs 100 lb., and upon distinctly specifying this standard, as otherwise an unscrupulous contractor may adopt for his unit the volume of cement very loosely measured, and thus produce too lean a concrete. Moreover, without a standard there is no means of comparing the concrete in different structures or the results of different experiments. It is even inaccurate to state that proportions shall be based on packed or on loose measurement of cement, for either of these terms is very elastic. The authors have personally known engineers to place the volume of a barrel of packed cement all the way from 3.1 to 3.8 cu. ft., corresponding to a variation in weight of from 123 to 100 lb. per cubic foot, while loose measurement, on the other hand, is variously fixed at from 3.8 to 4.5* cu. ft. to the barrel, or 100 to 84½ lb. per cubic foot. The extreme actual variation is therefore from 3.1 to 4.5 cu. ft. per barrel, or 123 to 84½ lb. per cubic foot. Proportions 1:3:6 in the first case would require 1 bbl. or 376 lb. cement to 9.3 cu. ft. of sand and 18.6 cu. ft. of gravel; in the last case, proportions 1:3:6 would stand for 1 bbl. or 376 lb. cement to 13.5 cu. ft. of sand and 27 cu. ft.

*This value is given by one engineer in Proceedings Association of Railway Superintendents of Bridges and Buildings, 1900, p. 212.

of gravel. In other words, concrete mixed 1 : 3 : 6 by one man may be called 1 : 4½ : 8½ by another.*

It may be contended that this variation is of little moment provided the unit is distinctly stated. The fact is, however, that it is customary in discussing a piece of work to give the proportions of materials without stating the unit selected, and many records giving tests of strength of concrete do not even specify the units used in proportioning the ingredients. It is especially confusing also, to a contractor who is not very careful in

Tests of Capacity of Portland Cement Barrels and Weight of Contents.

(Tabulated by the authors from measurements of Boston Transit Commission, 1896, Howard A. Carson, Chief Engineer.) (*See p.* 219.)

No. of barrels tested results averaged	Brand	Height between heads	Average diameter of barrel	Average horizontal area	Capacity of barrel between heads	Depression of cement below head	Volume of depression	Volume of cement per barrel			Net weight of cement per barrel		Weight per cubic foot				Weight of barrel
								Packed	Loose	Shaken*	Before dumping	After dumping	Packed	Loose	Shaken	Sifted	
		ft.	ft.	sq.ft.	cu.ft.	ft.	cu.ft.	cu. ft.	cu. ft.	cu. ft.	lb.	lb.	lb.	lb.	lb.	lb.	lb.
5	A	2.12	1.437	1.622	3.446	0.17	0.235	3.21	3.75	3.432	377.4	376.9	117.5	100.5	109.4	90.6	21.1
6	B	2.19	1.430	1.605	3.495	0.12	0.171	3.35	4.17		381.0		113.8	91.4			29.0
3	C	2.07	1.412	1.571	3.249	0.07	0.096	3.15	4.05		387.0		112.8	94.2			22.7
5	D	2.01	1.407	1.554	3.123	0.07	0.093	3.03	3.99	3.522	373.2	371.4	123.2	93.2	105.5		25.6
6	E	2.08	1.403	1.546	3.219	0.04	0.059	3.16	4.19		374.2		118.4	89.2			24.3
1	F	2.13	1.38	1.496	3.186	0.03	0.039	3.15	4.27	3.695	378.0	378.0	120.1	88.5	102.3		22.0
5	G	2.01	1.46	1.662	3.327	0.10	0.148	3.21	4.06	3.598	370.7	370.2	115.7	91.4	102.9	80.3	23.3
Final Averages		2.09	1.42	1.579	3.292	0.09	0.120	3.18	4.07	3.562†	377.4	374.1†	118.8	92.6	105.1†	85.4†	24.0

NOTE.—A and B are American Cements; C. D E and F are German Cements; G is a Danish Cement; Paper weighs about 1 lb.
*Box rocked over bar.
†Partial averages, to be compared only with like brands.

reading specifications, to find that, say, 25% or 30% more cement than he had figured is required to a cubic yard of concrete. When considering this question, the authors were surprised to find that the sidewalk and paving specifications of fifteen of the largest cities in the United States failed to state the proportions by definite weight or volume, but gave the quantities simply in "parts," a few of them adding that the parts shall be "by measure" or "by exact measure."

Weight of Cement. Experiments by Mr. Howard A. Carson, for Boston Transit Commission, upon 31 barrels of Portland cement of

*For further data, see letter of Sanford E. Thompson to *Engineering News*, Nov. 12, 1903, p. 434.

American and foreign brands, furnish an interesting illustration of the difference in weight of the same cement in different stages of compactness. The results,* a summary of which is presented in the table on page 219, show a variation from 86 to 118 lb. in the average weights of the same cement, according as it was weighed sifted, or packed in a barrel, while the actual weight of one brand, the average of 5 barrels, was as high as 123 lb. per cubic foot as it came from Germany packed in a barrel.

From the experiments just described, the ratios of volume and weight of the same cements in different degrees of compactness are calculated by the authors as follows:

Ratio of volume of packed cement to capacity of barrel between heads	0.97
Ratio of volume packed to volume loose	0.78
Ratio of volume packed to volume shaken	0.88
Ratio of volume loose to volume shaken	1.13
Ratio of weight packed to weight loose	1.28
Ratio of weight packed to weight shaken	1.13
Ratio of weight packed to weight sifted	1.37

From the table it is evident that the selection of the volume of a barrel is arbitrary. The adopted volume of 3.8 cu. ft. is convenient for calculation because it assumes a cubic foot of cement to weigh approximately 100 lb.

THEORY OF A CONCRETE MIXTURE

The discussion and the formulas which follow relate to plastic mortars and plastic or medium concrete. While a small amount of water in mixing may result, with heavy ramming, in a concrete or mortar of less than average volume, in practice the volume is more apt to be increased by lack of water because of the less perfect mixture and the visible voids. The volume of set concrete or mortar produced by a very wet mixture is approximately the same as that of a plastic mixture, because nearly all of the surplus water is thrown to the surface and expelled by the settling of the solid materials. This the authors have repeatedly proved by experiment.

The frequently repeated assertion that a very wet mixture contains visible air voids because of the drying out of the water is incorrect. This may be proved by carefully pouring neat cement grout into a rectangular mold, one of whose sides is formed by a piece of glass. The surplus water is expelled, and the specimen after setting is dense and glassy with no visible voids. The large visible voids which sometimes occur in very wet

*Tabulated by Sanford E. Thompson in *Engineering News*, Oct. 4, 1900, p. 229.

concrete, similar in appearance to visible voids in dry concrete, are due to the grout running away from the stones, or to too violent agitation in placing.

The volume of fresh concrete or mortar produced by any mixture of cement and aggregate or aggregates is equal to the sum of the volumes of the *separate particles* of the cement, the sand, and the other dry materials, the water contained in the aggregate and added in mixing, and the small volume of air entrained between the particles. The volume of set mortar or concrete is not appreciably different from its compacted volume when fresh or green, except in very wet mixtures, which expel a portion of the water. The volumes of the *particles* of dry materials are termed *absolute volumes*, and it is important to note the distinction between the absolute volumes and the apparent volumes determined by measuring the materials. Absolute volumes are discussed on pages 135 to 139.

The fact that water actually occupies space in a mass of fresh concrete or mortar has been entirely ignored by many writers on the subject of concrete mixtures. As stated on page 216, the fineness of the sand and the moisture contained in it affect the volume of the resulting concrete or mortar. Mr. Feret has proved by experiments (cited on page 179) that fine sands require more water for gaging than coarse. This extra volume of water produces a mortar of less density and consequently less strength; even stones such as are found in gravel or coarse broken stone require a very small percentage of water.

FORMULAS FOR QUANTITIES OF MATERIALS AND VOLUMES

A concrete is therefore made up of solid grains of cement plus water required for the cement, plus solid grains of sand plus water required for the sand, plus solid stone particles plus water required for the stone, plus air voids. The last term, the *air voids*, represents the voids entrained by the sand, which may be considered as a function or percentage of the sand, and the voids due to imperfect mixing of the concrete materials, which may be considered a function or percentage of the stone. Accordingly the volume of a concrete mixture may be expressed as a rational formula, which is applicable to all concrete and mortar mixtures in which the voids of the coarse stone are filled with mortar. The formula (1) which follows is presented to illustrate the theory, but because of the variation in the coefficient with different sands and different proportions, formula (2), page 222, and formulas (3) to (8), which are based on average conditions, are suggested for practical use as sufficiently accurate for most purposes.

Let

c = absolute volume* of cement.

s = absolute volume* of sand.

g = absolute volume* of stone.

m = ratio of the absolute volume of the water plus air voids of the cement, to the absolute volume of cement.

n = ratio of the absolute volume of the water coating the grains of sand plus the air entrained in gaging it, to the absolute volume of sand.

p = ratio of the absolute volume of the water coating the stone particles plus the air voids due to imperfect mixing, to the absolute volume of stone..

W = volume of concrete produced.

In other words, these ratios, m, n, and p, represent the sum of the volumes occupied by the water required for the material in mixing plus the air, in terms of the respective volumes of cement, sand, and stone.

Then

$$W = c + mc + s + ns + g + pg$$

or

$$W = (1 + m)\,c + (1 + n)\,s + (1 + p)\,g \qquad (1)$$

The coefficient n is really composed of two variables, one depending upon the coarseness of the sand, and the other upon the ratio of cement to sand, since a lean mortar contains more air voids. It is possible to express this coefficient as a more complex term with this ratio as a factor, but by what appears to be a peculiar coincidence, experiments show that for ordinary bank sand the variation in voids caused by different proportions may be provided for by taking the cement and sand together; in other words, for different proportions of the same cement and sand, the sum of the water and the air voids in the mortar is approximately a constant. Where there is no sand, or where the stone and sand are mixed, formula (1) must be employed.

The more practical formula may be expressed as follows, employing similar notation to that given above, and letting

r = ratio of the absolute volume of the water plus the air entrained in gaging, to the absolute volume of cement plus sand,

then

$$W_1 = c + s + r\,(c + s) + g + pg$$

or

$$W_1 = (1 + r)\,(c + s) + (1 + p)g \qquad (2)$$

*Absolute volumes are defined on p. 135.

Substituting average values for r and p, which the authors have selected by analyzing the results of a number of exact records in the United States and Europe of the volumes of concrete and mortar, the formula becomes

$$W_1 = 1.34\,(c + s) + 1.08\,g \tag{3}$$

The comparison of this formula with actual experiments is shown on page 227. The formula may be readily reduced to practical working form if the characteristics of the cement, sand, and stone are known. The cement may be expressed in pounds by substituting for the absolute volume, c, the number of pounds of cement divided by its specific gravity (which may be taken as 3.1) times the weight of a cubic foot of water (62.3 lb.). It may also be expressed in barrels by substituting for the absolute volume, c, the number of barrels, B, multiplied by the net weight per barrel, 376 pounds, and divided, as above, by the specific gravity times the weight of a cubic foot of water [see formula (4)]. The terms relating to sand and stone may be expressed in pounds in a way similar to that just shown for cement, or they may be expressed in measured volume by substituting for the absolute volume, s or g, the measured volume, S or C, multiplied by the proportion of solid material contained in it. Expressing this algebraically, if

Q = quantity of concrete made with B barrels cement,
Q_1 = quantity of concrete made with one barrel cement,
B = number barrels cement,
B_1 = number barrels cement per cubic yard of concrete,
S = volume of loose sand in cubic feet,
S_1 = volume of loose sand in cubic yards per cubic yard of concrete,
G = volume of broken stone or gravel or cinders in cubic feet,
v = absolute voids in sand determined by weight method (p. 166),
v' = absolute voids in stone determined by weight method (p. 167),

then from formula (3), since $c = B \dfrac{376}{3.1 \times 62.3}$.

$$Q = \frac{1.34 \times 376}{62.3 \times 3.1} B + 1.34\,(1 - v)\,S + 1.08\,(1 - v')\,G$$

$$Q = 2.61\,B + 1.34\,(1 - v)\,S + 1.08\,(1 - v')\,G \tag{4}$$

The volume of concrete in cubic feet made by one barrel of cement, assuming that a cubic foot of average loose, moist sand contains 89 pounds of dry sand, and that its specific gravity dry is 2.65, is,

$$Q_1 = 2.61 + 0.723\,S + 1.08\,(1 - v')\,G \tag{5}$$

This formula is applicable to average concrete made with Portland cement of good quality, coarse bank sand measured loose and containing ordinary moisture, and any broken stone or gravel of known voids. Formula (5) has been used in compiling tables on pages 233 to 235, except in the first twelve proportions, which contain no sand.

If the volume of concrete made from a barrel of cement plus the sand and other aggregate which accompanies it is known, the number of barrels of cement per cubic yard is readily calculated. In formula (5); Q_1 represents the number of cubic feet of concrete made with one barrel cement, hence the number of barrels cement per cubic yard of concrete is 27 divided by Q_1

$$B_1 = \frac{27}{Q_1} \tag{6}$$

Assuming a cubic foot of average sand to contain 89 pounds of dry sand produces the formula employed in calculating tables on pages 230 to 232, and substituting in formula (6) the value of Q_1 from formula (5),

$$B_1 = \frac{27}{2.61 + 0.723\,S + 1.08\,(1 - v')\,G} \tag{7}$$

The formulas may be expressed in parts by volume (such as 1 : 2 : 4) by multiplying the coefficient of S and G by the assumed volume of a barrel, say by 3.8

Knowing the number of barrels of cement, B_1, per cubic yard of concrete, the number of cubic yards of sand per cubic yard of concrete, S_1, is evidently

$$S_1 = \frac{B_1 \times \text{quantity sand in cubic feet per barrel of cement}}{27} \tag{8}$$

The quantity of stone is similarly obtained.

If two or more coarse materials, such as broken stone and gravel, are used, they must be mixed in the selected proportions, before weighing, to determine their voids

In mortars of extremely fine sands the density $(c + s)$ is apt to be about 0.60 (see Feret's table, sand C, p. 136) and the coefficient of first term of formula (3) becomes $\frac{1.00}{0.60} = 1.67$ instead of 1.34. In plastic mortars of standard Ottawa sand the density $(c + s)$, by tests of the authors, averages about 0.71, hence the coefficient becomes $\frac{1.00}{0.71} = 1.41$ instead of 1.34. Substituting these values, or any others which may be obtained by

experiment, in formula (2), the working formulas which follow it may be readily deduced. It is evident from the variation in the coefficient with different sands, that the variation in volume of mortar and concrete obtained by different experimenters is due chiefly to the difference in the materials employed.

The coefficient of $(c + s)$ is also affected, though to a less degree, by the character of the cement, some cements requiring more water than others and therefore producing a greater bulk of paste for a given weight of cement.

In concrete mixtures of cement and coarse stone, with no sand or screenings, formulas (2) to (8) are inapplicable because apparently the air voids do not increase with the leanness of the mixture until the point is reached at which the paste fails to fill the voids in the stone. It is therefore necessary to go back to formula (1), page 222. Since s is zero, the formula becomes

$$W_2 = (1 + m)c + (1 + p)g \qquad (9)$$

An average value of $(1 + m)$ for a first class American Portland cement has been found by experiment to be 1.65. It varies with the quantity of water required to gage the cement to such a consistency that the voids will be filled, but no free water will exist upon the surface. The selected value, assuming 1% voids in the paste, corresponds to 20% of water by weight. The value of $(1 + p)$ is usually 1.04 to 1.08. An average formula for a concrete of cement and coarse stone may thus be taken as

$$W_2 = 1.65c + 1.08g \qquad (10)$$

which is readily reduced to practical forms by the method adopted in evolving formulas (4) to (8) from formula (3).

If the stone is a mixture of sand and gravel, or broken stone and screenings, the coefficient of g must be increased and a figure selected whose value depends upon the relative proportion of fine and coarse material.

TABLES AND CURVES OF QUANTITIES OF MATERIALS AND VOLUMES

Tables on pages 229 to 235 are calculated from formulas (5), (6), (8), and (9). These formulas are used not merely because of their theoretical worth, but because, as stated on pages 216 and 227, the results from them agree with actual experiment.

The values are average values of sufficient exactness for practical use, although, as suggested on pages 222 and 224, variations in the

quality of the materials largely affect the resulting volumes, especially of the mortar.

The tables on pages 231 and 234 are recommended for general use in determining the quantities of materials for concrete, or the volume of concrete made with known materials, and where the percentage of voids in the coarse aggregate is unknown the 45% columns should be adopted. The curves on page 228 are also in convenient form for practical use.

All except the first item in the table on page 229 and the first 12 items in tables on pages 230 to 235 are calculated from formulas (5), (6), and (8), page 223, with the assumption there outlined. The broken stone in the first twelve items in the concrete tables, pages 230 to 235, except where the voids are 40% or over, is assumed to contain fine material, and the coefficient selected for g, formula (9), varies from 1.08 for 50%, 45%, and 40% voids to 1.14 for 20% voids.

Use of Curves. The use of the curves on page 228 is best illustrated by the following examples:

Example 1. — Find quantities of materials required for 1 000 cubic yards 1:2½:5 concrete.

Solution. — Intersection of dotted horizontal line corresponding to 2½ barrels sand with dotted vertical line corresponding to 5 barrels stone falls on diagonal curve 1.30; hence, 1.30 barrels cement are required per cubic yard, or 1 300 barrels cement for 1 000 cubic yards concrete. From Note 4 of diagram $1\,300 \times 0.141 \times 2\frac{1}{2} = 460$ cubic yards sand will be required, and $1\,300 \times 0.141 \times 5 = 920$ cubic yards stone required.

Example 2. — Find number of barrels cement required for 1 000 cubic yards concrete in proportions one barrel cement to 9 cubic feet sand to 18 cubic feet stone.

Solution. — Intersection of full cross section horizontal line corresponding to 9 cubic feet sand with vertical line for 18 cubic feet stone gives 1.37 barrels cement per cubic yard or 1 370 barrels for 1 000 cubic yards concrete.

Example 3. — Find volume of concrete of Example 1 made from one barrel of cement.

Solution. — By Note 5 of diagram volume of concrete per barrel cement is 27 divided by the quantity of cement per cubic yard of concrete, or $\frac{27}{1.30} = 20.8$ cubic feet.

Comparison of Table Values with Actual Experiments. Comparatively few experimenters have recorded complete data with reference to the materials entering into their specimens of concrete and mortar. The most comprehensive records of this nature that have come to the knowledge of the authors are those by Mr. William B. Fuller,* which are tabulated in full on page 258, his proportions ranging from 1:0 to 1:6:10. The actual volumes obtained by him, having been found to agree closely with other carefully made experiments, are used in the determination of the constants employed in the above formulas and in compiling the tables and curves on pages 228 to 235. Volumes calculated from the formulas employing these constants agree with Mr. Fuller's tests with an average variation of 0.2 of 1% and a maximum variation of 6%.

Other records which have been compared with results calculated by our formulas, and with which they usually agree within less than 5% after making allowance for different materials and units, are those by Messrs. George W. Rafter,† Edwin Thacher,‡ J. E. Howard,§ E. Candlot,‖ and E. S. Wheeler,¶ C. A. Matcham,** E. S. Larned†† and Leonard Metcalf.††

Experiments by Mr. Edwin Thacher show the rammed volume of dry facing mortar (that is mortar mixed with a small proportion of water) to be about 12% less than the volume of slush mortar made from the same materials, and the quantity of cement per cubic yard to be correspondingly greater for the dry mortar.

The volume of mortar or concrete is affected by the character of the cement as well as by the sand and method of mixing, since some cements require more water and will make more paste to a unit weight of cement than others even of the same class. In one series of experiments, for example, 85 pounds of a certain first-class American Portland cement were required to make one cubic foot of paste, while for another standard American Portland cement of a different brand 107 pounds were required. Average values for wet or plastic mortars are given in the table on page 229.

*See page 261.

†Transactions American Society of Civil Engineers, Vol. XLII, p. 104.

‡Johnson's "The Materials of Construction," 1903, p. 610a.

§Tests of Metals, U. S. A., 1899, p. 786.

‖Ciments et Chaux Hydrauliques, 1898, p. 446.

¶Report Chief of Engineers, U. S. A., 1895, pp. 2922 to 2931.

**Engineering Record, April 15, 1905, p. 434.

††Personal Correspondence.

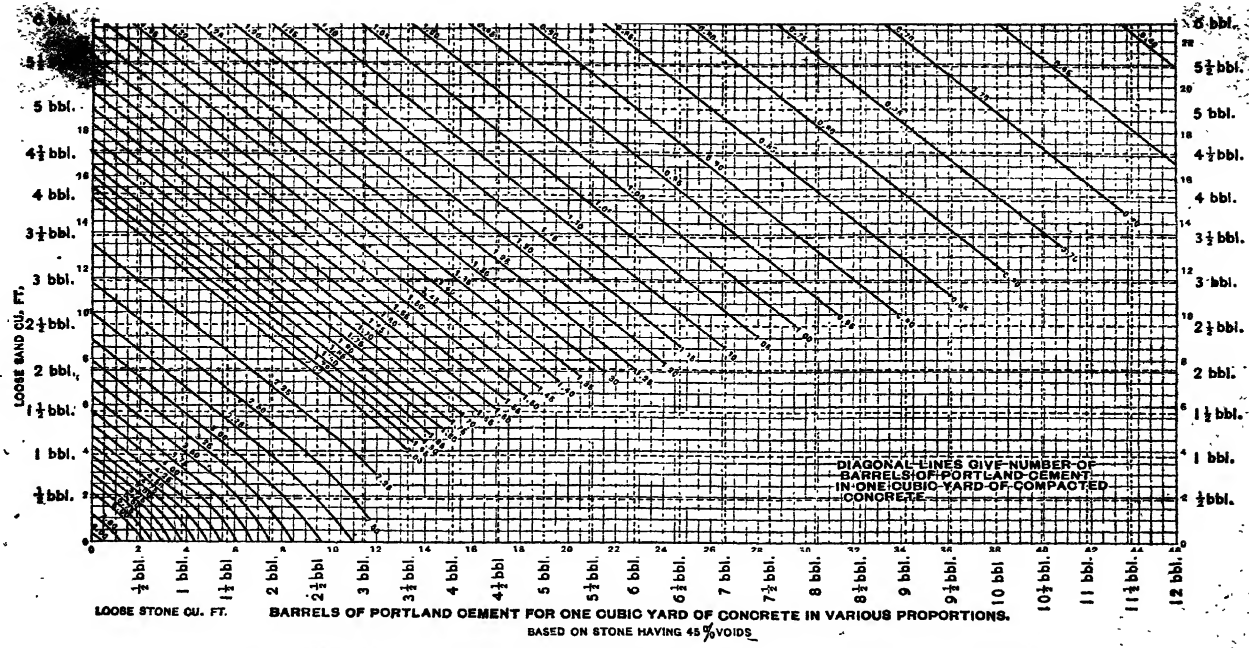


FIG. 76. (See important rules below, also examples on page 226, and formula (7) on page 224.)

1. ***Dotted cross section lines*** represent barrels or parts of sand and stone to one barrel (4 bags) Portland cement weighing 376 lb. and measuring (assumed) 3.8 cu. ft.

2. ***Full cross section lines*** represent cubic feet of sand and stone to one barrel (4 bags) Portland cement.

3. ***Diagonal lines*** represent number of barrels of Portland cement in one cubic yard of compacted concrete.

4. ***To find number of cubic yards of sand or stone*** per cubic yard of concrete, multiply number of barrels cement, as above, by 0.141 times the number of parts of sand or stone.

5. ***To find number of cubic feet of concrete,*** in any proportions, made from one barrel of cement, divide 27 by the number of barrels of cement per cubic yard, obtained as above.

MORTAR WITH ORDINARY COARSE BANK SAND

Volume of Plastic Mortar and Quantities of Materials per Cubic Yard. (See p 226)

Relative proportions by volume*		Volume of Compacted Plastic Mortar						Materials for 1 cu yd Compact Plastic Mortar Based on barrel of					
		from 1 cu. ft Cement, Based on Portland Cement weighing			from 1 bbl Cement, Based on barrel of			3.5 cu ft		***3.8 cu ft*** †		4 cu ft	
Cement	Sand	108 lbs per cu. ft	***100 lbs per cu. ft*** †	95 lbs per cu ft	3.5 cu ft	***3.8 cu ft*** †	4 cu ft	Packed Cement	Loose Sand	***Packed Cement***	***Loose Sand***	Packed Cement	Loose Sand
		cu ft	***cu ft***	cu ft	cu ft	***cu ft***	cu ft	bbl	cu yd	***bbl***	***cu yd***	bbl	cu yd
1	0	0.93	0.86	0.80	3.2	3.2	3.2	8.31		8.31		8.31	
1	½	1.12	1.06	1.02	3.9	4.0	4.1	6.92	0.46	6.73	0.47	6.61	0.49
1	1	1.48	1.42	1.38	5.2	5.4	5.5	5.22	0.68	5.01	0.71	4.88	0.72
1	1½	1.84	1.78	1.74	6.4	6.7	7.0	4.20	0.81	4.00	0.84	3.87	0.86
1	2	2.20	2.14	2.11	7.7	8.1	8.4	3.51	0.91	3.32	0.93	3.21	0.9[illegible]
1	2½	2.56	2.50	2.47	9.0	9.5	0.9	3.01	0.98	2.84	1.00	2.74	1.01
1	3	2.92	2.86	2.83	10.2	10.9	11.3	2.64	1.03	2.48	1.05	2.39	1.06
1	3½	3.28	3.23	3.19	11.5	12.2	12.8	2.35	1.06	2.20	1.08	2.12	1.10
1	4	3.64	3.50	3.55	12.8	13.6	14.2	2.12	1.10	1.98	1.11	1.90	1.13
1	4½	4.01	3.95	3.01	14.0	15.0	15.6	1.02	1.12	1.80	1.14	1.72	1.15
1	5	4.37	4.31	4.28	15.3	16.4	17.1	1.77	1.15	1.65	1.16	1.58	1.17
1	5½	4.73	4.67	4.64	16.6	17.7	18.5	1.63	1.16	1.52	1.18	1.46	1.19
1	6	5.09	5.03	5.00	17.8	19.1	20.0	1.52	1.18	1.41	1.19	1.[illegible]5	1.20
1	6½	5.45	5.39	5.36	19.1	20.5	21.4	1.41	1.10	1.32	1.21	1.26	1.21
1	7	5.81	5.76	5.72	20.3	21.[illegible]	22.9	1.33	1.21	1.23	1.21	1.18	1.22
1	7½	6.18	6.1	6.08	21.6	23.2	24.3	1.25	1.21	1.16	1.22	1.11	1.23
1	8	6.54	6.48	6.44	22.9	24.6	25.8	1.15	1.22	1.10	1.24	1.05	1.24

NOTE — Variations in the fineness of the sand and the cement and in the consistency of the mortar may affect the values by 10% in either direction

*Cement is packed by manufacturer sand loose

†***Use these columns ordinarily.***

MORTAR WITH VERY FINE SAND

Volume of Plastic Mortar and Quantities of Materials per Cubic Yard (See p 226)

Relative proportions by volume		Volume of Compacted Plastic Mortar						Materials for 1 cu yd Compact Plastic Mortar Based on barrel of					
		from 1 cu ft Cement, Based on Portland Cement weighing			from 1 bbl Cement, Based on barrel of			3.5 cu ft		*3.8 cu ft* †		4 cu ft	
Cement	Sand	108 lbs per cu ft	*100 lbs per cu ft* †	95 lbs per cu ft	3.5 cu ft	*3.8 cu ft* †	4 cu ft	Packed Cement	Loose Sand	*Packed Cement*	*Loose Sand*	Packed Cement	Loose Sand
		cu ft	*cu ft*	cu ft	cu ft	*cu ft*	cu ft	bbl	cu yd	*bbl*	*cu yd*	bbl	cu yd
1	½	1.26	1.19	1.15	4.4	4.5	4.6	6.16	0.40	6.01	0.42	5.91	0.44
1	1	1.62	1.56	1.51	5.7	5.9	6.0	4.78	0.62	4.59	0.65	4.48	0.66
1	1½	1.98	1.92	1.88	6.9	7.3	7.5	3.79	0.76	3.72	0.78	3.61	0.80
1	2	2.35	2.28	2.24	8.2	8.6	8.9	3.29	0.85	3.12	0.88	3.02	0.90
1	2½	2.71	2.65	2.51	9.5	10.0	10.4	2.85	0.92	2.69	0.95	2.60	0.96
1	3	3.08	3.01	2.97	10.8	11.4	11.9	2.51	0.98	2.37	1.00	2.28	1.01
1	3½	3.44	3.38	3.33	12.0	12.8	13.3	2.24	1.02	2.11	1.04	2.03	1.05
1	4	3.80	3.74	3.70	13.3	14.2	14.8	2.03	1.03	1.90	1.07	1.93	1.08
1	4½	4.17	4.10	4.06	14.6	15.6	16.2	1.85	1.08	1.74	1.10	1.67	1.11
1	5	4.53	4.47	4.43	15.9	16.9	17.7	1.70	1.10	1.59	1.12	1.53	1.13
1	5½	4.90	4.33	4.79	17.1	18.3	19.1	1.58	1.12	1.47	1.14	1.41	1.15
1	6	5.26	5.20	5.15	18.4	19.7	20.6	1.47	1.14	1.37	1.16	1.31	1.17

*Cement as packed by manufacturer, sand loose.

†*Use these columns ordinarily.*

Quantities of Materials for One Cubic Yard of Rammed Concrete.

Based on a Barrel of 3.5 Cubic Feet.

(See important foot-notes, also p. 225.)

PROPORTIONS BY PARTS			PROPORTIONS BY VOLUMES			Volume of mortar in terms of percentage of volume of stone	PERCENTAGES OF VOIDS IN BROKEN STONE OR GRAVEL														
							50%*			45%†			40%‡			30%§			20%§		
Cement	Sand	Stone	Packed Cement	Loose Sand	Loose Stone		Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone
			bbl.	cu. ft.	cu. ft.	%	bbl.	cu. yd.	cu. yd.	bbl.	cu. yd.	cu. yd.	bbl.	cu. yd.	cu. yd.	bbl.	cu. yd.	cu. yd.	bbl.	cu. yd.	cu. yd.
1		1	1		3.5	101	5.25		0.68	5.07		0.66	4.89		0.63	4.51		0.58	4.19		0.54
1		2	1		7.0	54	3.84		1.00	3.64		0.94	3.47		0.90	3.09		0.80	2.80		0.73
1		3	1		10.5	39				2.85		1.11	2.69		1.05	2.35		0.91	2.10		0.82
1		4	1		14.0	31										1.90		0.99	1.69		0.88
1		5	1		17.5	27										1.59		1.03	1.41		0.91
1		6	1		21.0	24										1.37		1.07	1.21		0.94
1		7	1		24.5	21													1.06		0.96
1		8	1		28.0	20													0.94		0.98
1		9	1		31.5	18													0.84		0.98
1		10	1		35.0	17													0.77		1.00
1		11	1		38.5	16													0.70		1.00
1		12	1		42.0	16													0.65		1.01
1	1	1½	1	3.5	5.2	104	3.37	0.44	0.65	3.26	0.42	0.63	3.15	0.41	0.61	2.95	0.38	0.57	2.78	0.36	0.54
1	1	2	1	3.5	7.0	78	3.02	0.39	0.78	2.89	0.38	0.75	2.78	0.36	0.72	2.58	0.33	0.67	2.41	0.31	0.62
1	1	2½	1	3.5	8.7	64	2.73	0.35	0.88	2.60	0.34	0.84	2.49	0.32	0.80	2.29	0.30	0.74	2.12	0.28	0.68
1	1	3	1	3.5	10.5	54	2.49	0.32	0.97	2.37	0.31	0.92	2.25	0.29	0.88	2.06	0.27	0.80	1.90	0.25	0.74
1	1½	2	1	5.2	7.0	95	2.64	0.51	0.68	2.55	0.49	0.66	2.46	0.47	0.64	2.30	0.44	0.60	2.16	0.42	0.56
1	1½	2½	1	5.2	8.7	78	2.42	0.47	0.78	2.32	0.45	0.75	2.23	0.43	0.72	2.07	0.40	0.67	1.93	0.37	0.62
1	1½	3	1	5.2	10.5	65	2.23	0.43	0.87	2.13	0.41	0.83	2.04	0.39	0.79	1.88	0.36	0.73	1.74	0.34	0.68
1	1½	3½	1	5.2	12.2	56	2.07	0.40	0.94	1.97	0.38	0.89	1.88	0.36	0.85	1.72	0.33	0.78	1.59	0.31	0.72
1	1½	4	1	5.2	14.0	50	1.93	0.37	1.00	1.83	0.35	0.95	1.74	0.34	0.90	1.59	0.31	0.82	1.46	0.28	0.76
1	1½	4½	1	5.2	15.7	45	1.81	0.35	1.05	1.71	0.33	0.99	1.62	0.31	0.94	1.47	0.28	0.86	1.35	0.26	0.78
1	1½	5	1	5.2	17.5	41	1.70	0.33	1.10	1.60	0.31	1.04	1.52	0.29	0.99	1.37	0.26	0.89	1.25	0.24	0.81
1	2	3	1	7.0	10.5	77	2.02	0.52	0.79	1.94	0.50	0.75	1.86	0.48	0.72	1.72	0.45	0.67	1.61	0.42	0.63
1	2	3½	1	7.0	12.2	67	1.89	0.49	0.85	1.80	0.47	0.81	1.73	0.45	0.78	1.59	0.41	0.72	1.48	0.38	0.67
1	2	4	1	7.0	14.0	59	1.77	0.46	0.92	1.69	0.44	0.88	1.61	0.42	0.83	1.48	0.38	0.77	1.37	0.35	0.71
1	2	4½	1	7.0	15.7	53	1.67	0.43	0.97	1.58	0.41	0.92	1.51	0.39	0.88	1.38	0.36	0.80	1.27	0.33	0.74
1	2	5	1	7.0	17.5	48	1.57	0.41	1.02	1.49	0.39	0.97	1.42	0.37	0.92	1.29	0.33	0.84	1.18	0.31	0.76
1	2	5½	1	7.0	19.2	44	1.49	0.39	1.06	1.41	0.36	1.00	1.34	0.35	0.95	1.21	0.31	0.86	1.11	0.29	0.79
1	2	6	1	7.0	21.0	41	1.42	0.37	1.10	1.34	0.35	1.04	1.27	0.33	0.99	1.14	0.30	0.89	1.04	0.27	0.81
1	2½	3	1	8.7	10.5	90	1.84	0.59	0.72	1.78	0.57	0.69	1.71	0.55	0.66	1.60	0.52	0.62	1.50	0.48	0.58
1	2½	3½	1	8.7	12.2	78	1.73	0.56	0.78	1.66	0.53	0.75	1.60	0.52	0.72	1.48	0.48	0.67	1.38	0.44	0.62
1	2½	4	1	8.7	14.0	68	1.63	0.52	0.85	1.56	0.50	0.81	1.50	0.48	0.78	1.38	0.44	0.72	1.28	0.41	0.66
1	2½	4½	1	8.7	15.7	61	1.55	0.50	0.90	1.47	0.47	0.86	1.41	0.45	0.82	1.29	0.42	0.75	1.20	0.39	0.70
1	2½	5	1	8.7	17.5	55	1.47	0.47	0.95	1.39	0.45	0.90	1.33	0.43	0.86	1.22	0.39	0.79	1.12	0.36	0.73
1	2½	5½	1	8.7	19.2	51	1.39	0.45	0.99	1.32	0.42	0.94	1.26	0.41	0.90	1.15	0.37	0.82	1.06	0.34	0.75
1	2½	6	1	8.7	21.0	47	1.33	0.43	1.03	1.26	0.41	0.98	1.20	0.39	0.93	1.09	0.35	0.85	1.00	0.32	0.78
1	2½	6½	1	8.7	22.7	44	1.27	0.41	1.07	1.20	0.39	1.01	1.14	0.37	0.96	1.03	0.33	0.87	0.94	0.30	0.79
1	2½	7	1	8.7	24.5	41	1.22	0.39	1.11	1.15	0.37	1.04	1.09	0.35	0.99	0.98	0.32	0.89	0.90	0.29	0.82
1	3	4	1	10.5	14.0	77	1.52	0.59	0.79	1.46	0.57	0.76	1.40	0.54	0.73	1.30	0.50	0.67	1.21	0.47	0.63
1	3	4½	1	10.5	15.7	69	1.44	0.56	0.84	1.38	0.54	0.80	1.32	0.51	0.77	1.22	0.47	0.71	1.13	0.44	0.66
1	3	5	1	10.5	17.5	62	1.37	0.53	0.89	1.31	0.51	0.85	1.25	0.48	0.81	1.15	0.45	0.75	1.07	0.42	0.69
1	3	5½	1	10.5	19.2	57	1.31	0.51	0.93	1.25	0.48	0.89	1.19	0.46	0.85	1.09	0.42	0.78	1.01	0.39	0.72
1	3	6	1	10.5	21.0	53	1.25	0.48	0.97	1.19	0.46	0.93	1.13	0.44	0.88	1.03	0.40	0.80	0.95	0.37	0.74
1	3	6½	1	10.5	22.7	49	1.20	0.47	1.01	1.14	0.44	0.96	1.08	0.42	0.91	0.98	0.38	0.82	0.90	0.35	0.76
1	3	7	1	10.5	24.5	46	1.15	0.45	1.04	1.09	0.42	0.99	1.03	0.40	0.93	0.94	0.36	0.85	0.86	0.33	0.78
1	3	7½	1	10.5	26.2	43	1.11	0.43	1.08	1.05	0.41	1.02	0.99	0.38	0.96	0.90	0.35	0.87	0.82	0.32	0.80
1	3	8	1	10.5	28.0	40	1.06	0.41	1.10	1.01	0.39	1.05	0.95	0.37	0.99	0.86	0.33	0.89	0.78	0.30	0.81
1	4	5	1	14.0	17.5	77	1.22	0.63	0.79	1.17	0.61	0.76	1.12	0.58	0.73	1.04	0.54	0.67	0.97	0.50	0.63
1	4	6	1	14.0	21.0	65	1.12	0.58	0.87	1.07	0.55	0.83	1.02	0.53	0.79	0.94	0.49	0.73	0.87	0.45	0.68
1	4	7	1	14.0	24.5	56	1.04	0.54	0.94	0.99	0.51	0.90	0.94	0.49	0.85	0.86	0.44	0.78	0.80	0.41	0.73
1	4	8	1	14.0	28.0	50	0.97	0.50	1.01	0.92	0.48	0.95	0.87	0.45	0.90	0.80	0.41	0.83	0.73	0.38	0.76
1	4	9	1	14.0	31.5	45	0.91	0.47	1.06	0.86	0.44	1.00	0.81	0.42	0.94	0.74	0.38	0.86	0.68	0.35	0.79
1	4	10	1	14.0	35.0	41	0.85	0.44	1.10	0.81	0.42	1.05	0.76	0.39	0.98	0.69	0.36	0.89	0.63	0.33	0.82
1	5	10	1	17.5	35.0	48	0.79	0.51	1.02	0.75	0.49	0.97	0.71	0.46	0.92	0.65	0.42	0.84	0.59	0.38	0.76
1	6	12	1	21.0	42.0	46	0.67	0.52	1.04	0.63	0.49	0.98	0.60	0.47	0.93	0.54	0.42	0.84	0.50	0.39	0.78

NOTE. — Variations in the fineness of the sand and the compacting of the concrete may affect the quantities 10% in either direction.

*Use 50% columns for broken stone screened to uniform size.
†Use 45% columns for average conditions and for broken stone with dust screened out.
‡Use 40% columns for gravel or mixed stone and gravel.
§Use these columns for scientifically graded mixtures.

USE THIS TABLE ORDINARILY.

Quantities of Materials for One Cubic Yard of Rammed Concrete. Based on a Barrel of 3.8 Cubic Feet.

(See important foot-notes, also p. 225.)

Proportions by Parts			Proportions by Volume			Volume of mortar in terms of percentage of volume of stone	Percentages of Voids in Broken Stone or Gravel														
							50%*			*45%*†			40%‡			30%§			20%§		
Cement	Sand	Stone	Packed Cement	Loose Sand	Loose Stone		Cement	Sand	Stone	*Cem'nt*	*Sand*	*Stone*	Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone
			bbl	cu. ft.	cu. ft.	%	bbl	cu. yd.	cu. yd.	*bbl*	*cu. yd.*	*cu. yd.*	bbl	cu. yd.	cu. yd.	bbl	cu. yd.	cu. yd.	bbl	cu. yd.	cu. yd.
1		1	1		3.8	94	5.09		0.72	4.90		0.69	4.73		0.67	4.33		0.61	4.02		0.57
1		2	1		7.6	51	3.67		1.03	3 48		0.98	3.30		0.93	2.93		0.82	2.65		0.75
1		3	1		11.4	36				2.69		1.14	2.54		1.07	2.22		0.94	1.98		0.84
1		4	1		15.2	29										1.78		1.00	1.58		0.89
1		5	1		19.0	25										1.49		1.05	1.31		0.92
1		6	1		22.8	22										1.28		1.08	1.12		0.95
1		7	1		26.6	20													0.98		0.97
1		8	1		30.4	19													0.87		0.98
1		9	1		34.2	18													0.78		0.99
1		10	1		38.0	17													0.71		1.00
1		11	1		41.8	16													0.65		1.01
1		12	1		45.5	15													0.60		1.01
1	1	1½	1	3.8	5.7	99	3.19	0.45	0.67	3.08	0.43	0.65	2.97	0.42	0.63	2.78	0.39	0.50	2.62	0.37	0.55
1	1	2	1	3.8	7.6	75	2.85	0.40	0.80	2.73	0.38	0.77	2.62	0.37	0.74	2.43	0.34	0.68	2.26	0.32	0.64
1	1	2½	1	3.8	9.5	61	2.57	0.36	0.90	2.45	0.34	0.86	2.34	0.33	0.82	2.15	0.30	0.76	1.99	0.28	0.70
1	1	3	1	3.8	11.4	51	2.34	0 33	0.99	2.22	0.31	0.94	2.12	0.30	0.90	1.03	0.27	0.82	1.77	0.25	0.75
1	1½	2	1	5.7	7.6	93	2.49	0.53	0.70	2.40	0.51	0.68	2.31	0.49	0.65	2.16	0.46	0.61	2.03	0.43	0.57
1	1½	2½	1	5.7	9.5	76	2.27	0.48	0.80	2.18	0.46	0.77	2.00	0.44	0.74	1.94	0.41	0.68	1.80	0.38	0.63
1	1½	3	1	5.7	11.4	64	2.09	0.44	0.88	2.00	0.42	0.84	1.01	0.40	0.81	1.76	0.37	0.74	1.63	0.34	0.69
1	1½	3½	1	5.7	13.3	55	1.04	0.41	0.96	1.84	0.39	0.91	1.76	0.37	0.87	1.61	0.34	0.79	1.48	0.31	0.73
1	1½	4	1	5.7	15.2	49	1.80	0.38	1.01	1.71	0.36	0.96	1.63	0.34	0.92	1.48	0.31	0.83	1.36	0.29	0.77
1	1½	4½	1	5 7	17.1	44	1.60	0.36	1.07	1.60	0 34	1.01	1.51	0.32	0.96	1.37	0.29	0.87	1.25	0.26	0.79
1	1½	5	1	5.7	19.0	40	1.50	0.34	1.12	1.50	0 32	1.06	1.42	0.30	1.00	1.28	0.27	0.90	1.17	0.25	0.82
1	2	3	1	7.6	11.4	75	1.89	0.53	0.80	1.81	0.51	0.76	1.74	0.49	0.74	1.61	0.45	0.68	1.50	0.42	0.63
1	2	3½	1	7.6	13.3	65	1.76	0.49	0.87	1.68	0.47	0.83	1.61	0.45	0.79	1.48	0.42	0.73	1.38	0.39	0.68
1	2	4	1	7.6	15.2	57	1.65	0.46	0.93	1.57	0 44	0.88	1.50	0.42	0.84	1.38	0.39	0.78	1.27	0.36	0.72
1	2	4½	1	7.6	17.1	51	1.55	0.44	0.98	1.48	0.42	0.94	1.41	0.40	0.89	1.28	0.36	0.81	1.18	0.33	0.75
1	2	5	1	7.6	19.0	47	1.47	0.41	1.03	1.39	0.39	0.98	1.32	0.37	0.03	1.20	0.34	0.84	1.10	0.31	0.77
1	2	5½	1	7 6	20 9	43	1.30	0.39	1.08	1.31	0.37	1 01	1.25	0.35	0.97	1.13	0.32	0.87	1.03	0.29	0.80
1	2	6	1	7.6	22.8	40	1.32	0.37	1.11	1.25	0.35	1.06	1.18	0.33	1.00	1.06	0.30	0.89	0.97	0.27	0.82
1	2½	3	1	9 5	11.4	87	1.72	0.61	0.73	1.66	0.58	0.70	1.60	0.56	0.68	1.49	0.52	0.63	1.40	0.40	0.59
1	2½	3½	1	9.5	13.3	75	1.62	0.57	0.80	1.55	0.55	0.76	1.49	0.52	0.73	1.38	0.49	0.68	1.29	0.45	0.64
1	2½	4	1	9.5	15.2	66	1.52	0.54	0.86	1.46	0.51	0.82	1.40	0.49	0.79	1.29	0.45	0.73	1.19	0.42	0.67
1	2½	4½	1	9.5	17.1	60	1.44	0.51	0.91	1.37	0.48	0.87	1.31	0.46	0.83	1.20	0.42	0.76	1.11	0.39	0.70
1	2½	5	1	9.5	19.0	54	1.37	0.48	0.06	1.30	0.46	0.92	1.24	0.44	0.87	1.13	0.40	0.80	1.04	0.37	0.73
1	2½	5½	1	9.5	20.9	49	1.30	0.46	1.01	1.23	0.43	0.95	1.17	0.41	0.91	1.07	0.38	0.83	0.98	0.34	0.76
1	2½	6	1	9.5	22.8	46	1.24	0.44	1.05	1.17	0.41	0.99	1.11	0.39	0.94	1.01	0.36	0 85	0.02	0.32	0.78
1	2½	6½	1	9.5	24.7	42	1.18	0.42	1.08	1.12	0.39	1.02	1.06	0.37	0.97	0.96	0.34	0.88	0.88	0.31	0.80
1	2½	7	1	9.5	26.6	40	1.13	0.40	1.11	1.07	0.38	1.05	1.01	0.36	0.99	0.91	0.32	0.90	0.83	0.29	0.82
1	3	4	1	11.4	15.2	76	1.42	0.60	0.80	1.36	0.57	0.77	1.30	0.55	0.73	1.21	0.51	0.68	1.12	0.47	0.63
1	3	4½	1	11.4	17.1	68	1.34	0.57	0.85	1.28	0.54	0.81	1.23	0.52	0.78	1.13	0.48	0.72	1.05	0.44	0.66
1	3	5	1	11.4	19.0	61	1.28	0.54	0.90	1.22	0.52	0.86	1.17	0.49	0.82	1.07	0.45	0.75	0.99	0.42	0.70
1	3	5½	1	11.4	20.9	56	1.22	0.52	0.94	1.16	0.49	0.90	1.11	0.47	0.86	1.01	0.43	0.78	0.93	0.39	0.72
1	3	6	1	11.4	22.8	52	1.16	0.40	0.98	1.11	0 47	0.94	1.05	0.44	0.89	0.96	0.41	0.81	0.88	0.37	0.74
1	3	6½	1	11.4	24.7	48	1.12	0.47	1.02	1.06	0.45	0.97	1.01	0.43	0.92	0.92	0.39	0.84	0.84	0.35	0.77
1	3	7	1	11.4	26.6	45	1.07	0.45	1.05	1.01	0.43	0.99	0.06	0.40	0.95	0.87	0.37	0.86	0.80	0.34	0.79
1	3	7½	1	11.4	28.5	42	1.03	0.44	1.09	0.97	0.11	1.02	0.92	0.39	0.97	0.83	0.35	0.88	0.76	0.32	0.80
1	3	8	1	11.4	30.4	40	0.99	0.42	1.11	0.93	0.39	1.05	0.88	0.37	0.99	0.80	0.34	0.90	0.73	0.31	0.82
1	4	5	1	15.2	19.0	76	1.13	0.64	0.80	1.08	0.61	0.76	1.04	0.59	0 73	0.96	0.54	0.68	0.90	0.51	0.63
1	4	6	1	15.2	22.8	64	1.04	0.59	0.88	0 99	0.56	0.84	0.95	0.54	0.80	0.87	0.49	0.73	0.81	0.46	0.68
1	4	7	1	15.2	26.6	55	0.96	0.54	0.95	0.92	0.52	0.91	0.88	0.50	0.87	0.80	0.45	0.79	0.74	0.42	0.73
1	4	8	1	15.2	30.4	49	0.90	0.51	1.01	0.85	0.48	0.96	0.81	0.46	0.91	0.74	0.42	0.83	0.68	0.38	0.77
1	4	9	1	15.2	34.2	44	0.84	0.47	1.06	0 80	0.45	1.01	0.76	0.43	0.96	0.68	0.38	0.86	0.63	0.35	0.80
1	4	10	1	15.2	38.0	40	0 79	0.44	1.11	0.75	0.42	1.06	0.71	0.40	1.00	0.64	0.36	0.90	0.58	0.33	0.82
1	5	10	1	19.0	38.0	47	0.73	0.52	1.03	0.69	0.49	0.97	0.66	0.46	0.93	0.60	0.42	0.84	0.55	0.39	0.77
1	6	12	1	22.8	45.5	46	0.62	0.52	1.04	0.58	0.49	0.98	0.56	0.47	0.94	0.50	0.42	0.84	0.46	0.39	0.78

NOTE. — Variations in the fineness of the sand and the compacting of the concrete may affect the quantities by 10% in either direction.

*Use 50% columns for broken stone screened to uniform size.

†***Use 45% columns for average conditions*** and for broken stone with dust screened out.

‡Use 40% columns for gravel or mixed stone and gravel.

§Use these columns for scientifically graded mixtures.

Quantities of Material for One Cubic Yard of Rammed Concrete.

Based on a Barrel of 4 Cubic Feet.

(See important foot-notes, also p. 225.)

Proportions by Parts			Proportions by Volumes			Volume of mortar in terms of percentage of volume of stone	Percentages of Voids in Broken Stone or Gravel														
							50%*			45%†			40%‡			30%§			20%§		
Cement	Sand	Stone	Packed Cement	Loose Sand	Loose Stone		Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone	Cement	Sand	Stone
			bbl	cu. ft.	cu. ft.	%	bbl	cu. yd.	cu. yd	bbls	cu. yd.	cu. yd.	bbl	cu. yd.	cu. yd.	bbl	cu. yd.	cu. yd.	bbl	cu. yd.	cu. yd.
1		1	1		4	89	4.90		0.74	4.80		0.71	4.62		0.69	4.23		0.63	3.91		0.58
1		2	1		8	40	3.57		1.06	3.37		1.00	3.20		0.95	2.84		0.84	2.56		0.76
1		3	1		12	35				2.60		1.16	2.45		1.09	2.13		0.95	1.90		0.84
1		4	1		16	28										1.71		1.01	1.51		0.89
1		5	1		20	24										1.43		1.06	1.26		0.93
1		6	1		24	22										1.22		1.08	1.07		0.95
1		7	1		28	20													0.94		0.98
1		8	1		32	18													0.83		0.98
1		9	1		36	17													0.75		1.00
1		10	1		40	16													0.68		1.01
1		11	1		44	15													0.62		1.01
1		12	1		48	15													0.57		1.01
1	1	1½	1	4	6	96	3.08	0.46	0.68	2.97	0.14	0.66	2.87	0.42	0.64	2.69	0.40	0.60	2.53	0.38	0.56
1	1	2	1	4	8	73	2.74	0.41	0.81	2.63	0.39	0.78	2.52	0.37	0.75	2.33	0.34	0.69	2.17	0.32	0.64
1	1	2½	1	4	10	59	2.47	0.37	0.91	2.35	0.35	0.87	2.25	0.33	0.83	2.06	0.31	0.76	1.90	0.28	0.71
1	1	3	1	4	12	50	2.25	0.33	1.00	2.13	0.32	0.95	2.03	0.30	0.00	1.85	0.27	0.82	1.70	0.25	0.76
1	1½	2	1	6	8	92	2.39	0.53	0.71	2.30	0.51	0.68	2.22	0.49	0.66	2.07	0.46	0.61	1.94	0.43	0.58
1	1½	2½	1	6	10	74	2.18	0.48	0.81	2.00	0.46	0.77	2.01	0.45	0.74	1.86	0.41	0.69	1.73	0.38	0.64
1	1½	3	1	6	12	62	2.01	0.45	0.80	1.91	0.42	0.85	1.83	0.41	0.81	1.68	0.37	0.75	1.56	0.35	0.69
1	1½	3½	1	6	14	54	1.86	0.41	0.96	1.77	0.39	0.92	1.68	0.37	0.87	1.54	0.34	0.80	1.42	0.32	0.74
1	1½	4	1	6	16	48	1.73	0.38	1.03	1.64	0.36	0.97	1.56	0.35	0.92	1.42	0.32	0.84	1.30	0.29	0.77
1	1½	4½	1	6	18	43	1.62	0.36	1.08	1.53	0.34	1.02	1.45	0.32	0.97	1.31	0.29	0.87	1.20	0.27	0.80
1	1½	5	1	6	20	39	1.52	0.34	1.13	1.43	0.32	1.06	1.35	0.30	1.00	1.22	0.27	0.90	1.11	0.25	0.82
1	2	3	1	8	12	74	1.81	0.54	0.80	1.74	0.52	0.77	1.67	0.50	0.74	1.54	0.46	0.68	1.44	0.43	0.64
1	2	3½	1	8	14	64	1.69	0.50	0.88	1.61	0.48	0.83	1.54	0.46	0.80	1.42	0.42	0.74	1.31	0.39	0.68
1	2	4	1	8	16	56	1.58	0.47	0.94	1.51	0.45	0.80	1.44	0.43	0.85	1.32	0.39	0.78	1.21	0.36	0.72
1	2	4½	1	8	18	51	1.49	0.44	0.99	1.41	0.42	0.94	1.34	0.40	0.89	1.23	0.36	0.82	1.13	0.34	0.75
1	2	5	1	8	20	46	1.40	0.42	1.04	1.33	0.39	0.98	1.26	0.37	0.93	1.15	0.34	0.85	1.05	0.31	0.78
1	2	5½	1	8	22	42	1.33	0.39	1.08	1.26	0.37	1.03	1.10	0.35	0.97	1.08	0.32	0.88	0.98	0.29	0.80
1	2	6	1	8	24	39	1.26	0.37	1.12	1.10	0.35	1.06	1.13	0.34	1.00	1.02	0.30	0.91	0.93	0.28	0.83
1	2½	3	1	10	12	86	1.65	0.61	0.73	1.50	0.59	0.71	1.53	0.57	0.68	1.42	0.52	0.63	1.33	0.49	0.59
1	2½	3½	1	10	14	75	1.55	0.57	0.80	1.48	0.55	0.77	1.42	0.52	0.74	1.32	0.49	0.68	1.23	0.46	0.64
1	2½	4	1	10	16	66	1.46	0.54	0.87	1.39	0.51	0.82	1.33	0.49	0.79	1.23	0.46	0.73	1.14	0.42	0.68
1	2½	4½	1	10	18	59	1.38	0.51	0.92	1.31	0.48	0.87	1.25	0.46	0.83	1.15	0.43	0.77	1.06	0.39	0.71
1	2½	5	1	10	20	54	1.31	0.48	0.97	1.24	0.46	0.92	1.18	0.44	0.87	1.08	0.40	0.80	0.99	0.37	0.73
1	2½	5½	1	10	22	49	1.24	0.46	1.01	1.18	0.44	0.96	1.12	0.41	0.91	1.02	0.38	0.83	0.93	0.34	0.76
1	2½	6	1	10	24	45	1.18	0.44	1.05	1.12	0.41	1.00	1.06	0.39	0.94	0.96	0.36	0.85	0.88	0.33	0.78
1	2½	6½	1	10	26	42	1.13	0.42	1.09	1.07	0.40	1.03	1.01	0.37	0.97	0.92	0.34	0.89	0.84	0.31	0.81
1	2½	7	1	10	28	39	1.08	0.40	1.12	1.02	0.38	1.06	0.96	0.36	1.00	0.87	0.32	0.90	0.79	0.29	0.82
1	3	4	1	12	16	75	1.35	0.60	0.80	1.30	0.58	0.77	1.25	0.56	0.74	1.15	0.51	0.68	1.08	0.48	0.64
1	3	4½	1	12	18	67	1.28	0.57	0.85	1.23	0.55	0.82	1.18	0.52	0.79	1.08	0.48	0.72	1.01	0.45	0.67
1	3	5	1	12	20	60	1.22	0.54	0.90	1.16	0.52	0.86	1.11	0.49	0.82	1.02	0.45	0.76	0.94	0.42	0.70
1	3	5½	1	12	22	55	1.16	0.52	0.95	1.11	0.49	0.90	1.06	0.47	0.86	0.97	0.43	0.79	0.89	0.40	0.72
1	3	6	1	12	24	50	1.11	0.49	0.99	1.06	0.47	0.94	1.01	0.45	0.90	0.92	0.41	0.82	0.84	0.37	0.75
1	3	6½	1	12	26	48	1.06	0.47	1.02	1.01	0.45	0.97	0.96	0.43	0.92	0.87	0.39	0.84	0.80	0.36	0.77
1	3	7	1	12	28	44	1.02	0.45	1.06	0.97	0.43	1.01	0.92	0.41	0.95	0.83	0.37	0.86	0.76	0.34	0.79
1	3	7½	1	12	30	42	0.98	0.44	1.09	0.93	0.41	1.03	0.88	0.39	0.98	0.79	0.35	0.88	0.73	0.32	0.81
1	3	8	1	12	32	39	0.94	0.42	1.11	0.89	0.40	1.05	0.84	0.37	1.00	0.76	0.34	0.90	0.69	0.31	0.82
1	4	5	1	16	20	75	1.08	0.64	0.80	1.03	0.61	0.76	0.90	0.50	0.73	0.92	0.55	0.68	0.86	0.51	0.64
1	4	6	1	16	24	63	0.99	0.59	0.88	0.95	0.56	0.84	0.91	0.54	0.81	0.83	0.49	0.74	0.77	0.46	0.68
1	4	7	1	16	28	55	0.92	0.54	0.95	0.88	0.52	0.91	0.83	0.49	0.86	0.76	0.45	0.79	0.70	0.42	0.73
1	4	8	1	16	32	48	0.86	0.51	1.02	0.81	0.48	0.96	0.77	0.46	0.91	0.70	0.42	0.83	0.64	0.38	0.76
1	4	9	1	16	36	43	0.80	0.47	1.07	0.76	0.45	1.01	0.72	0.43	0.96	0.65	0.39	0.87	0.60	0.36	0.80
1	4	10	1	16	40	40	0.75	0.44	1.11	0.71	0.42	1.05	0.67	0.40	0.99	0.61	0.36	0.90	0.55	0.33	0.81
1	5	10	1	20	40	47	0.70	0.52	1.04	0.66	0.49	0.98	0.63	0.47	0.93	0.57	0.42	0.84	0.52	0.38	0.77
1	6	12	1	24	48	46	0.59	0.52	1.05	0.56	0.50	1.00	0.53	0.47	0.94	0.48	0.43	0.85	0.44	0.39	0.78

NOTE. — Variations in the fineness of the sand and the compacting of the concrete may affect the quantities 10% in either direction.

*Use 50% columns for broken stone screened to uniform size.

†Use 45% columns for average conditions and for broken stone with dust screened out.

‡Use 40% columns for gravel or mixed stone and gravel.

§Use these columns for scientifically graded mixtures

Volume of Concrete Based on a Barrel of 3.5 Cubic Feet.

(See important foot notes, also p 225)

Proportions by Parts: Cement	Proportions by Parts: Sand	Proportions by Parts: Stone	Proportions by Volume: Cement bbl	Proportions by Volume: Sand cu ft	Proportions by Volume: Stone cu ft	Volume of mortar in terms of percentage of volume of stone %	Average volume of rammed concrete made from one barrel cement — Percentages of Voids in Broken Stone or Gravel: 50%* cu ft	45%† cu ft	40%‡ cu ft	30%§ cu ft	20%§ cu ft
1		1	1		3.5	101	5.1	5.3	5.5	6.0	6.4
1		2	1		7.0	54	7.0	7.4	7.8	8.7	9.6
1		3	1		10.5	39		9.5	10.0	11.5	12.8
1		4	1		14.0	31				14.2	16.0
1		5	1		17.5	27				17.0	19.2
1		6	1		21.0	24				19.7	22.4
1		7	1		24.5	21					25.6
1		8	1		28.0	20					28.8
1		9	1		31.5	18					32.0
1		10	1		35.0	17					35.2
1		11	1		38.5	16					38.4
1		12	1		42.0	16					41.6
1	1	1½	1	3.5	5.2	104	8.0	8.3	8.6	0.1	0.7
1	1	2	1	3.5	7.0	78	8.0	0.3	9.7	10.5	11.2
1	1	2½	1	3.5	8.7	64	9.9	10.4	10.8	11.8	12.[illegible]
1	1	3	1	3.5	10.5	54	10.8	11.4	12.0	13.1	14.2
1	1½	2	1	5.2	7.0	05	10.2	10.6	11.0	11.7	12.5
1	1½	2½	1	5.2	8.7	78	11.2	11.6	12.1	13.0	14.0
1	1½	3	1	5.2	10.5	65	12.1	12.7	13.2	14.4	15.5
1	1½	3½	1	5.2	1[illegible].2	56	13.0	13.7	14.4	15.7	17.0
1	1½	4	1	5.2	14.0	50	14.0	14.8	15.5	17.0	18.5
1	1½	4½	1	5.2	15.7	45	14.0	15.8	16.6	18.3	20.0
1	1½	5	1	5.2	17.5	41	15.9	16.8	17.8	20.0	21.6
1	2	3	1	7.0	10.5	77	1[illegible].4	13.0	14.5	1[illegible].6	16.8
1	2	3½	1	7.0	12	67	14.3	15.0	15.6	17.0	18.3
1	2	4	1	7.0	14.0	50	15.3	16.0	16.8	18.3	10.8
1	2	4½	1	7.0	15.7	53	16.2	17.0	17.0	10.6	21.3
1	2	5	1	7.0	17.5	48	17.1	18.1	10.0	20.0	22.8
1	2	5½	1	7.0	19.[illegible]	14	18.1	19.1	20.2	22.2	24.3
1	2	6	1	7.0	21.0	41	19.[illegible]	20.2	21.[illegible]	23.6	25.8
1	2½	3	1	8.7	10.5	92	14.6	15.2	15.8	16.9	18.0
1	2½	3½	1	8.7	12.2	78	15.6	16.2	16.0	18.2	19.6
1	2½	4	1	8.7	14.0	[illegible]	16.5	17.3	18.0	19.6	21.1
1	2½	4½	1	8.7	15.7	61	17.5	18.3	19.2	20.0	22.6
1	2½	5	1	8.7	17.5	55	18.4	10.4	20.3	22.2	24.1
1	2½	5½	1	8.7	19.2	51	19.4	20.4	21.4	23.5	25.6
1	2½	6	1	8.7	21.0	47	20.3	21.4	22.6	24.8	27.1
1	2½	6½	1	8.7	22.7	44	21.2	22.5	23.7	26.2	28.6
1	2½	7	1	8.7	24.5	41	2[illegible].2	23.[illegible]	24.8	27.5	30.1
1	3	4	1	10.5	14.0	77	17.8	18.5	19.3	20.8	22.3
1	3	4½	1	10.5	15.7	69	18.7	19.6	20.4	22.1	23.8
1	3	5	1	10.5	17.5	62	19.7	20.6	21.6	23.4	25.3
1	3	5½	1	10.5	19.2	57	20.6	21.7	22.7	24.8	26.8
1	3	6	1	10.5	21.0	53	21.6	22.7	23.8	26.1	28.4
1	3	6½	1	10.5	22.7	49	22.5	23.7	25.0	27.4	29.9
1	3	7	1	10.5	24.5	46	23.5	24.8	26.1	28.7	31.4
1	3	7½	1	10.5	26.2	43	24.4	25.8	27.2	30.1	32.9
1	3	8	1	10.5	28.0	40	25.3	26.9	28.4	31.4	34.4
1	4	5	1	14.0	17.5	77	22.2	23.2	24.1	26.0	27.9
1	4	6	1	14.0	21.0	65	24.1	25.2	26.4	28.6	30.9
1	4	7	1	14.0	24.5	56	26.0	27.3	28.6	31.3	33.9
1	4	8	1	14.0	28.0	50	27.9	30.4	30.9	33.9	36.9
1	4	0	1	14.0	31.5	45	29.8	31.5	33.2	36.6	40.0
1	4	10	1	14.0	35.0	41	31.7	33.6	35.4	39.2	43.0
1	5	10	1	17.5	35.0	48	34.2	36.1	38.0	41.8	45.5
1	6	12	1	21.0	42.0	46	40.[illegible]	42.8	45.0	49.6	54.1

NOTE. — Variations in the fineness of the sand and the compacting of the concrete may affect the volumes by 10% in either direction

*Use 50% column for broken stone screened to uniform size

†Use 45% column for average conditions and for broken stone with dust screened out.

‡Use 40% column for gravel or mixed stone and gravel.

§Use these columns for scientifically graded mixtures

USE THIS TABLE ORDINARILY.

Volume of Concrete Based on a Barrel of 3.8 Cubic Feet.

(See important foot-notes, also p. 225.)

PROPORTIONS BY PARTS			PROPORTIONS BY VOLUME			Volume of mortar in terms of percentage of volume of stone.	AVERAGE VOLUME OF RAMMED CONCRETE MADE FROM ONE BARREL CEMENT — Percentages of Voids in Broken Stone or Gravel				
Cement	Sand	Stone	Cement	Sand	Stone		50%*	***45%†***	40%‡	30%§	20%§
			bbl.	cu. ft.	cu. ft.	%	cu. ft.	***cu. ft.***	cu. ft.	cu. ft.	cu. ft.
1		1	1		3.8	94	5.3	5.5	5.7	6.2	6.7
1		2	1		7.6	51	7.4	7.8	8.2	9.2	10.2
1		3	1		11.4	36		10.0	10.6	12.2	13.6
1		4	1		15.2	29				15.2	17.1
1		5	1		19.0	25				18.2	20.6
1		6	1		22.8	22				21.1	24.0
1		7	1		26.6	20					27.5
1		8	1		30.4	19					31.0
1		9	1		34.2	18					34.4
1		10	1		38.0	17					37.9
1		11	1		41.8	16					41.4
1		12	1		45.5	15					44.8
1	1	1½	1	3.8	5.7	90	8.5	8.8	9.1	9.7	10.3
1	1	2	1	3.8	7.6	75	9.5	9.9	10.3	11.1	11.9
1	1	2½	1	3.8	9.5	61	10.5	11.0	11.5	12.6	13.6
1	1	3	1	3.8	11.4	51	11.5	12.2	12.8	14.0	15.2
1	1½	2	1	5.7	7.6	63	10.8	11.3	11.7	12.5	13.3
1	1½	2½	1	5.7	9.5	76	11.0	12.4	12.9	13.9	15.0
1	1½	3	1	5.7	11.4	64	12.9	13.5	14.1	15.4	16.6
1	1½	3½	1	5.7	13.3	55	13.9	14.6	15.4	16.8	18.2
1	1½	4	1	5.7	15.2	40	15.0	15.8	16.6	18.2	19.9
1	1½	4½	1	5.7	17.1	44	16.0	16.9	17.8	19.7	21.5
1	1½	5	1	5.7	19.0	40	17.0	18.0	19.1	21.1	23.2
1	2	3	1	7.6	11.4	75	14.3	14.9	15.5	16.7	18.0
1	2	3½	1	7.6	13.3	65	15.3	16.0	16.8	18.2	19.6
1	2	4	1	7.6	15.2	57	16.3	17.2	18.0	19.6	21.3
1	2	4½	1	7.6	17.1	51	17.4	18.3	19.2	21.0	22.9
1	2	5	1	7.6	19.0	47	18.4	19.4	20.4	22.5	24.5
1	2	5½	1	7.6	20.9	43	19.4	20.5	21.7	23.9	26.2
1	2	6	1	7.6	22.8	40	20.4	21.7	22.9	25.4	27.8
1	2½	3	1	9.5	11.4	87	15.7	16.3	16.9	18.1	19.3
1	2½	3½	1	9.5	13.3	75	16.7	17.4	18.1	19.6	21.0
1	2½	4	1	9.5	15.2	66	17.7	18.5	19.3	21.0	22.6
1	2½	4½	1	9.5	17.1	60	18.7	19.6	20.6	22.4	24.3
1	2½	5	1	9.5	19.0	54	19.8	20.8	21.8	23.9	25.9
1	2½	5½	1	9.5	20.9	49	20.8	21.9	23.0	25.3	27.6
1	2½	6	1	9.5	22.8	46	21.8	23.0	24.3	26.7	29.2
1	2½	6½	1	9.5	24.7	42	22.8	24.2	25.5	28.2	30.8
1	2½	7	1	9.5	26.6	40	23.9	25.3	26.7	29.6	32.5
1	3	4	1	11.4	15.2	76	19.1	19.9	20.7	22.4	24.0
1	3	4½	1	11.4	17.1	68	20.1	21.0	21.9	23.8	25.6
1	3	5	1	11.4	19.0	61	21.1	22.1	23.2	25.2	27.2
1	3	5½	1	11.4	20.9	50	22.1	23.3	24.4	26.7	28.9
1	3	6	1	11.4	22.8	52	23.2	24.4	25.6	28.1	30.6
1	3	6½	1	11.4	24.7	48	24.2	25.5	26.9	29.5	32.2
1	3	7	1	11.4	26.6	45	25.2	26.7	28.1	31.0	33.8
1	3	7½	1	11.4	28.5	42	26.2	27.8	29.3	32.4	35.5
1	3	8	1	11.4	30.4	40	27.3	28.9	30.6	33.8	37.1
1	4	5	1	15.2	19.0	76	23.9	24.9	25.9	28.0	30.0
1	4	6	1	15.2	22.8	64	25.9	27.2	28.4	30.8	33.3
1	4	7	1	15.2	26.6	55	28.0	29.4	30.8	33.7	36.6
1	4	8	1	15.2	30.4	49	30.0	31.7	33.3	36.6	39.9
1	4	9	1	15.2	34.2	44	32.1	33.9	35.8	39.4	43.1
1	4	10	1	15.2	38.0	40	34.1	36.2	38.2	42.3	46.4
1	5	10	1	19.0	38.0	47	36.9	38.9	41.0	45.1	49.2
	6	12	1	22.8	45.5	46	43.7	46.2	48.6	53.6	58.5

NOTE. — Variations in the fineness of the sand and the compacting of the concrete may affect the volumes by 10% in either direction.

*Use 50% column for broken stone screened to uniform size.

†***Use 45% column for average conditions*** and for broken stone with dust screened out.

‡Use 40% column for gravel or mixed stone and gravel.

§Use these columns for scientifically graded mixtures.

Volume of Concrete Based on a Barrel of 4 Cubic Feet.

(See important foot-notes, also p. 225.)

PROPORTIONS BY PARTS			PROPORTIONS BY VOLUME			Volume of mortar in terms of percentage of volume of stone	AVERAGE VOLUME OF RAMMED CONCRETE MADE FROM ONE BARREL CEMENT — Percentages of Voids in Broken Stone or Gravel				
Cement	Sand	Stone	Cement	Sand	Stone		50%*	45%†	40%‡	30%§	20%§
			bbl.	cu. ft.	cu. ft.	%	cu. ft.	cu. ft.	cu. ft.	cu. ft.	cu. ft.
1		1	1		4	80	5.4	5.6	5.8	6.4	6.9
1		2	1		8	40	7.6	8.0	8.4	9.5	10.5
1		3	1		12	35		10.4	11.0	12.7	14.2
1		4	1		16	28				15.8	17.8
1		5	1		20	24				18.9	21.5
1		6	1		24	22				22.1	25.1
1		7	1		28	20					28.8
1		8	1		32	18					32.4
1		9	1		36	17					36.1
1		10	1		40	16					39.7
1		11	1		44	15					43.4
1		12	1		48	15					47.0
1	1	1½	1	4	6	96	8.8	9.1	9.4	10.0	10.7
1	1	2	1	4	8	73	0.8	10.3	10.7	11.6	12.4
1	1	2½	1	4	10	59	10.9	11.5	12.0	13.1	14.2
1	1	3	1	4	12	50	12.0	12.7	13.3	14.6	15.0
1	1½	2	1	6	8	92	11.3	11.7	12.2	13.0	13.9
1	1½	2½	1	6	10	74	12.4	12.9	13.5	14.5	15.6
1	1½	3	1	6	12	62	13.5	14.1	14.8	16.0	17.3
1	1½	3½	1	6	14	54	14.5	15.3	16.0	17.6	19.1
1	1½	4	1	6	16	48	15.0	16.5	17.3	19.1	20.8
1	1½	4½	1	6	18	43	16.7	17.7	18.6	20.6	22.5
1	1½	5	1	6	20	30	17.8	18.9	19.0	22.1	24.3
1	2	3	1	8	12	74	14.0	15.6	16.2	17.5	18.8
1	2	3½	1	8	14	64	16.0	16.7	17.5	19.0	20.5
1	2	4	1	8	16	56	17.1	17.9	18.8	20.5	22.3
1	2	4½	1	8	18	51	18.1	19.1	20.1	22.0	23.9
1	2	5	1	8	20	46	19.2	20.3	21.4	23.5	25.7
1	2	5½	1	8	22	42	20.3	21.5	22.7	25.1	27.4
1	2	6	1	8	24	39	21.4	22.7	24.0	26.6	29.2
1	2½	3	1	10	12	86	16.3	17.0	17.6	18.9	20.2
1	2½	3½	1	10	14	75	17.[illegible]	18.2	18.9	20.5	22.0
1	2½	4	1	10	16	66	18.[illegible]	19.4	20.2	21.9	23.7
1	2½	4½	1	10	18	59	19.6	20.6	21.5	23.5	25.4
1	2½	5	1	10	20	54	20.7	21.8	22.8	25.0	27.2
1	2½	5½	1	10	22	49	21.8	22.9	24.1	26.5	28.0
1	2½	6	1	10	24	45	22.8	24.1	25.4	28.0	30.6
1	2½	6½	1	10	26	42	23.9	25.3	26.7	20.5	32.3
1	2½	7	1	10	28	39	25.0	26.5	28.0	31.0	34.0
1	3	4	1	12	16	75	20.0	20.8	21.7	23.4	25.1
1	3	4½	1	12	18	67	21.0	22.0	23.0	24.9	26.8
1	3	5	1	12	20	60	22.1	23.2	24.3	26.4	28.6
1	3	5½	1	12	22	55	23.2	24.4	25.6	28.0	30.3
1	3	6	1	12	24	50	24.3	25.6	26.9	29.5	32.1
1	3	6½	1	12	26	48	25.4	26.8	28.2	31.0	33.8
1	3	7	1	12	28	44	26.4	27.9	29.4	32.5	35.5
1	3	7½	1	12	30	42	27.5	29.1	30.8	34.0	37.2
1	3	8	1	12	32	39	28.6	30.3	32.0	35.5	39.0
1	4	5	1	16	20	75	25.0	26.1	27.2	29.3	31.5
1	4	6	1	16	24	63	27.2	28.5	29.8	32.4	35.0
1	4	7	1	16	28	55	29.3	30.8	32.4	35.4	38.4
1	4	8	1	16	32	48	31.5	33.2	34.9	38.4	41.9
1	4	9	1	16	36	43	33.6	35.6	37.5	41.4	45.3
1	4	10	1	16	40	40	35.8	38.[illegible]	40.1	44.4	48.8
1	5	10	1	20	40	47	38.7	40.[illegible]	43.0	47.3	51.7
1	6	12	1	24	48	46	45.9	48.5	51.1	56.3	61.4

NOTE. — Variations in the fineness of the sand and the compacting of the concrete may affect the volumes by 10% in either direction.

*Use 50% column for broken stone screened to uniform size.

†Use 45% column for average conditions and for broken stone with dust screened out.

‡Use 40% column for gravel or mixed stone and gravel.

§Use these columns for scientifically graded mixtures.

MATERIALS FOR RUBBLE CONCRETE.

Quantities of Materials for One Cubic Yard Based on a Barrel of 3.8 Cubic Feet.
(See important footnotes, also pp. 238 and 296.)

Percentage of Rubble in Total Volume of Concrete.	Proportions of Plain Concrete by Parts			Proportions of Plain Concrete by Volume			Percentages of Voids in Broken Stone or Gravel.											
							50%*			45%†			40%‡			30%§		
	Cement	Sand	Stone	Packed Cement bbl	Loose Sand cu. ft.	Loose Stone cu. ft.	Cement bbl.	Sand cu. yd.	Stone cu. yd.	Cement bbl.	Sand cu. yd.	Stone cu. yd.	Cement bbl.	Sand cu. yd.	Stone cu. yd.	Cement bbl.	Sand cu. yd.	Stone cu. yd.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	1	2	3	1	7.6	11.4	1.51	0.43	0.64	1.45	0.41	0.61	1.39	0.39	0.59	1.29	0.36	0.54
	1	2	4	1	7.6	15.2	1.32	0.37	0.74	1.25	0.35	0.70	1.20	0.34	0.67	1.10	0.31	0.62
	1	2	5	1	7.6	19.0	1.18	0.33	0.82	1.11	0.31	0.78	1.06	0.30	0.74	0.96	0.27	0.67
	1	2½	4	1	9.5	15.2	1.22	0.43	0.69	1.17	0.41	0.66	1.12	0.39	0.63	1.03	0.36	0.58
20%	1	2½	5	1	9.5	19.0	1.10	0.38	0.77	1.04	0.37	0.74	0.99	0.35	0.70	0.90	0.32	0.64
	1	2½	6	1	9.5	22.8	0.99	0.35	0.84	0.94	0.33	0.79	0.89	0.31	0.75	0.81	0.29	0.68
	1	3	5	1	11.4	19.0	1.02	0.43	0.72	0.98	0.42	0.69	0.94	0.39	0.66	0.86	0.36	0.60
	1	3	6	1	11.4	22.8	0.93	0.39	0.78	0.89	0.38	0.75	0.84	0.35	0.71	0.77	0.33	0.65
	1	3	7	1	11.4	26.6	0.86	0.36	0.84	0.81	0.34	0.79	0.77	0.32	0.76	0.70	0.30	0.69
	1	2	3	1	7.6	11.4	1.32	0.37	0.56	1.27	0.36	0.53	1.22	0.34	0.52	1.13	0.32	0.48
	1	2	4	1	7.6	15.2	1.15	0.32	0.65	1.10	0.31	0.62	1.05	0.29	0.59	0.97	0.27	0.55
	1	2	5	1	7.6	19.0	1.03	0.29	0.72	0.97	0.27	0.69	0.92	0.26	0.65	0.84	0.24	0.59
	1	2½	4	1	9.5	15.2	1.06	0.38	0.60	1.02	0.36	0.57	0.98	0.34	0.55	0.90	0.32	0.51
30%	1	2½	5	1	9.5	19.0	0.96	0.34	0.67	0.91	0.32	0.64	0.87	0.31	0.61	0.79	0.28	0.56
	1	2½	6	1	9.5	22.8	0.87	0.31	0.74	0.82	0.29	0.69	0.78	0.27	0.66	0.71	0.25	0.60
	1	3	5	1	11.4	19.0	0.90	0.38	0.63	0.85	0.36	0.60	0.82	0.34	0.57	0.75	0.32	0.53
	1	3	6	1	11.4	22.8	0.81	0.34	0.69	0.78	0.33	0.66	0.74	0.31	0.62	0.67	0.29	0.57
	1	3	7	1	11.4	26.6	0.75	0.32	0.74	0.71	0.30	0.69	0.67	0.28	0.67	0.61	0.26	0.60
	1	2	3	1	7.6	11.4	1.13	0.32	0.48	1.09	0.31	0.46	1.04	0.29	0.44	0.97	0.27	0.41
	1	2	4	1	7.6	15.2	0.99	0.28	0.56	0.94	0.26	0.53	0.90	0.25	0.50	0.83	0.23	0.47
	1	2	5	1	7.6	19.0	0.88	0.25	0.62	0.83	0.23	0.59	0.79	0.22	0.56	0.72	0.20	0.50
	1	2½	4	1	9.5	15.2	0.91	0.32	0.52	0.88	0.31	0.49	0.84	0.29	0.47	0.77	0.27	0.44
40%	1	2½	5	1	9.5	19.0	0.82	0.29	0.58	0.78	0.28	0.55	0.74	0.26	0.52	0.68	0.24	0.48
	1	2½	6	1	9.5	22.8	0.74	0.26	0.63	0.70	0.25	0.59	0.67	0.23	0.56	0.61	0.22	0.51
	1	3	5	1	11.4	19.0	0.77	0.32	0.54	0.73	0.31	0.52	0.70	0.29	0.49	0.64	0.27	0.45
	1	3	6	1	11.4	22.8	0.70	0.29	0.59	0.67	0.28	0.56	0.63	0.26	0.53	0.58	0.25	0.49
	1	3	7	1	11.4	26.6	0.65	0.27	0.63	0.61	0.26	0.59	0.58	0.24	0.57	0.52	0.22	0.52
	1	2	3	1	7.6	11.4	0.94	0.27	0.40	0.90	0.26	0.38	0.87	0.24	0.37	0.80	0.22	0.34
	1	2	4	1	7.6	15.2	0.82	0.23	0.46	0.78	0.22	0.44	0.75	0.21	0.42	0.69	0.20	0.39
	1	2	5	1	7.6	19.0	0.74	0.20	0.52	0.70	0.20	0.49	0.66	0.18	0.46	0.60	0.17	0.42
	1	2½	4	1	9.5	15.2	0.76	0.27	0.43	0.73	0.26	0.41	0.70	0.24	0.40	0.64	0.22	0.36
50%	1	2½	5	1	9.5	19.0	0.68	0.24	0.48	0.65	0.23	0.46	0.62	0.22	0.44	0.56	0.20	0.40
	1	2½	6	1	9.5	22.8	0.62	0.22	0.52	0.58	0.20	0.50	0.56	0.20	0.47	0.50	0.18	0.42
	1	3	5	1	11.4	19.0	0.64	0.27	0.45	0.61	0.26	0.43	0.58	0.24	0.41	0.54	0.22	0.38
	1	3	6	1	11.4	22.8	0.58	0.24	0.49	0.56	0.24	0.47	0.52	0.22	0.44	0.48	0.20	0.40
	1	3	7	1	11.4	26.6	0.54	0.22	0.52	0.56	0.22	0.49	0.48	0.20	0.48	0.44	0.18	0.43
	1	2	3	1	7.6	11.4	0.76	0.21	0.32	0.72	0.20	0.30	0.70	0.20	0.30	0.64	0.18	0.27
	1	2	4	1	7.6	15.2	0.66	0.18	0.37	0.63	0.18	0.35	0.60	0.17	0.34	0.55	0.16	0.31
	1	2	5	1	7.6	19.0	0.58	0.16	0.41	0.56	0.16	0.39	0.53	0.15	0.37	0.48	0.14	0.34
	1	2½	4	1	9.5	15.2	0.61	0.22	0.34	0.58	0.20	0.33	0.56	0.20	0.32	0.52	0.18	0.29
60%	1	2½	5	1	9.5	19.0	0.55	0.19	0.38	0.52	0.18	0.37	0.50	0.18	0.35	0.45	0.16	0.32
	1	2½	6	1	9.5	22.8	0.50	0.18	0.42	0.47	0.16	0.40	0.44	0.16	0.38	0.40	0.14	0.34
	1	3	5	1	11.4	19.0	0.51	0.22	0.36	0.49	0.21	0.34	0.47	0.20	0.33	0.43	0.18	0.30
	1	3	6	1	11.4	22.8	0.46	0.20	0.39	0.44	0.19	0.38	0.42	0.18	0.36	0.38	0.16	0.32
	1	3	7	1	11.4	26.6	0.43	0.18	0.42	0.40	0.17	0.40	0.38	0.16	0.38	0.35	0.15	0.34

NOTE.—Variations in the fineness of the sand and the compacting of the concrete may affect the quantities by 10% in either direction.
* Use 50% columns for broken stone screened to uniform size.
† Use 45% columns for average conditions and for broken stone with dust screened out.
‡ Use 40% columns for gravel or mixed stone and gravel.
§ Use 30% columns for scientifically graded mixtures.

VOLUME OF RUBBLE CONCRETE

Based on a Barrel of 3.8 Cubic Feet (see important footnotes, also pp. 238 and 296).

PERCENTAGE OF RUBBLE IN TOTAL VOLUME OF CONCRETE.	PROPORTIONS OF PLAIN CONCRETE BY PARTS.			PROPORTIONS OF PLAIN CONCRETE BY VOLUME.			AVERAGE VOLUME OF RUBBLE CONCRETE MADE FROM ONE BARREL CEMENT. Percentages of Voids in Broken Stone or Gravel.			
	Cement.	Sand.	Stone.	Cement. bbl.	Sand. cu. ft.	Stone. cu. ft.	50%* cu. ft.	45%† cu. ft.	40%‡ cu. ft.	30%§ cu. ft.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
20%	1	2	3	1	7.6	11.4	17.9	18.6	19.4	20.9
	1	2	4	1	7.6	15.2	20.4	21.5	22.5	24.5
	1	2	5	1	7.6	19.0	23.0	24.2	25.5	28.1
	1	2½	4	1	9.5	15.2	22.0	23.1	24.1	26.2
	1	2½	5	1	9.5	19.0	24.8	26.0	27.2	29.9
	1	2½	6	1	9.5	22.8	27.3	28.8	30.4	33.4
	1	3	5	1	11.4	19.0	26.4	27.6	29.0	31.4
	1	3	6	1	11.4	22.8	29.0	30.5	32.0	35.1
	1	3	7	1	11.4	26.6	31.5	33.4	35.1	38.8
30%	1	2	3	1	7.6	11.4	20.4	21.3	22.2	23.9
	1	2	4	1	7.6	15.2	23.3	24.6	25.7	28.0
	1	2	5	1	7.6	19.0	26.3	27.7	29.2	32.1
	1	2½	4	1	9.5	15.2	25.3	26.4	27.6	30.0
	1	2½	5	1	9.5	19.0	28.3	29.7	31.2	34.2
	1	2½	6	1	9.5	22.8	31.2	32.9	34.7	38.2
	1	3	5	1	11.4	19.9	30.2	31.6	33.2	36.0
	1	3	6	1	11.4	22.8	33.2	34.9	36.6	40.2
	1	3	7	1	11.4	26.6	36.8	38.2	40.2	43.0
40%	1	2	3	1	7.6	11.4	23.8	24.8	25.8	27.8
	1	2	4	1	7.6	15.2	27.2	28.7	30.0	32.7
	1	2	5	1	7.6	19.0	30.7	32.3	34.0	37.5
	1	2½	4	1	9.5	15.2	29.5	30.8	32.2	35.0
	1	2½	5	1	9.5	19.0	33.0	34.7	36.3	39.8
	1	2½	6	1	9.5	22.8	36.3	38.4	40.5	44.5
	1	3	5	1	11.4	19.0	35.2	36.8	38.7	42.0
	1	3	6	1	11.4	22.8	38.7	40.7	42.7	46.8
	1	3	7	1	11.4	26.6	42.0	44.5	46.8	51.7
50%	1	2	3	1	7.6	11.4	28.6	29.8	31.0	33.4
	1	2	4	1	7.6	15.2	32.6	34.4	36.0	39.2
	1	2	5	1	7.6	19.0	36.8	38.8	40.8	45.0
	1	2½	4	1	9.5	15.2	35.4	37.0	38.6	42.0
	1	2½	5	1	9.5	19.0	39.6	41.6	43.6	47.8
	1	2½	6	1	9.5	22.8	43.6	46.0	48.6	53.4
	1	3	5	1	11.4	19.0	42.2	44.2	46.4	50.4
	1	3	6	1	11.4	22.8	46.4	48.8	51.2	56.2
	1	3	7	1	11.4	26.6	50.4	53.4	56.2	62.0
60%	1	2	3	1	7.6	11.4	35.8	37.2	38.8	41.8
	1	2	4	1	7.6	15.2	40.8	43.0	45.0	49.0
	1	2	5	1	7.6	19.0	46.0	48.5	51.0	56.3
	1	2½	4	1	9.5	15.2	44.3	46.3	48.3	52.5
	1	2½	5	1	9.5	19.0	49.5	52.0	54.6	59.8
	1	2½	6	1	9.5	22.8	54.5	57.5	60.8	66.8
	1	3	5	1	11.4	19.0	52.8	55.3	58.0	63.0
	1	3	6	1	11.4	22.8	58.0	61.0	64.0	70.3
	1	3	7	1	11.4	26.6	63.0	66.8	70.3	77.5

NOTE:—Variations in the fineness of the sand and the compacting of the concrete may affect the quantities by 10% in either direction.

*Use 50% column for broken stone screened to uniform size.

†Use 45% column for average conditions and for broken stone with dust screened out.

‡Use 40% column for gravel or mixed stone and gravel.

§Use 30% column for scientifically graded mixtures.

TABLES OF RUBBLE CONCRETE

The tables on pages 236 and 237 give the quantities of materials and the volumes of concrete mixed in different proportions and with different percentages of rubble. The values are made up as described on pages 298 and 299, where illustrations are given of the methods of computing the cost.

The percentages of rubble are based on the ratio of the volume of the concrete after it is laid to the actual volume of the large stone contained in it. In other words, it is the percentage of the finished concrete occupied by the large stone.

CHAPTER XIII

PREPARATION OF MATERIALS FOR CONCRETE

The various operations relating directly to the laying of concrete are discussed in detail in this and several succeeding chapters. While the selection of the special methods and machinery, which are described at length in the succeeding chapters, are determined by local conditions, certain general principles apply to all classes of work. The preparation of the materials relates to the storing of cement, the screening of sand and gravel, and the crushing of stone.

STORING CEMENT

Portland cement is not injured by storing in a dry place for an indefinite length of time; in fact, contrary to former belief, instead of deteriorating, the quality is often improved by storage. Cement manufacturers when rushed with orders sometimes ship material which, not being sufficiently air-slaked, contains free lime that exposure to air may change to a hydrate and thus render harmless.

Recognition of the fact that exposure to dry atmosphere does not injure cement has led to packing it in bags instead of in barrels, thus saving both the cost of the barrel and the extra freight upon it. If, however, the work is in a damp location, as in marine construction, barrel shipments are advisable.

The economy of storing the cement as near as possible to the mixing platform or mixing machine is obvious, but since, on the other hand, it is more easily handled and is always less in volume than sand and stone, these should be given the preference in the matter of location.

SCREENING SAND AND GRAVEL

The three most common methods of screening are (1) by hand, that is, by throwing shovelfuls of the material on to an inclined screen, (2) by dumping or hoisting the material on to a fixed inclined screen, (3) by a revolving screen.

Cost of Hand Screening. The cost of hand screening depends upon the total amount of material handled rather than upon the quantity of sand or gravel produced. A material most of whose particles run through the screen can be most cheaply screened, because the screen can be moved,

or arranged over a hole, while if a large proportion of the particles are caught they must be shoveled from the foot of the screen.

An average laborer, properly superintended, will throw about 24 cu. yd. of material against a screen in a ten-hour day, but in estimating the cost, allowance must be made for shoveling the material out of the way, moving screen, and superintendence.

The following are approximate costs of screening sand and gravel by hand under ordinary conditions. The prices are from actual records on a number of jobs and are based on labor at $1.50 for ten hours, with a suitable allowance for superintendence and contractor's profit. The minimum prices apply to first-class men.

	Average cost per cu. yd.	Minimum cost per cu. yd.
Screening sand, coarse stuff wasted	$0.11	$0.08
Screening gravel to remove large stones	0.15	0.10
Screening gravel to remove sand, sand wasted	0.24	0.17
Screening gravel coarse, and fine stuff, both measured	0.18	0.12

If laborers are working alone with no foreman in sight, as is often the case on concrete work, 50% should be added to the average costs.

Inclined Screen fed by Carts, Derrick Buckets, or Endless Chain. The slope of an elevated screen may vary from 35° to 45° from the horizontal, according to the character of the material. Coarser screens are required to pass material of a certain size than for hand screening.

At the new Cambridge Bridge, Boston, the contractors employed a screen about 15 feet long, hinged at the top so that the slope could be varied to suit the material. A hopper located above the screen fed on to a 3-inch bar screen, consisting of parallel iron bars about 3 inches apart, supported by iron cross pieces about 5 inches apart. The stones too large for the concrete ran down this coarse screen, and rolled off one side, while the remainder of the material fell through it on to a screen with 1-inch by ¾-inch mesh, which separated the medium gravel from the sand.

On another large job in Everett, Mass., where an inclined screen was fed by a bucket elevator supplied by carts, 300 to 350 cu. yd. of sand and gravel were screened in ten hours, and an even larger quantity could have been handled had it been supplied with absolute regularity.

The cost of screening by this method depends both upon local conditions and the quantity screened. The average cost may be assumed to be from 4 to 8 cents per cubic yard when large quantities of sand or gravel are handled at once.

Rotating Screens. Rotating screens, cylindrical or hexagonal in shape, although most frequently employed for separating crushed stone

(see p. 245), are also adapted, if power is available, for separating sand from gravel, or for separating gravel into several sizes to remix in the theoretical proportions required for a dense, impervious concrete.

While the first cost of a rotating screen is more than that of an inclined screen, less elevation is required and it may be fed with a bucket conveyor.

A plant for ordinary concrete made from two aggregates, sand and gravel, requires a screen with only two sizes of mesh, the smaller about ⅜-inch and the larger 2, 2½ or 3-inch mesh, as desired. Often no screening is required except to remove the sand, as a few large stones do no harm. The screen may be about 3 feet in diameter by 12 feet in length.

The present tendency, for concrete which is to be subjected to severe stress or to water pressure, is to require more scientific proportioning by separating the aggregate into several sizes and remixing them so as to produce the greatest density. This separation may be accomplished in practice by adding more sections, and thus lengthening the screen, or by employing a double cylinder, which occupies about half the space of a single cylinder.

The inner cylinder of a double cylinder screen is composed of two or more sections of different sized mesh, and the outer cylinder is composed of two or more corresponding sections which are entirely separate from each other so that each may discharge into a separate bin. Each outer section has a finer mesh than the corresponding section of the inner cylinder. The material, after passing through a section of the inner cylinder, falls upon the outer wire and is again separated, the part which is caught rolling out through an annular opening into one bin and the remainder passing through the mesh into another bin.

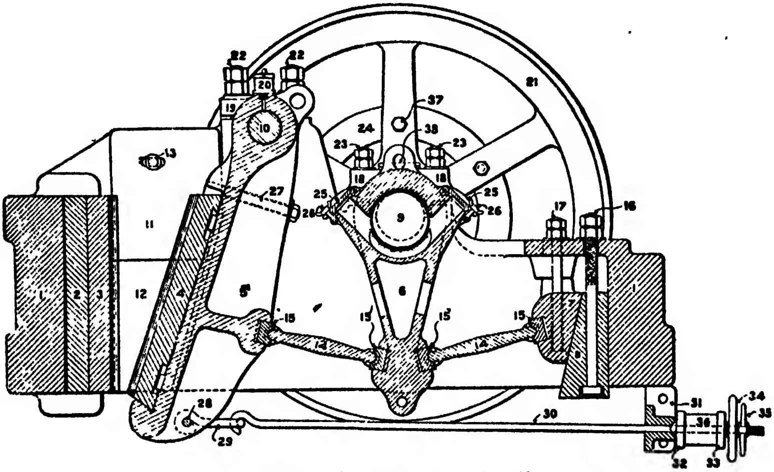
STONE CRUSHING

The crushing of stone for concrete must be approached from a different standpoint than the preparation of material for macadam paving, although the costs will not vary materially from those of a well-arranged portable crushing plant used on road construction.

For city or town macadam paving, where a suitable ledge is available, it is possible to establish a fixed plant with stationary engine, large stone bins, and economical machinery for handling cars, so that the stone can be hauled over a system of movable tracks directly from the ledge to the crusher, while for country road building the plant is arranged with a view to its portability, sometimes even resting on wheels.

For concrete work a plant intermediate in style between these is usually required. Its design is governed by the local conditions and by the quan-

tity of concrete to be made. In some cases where the concrete is laid in excavation it is possible to locate the crusher on the bank, and allow the stone to pass by gravity on to and through an inclined screen, or, if "crusher run" is used, to fall directly into a pile below. Generally the stone from the crusher must be taken by bucket or belt conveyors to bins, located, if possible, above the concrete mixer, or where the stone can be conveniently conveyed to the mixer without shoveling.



NAME AND NUMBER OF PARTS

1 Main Frame
2 Round Back
3 Fixed Jaw Plate
4 Swing Jaw Plate
5 Swing Jaw
6 Pitman
7 Toggle Block
8 Wedge
9 Eccentric Shaft
10 Swing Jaw Shaft
11 Upper Half Cheek Plate
12 Lower Half Cheek Plate
13 Bolt for Cheek Plate
14 Toggle
15 Toggle Bearing
16 Bolt for Wedge
17 Bolt for Toggle Block
18 Cover for Main Bearin
19 Cover for Swing Jaw Shaft
20 Grease Cup
21 Balance Wheel
22 Bolt for Swing Jaw Shaft Cover
23 Bolt for Main Bearing
24 Pulley
25 Grease Box Cover
26 Bolt and Thumb Screw
27 Bolt for Swing Jaw Plate
28 Shackle Pin
29 Spring Rod Shackle
30 Spring Rod
31 Spring Bar
32 Washer
33 Washer
34 Hand Wheel
35 Thumb Nut
36 Rubber Spring
37 Bolt for Pulley
38 Grease Box Cover on Main Bearing

FIG. 77.—Jaw Crusher. (*See p.* 242.)

Stone Crushers. Stone crushers are of two general types, jaw crushers and gyratory crushers.

The size of a jaw crusher is designated by the opening into which the stone is introduced. A 16 by 10-inch crusher has jaws 16 inches in width, and the space between the two jaws at the top is 10 inches. A "duplex" crusher has two pairs of jaws operated by the same shaft, but working alternately by means of different eccentrics. Single jaw crushers range in size from 3 by 1½ inches to 36 by 24 inches.

The operation of a typical jaw crusher is shown in Fig. 77. One of the jaws is fixed, and the other is hinged at the top, and swung back and forth

through a very small arc. The motion is imparted by the eccentric shaft, which, in revolving, raises and lowers the "pitman," whose lower end is connected by toggles with the lower end of the movable jaw. The size of the stone passing through the jaws, that is, the size of the largest particles, is regulated by the opening at the bottom of the swing jaw, which is changed by using longer or shorter toggles.

The capacity of any crusher,— that is, the quantity of broken stone which it will turn out per hour or per day — is dependent not only upon the size of the crusher, but upon the texture of the stone and the sizes of the largest particles. From the following catalogue capacities for a 16 by 10-inch jaw crusher per day of ten hours, it may be inferred that the quantity turned out is nearly in the ratio of the sizes of the stones.

120	tons	crushed	to	2½	-inch	size
100	"	"	"	2	"	"
80	"	"	"	1½	"	"
60	"	"	"	1	"	"

In estimating the actual daily output of a crusher, — and this is in fact true for most machinery, — all catalogue figures are likely to be misleading because they are based on maximum capacity with continuous feeding, while in practice there are likely to be unavoidable delays. An average day's work of ten hours, — based on actual records obtained by the authors from a number of jobs, — for a 15 by 9-inch crusher set for 2½-inch stone, with a small percentage of tailings, may be taken at 65 cu. yd. or, say, 78 tons, in ten hours. This estimate applies to continuous running of the crusher, allowing only for occasional unavoidable delays.*

A section of a gyratory crusher, which is adapted for more stationary plants, is shown in Fig. 78, page 244. It consists essentially of a cone with a gyratory motion within an inverted conical chamber or shell. The size of the crusher is determined by the width of the opening between the top of the cone and the shell, and the circumference. The gyratory motion of the cone shaft is produced by an eccentric keyed to its lower end. As the shaft revolves, the cone is given a kind of a rocking motion which continually directs it toward, and then away from, different portions of the shell. The size of the broken stone is regulated by raising or lowering the cone on the shaft.

For a concrete plant producing 200 cubic yards per day, manufacturers recommend a No. 4 gyratory crusher with openings 8 x 27 inches.

The horse-power required to drive a crusher and its attendant machinery

*The Annual Report of the Newton, Mass., City Engineer for 1891 gives interesting data on detail costs of stone crushing, a portion of which are here summarized on page 249.

varies largely with the material handled. It is advisable to make ample allowance above the figures given in manufacturers' catalogues. It is, also, economical to use a wider and heavier belt than is generally specified,

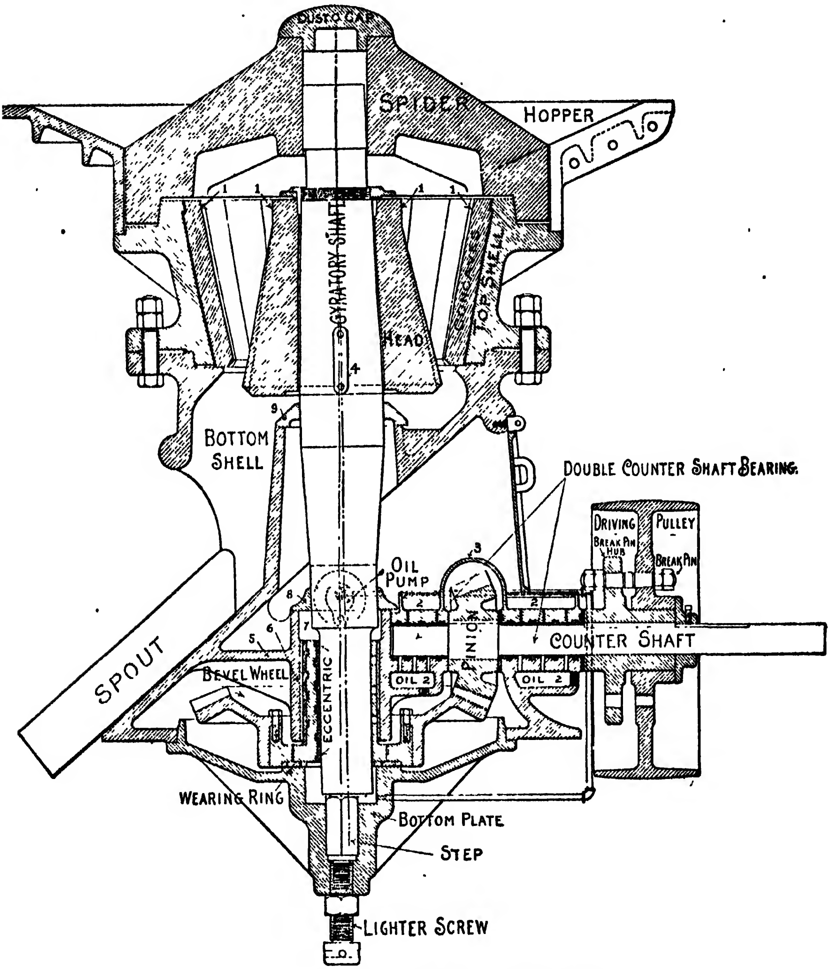


FIG. 78.—Gyratory Crusher. (See p. 243.)

in order to avoid delays and shutdowns. When ordering almost any kind of machinery the authors make it a practice to require a wider and heavier pulley than the standard width. It is wise to make a pulley at least 2 inches wider than the belt which is to be run upon it.

Crusher Screens and Bins. A typical design, by Mr. Earle C. Bacon, for bins suitable for a plant where the concrete mixer or mixing platform is located at a distance from the crusher is shown in Fig. 79. With slight changes they may be arranged to discharge into hoppers over a concrete mixer. The dimensions of timber employed in the construction may be used as a basis for bins of other sizes.

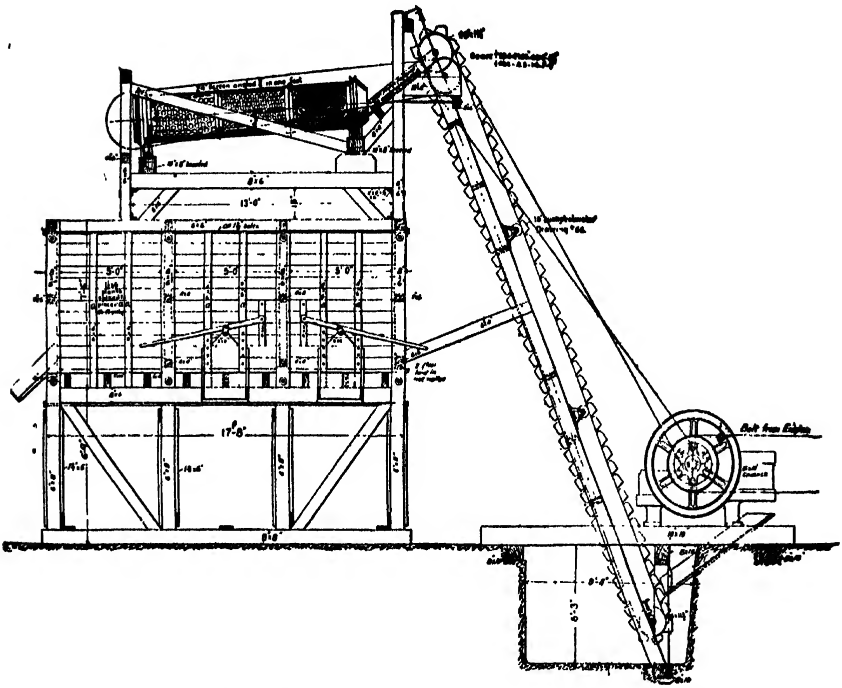


FIG. 79.—Small Crushing Plant with Elevator, Screen, and Portable Bins. (See p. 245.)

A safe slope for the bottom of stone bins is 45°, although if lined with sheet iron this may be decreased to 35° or 40°.

Screens for broken stone as shown in Fig. 80, page 246, are usually made in sections varying in length from 3 to 5 feet, so that they can be bolted together and give as many divisions of sizes as are required. The diameters vary from 24 to 48 inches. The mesh of a rotating screen should be about 20% smaller in diameter than the required size for the stone, as there is more or less wear on the screen, which enlarges the holes, and this allowance will also assist in excluding the oblong pieces whose longest dimen-

sion is above the limit. For concrete, unless two or more sizes of stone are mixed, no more than two sizes of mesh are required, one, ¼-inch to remove the dust, and the other, 2, 2½, or 3-inch to remove the coarse stuff. Often it is necessary only to remove the dust which may then be used as sand.

Stone Bin Gates. A gate designed by Mr. C. S. MacHenry, of the Greene Consolidated Copper Co., has proved extremely satisfactory for cutting off the flow of materials of the nature of broken stone, gravel, and sand. A detail drawing of this is shown in Fig. 81.

Cost of Stone Crushing. The cost of stone crushing is so dependent

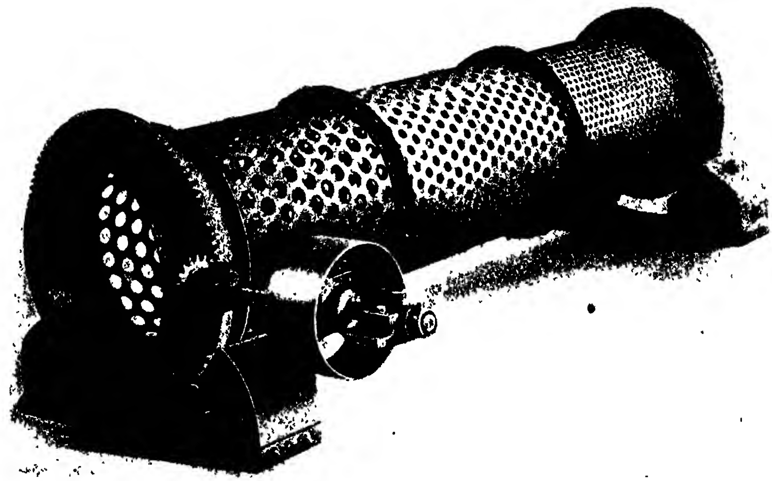


FIG. 80 — Rotating Screen. (See p. 245.)

upon local conditions and upon the character of the rock, that only approximate estimates based upon actual experience can be given. There are, in general, two classes of work, — one where the rock is blasted from a ledge near at hand, and the other where the crushers are supplied with boulders or other loose rock. The gang at the crusher is similar in both cases, and the chief difference in operation is the extra gang for drilling and breaking up the stone in the ledge. On the other hand, usually more permanent, and therefore more economical, arrangements for hauling the stone can be made in ledge excavation than when the stone is obtained from various sources.

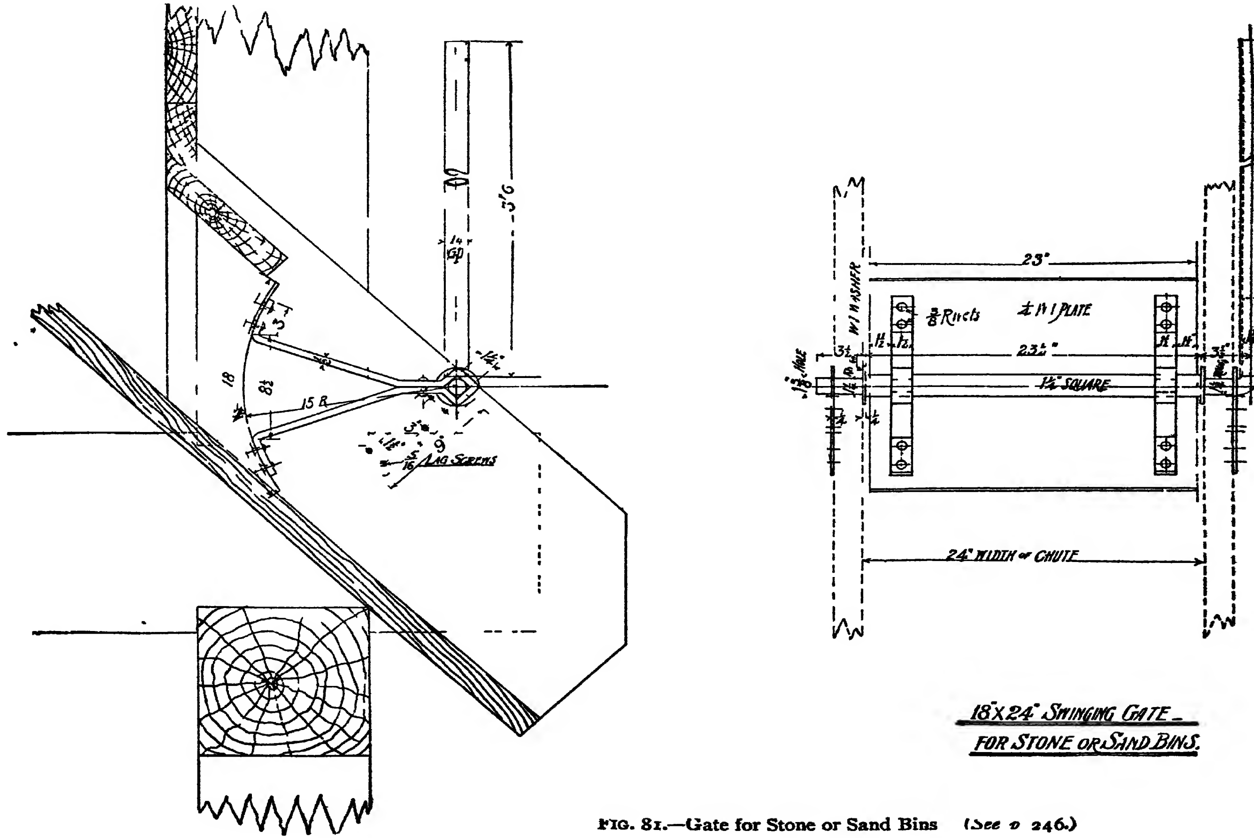


FIG. 81.—Gate for Stone or Sand Bins (See p. 246.)

A typical gang* for operating a 15 by 9-inch crusher, turning out, say, 65 cubic yards of broken stone in ten hours, is as follows:

One foreman.
One engineman.
Two men feeding crusher.
One other man at crusher on odd work.
Three men loading stone into carts to supply crusher.
Two single carts with one teamster hauling stone to crusher.

The number of teams required to haul stone to crusher depends, of course, upon the length of haul. Sometimes additional men will be needed to pass the stone to the men feeding the crusher; on the other hand, if the stone is dumped directly into a hopper above the crusher so as not to require handling, two men are capable of supplying a crusher whose capacity is 200 cubic yards per day.

The labor of drilling a ledge obviously depends upon the quality and seaminess of the rock and the depth of the holes. Under ordinary conditions, a steam drill with two men can be counted upon to loosen considerably more rock than can be handled by a 15 by 9-inch crusher. The cost of barring out and sledging the blasted rock may be estimated on the basis of about 10 cubic yards (measured after crushing) per man per day of ten hours. If the crusher is a large one, say a No. 6 rotary (11 by 36 in.), a man will bar and sledge about double this quantity because it does not need to be broken so fine. The figures are averaged by the authors from actual observed speeds on a number of jobs.

In estimating the cost of crushing stone, the original cost of the plant is an important item. The allowance for this per yard of rock is dependent upon the length of time the plant is to be operated, and the probable value of the machinery when the work is complete, as well as upon the interest on the investment and the cost of repairs. A plant similar to that shown in Fig. 79, page 245, with a 16 by 10-inch jaw crusher, may be estimated to cost from $2,000 to $2,500.†

A very careful analysis of the actual cost of crushing stone for macadam in a large gyratory crusher was made by Mr. Albert F. Noyes, City Engineer of Newton, Mass. His prices are based on common labor at $1.75 per day of nine hours, drill men at $3.00, drill helpers at $1.75, engineman for crusher at $2.00, and two one-horse carts with driver at $5.00. The detail costs per cubic yard of crushed stone were as follows:

*Actual gang employed on a concrete contract for the Metropolitan Water Works, Mass.
†Estimated by Earle C. Bacon.

*Cost per cubic yard of Quarrying and Crushing Hard Green Trap at Newton, Mass.**

Labor of steam drilling	$0.092
Coal, oil, waste, powder, drilling and repairs for drilling and blasting	0.084
Sharpening drills and tools	0.069
Breaking stone for crusher	0.279
Filling carts with rough stone	0.098
Carting stone to crusher	0.072
Feeding crusher	0.053
Engineman of crusher	0.031
Coal, oil, and waste for crusher	0.079
Repairs	0.041
Total cost per cubic yard of crushed stone	$0.898

The total cost of crushing in a jaw crusher conglomerate ledge stone drilled by hand, Mr. Noyes gives as $1.113 per cubic yard; of trap cobble stone wheeled to crusher in barrows, as $0.445 per cubic yard; and of granite cobble stone hauled in carts, as $0.372 per cubic yard.

These costs, which, as well as the wages paid per day, must be taken into account when estimating under other conditions, are based upon an output per hour of 7.7 cubic yards hard green trap, 8.9 cubic yards conglomerate ledge, 11.8 cubic yards trap cobble stone, and 9 cubic yards granite cobble stone.†

Data on Broken Stone. Broken stone is often sold by weight instead of by the cubic yard, because of the variation in volume due to handling or transporting. A cubic yard of broken trap stone may vary in weight from 2 400 to 2 700 pounds.‡ If measured after carting some distance, broken stone will weigh about 10% heavier per cubic yard than at the crusher, because of the settling. The authors have found by repeated measurements that 100 pounds per cubic foot is a fair average weight for screened trap rock after it has been shaken down by hauling, although when measured loose in a small measure an average weight is about 90 pounds. Crusher run stone is about 10% heavier than this because it contains less voids. Stones having lower specific gravities than trap are correspondingly lighter in weight.§

On macadamized or paved roads, if no steep hills are to be encountered, two horses will haul from 6 000 to 7 000 pounds of broken stone to a load. Very high side boards are of course necessary to carry this quantity.

*Annual Report of City Engineer for 1891.

†Cost per cubic yard of stone crushing for pavement in various towns is given in Report Mass. Highway Commission, 1895, p. 38, and further data in *Engineering News*, March 27, 1902, p. 258, and Jan. 15, 1903, p. 55.

‡For data on weights, see article by W. E. McClintock in Journal Association Engineering Societies, Vol. XI., p. 424.

§See table, p. 163.

Numbers are used to designate the sizes of stone on road construction, and stone bought from a crusher is likely to be sold in this way. In such cases it must be borne in mind that these numbers are of local significance. Some plants call their finest product, including dust, No. 1 stone, while others commence to number from their coarsest size or tailings.

WASHING SAND AND STONE

Gravel frequently requires washing to remove the coating of clay or loam from the pebbles Crushed stone may require removal of the dust. Sand sometimes has too much silt to produce a strong concrete, or may contain vegetable matter (see p 154b) which renders it absolutely unfit for concrete. Washing also may be employed to assist in the separation of aggregates into the sizes required for accurate proportioning

The most satisfactory plan for washing appears to be to wash the material down a trough over screens in the bottom of the trough, or against and through screens inclined in the opposite direction from the trough Screens with round punched holes are better for this purpose than wire mesh

Bellows Falls Canal Company's Plant. The method used by the Aberthaw Construction Company for washing both the crushed stone and gravel consisted of shoveling the material from an elevated platform into inclined chutes over the upper end of which were placed eight 1 inch pipes with their lower ends hammered together to form a spray The water from these pipes washed the gravel and stone down the chute into storage bins below, the dirty water passing through screens near the bottom of the chute into troughs lined with tarred paper which carried it away For washing stone or gravel, ¼ inch screens were used, and for sand, No 20 mesh screens, the latter requiring frequent cleats to support the wire cloth

Rockingham Power Company Washing Plant.* In this plant the gravel was dumped as it came from the pit into hoppers forming the upper end of an inclined sluice carried on a light pole trestle Enough water was then drawn from an elevated tank to float the gravel down the chute to the lower end which terminated in an inclined screen with ½ inch mesh. The water and sand passed through the screen into hoppers below, while the pebbles rolled along the screen and passed over the end into a gondola car The water overflowing the sides of the sand hopper carried off the loam and lighter material while the sand settled, and when the hopper was filled it could be drawn off into cars beneath.

* Engineering-Contracting, May, 13, 1908, p 292.

CHAPTER XVI

MIXING CONCRETE

The method employed for mixing concrete is immaterial, provided the result is a homogeneous mass of the required uniform consistency, containing the various aggregates and cement in proper proportions. If the color of the mass is not absolutely uniform, that is, if uncoated particles of sand or stone are visible, if masses of stones are separate from the mortar, or if some portions of the mortar are dryer than others, the mixing has not been thorough.

Hand vs. Machine Mixing. First-class concrete may be produced, with careful superintendence, by either hand or machine-mixing.

The relative cost of the two methods depends entirely upon circumstances, and must be estimated for each individual case If the job is a small one, so that the cost of erecting the plant plus the interest and depreciation, divided by the number of cubic yards to be made, is a large item, or if frequent moving is required, concrete may be and often is mixed cheaper by hand than by machinery. The information which follows concerning both methods will serve as a guide for comparison in special cases.

MIXING CONCRETE BY HAND

The methods employed by different engineers and contractors for handling the materials and arranging the men are nearly as varied with hand-mixed as with machine-mixed concrete Concrete mixing is seemingly so simple an operation that it is often neglected by the inspector, and poor workmanship escapes detection.

The inspector should lay the greatest stress upon (*a*) exact measurement of the gravel or broken stone, (*b*) thorough mixture of the cement and sand, (*c*) thorough mixture of the mass, and (*d*) care in dumping the concrete into place. The quantity of water used in the mixing and the proper ramming or puddling of the concrete in place are equally important but are less likely to be overlooked.

In proportioning the ingredients, it is poor economy to make allowance for insufficient mixing or improper handling of the materials. The additional cement will be much more expensive than the extra time expended by laborers in securing a homogeneous mixture.

In the first place the mixing platform should be located as near the work

as possible, and so situated that the coarse materials can be conveniently dumped on one side of it and the sand on the other. It should be not less than 15 to 20 feet square if all the work is to be done upon it, and except for a very small job should be of 2-inch plank, planed one side, spiked to, say, 2 by 4-inch stringers about 5 feet apart, so that it can be moved from place to place as required. A 2 by 3-inch strip around the edge will prevent loss of material. If the sand and cement are made into a mortar before mixing with the stone, the platform may be narrower and a mortar box employed in addition.

Methods of Measuring Material. Cement should invariably be measured by weight. In practice this is accomplished not by weighing on scales but by counting packages, since bags or barrels of cement have standard weights.*

The volumes of sand and stone or other aggregate should be distinctly stated in the proportions in terms of the number of cubic feet of each material to a barrel of cement, or else by parts, coupled with the explanation that one part, or barrel, represents a definite volume, such as 3.8 cubic feet. In specifications where the proportions are given by parts with no unit of measurement, the contractor undoubtedly has the legal right to base the volumes of aggregate on the loose measurement of cement, hence the necessity of exact statement of units, as prescribed on page 217.

The sand measure preferred by the authors is a bottomless box similar to the gravel box shown in Fig. 5, page 18, having a depth of about 6 inches, and other dimensions determined by the required volume. The filling of cement barrels or half-barrels with sand is a slower and less accurate process. If the sand cannot be conveniently unloaded close to the measuring platform, it may be measured in a barrow or other wheeled vehicle so constructed that it can be accurately leveled off after filling. For rough measurement ordinary contractors' barrows, whose approximate "large" capacities are given on page 9, are suitable. If more exact quantities are required, however, it takes only a few more seconds to dump the sand from the barrows into a bottomless box.

For gravel or broken stone a bottomless box about 8 or 9 inches deep, shown in Fig. 5, page 18, is a convenient measure. Special barrows built to exact dimensions are more exact measures than ordinary contractors' barrows and, in some cases, than the bottomless box, because an unscrupulous contractor can more easily heap the material in the latter when the inspector's back is turned. Cement barrels are accurate measures, but time is wasted in lifting the shovels when filling, and in dumping them.

*See page 2.

A measuring barrow car,* built so that it can be handled with a derrick, is sometimes convenient.

Hand Mixing. A detailed description of one of the best ways to mix concrete by hand is given in Chapter II for the benefit of those not familiar with concreting. It is the general opinion of concrete experts that the particular order adopted for mixing the materials has little effect upon the strength of the concrete, provided the materials are turned a sufficient number of times to incorporate them thoroughly. Some engineers prefer to make the cement and sand into a mortar, while others do not add the water until the final turning. The authors have seen excellent work produced by both methods, but prefer the latter chiefly because shoveling the mortar on to the stone involves more labor than handling the dry mixed cement and sand; in fact, comparative tests show that it costs less to mix the cement and sand dry, shovel the mixture on to the stone and mix three times, than to make a mortar, shovel it on to the stone and mix only twice.

Methods variously employed, the first of which is described in detail on page 21, are outlined as follows:

(1) Cement and sand mixed dry and shoveled on to the stone or gravel leveled off, and wet as the mass is turned.

(2) Cement and sand mixed dry, and the stone or gravel dumped on top of it, leveled off, and wet as the mass is turned.

(3) Cement and sand mixed with water into a mortar which is shoveled on to the gravel or stone, and the mass turned with shovels.

(4) Cement and sand mixed with water into a mortar, the gravel or stone spread on top of it, and the mass turned with shovels.

(5) Gravel or stone, sand, and cement, spread in successive layers, mixed slightly and shoveled into a circle or crater, water poured into the center, and the mass mixed with shovels and hoes.

The last method is applicable only where a small amount of concrete is to be mixed on the ground with no mixing platform or mortar box.

Sand and cement must never be mixed up in advance, as lime and sand are often mixed, because the natural moisture which all sands contain will make the cement set and cake.

The systematic arrangement of the men in pairs, as described on page 21, and insistence upon their shoveling from the bottom of the pile and then turning their shovels completely over, are essentials for thoroughly mixed concrete. In the final wet mixing the materials should be turned in this way two or three times.

For wetting the concrete some engineers specify spraying with the hose,

*See illustration in *Engineering News*, April 23, 1896, p. 268

but in practice there appears to be no special advantage in this over ordinary galvanized iron buckets, while with the latter the quantity can be gaged more accurately by filling the required number of buckets in advance. Nearly all the water can be poured on the dry materials before commencing to turn, and the remainder used to wet up occasional dry spots.

The quantity of water is regulated according to the appearance of the concrete after placing. In a thin wall the water will rise to the surface through successive layers so that the first batches in a day's work require the most water. Whatever the quantity, it should be thoroughly incorporated with the other ingredients, and the amount which can be thus incorporated may sometimes be taken as the allowable limit in hand-mixing. The best consistency for different classes of concrete is discussed on page 279.

Distribution of Mixing Gang. Whatever the methods of mixing, the chief requisites for economy are such an arrangement of the gang that each man will have definite duties, and that the number of men on one set of operations will perform their work in the same length of time required by another set of men to perform a different operation or set of operations. A gang should be as large as practicable in order to lessen the cost of superintendence and the general expense.

The best plan, where the size of the gang can be regulated to suit, is to give each man a single operation to perform. For example, let one man or set of men wheel and measure all the sand, let another set of men mix the sand and cement, let a third set be continually employed measuring the gravel or stone; a fourth mixing the mass, while one or two of their number supply water; a fifth filling the barrows and wheeling the concrete to place, and still another set leveling the concrete and ramming or puddling.

It is generally economical to have two batches of concrete in preparation at once, although one set of men usually can measure and mix the sand and cement for two mixing gangs. While one batch of concrete is being shoveled to place or wheeled in barrows, the other batch either in a different location on the same platform or on a separate platform, may be spread and mixed.

The method of handling a small gang is described on page 21. The arrangement of gangs on two well managed actual jobs is illustrated in the following outline:

(1) Gang on a core wall for a dike where the sand and cement were mixed dry and spread on to the stone, then wet as the mass was turned.

The large mixing platform was located 30 to 50 feet distant from the excavation, and the concrete was handled in wheelbarrows.

One foreman.

One man wheeling sand to measuring box.

Two men, working alternately at the two ends of the mixing platform, opening cement, and mixing sand and cement dry.

Three or four men, working alternately at each end of platform, shoveling gravel into bottomless boxes.

Six men working alternately at each end of platform, mixing concrete (turning it three times).

Two men handling water.

Four men wheeling concrete, each filling his own barrow.

Four men leveling and ramming.

The average quantity of concrete in proportions 1 : 2 : 5 laid by this gang per day of ten hours was about 65 batches or 47 cubic yards, with a maximum of about 90 batches or 65 cubic yards.

(2) Gang for a 6-inch foundation for a street pavement where the sand and cement were made into a mortar and spread on to the stone, and where two mixing platforms were used, one on each side of the street, with a mortar box between them.

One foreman.

Two men mixing mortar in one mortar box.

Four men shoveling stone alternately into two measuring boxes.

Four men working alternately on the two mixing platforms, spreading mortar on stone, mixing concrete and shoveling to place.

Three men leveling and ramming concrete and also assisting to shovel to place.

One man carrying water and doing other odd work.

The total quantity of concrete in proportions 1 : 2 : 5 laid per day of ten hours averaged from 40 to 46 batches or 29 to 33 cubic yards per day for the gang. The gang was not quite up to the average, for under given conditions they ought to have turned out regularly 34 cubic yards per day of ten hours.

Approximate costs of concrete mixing are discussed on page 25

MIXING BY MACHINERY

On all large contracts machinery for mixing concrete is universally replacing hand labor. The economy of this usually is due as much to the appliances introduced for handling the raw materials and the concrete

as to the saving in the actual labor of mixing. Any arrangement which requires the measuring and spreading of materials by shovelers before entering the mixer results simply in saving the process of hand turning of the concrete and the labor of shoveling it into the vehicle, and this saving is partly balanced by the cost of maintaining and operating the mixer. On a small job this last item almost invariably exceeds the saving in hand labor and renders the expense with the machine greater than without it.

The design of the appliances or plant for handling the materials, and to some extent the selection of the type of mixer, depends upon local conditions, the quantity to be mixed per day, and the total volume of concrete. For a large mass of concrete masonry it is evident that it pays to invest a considerable sum in machinery to reduce the number of men and horses, but if for any reason only a small quantity, we will say not over 50 cubic yards, can be deposited in a day, the cost of expensive machinery cuts a very large figure and hand labor is generally cheaper. In estimating the interest on the cost of the plant which must be charged against a cubic yard of concrete, instead of dividing the interest per day by the usual daily output, the interest for the year must be divided by the total amount of concrete to be laid in the year. In other words, allowance must be made for the days when inclement weather prevents work. To find the depreciation, the value of the entire plant when new, minus its value after the job is completed, is divided by the total number of yards of concrete. Some of the other running expenses, such as the wages of the engineman, may continue from day to day whether or not any concrete is being laid.

Concrete Mixers. An effective concrete mixer not only stirs the mass, which may tend to separate the light and heavy particles, but cuts it again and again, and repeatedly transfers the materials from one part of the machine to another, so that in whatever order they are introduced, the product will be homogeneous. Continuous turning alone does not accomplish the result so quickly or thoroughly as the more complicated motions. The appearance of the concrete as it falls from the mixer will often distinguish the better of two machines.

The larger the machine, the more economical it will be, provided the arrangements for supplying it with material and conveying the concrete to the work permit running at full capacity.

Concrete mixers are of two general classes: (1) continuous mixers into which the materials are fed constantly, usually by shovelfuls, and from which the concrete is discharged in a steady stream, and (2) batch mixers, designed to receive at one charge, say, a barrel or a bag of cement with its proportionate volume of sand and stone, and after mixing to discharge it

in one mass. It is impossible to separate these two classes very distinctly because many of the machines are adapted to either continuous or batch mixing.

The authors are opposed, as a rule, to the use of continuous mixers, unless the materials are measured and fed mechanically, because of the difficulty of uniform feeding. When the ingredients are measured out by hand, spread in layers one above another, and then, starting at one edge, are shoveled into the mixer, the proportions of the materials in the resulting concrete are regulated by the thickness of the layers of the different ingredients rather than by the dimensions of the measuring barrels or boxes. If in one portion of the pile the layer of cement is thicker than in another, the resulting concrete will be proportionally richer. With batch mixers all the materials enter the machine at once; the homogeneity of the product depends upon the character and length of time of mixing rather than upon the care exercised by the laborers in feeding, and less inspection is necessary.

The regulation of the water supply in machine-mixing as in hand-mixing is largely a matter of judgment. Even if the materials were all supplied under absolutely uniform conditions, the same volume of water would not produce from each batch a concrete of uniform consistency, because, as the concrete is laid, the water works up through from one layer to the next, so that more water may be necessary early in the morning than later in the day. It is well, nevertheless, to roughly measure the quantity each time, varying the amount from batch to batch as the condition of the materials and the state of the mass require.

The selection of the type of mixer is often governed by local conditions. If, for example, there is to be a large quantity of concrete, and the machinery can be located at one place, a stationary machine, mounted perhaps on timber framework, with derricks, elevators, or belts, to raise the materials, may be economical. On running work, like a conduit or retaining wall, more portable machines are required, while for thin layers, like pavement foundations, if any machine is used it must be very light or easily moved. If stone for the aggregate is to be broken on the spot, a stationary plant may be built, or the stone may be hauled from the crusher bin to the mixer. In some cases the conformation of the ground will permit of dropping the materials into or through the machine by gravity. Frequently the volume of concrete to be laid is limited by the construction of forms, and a machine of small size is sufficient.

Mixers may be classified in three general types:

Rotating mixers.

Paddle mixers.
Gravity mixers.

Rotating or rotary mixers, as they are usually termed, sometimes mix the materials by simply tumbling them in an oblong or cubical box, and in other cases by throwing them against the deflectors, blades, or plows.

The cubical box is one of the simplest forms of rotating mixers, and formerly was used largely on extensive concrete construction, but is now giving place to modified forms which permit more thorough mixing and the inspection of the material during mixing. The cubical box is of steel, generally mounted on a timber frame similar to the plan in Fig. 94, page 272. The shaft for revolving it runs through two opposite corners and consists of a perforated hollow tube which supplies the water. The measured materials are dropped in from above through a hinged door in the side of the mixer, and the machine after some twelve or fifteen revolutions is stopped, the door is opened, and the concrete dropped into carts or cars When most of the concrete is out, the box is revolved once again to empty it more completely. The mixer itself is inexpensive, but the cost of erection and of raising the stone and sand often renders it less economical than more expensive machines.

Cube mixers are also made on a frame and geared so that they may rotate while filling and dumping as illustrated in Fig. 85.

The rotating mixers illustrated in Figs. 82 and 83 which contain deflectors, or blades, are usually mounted by the manufacturers upon a suitable frame, although in certain cases it is preferable to construct special timber framework, so that materials may be introduced and the concrete taken away more economically. The larger machines of this type are so constructed that the materials can be introduced from derrick buckets, carts, or barrows. The rotating of the drum tumbles the material and also throws it against the mixing blades which cut and throw it from side to side. Most of these machines can be dumped while running, by tilting either them or their chutes. They are also provided with hoppers as shown in Fig. 83, or with loading skips or trays, operated by the engine that runs the mixer, which lift the materials from the ground up to the charging hopper as in Figs. 82 and 85.

A different style of rotary machine is shown in Fig. 84. It consists of an open revolving pan in which are stationary plows which mix the concrete. The outlet is through trap doors in the bottom.

Of the paddle mixers, those adapted to mix a batch at a time can be more surely depended upon to produce good concrete than the continous machines. Fig. 86 shows a duplex paddle mixer to be placed upon a

raised platform and fed by hand wheelbarrows or derrick buckets. The mixing paddles, on two shafts, revolve in opposite directions, and the concrete falls through a trap door in the bottom of the machine into carts, cars, or wheelbarrows, or upon a platform whence it is shoveled to place.

The continuous paddle mixer with a single shaft and an open end is sometimes used for a volume of concrete ranging from 75 to 150 cubic yards per day. Care should be taken that the materials are thrown in near enough the upper end to be thoroughly mixed. The water is usually fed near the middle of the machine so that the materials are first partially mixed dry. They may be measured by shovelfuls, or by spreading in layers before shoveling into the mixer, or by automatic machinery which feeds the cement and each aggregate in the proper proportions.

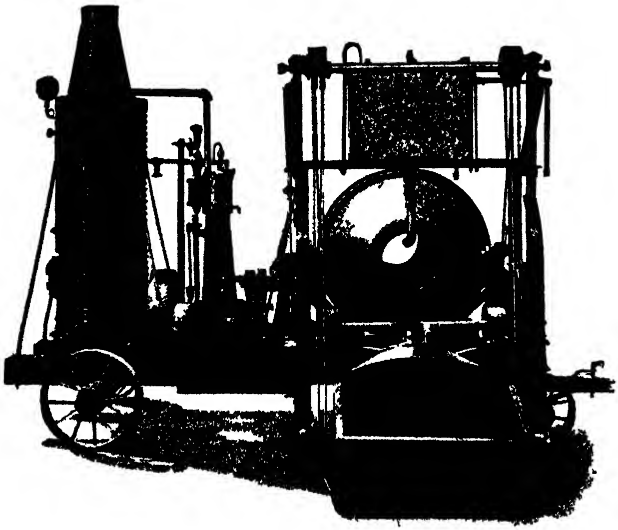


Fig 82 --Rotary Mixer (*See p* 258)

Measuring the materials by shovelfuls would seem at first thought likely to give a poorer quality of concrete than measuring in boxes or barrels, but with a properly trained gang and periodic checking of the number of barrels of cement to a given volume of concrete, fair results may be obtained. At the Charlestown Bridge piers in Boston (see Fig 92, p. 270), the contractors, by changing off the men who shoveled into the mixer so as to give them light work half the time, turned out (by steady work) concrete at the rate of about 17 cubic yards per hour. Each feeding gang consisted of five men, three shoveling gravel, one shoveling sand,

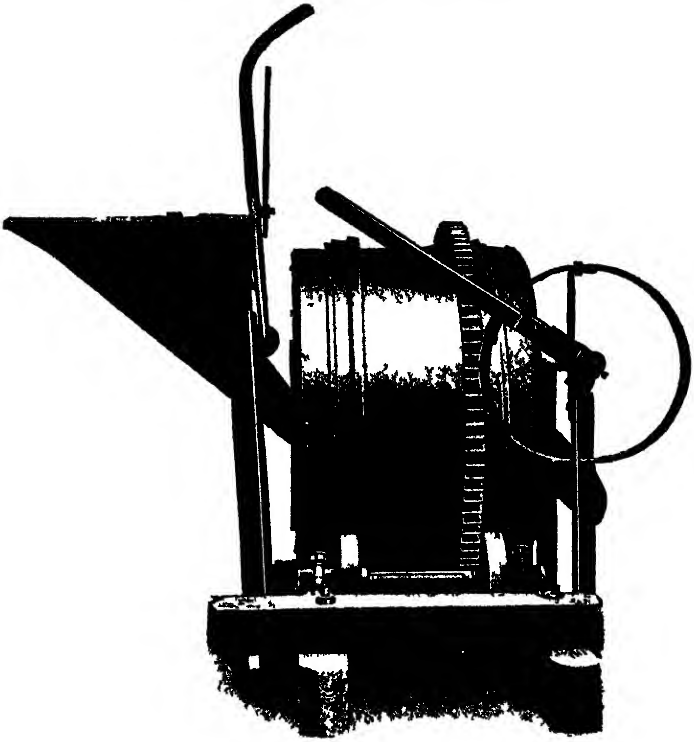


FIG. 83.—Rotary Mixer. (See p. 258.)

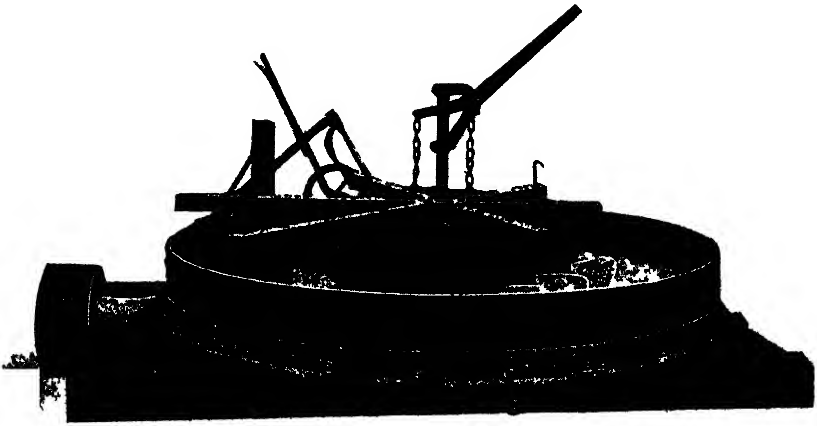


FIG. 84.—Revolving Pan Mixer. (See p. 258.)

and one shoveling cement, the size of shovels being so arranged that when all worked together the proper proportions were introduced. The two gangs changed off every half-hour.

Fig. 85.—Rotary Cube Mixer. (See p. 255)

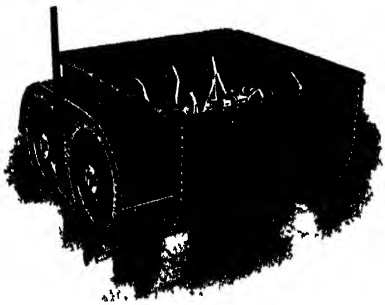


Fig. 86.—Duplex Paddle Mixer. (See p. 259)

When the materials are measured and spread in layers before shoveling into the mixer, the machine should be below the measuring platform, and two gangs of men employed, one on each side of the machine, so that one

batch may be prepared while another is entering the mixer. This seems like a very simple requirement, yet the authors have often seen a single gang measure out the materials on the ground while the machine stood idle, and then lift them to a height of perhaps 3 or 4 feet, while the mixed concrete fell to the ground to be shoveled into barrows. With such an arrangement, hand-mixing is cheaper than machine-mixing.

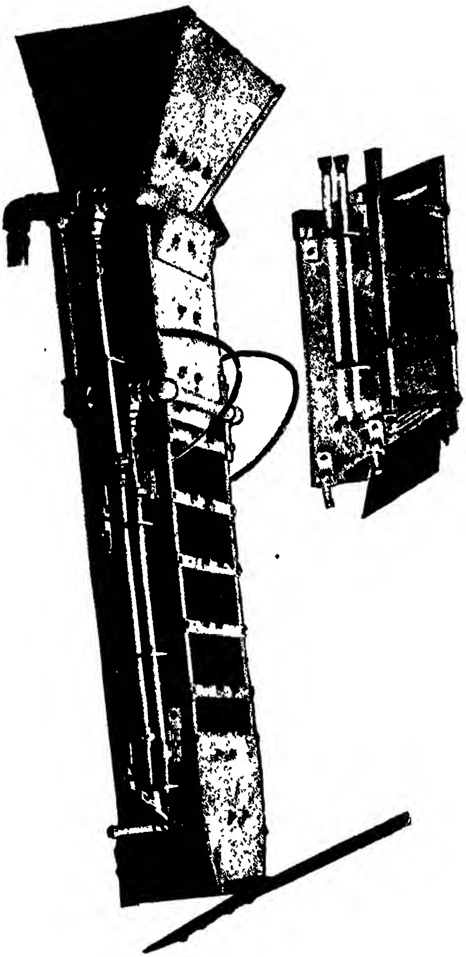


FIG. 87.—Gravity Mixers. (*See p.* 263.)

Gravity machines, properly so-called, require no power, the materials being mixed by striking obstructions which throw them together in their descent through the machine. A gravity concrete mixer is illustrated in Gillmore's "Treatise on Limes, Hydraulic Cements and Mortars,"* first published in 1863. In this machine the concrete fell into successive hoppers opened and closed by hand-levers.

A well-known modern type of the gravity machine, shown in Fig. 87, may be increased in length from 4 to 10 feet by adding different sections. In falling through the slanting tube the materials are thrown by the deflectors on the sides and the curved back — the deflectors also acting as tables upon which the stones are coated with mortar—against several series of iron rods which mix them violently together. The inventor claims that by this violence the cement is pounded into the fractures and indentations of the sand and stone so as to increase

*Page 229.

the strength of the concrete produced. The materials generally are measured in layers on a platform above the machine and fed by shovels, but may be fed by a tipping box or by a derrick bucket. In the latter case the mixer becomes practically a batch machine.

Another gravity mixer is illustrated in Fig. 88. Four cone-shaped

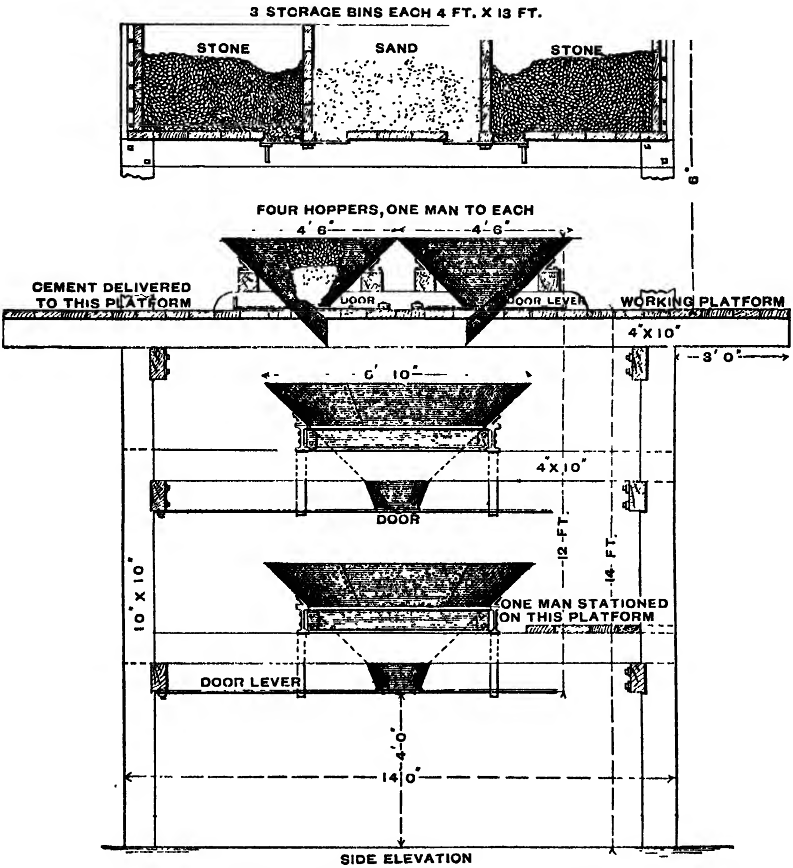


FIG. 88.—Gravity Mixer. (*See p.* 263.)

hoppers at the top of the machine receive the materials in layers, with the cement at the bottom and the coarsest material at the top. From these, on the opening of gates, the mixture falls into a single cone below, and thence at the will of an operator into a still lower cone, whence it drops into the car or other receptacle. The same type of mixer is used

with a derrick so that the mixing progresses while the material is being swung to place. A series of cone-shaped buckets, telescoping each other when at rest, are connected with chains so that the concrete materials may be placed in the upper one and will fall when this is raised through an opening in the bottom to the next, and so on to the lowest bucket, from which it is dumped into the work by the operator at the place where it is needed.

Portable Concrete Mixing Machinery. Nearly all the types of concrete mixers described are made, at least in their smaller sizes, so that they can be readily transported from one part of a job to another. A few of them are adapted for such work as laying a thin foundation for street paving, while the heavier machines are sometimes arranged upon cars running on a track, so that the concrete can be dropped directly into place from the mixer, or conveyed to place by an endless belt.

On the Chicago & Western Indiana R. R.* a train was made up for preparing and depositing concrete for retaining walls. Three or four cars carried the stone, sand, and cement, and from these the materials were conveyed by wheelbarrows to the mixing car, where the sand and stone were measured, dumped into the mixer, and thence on to a belt conveyor mounted upon a swinging steel boom like a derrick boom, which deposited at any point within derrick swing. The train was hauled by the winding drum on the same engine which operated the mixer, a cable running ahead to an anchor or "dead-man" in the ground.

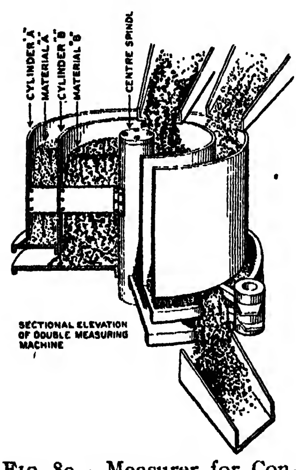


FIG. 89.—Measurer for Concrete Materials. (*See p.* 265.)

In building a dam at Chaudiere Falls, P. Q.,† tracks were laid just above and below the site of the dam and parallel to it, and a traveling platform containing the mixer was constructed so as to straddle the dam. The mixer discharged the concrete into the upper end of a tube fitted with a lower telescoping section, so that it could be deposited directly on any part of the dam.

Automatic Measurers for Concrete Materials. The accurate measuring of concrete materials by mechanical means has not been extensively developed. One difficulty, if methods of volumes are employed, lies in the inaccuracy of measuring cement by volume.

**Engineering News*, Feb. 28, 1901, p. 149.
†*Engineering News*, May 7, 1903, p. 403.

One patented device consists of several drums, one for each material placed directly under the bins containing the cement, sand and stone, and rotating upon the same horizontal shaft. The quantity of each material is regulated by the position of the gates in the bins and by the speed of rotation.

Another machine delivers the different materials through separate troughs containing Archimedean screws.

Another type of measuring machine, the working of which is illustrated in Fig. 89, consists of one or more bottomless storage cylinders, from under which the material flows out on to revolving discs or tables, and is peeled off by stationary adjustable knives which rest upon the discs and project into each material a distance determined by the quantity of each required.

A partially automatic measuring arrangement was employed on one section of the Boston Subway, in 1896. Each material fell into a closed chute arranged with gates at such distances apart as to enclose the required volume, whence it dropped into a hopper above the mixer.

Proportioning by Weight. Attention has been called on page 217 to the fact that not only cement, but also sand, stone, and gravel, can be more accurately proportioned by weighing than by volume measurement. When a large amount of concrete is to be mixed, it is possible to arrange apparatus for weighing each material in such a way that less labor will be required than for proportioning by volume. The first cost of the scales may often be more than counterbalanced by the accuracy in proportioning, which permits of leaner mixtures, while at the same time greater uniformity is assured.

In view of these facts, the authors predict that engineers will gradually recognize the advantage of proportioning by weight. In most cases excessive cost may prohibit the use of standard scales, but if the materials are accurately screened and subdivided, the relative weights of each on the same job will be so nearly constant that the weighing can be performed by a simple system of counterweights and levers. With properly constructed gates to the bins it might be possible to arrange for their automatic closing after the required weight of each material had been received in the hopper.

Measurements by weight are employed to excellent advantage by Warren Brothers Company at their various plants where the materials, which consist of stone, sand, and binding material, are prepared for their bituminous macadam pavement. Eight bins containing aggregates of different coarseness drop their materials through gates into a hopper which forms the platform of the scales and is located directly above the mixer. The scale-beam is compound, with as many arms as there are ingredients to be weighed, and each of the arms has a sliding weight and a stop so arranged that the sliding weight can be moved only to the point on the beam which will balance the required weight of one of the materials. When the

sliding weights are all at zero and the hopper is empty, the scale balances. The weight on one of the arms is moved out by the laborer who operates the apparatus until it comes to the stop fixed at the point corresponding to the weight of the material to be used from a certain bin. The gate of this bin is opened, and the material allowed to run into the hopper until the scale balances. The weight on the next lever is then slid out, and the second material deposited in like manner upon the first. When all the materials are thus weighed, the entire mass is dropped into the mixer below.

Measuring Water. The water for each batch of concrete should be measured. The quantity of water used in different batches must be varied occasionally because of the conditions of the materials, but even in such cases the amount can be regulated best by measurement. A tank with a float connected with an indicator on the outside is easily constructed.

CONCRETE PLANTS

The design of the plant for handling the raw materials and the concrete usually has more to do with an economical production than the type of the mixing machine. The plant should be drawn or sketched on paper and accurate estimates made of its cost and the expense of operation, so as to determine whether the volume of concrete is sufficiently large to warrant its installation. The authors have occasionally seen expensive machinery, which could not be readily transported to another job, installed on a section of work where, because of the small total volume of concrete and on account of its distribution, hand-mixing was really more economical.

It is evident that the arrangement of any plant must be determined by local conditions, such as the contour of the ground, the distance from which the raw materials are transported, and the class of construction. A description of several plants, successful and economical in operation, may afford suggestions for other work. The illustrations are intended to show the arrangement of the gang and conveying machinery rather than the type of mixer.

Platform over Mixer. A common practice with mixers of various types, where the conformation of the ground permits, and where the quantity to be laid does not warrant the introduction of bins or machinery for handling the aggregate, is to locate the platform for measuring materials directly above the mixer. When ready they are shoveled through a hole in the planking into the machine. One gang of men can measure and spread the materials for a batch while another is shoveling it in. If the mixer is run as a batch machine, the materials may be measured directly into a hopper above it.

A satisfactory arrangement for a stationary batch mixer is illustrated in Fig. 90. The bin above the hopper is divided into two compartments for the sand and stone, and these are measured by feeding them to definite heights in the hopper, while the cement is dumped into the chute in front.

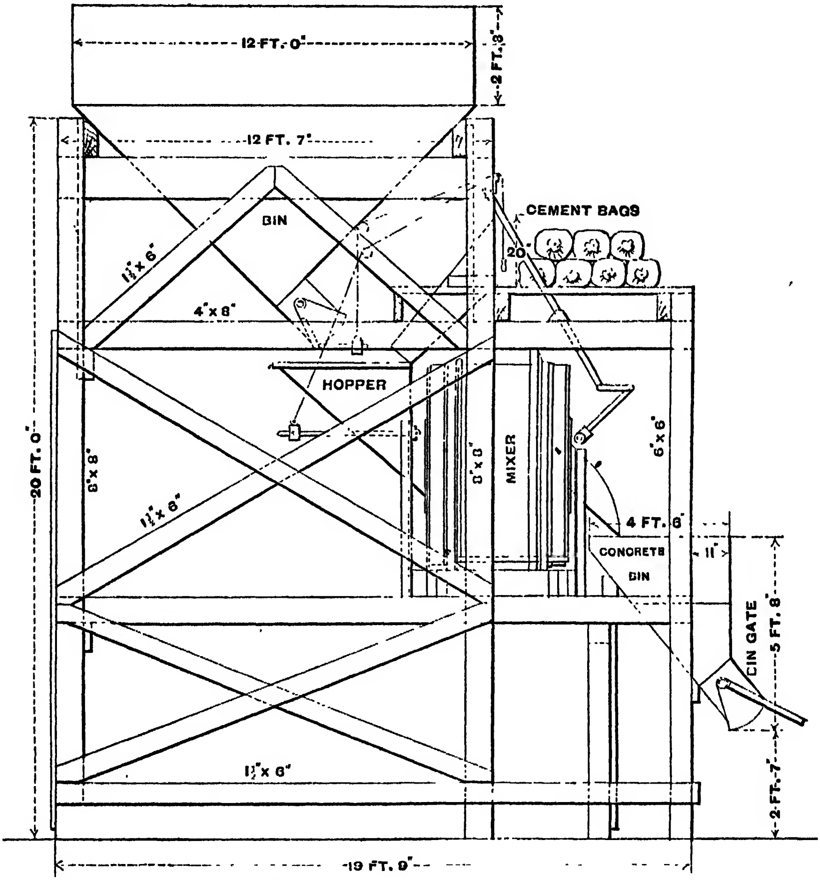


Fig. 90.—Stationary Mixing Plant with a one-yard Rotary Batch Mixer. (*See p.* 267.)

Building Construction. The concrete for building construction may be elevated in buckets running in a light timber frame or on steel guides as shown in Fig. 91.

A Central Plant. The establishment of a central plant from which the mixed concrete may be hauled to various points as required may be economical in some cities or large towns. This plan has been adopted in St. Louis, Mo.,* for concrete, and is employed in many places for tar and asphalt

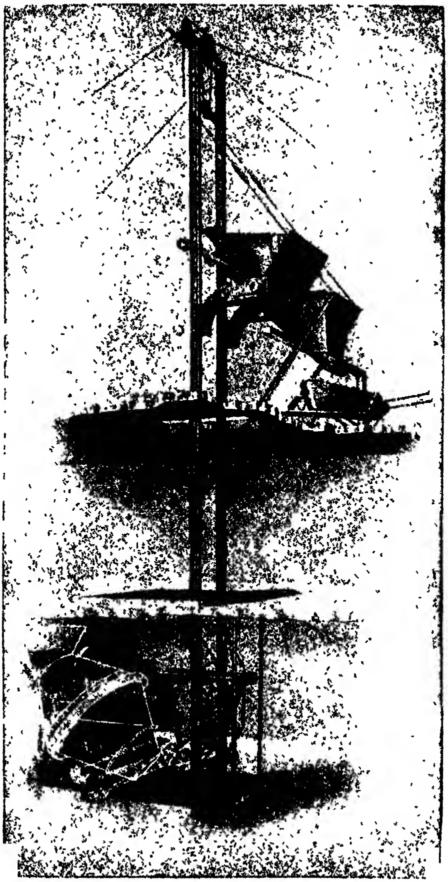


Fig. 91.—Automatic Dumping Concrete Elevator. (*See p.* 267.)

paving. The plant may be located at a gravel bank or stone crusher, or near a railroad siding, permanent machinery provided which will mix the concrete at a much lower cost than could be done by hand-mixing, and the concrete hauled in carts to the work at but slightly higher cost than the hauling of the dry materials. Most Portland cement con-

**Engineering News*, March 10, 1904, p. 231.

crete will not be injured (see page 157) if laid within an hour or two after mixing.

Charlestown Bridge Pier. An economical handling of materials and concrete, where the only machinery was the concrete mixer, is shown in Fig. 92, which illustrates the building of the foundation for the draw pier of the Charlestown Bridge, Boston.* The gravel and sand were brought on scows and deposited so near to the mixer as to require only a short throw or wheelbarrow haul, and were then measured by shovelfuls, as described on page 259. Eight wheelbarrow men, in single file, conveyed the concrete from the paddle mixer, which is shown just to the right of the central mast, along the circular run, then on to the turn-table to the chute for depositing it under water. The entire gang consisted of some thirty-five men, and when working steadily they laid at the rate of about 170 cubic yards of concrete in ten hours, which may be considered a maximum output for a machine of this character, the more usual quantity being from 75 to 100 cubic yards per day of ten hours. The method of depositing concrete from the chute is described on page 303.

Harvard Stadium.† At the Harvard Stadium the builders, the Aberthaw Construction Company, erected a movable tower on each side of the site, and the buckets of concrete and the seat slabs‡ were then taken from cars and conveyed by the cable suspended between the towers to the point where they were needed.

Chicopee River Dam. In mixing concrete for a dam across the Chicopee River in Massachusetts, the contractors utilized a portion of the excavation by locating their mixer against a bank and building out over it a covered platform containing the hopper from which the materials could be dropped directly into the mixer. Stone from the excavation was crushed and elevated to storage bins, whence it was hauled by carts holding exactly the quantity required for a batch, and dumped directly into the hopper above the mixer. The sand was measured and wheeled to the hopper in an iron vehicle consisting of a bucket set on two large wheels which dumped into the hopper by rotating on its axis. The cement was emptied on top of the sand. One batch was mixing in the machine while another was being emptied into the hopper, and thus twenty batches could be handled per hour. The concrete was dumped from the mixer into carts which conveyed it to the dam.

Cambridge Electric Light Station. A portable mixing plant em-

*Sixth Annual Report Boston Transit Commission, 1900.

†See Frontispiece.

‡See chapter xxiv.

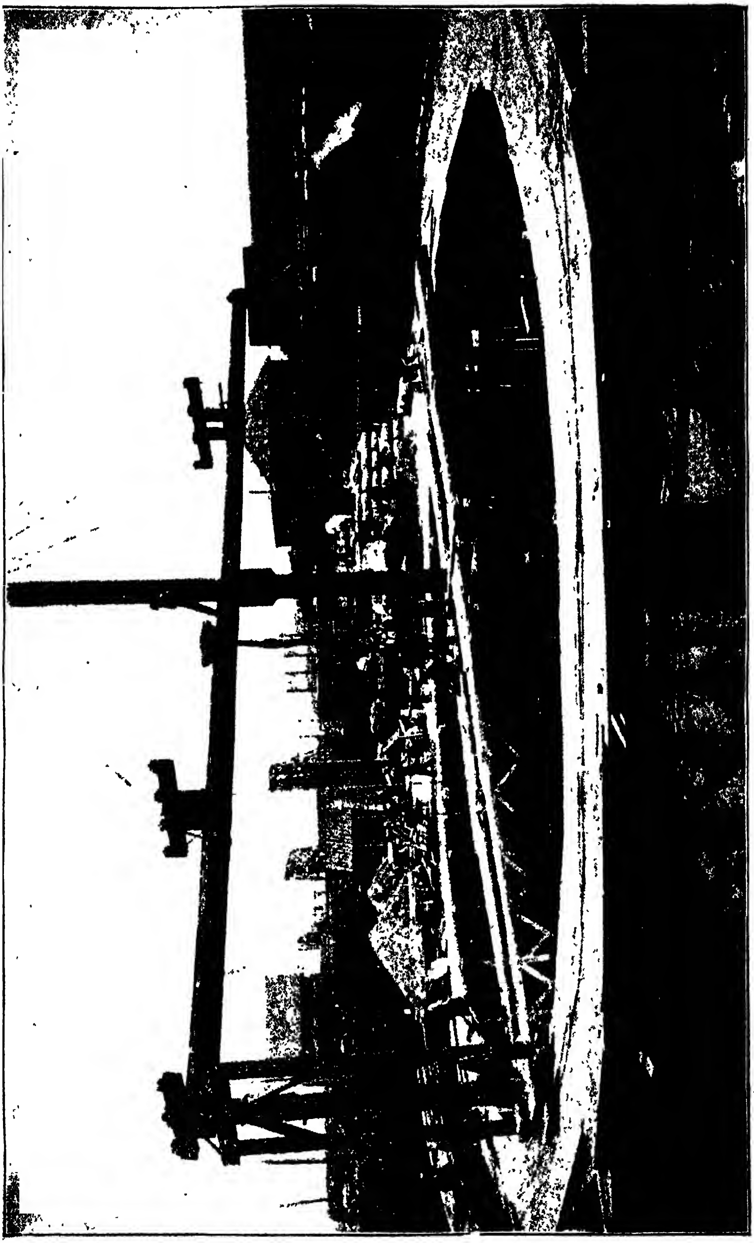


FIG. 92.—Depositing Concrete of Draw Foundation Pier, Charlestown Bridge. (See p. 269.)

ployed on the Cambridge (Mass.) Electric Light Station is shown in Fig. 93. The special feature of the arrangement is the framework containing the mixer. This may be taken up by the derrick, which also supplies it with raw materials, and moved in a few minutes to any other position within derrick swing, so that the concrete can be dropped from the mixer close to or directly upon the place where it is required.

East Boston Tunnel. For measuring materials brought in cars to the work, the contractors for one of the entrance sections of the East

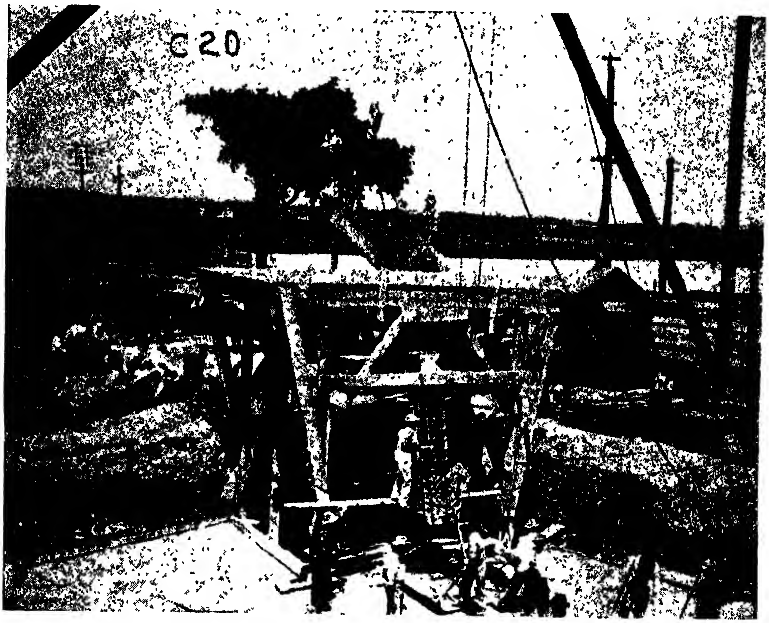


FIG. 93.—Portable Mixing Plant. (*See p.* 271.)

Boston Tunnel employed a derrick bucket. The stone was first filled in to a height determined by a gage, then the sand was shoveled on top of it and struck off with a different gage, and finally the required number of bags of cement emptied on top of the sand. The bucket was taken by a derrick and dumped into a duplex mixer.

Cambridge Bridge Piers. When the quantity of concrete to be laid warrants the installation of the necessary machinery, economy requires that the stone and sand shall not be handled at all by laborers. If the stone is crushed on the spot, it may be raised to bins above the mixer

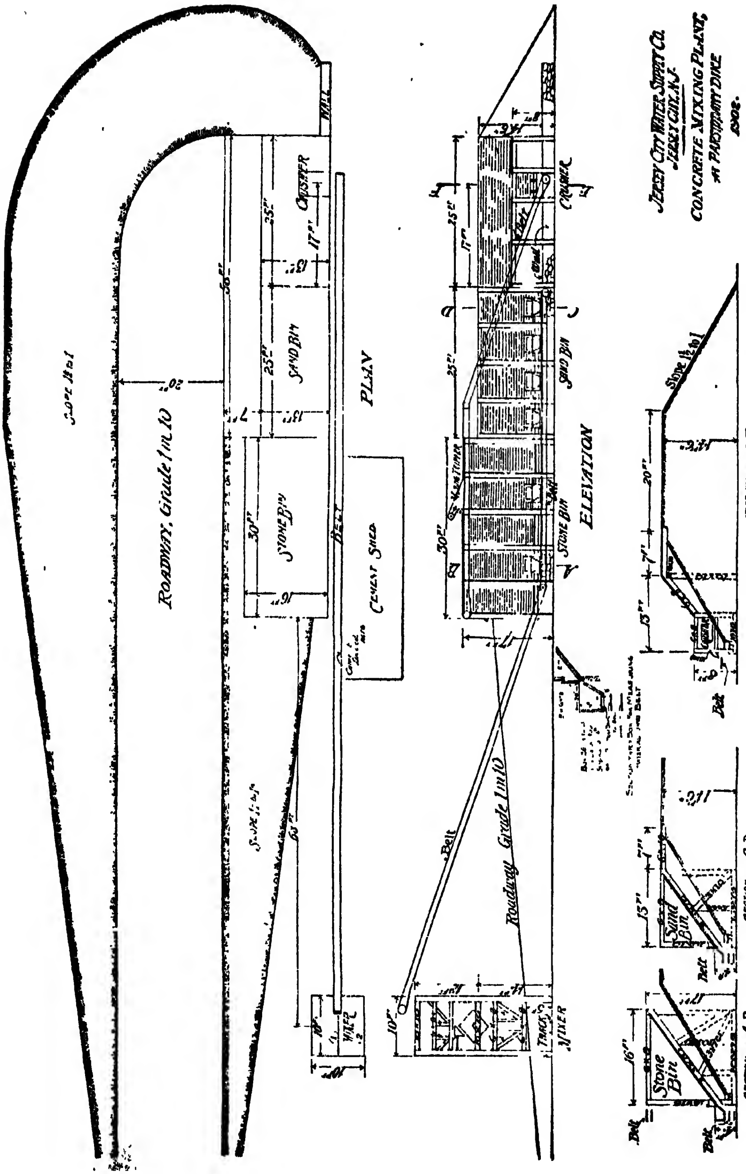


FIG. 94.—Mixing Plant Employing Belt Conveyor. (See p. 273.)

by bucket elevators or belt conveyors, while a similar plan for elevating the material may sometimes be advantageously followed where gravel is used. In building the substructure of the Cambridge Bridge, Boston, Mass.,* the concrete plant was located on a pier resting on piles. The gravel for the concrete was dredged from the harbor and dumped from scows into the water close to the pier. An "orange peel" bucket, operated from a dredging machine on a scow, lifted the gravel, and dropped it into a hopper whence it ran by gravity upon the combination inclined screen described on page 240, which separated the sand, pebbles, and the coarse waste material. Bucket elevators raised the sand and pebbles to bins above the mixer, and from the bins, which were V shaped, the materials fell by gravity into the measuring hoppers These were arranged in two sets, an essential requirement for maximum output, so that one batch could be measured while another was being dropped into the mixer The barrels of cement were brought from the cement shed by a horizontal endless chain, opened on the ground under the mixer, and then three barrels, enough for one batch, were raised at one time by a bucket elevator to one of the hoppers over the mixer

Williamsburg Bridge Pier. A method of measuring the materials in cars was adopted in building one of the anchorages of the East River Bridge, New York The cement and sand were stored in bins, and fell by gravity into cars whose capacities were equal, respectively, to the volume of stone and sand required for a batch Between the tracks upon which these cars ran were two holes in the ground into each of which could be lowered a box of sufficient size to hold one batch of the broken stone, sand, and cement By tipping the measuring car the broken stone was dumped into the box, the sand fell from another car through a trap door, and the cement was dumped in from the bags. After filling, the box was raised by a derrick and dumped into the mixer.

Parsippany Dike. An endless rubber belt furnishes an excellent means for handling concrete raw materials in a stationary plant The width of the belt should be not less than 18 inches and the slope no greater than about 22°, which corresponds to 2½ feet horizontal to one foot vertical. Idlers for giving the proper V shape to the belt were placed at proper intervals.

The plan in Fig. 94, page 272, shows the design by Mr William B Fuller of a plant used at the Parsippany Dike of the Jersey City Water Supply Co., N. J. The sand was brought to the bins and the stone to

*For full description see article by Sanford E. Thompson in *Engineering News*, Oct 17, 1901, p. 282.

the crusher in wagons. A belt conveyor delivered the crushed stone to the bins. At the outlet of each bin a measuring hopper (shown in a detail section, in Fig. 94), containing about 8 cubic feet, received the sand or stone from the bin, and at the ring of a bell the proper quantity of each material for one batch of concrete was dropped upon the conveying belt. The cement was emptied from bags on top of the sand and stone as they were carried past the cement shed. The bin over the mixer had two hoppers. As soon as a batch was delivered to hopper No. 1, the bell was rung again and another batch started into hopper No 2, and while this was filling No. 1 batch was dumped into the mixer.

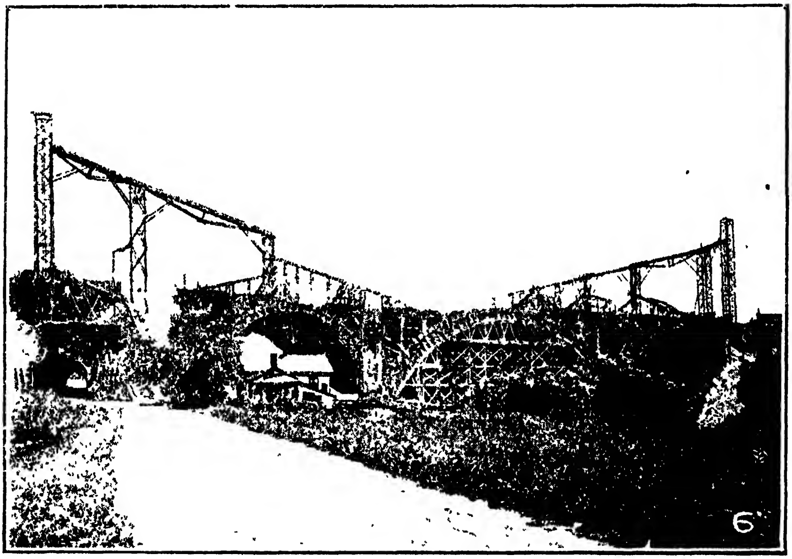


Fig. 95. Mixing plant at Painesville Bridge. (*See p.* 275.)

Blackwell's Island Bridge Piers. At a plant of somewhat similar design built for the piers of the Blackwell's Island Bridge, N. Y., the sand and stone were measured in cars running on a track below the bins. so that they could be moved from one gate to another and discharged at any point through trap doors on to the belt between the rails. The stone was carried up from the crusher by another belt to the top of the bins, where it fell off the belt on to an inclined screen, and rolled into a bin, while the dust, passing through, dropped on to another short belt which carried it to another bin to be used as sand.

Jerome Park Reservoir.* During the construction of the reservoir at Jerome Park, New York City, in 1906, the concreting of the large bottom and slope areas was systematically arranged by using a number of medium sized rotary batch mixers, each with a separate gang with wheelbarrows. The mixers were moved from time to time.

Chalmette Docks at New Orleans.† The concrete for the slip walls of the Chalmette Docks, New Orleans, was handled and mixed by a portable plant on standard gage tracks, consisting of a flat car with a 2-cubic yard hopper at each end which supplied sand and gravel to inclined belt conveyors. These discharged into a 3-cubic yard hopper with an undercut gate placed above a ¾-cubic yard rotary mixer at the center of the car. Cement was supplied the mixer by hand from a storage platform on the side of the car, and water, from a pipe laid along the wall with hose connection at convenient intervals.

Painesville Bridge. A unique method of handling concrete at the Painesville Bridge of the L. S. & M. S. R. R., completed in 1909, is illustrated in Fig. 95. Concrete was elevated in towers at each end of the bridge and flowed in movable spouts by gravity to place.

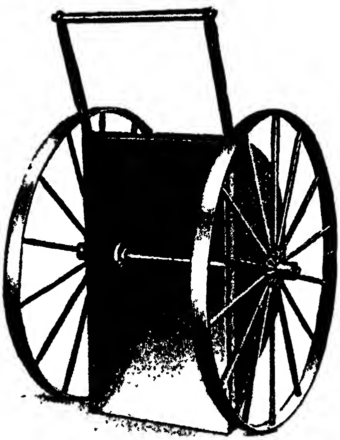


Fig. 96.—Two-wheeled Concrete Car. (*See p.* 277.)

* *Engineering News*, Sept. 21, 1905, p. 298.
† *Engineering Record*, July 29, 1906, p. 88.

CHAPTER XV

DEPOSITING CONCRETE

The actual handling and placing of the concrete after it has been mixed, and the construction of forms for ordinary mass work, are treated in this chapter. Forms for building construction and conduit construction are illustrated in subsequent chapters on these subjects.

Since the introduction of concrete into engineering construction, the opinions of engineers regarding the best methods of placing it have completely changed. For water-tight work or for the strongest construction it is now recognized that the concrete should resemble as nearly as possible one single solid mass of stone with no joints, and it is the usual practice, although not universal, to specify a "quaking," jelly-like consistency, while many authorities go still further and require water enough to be "mushy" or sloppy. Formerly, for all classes of work, concrete was mixed but slightly more moist than damp earth and laid in alternate blocks 6 to 12 inches thick. Then, after hardening, the forms were removed, and the spaces between filled in.

HANDLING AND TRANSPORTING CONCRETE

In handling and transporting concrete, it is essential to prevent the separation of the stones from the mortar. In hand-mixed concrete, especially for thin walls requiring the stuff to be carried in buckets, there is a tendency to allow the stones to separate on the mixing platform so that a lot of them fall together when cleaning up the last shovelfuls.

With the modern slow-setting cement, and in view of the accepted belief that some time may elapse after mixing without injury to the work, there is less difficulty than formerly in handling the concrete, and it can be readily transported to a considerable distance. Moreover, a wet mixture is much easier to handle, because the stones do not so readily separate from the mass.

The usual vehicle for transporting hand-mixed concrete is a wheelbarrow. For machine-mixed concrete, derricks are suitable if the mass is concentrated near the mixer, otherwise cars running on a track, or in some cases wagons, afford a means of conveyance. A combination of car and derrick work is readily effected by using flat cars with derrick buckets or trays upon them. Galvanized iron buckets are sometimes useful when

building by hand a high, thin wall. A bucket elevator is a poor contrivance for elevating concrete. The mortar sticks to the buckets and the ingredients of the concrete separate as it is thrown from them.

Volume and Weight of Loose Concrete. The volume and weight of loose concrete is of importance in designing the implements or vehicles for transporting it and in estimating the quantities which can be handled under different conditions. The weight of well-proportioned concrete after setting, as stated on page 3, generally ranges from 143 to 155 lb. per cubic foot. When green, it will weigh, after ramming, slightly more than this, say from 150 to 160 lb. The weight per cubic foot loose, that is, in the vehicle which transports it from the mixer to place, depends largely upon the consistency. If mixed very wet, it will settle down to very nearly the volume it has after it is placed, perhaps within 5% of it; but if of dry consistency, the volume of the rammed mass is apt to be as much as 25% less than the loose. A fair average weight of loose concrete may be estimated, then, at about 140 lb. per cubic foot, or 1.9 tons per cubic yard, when mixed wet, and 120 lb. per cubic foot, or 1.6 tons per cubic yard, when mixed dry. The weights and volumes vary, of course, with the proportions used in the mixture and the specific gravity of the stone in the aggregate, but for rough estimates these figures are sufficiently accurate. The volumes of loose mixed concrete required for a cubic yard of rammed concrete, based on the above percentages, are 28 cu. ft. of a very wet mixture and 36 cu. ft. of a dry mixture.

The volume of concrete contained in an iron wheelbarrow load of average size is 1.9 cu. ft. place measurement. A large load is about 2.2 cu. ft. place measurement. Special concrete barrows are also made with a capacity up to 6 cu. ft. (see Fig. 96, p. 275). Further data is given in Chapter I.

A single cart on ordinary construction roads will carry about half a batch of concrete of average proportions, which may be assumed as 1 barrel cement to 2½ barrels sand to 5 barrels stone, while with a properly constructed cart which will not overflow or leak, 50% more than this, or about three-quarters of a batch, can be drawn over macadam and paved streets.

DEPOSITING CONCRETE ON LAND

The methods which may be selected for depositing concrete depend largely upon its consistency. If mixed wet, it can be dropped vertically to any depth or passed through an inclined trough or chute. On the other hand, the stones in a dry mixture, that is, of damp earth consistency, will separate from the mortar on the slightest provocation.

To prevent the ingredients separating when passing down an incline, if the mixture is not plastic enough to prevent the stones running away from

the mortar, a pipe with a hopper top and composed of two or more telescoping sections about 15 inches in diameter is often employed. In such a case, the pipe must be often moved or the material shoveled away immediately, to prevent its forming a high cone. Sometimes it is convenient to run the lower end of the pipe into a hopper with a gate at its mouth, so that the concrete may be drawn out into a vehicle, while the pipe and hopper are kept continually full.*

The illustration in Fig. 97 shows at how flat a slope concrete of very

FIG. 97.—Depositing Concrete through a Trough. (See p. 278.)

wet consistency will run through an open trough. The picture is an actual construction photograph of the Jersey City Water Supply Conduit, and shows the concrete flowing directly from the mixer to the crown of the arch. Mr. William B. Fuller, the engineer, states that when the concrete is mixed of exactly the consistency he likes, it will easily run through an iron trough 15 inches wide by 4 inches deep, set on a slope of 8 feet horizontal to 1 foot vertical.

For water-tight work or for maximum strength the concrete should be

*Engineering News, Dec. 25, 1902, p. 537.

placed so as to form a monolith. To do this on a large structure two or three shifts are employed in twenty-four hours, so that no portion of the mass commences to set until fresh concrete has been laid on top of it. In a large reservoir wall at Little Falls, New Jersey, built *en masse* to sustain 40 feet head of water, the only point where the moisture appeared on the surface was at a layer where the work was stopped for one hour at noon. In most structures it is possible to divide the work into sections, each of which is a monolith. Monolithic construction is necessary for columns, beams and floors.

A tipping car for conveying concrete on a track and dumping it into place is shown in Fig. 98.

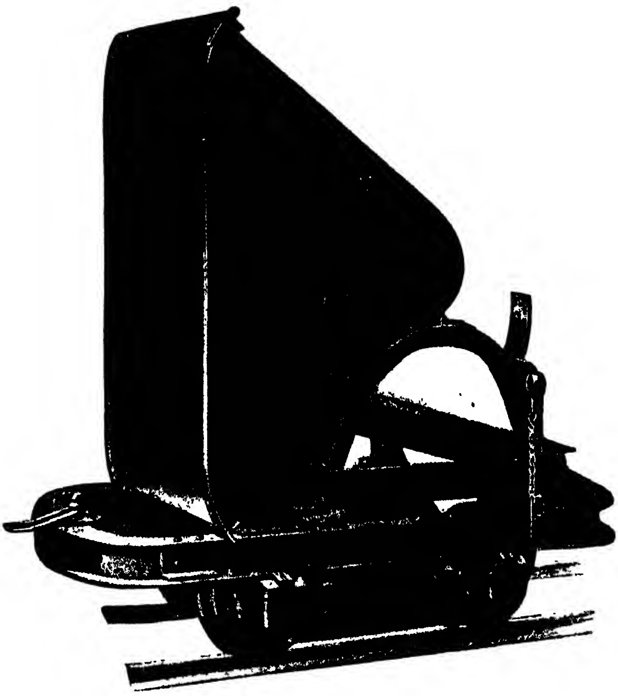


FIG. 98. Dumping Car. (See p. 279.)

In a thin wall or a structure requiring especial care, such as a tank, it may be advisable to shovel the concrete from the wheelbarrows. Stones which tend to separate can be thus mixed in with the mortar in the wheelbarrow and a very thin layer formed in the molds, so that even if the concrete is mixed very thin the mortar cannot run off from the stones.

CONSISTENCY OF CONCRETE

The terms for specifying the consistency, or degree of plasticity, of freshly mixed concrete are variously used by different engineers. In this

treatise the term *dry mixture* is applied to concrete of the consistency of damp earth, from which the water rises to the surface only after prolonged ramming, and then simply in a glistening film. A *medium* or *quaking mixture* means a tenacious, jelly-like consistency, which shakes on ramming. A *very wet* or *mushy mixture* is one which will not support the weight of a man and into which an ordinary rammer will sink of its own weight; it will run off a shovel unless shoveled very quickly, and will spread out and settle to a level surface after wheeling about 25 feet in a wheelbarrow.

The proper consistency, or wetness, of concrete is a disputed point among engineers, some still holding to the very dry mixture, while others prefer one nearly as liquid as grout. As a result of a series of tests and of practical experience, the authors advocate varying the consistency according to the class of work, and present the following general conclusions:

Medium or quaking concrete is adapted for ordinary mass concrete, such as foundations, heavy walls, large arches, piers, and abutments.

Very wet or mushy concrete is suitable for rubble concrete and for reinforced concrete, such as thin building walls, columns, floors, conduits, and tanks.

Dry concrete may be employed in dry locations for mass foundations which must withstand severe compressive strain within one month after placing, provided it is carefully spread in layers not over 6 inches thick and is thoroughly rammed.

The experiments of the authors show that while dry concrete, very carefully mixed and rammed, is stronger on short time tests, medium mixtures will attain nearly equal strength in six months' time. One of the arguments against very dry mixtures is the difficulty of obtaining a uniform consistency. Occasional batches will invariably be too dry, and it is impossible with ordinary care in placing and ramming to avoid visible voids or pockets of stone which form weak places and allow the penetration of water.

The 1903 specifications of the American Railway Engineering and Maintenance-of-Way Association are as follows:

The concrete shall be of such consistency that when dumped in place it will not require tamping; it shall be spaded down and tamped sufficiently to level off and will then quake freely like jelly, and be wet enough on top to require the use of rubber boots by the workmen.

A very wet mixture is more suitable for rubble concrete or concrete rubble because the large stones more readily settle into place and bed themselves In thin walls very wet concrete can be more easily "joggled"

into position so as to conform to the molds and give a smooth surface. The use of a mixture sufficiently wet to flow under and around metal reinforcement has been found by Prof. Charles L. Norton (see p. 328) to be one of the essentials for the preservation of the metal.

Stone pockets may occur even with very wet concrete because of the mortar running away from the stones. This may appear an imaginary danger to many users of concrete who have never employed a very wet consistency, but the authors have seen concrete mixed with too much water, which after setting and the removal of the forms had the appearance of being mixed too dry. In their opinion, however, the limit of wetness for many classes of work is not reached until there is so much water that with ordinary care in hand-mixing it cannot be made to incorporate with the other materials.

FIG. 99.—Rammers for Dry Concrete. (*See p. 281.*)

RAMMING OR PUDDLING

The method of compacting the concrete or forcing out the air after placing, and the kind of tools to employ for this, depend upon the consistency of the material.

In concrete mixed with a small amount of water the thickness of layers is usually specified at 6 to 10 inches, the former being the most common, but with a very wet or mushy concrete 12 to 15 inches may be placed at once, the chief object being to expel bubbles of air by puddling or joggling. In using very wet concrete there is danger of too much ramming, which results in wedging the stones together and forcing the finer material, the sand and cement, to the surface.

The style of rammers ordinarily used for dry mixed or medium concrete are similar to the forms shown in Fig. 99. The style on the left of the figure is the ordinary type, and on the right is a style convenient for use close to the forms.

The rammer shown in Fig. 100, page 282, which weighs about 8 pounds, is the design of Mr. William B. Fuller for very wet or mushy concrete. The handle may be lengthened, as shown, by screwing a pipe coupling on to the wood.

A "post-hole" tamping bar with iron shoe, shown in Fig. 101, has been successfully used by the authors for mushy concrete. A piece of 2 by

3-inch studding cut to the required length and smoothed off so as to be readily grasped by the hands is also a serviceable tool.

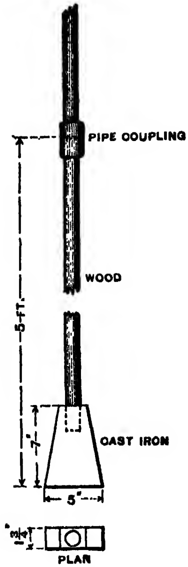


FIG. 100.—Rammer for Mushy Concrete. (See p. 281.)

A pneumatic rammer built on the principle of a pneumatic riveting machine, as illustrated in Fig. 102, has been used upon dry mixed concrete with fair success.

Mr. Rafter and Mr. Daniel F. Fulton have designed a rammer based on the principle of the steam drill which is arranged upon a traveling carriage resting upon cross girders which run on tracks. A speed of from 400 to 600 strokes per minute may be maintained with from 4 to 5 horse-power. For ramming street pavements, it should cover 600 to 800 linear feet of a street 30 to 40 feet wide.

Mr. Clarence R. Neher, an advocate of wet concrete, replies to an inquiry of the authors in regard to rammers, as follows:

I am governed so much by conditions that I use no standard tool, the principle being to use a wedge-shaped rammer of some kind. For the face of the work nothing appears much better than a common spade. This is useful in pushing back stones that have separated from the mass, and also can be used to select the softer and finer portions of the mass and place at the face, while working the spade up and down along the face until it is thoroughly filled. Care must be taken not to pry with the spade, as it will spring the form outward unless excessively strong.

In narrow forms where a man cannot stand in the concrete, a piece of 2-inch by 3-inch scantling,—with the upper portion rounded to make a convenient grip and the tamping end wedge-shaped, — of a length determined by the depth of the form, is convenient and cheap.

In heavy mass work I prefer this same form of rammer to the ordinary type, and thoroughly incorporate the different deposits together, avoiding as much as possible a smooth, flat finish, so frequently insisted on. I consider the use of the term "layers" as describing just what you do not want. I deposit as much concrete in a form as the rammer will penetrate and enter into the deposit below. The

FIG. 101.—Rammer for Mushy Concrete. (See. p. 281.)

amount will thus be governed by the size of the form and method of filling.

In elevator foundations we have filled columns 3 feet by 11 feet by 22 feet high in five hours, dumping 14 cubic feet at a time, and not trimming, but shoving the rammer through the mass. The work is absolutely free from voids.

Labor of Ramming. The number of men required for leveling and ramming concrete depends upon the thickness of the wall and the consistency of the mass.

In the table of concrete data in Chapter I, page 9, we have specified 11 cubic yards as the work of an average man in ten hours, including both leveling the material as it is dumped from barrows and the actual ramming. This figure is based upon actual records of a large number of jobs where the concrete was laid of the medium consistency most commonly employed in ordinary mass work. Similarly, a large day's work is placed at 16 cubic yards. Mr. George W. Rafter writes the authors that 4 cubic yards is about an average day's work for an Italian laborer on dry mixed concrete. Mr. Neher estimates for ordinary conditions 10 to 15 cubic yards of wet concrete per man per day with an average of about 12 cubic yards per ten-hour day. Mr. Fuller, who employs a still wetter mixture, considers 25 to 50 cubic yards a day's work for a man joggling.

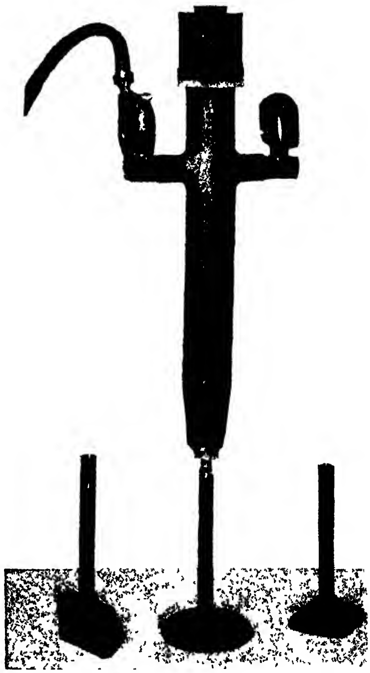


FIG. 102.—Pneumatic Concrete Rammer. (*See p.* 282.)

On the author's basis of 11 cubic yards per day, the average cost of leveling and ramming mass concrete with labor at $1.50 per day, allowing for superintendence and contractor's profit, is about 18 cents per cubic yard. For a 4 or 6-inch wall the cost may be two or three times this figure.

BONDING OLD AND NEW CONCRETE

In a foundation or other structure where the strain is chiefly compressive, the surface of the concrete laid on the previous day should be cleaned and wet, but no other precaution is necessary. Joints in walls or in other locations liable to tensile stress are coated with mortar, which should be richer in cement than the mortar in the concrete, proportions 1:2 being commonly used.

Some engineers spread the cement dry upon the wetted surface of the old concrete, while others make it into a mortar; the latter method is necessary in many cases to seal the joints between the top of the old concrete and the bottom of the raised forms.

The adhesive strength of cement or concrete is much less than its cohesive strength, hence in building thin walls for a tank or other work which must be water-tight, the only sure method is to lay the structure as a monolith, that is, without joints. If the wall is to withstand water pressure and cannot be built as a monolith, both horizontal and vertical joints must be first thoroughly cleaned of all dirt and "laitance" or powdery scum, wet, and then covered with a very thin layer of either neat cement or 1:1 mortar, according to the nature of the work. As an added precaution, one or more square or V-shaped sticks of timber, say 4 or 6 inches on an edge, may be imbedded in the surface, or placed vertically at the end of a section, of the last mass of concrete laid each day. In some instances large stones have been partially imbedded in the mass at night for doweling the new work next day.

In the New York Subway, work was commenced with no provision for bonding horizontal layers, but it was soon found that more or less seepage occurred, and in one case where a large arch was torn down the division line between two days' work was distinctly seen. Accordingly, at the end of each day's concreting a tongue-and-grooved joint was formed by a piece of timber 4 inches square partly imbedded in the top layer. This was removed before resuming work.

Roughening the surface after ramming or before placing the new layer will aid in bonding the old and new concrete.

Acid* is sometimes used for cleaning and roughening the surface of the set concrete. The acid must be thoroughly washed off before placing the new concrete or mortar.

In reinforced concrete, joints should be made so as to least affect the strength. In columns, joints should be made at lower surface of girder or at bottom of haunch, if any. In a floor system, or in reinforced walls resisting pressure, it is best to make the joints perpendicular to the surfaces at or near the center of the span.

* See U. S. Letters Patent No. 800942.

CONTRACTION JOINTS

Temperature changes are apt to produce contraction in concrete in air because in temperate climates most concrete is laid during the warm season. Moreover, it is generally recognized that while setting and hardening in air, concretes and mortars contract for a period.

It is probable that this contraction may be due, in part at least, to the cooling of the cement, which when setting attains a high temperature.* This is further evidenced by the fact that cracks in a thin building wall, 4 or 6 inches thick, open up within a few weeks after being placed, while heavier walls may not crack for several months. The concrete in the interior of a mass like a large dam cools very slowly, and records at the Boonton, N. J., dam indicate that the contraction cracks continue to increase in width for several years. The interior of a large mass like this is but slightly affected by atmospheric changes, and the cracks are but slightly wider in winter than in summer. In the Boonton Dam† no cracks were discovered during the first winter, but in the second and third winter seasons numerous vertical cracks developed. During the fourth and fifth winters all these cracks re-opened, but no new ones appeared. It was noticed that the cracks which were largest during one winter might be smaller the next, and be exceeded in width by some which were smaller the previous season. Approximate measurements gave: seventeen main cracks, 2.5 inches; sixteen smaller cracks averaging $\frac{1}{32}$ inch, 0.5 inch; thirty-three half cracks, averaging $\frac{1}{32}$ inch, 0.5 inch; with a sum total of 3.5 inches for a length of 2150 feet of masonry. The main cracks occurred at quite regular intervals of about 100 feet except near the ends of the dam. It was apparent that proportionally more cracks developed in that portion of the dam in which the masonry was laid during the warmer months.

Special measurements made upon a retaining wall along the Boston and Albany Railroad tracks at Newton Highlands, showed that for a length of wall of 673 feet the total contraction for a given period amounted to $1\frac{7}{16}$ inches. The range of temperature of the wall during this time was about 30°, which corresponds closely to the theoretical range necessary to produce the contractions, for assuming the coefficient of expansion to be 0.0000055, as given on a succeeding page, the range should be $32\frac{1}{2}$°.

Measurements were made by one of the authors of widths of opening of contraction joints in a long warehouse in Cincinnati, and found to agree almost exactly with that which would be expected by the range in temperature.

In an ordinary wall, if no cracks occur after nine months' setting there is apt to be no further danger, although after joints once form they will vary in width with the variations in temperatures.

*See page 130.
† Transactions American Society Civil Engineers, Vol. LXIII, 1909.

Contraction in concrete walls is provided for by forming joints at intervals to divide the wall into separate sections, and confine the cracks to straight lines, or else by reinforcing with sufficient steel to withstand shrinkage. The use of steel reinforcement is treated under Retaining Walls in Chapter XXVI.

Joints in vertical walls may be made simply by placing a temporary dam between the molds to remain until the concrete has set, when it is removed and the next section is filled in. To be sure of clear-cut cracks, however, it is necessary to insert non-adhesive material, as indicated below. In a reinforced wall rods may be run through holes in the dam if it is desired to tie the two sections together. If the old work has thoroughly set and the rods project only a few inches into the new, the adhesion between the old and new work will be so slight that a joint which will open as the concrete shrinks will be formed at the desired point. For bonding the two sections, a V-shaped groove may be molded into the part first laid. or alternate courses may be lapped or toothed out.

As a rule only contraction joints need to be provided, since expansion merely compresses the concrete. Sometimes, however, as in a long wall with recesses or in a reservoir floor with a channel in the middle, the expansion may cause a break at the angle. In such cases, water-tight joints may be made by leaving slits about ½-inch wide and filling them with a plastic material, one of the best for this purpose being pure asphalt of medium hardness. Lime dust is sometimes mixed with the asphalt. Another way of forming a joint is to insert two or more thicknesses of roofing paper.

In building the concrete filter tanks at Little Falls, N. J., which are 15 by 24 feet in horizontal area and rest upon concrete girders, the walls of adjoining tanks were laid on different days, and thus kept separate from each other. Contraction is provided for in each tank by sloping the ledges on which its walls rest, so that, in case of contraction, they will slide without cracking.

At the same plant* occasional expansion wells or vertical openings were built the entire height of the 40-foot retaining wall, to confine cracks to these places, and later, in cold weather, when the cracks were furthest open, these wells were filled with concrete.

From practical experience it appears that heavy walls require fewer contraction joints than light ones. In concrete retaining wall construction in Chicago† joints formed every 50 or 60 feet opened up quite noticeably in cold weather. Where the walls were of small cross-section a hair crack appeared half-way between the joints, tending to show that in thin walls joints should be provided about every 30 feet.

* Transactions American Society of Civil Engineers, Vol. L, p. 406.

† "The Coefficient of Expansion of Concrete," Journal Western Society of Engineers, Vol. VI, p. 549; republished in *Engineering News*. Nov 21. 1901. p. 380.

By properly distributed reinforcement, cracks may be made so small as to be unnoticeable. (See Chapter XXI.)

The Harvard Stadium, 575 feet in net length or 1390 feet measured around the U, which is illustrated in our frontispiece, is an example of the possibility of providing sufficient steel to withstand the contraction due to hardening and temperature changes.

Prof. William D. Pence, by very careful experiments at Purdue University, in 1899 to 1901,* determined the coefficient of expansion of concrete in air from changes of temperature to be 0.0000055 per each degree Fahrenheit. He experimented with Portland cement concrete mixed in proportions 1 : 2 : 4 broken stone and 1 : 2 : 4 gravel. The apparatus was designed to give extremely accurate results, and the variation in the coefficient of expansion in the different tests was from 0.0000052 to 0.0000057 per degree Fahrenheit. Two brands of Portland cement were employed, and in the broken stone concrete, two different stones. The average result for the gravel concrete was 0.0000054 per degree Fahrenheit, and for the broken stone concrete 0.0000055 per degree Fahrenheit. Prof. Pence concludes that "the coefficient of expansion of concrete is about 0.0000055 per degree Fahrenheit. (This value is conveniently remembered as five zeros fifty-five.)" The coefficient of expansion of the limestone used in a part of the tests was the same as that of the concrete made from it. Experiments† under the direction of Prof. Hallock at Columbia University gave 0.00000561 as coefficient for 1 : 2 mortar and 0.00000655 for 1 : 3 : 5 concrete. Prof. Burr calls attention to the similarity of this to the coefficient of linear thermal expansion of steel, which is about 0.0000066 per degree Fahrenheit. This fact is of great practical value to the engineer in the construction of reinforced concrete because it shows that the concrete and steel will be similarly affected by temperature changes.

A coefficient of 0.0000055 corresponds to a contraction of $\frac{1}{3}$ inch in 100 feet for 50° Fahrenheit fall in temperature.

The effect of hardening upon the volume, although less definitely determined, has been experimented upon by Prof. Bauschinger,§ of Munich, and Prof. George F. Swain,‖ of the Massachusetts Institute of Technology. As a result, the Committee on Cements of the American Society of Civil Engineers in 1887 reached the following conclusions:¶

First. Cement mortars hardening in air diminish in linear dimensions at least to the end of twelve weeks, and in most cases progressively.

* "The Coefficient of Expansion of Concrete," Journal Western Society of Engineers, Vol. VI, p. 549; republished in *Engineering News*, Nov. 21, 1901, p. 380.

† Burr's "Materials of Engineering," 1903, p. 378.

§ Transactions American Society of Civil Engineers, Vol. XV, p. 722.

‖ Transactions American Society of Civil Engineers, Vol. XVII, p. 213.

¶ Transactions American Society of Civil Engineers, Vol. XVII, p. 214.

Second. Cement mortars hardening in water increase in like manner but to a less degree.

Third. The contractions and expansions are greatest in neat cement mortars.

Among further conclusions of the committee given in this report it is stated that experiments show the contraction of neat cement in air at the end of twelve weeks to be from 0.14 to 0.32%, and of 1: 1 mortar, 0.08 to 0.17%. Although these values are corroborated by Bauschinger's* experiments on Portland cement mortars, the results of which also indicate nearly the same contraction for leaner mortars as for 1: 1, further data upon the action of concrete made of modern Portland cement is required before accepting the figures as applicable to this. Considère† gives 0.03% to 0.05% shrinkage for lean mortars corresponding to a contraction of about ½ inch in a wall 100 feet long. These various conclusions show that cracks in a newly laid concrete wall are due in part to contraction in setting. In fact, it has been noticed that joints open up in new concrete before it has been affected by external temperature.

It must be borne in mind that this action during hardening has nothing to do with the temperature of the atmosphere, and does not vary with it, but is in addition to the effects of temperature changes. It is possible, however, as suggested on page 285, that the shrinkage may be due in part to the cooling down from the heat evolved when the cement sets.

FACING CONCRETE WALLS

Exposed concrete walls had best not be plastered. It is a needless expense, and the results in variable climates are unsatisfactory. It is difficult to apply cement mortar uniformly to the face of hardened concrete, and it is apt to crack off and discolor, especially if the concrete behind it is porous enough for the water to penetrate it. For waterproofing walls not exposed to the atmosphere, cement plaster is sometimes serviceable, as described on page 341.

Mortar for patching irregularities and pockets, which will occasionally occur in the best work, and for filling holes, must contain the same proportions of cement and sand as the concrete, or it will set a different color.

The treatment of the face of concrete is determined by the character of the structure. A fair surface, suitable for work which is not exposed to view, and even for sheds or other buildings where the appearance need not be regarded, has been obtained by the authors on 4-inch and 6-inch walls

*Transactions American Society of Civil Engineers, Vol. XV, p. 722.

†Considère's Reinforced Concrete, 1903, p. 87.

by using merely a very wet mixture of cement, sand and gravel, with care in placing and puddling so that none of the stones, many of which were 2 inches in diameter, collected in pockets against the forms. Such treatment will result in a sandy finish, showing the joints in the forms less than a smoother one.

FIG. 103.—Face Cutter. (*See p.* 289.)

To produce a smooth mortar surface, a thin tool like a spade or an ice cutter, shown in Fig. 103, may be thrust down next to the molds as the concrete is placed, so as to force the stones back from the face and allow the mortar to cover every stone, care being taken not to pry the molds.

One of the best methods of finishing for a large smooth surface is to spade or cut the faces as described, and then after the forms are removed to pick them with a hand tool, shown in Fig. 104, or a pneumatic tool adapted for the purpose. The Harvard University Stadium, illustrated in our frontispiece, is finished in this way, and the photograph in Fig. 105 shows a near view of the surface. On the left is the concrete showing the impressions of the plank forms, and on the right is the finished surface. If this picking is performed by hand, it is done by a common laborer. The surface he will cover per day depends upon the hardness of the concrete. It must not be too green or the tool will loosen the stones, while if set very hard the labor is unnecessarily great. On the average, a man may be expected to cover about 50 square feet per day of ten hours. The picks require frequent, at least daily, sharpening. For the best appearance, the size of stone in the concrete should be limited to about ¾ inch to one inch. This method of picking was employed by Mr. E. L. Ransome in the construction of the Pacific Borax Works in New Jersey. A pneumatic tool suitable for this work is made with a circular end containing a number of points, using which a man should cover 400 to 500 square feet per day.

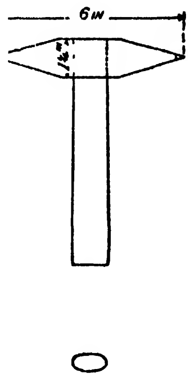
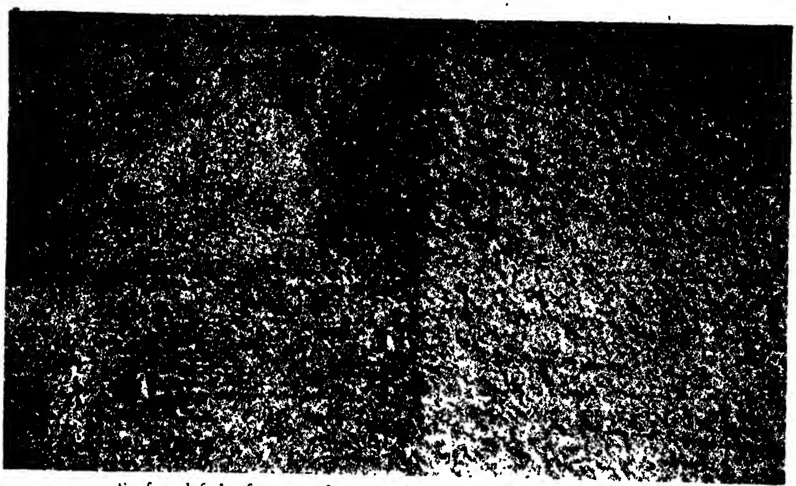


FIG. 104.—Pick for Facing Concrete. (*See p.* 289.)

Mr. C. R. Neher* states that with labor at $1.50 per day bush-hammering will cost less than 1½ cents per square foot.

A surface of washed concrete is shown in the photograph, Fig. 106.

*Journal Association of Engineering Societies, Jan., 1902, p. 41



Surface left by forms is shown on left and picked surface on right.
FIG. 105.—Surface of "Picked" Concrete. (See p. 289.)

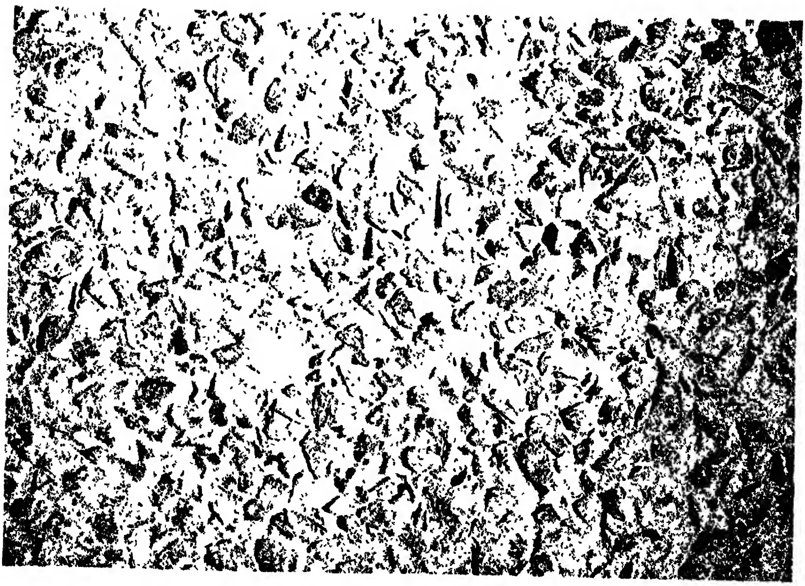


FIG. 106.—Surface of Washed Concrete. (See p. 289.)

p. 290. This finish, used by Mr. Henry H. Quimby* for surfacing concrete bridges in Philadelphia, is obtained by hand or with a hose. Hand methods are usually preferable because of the difficulty of applying the hose at exactly the right stage of hardening. In either case the forms must be removed as soon as the concrete is sufficiently hard,—a period varying from 6 hours to 2 or 3 days, according to the character of the cement and the weather,— and the washing done immediately. For washing by hand, a plasterer's float, or a small board 1 by 3 by 6 inches, is used and the cutting is done by sand rolled between the board and the wall, with plenty of water. The concrete face after this process may sometimes be too green for rinsing clean, when the final cleaning is deferred for a few hours. Mr. Quimby states that a laborer should wash and clean 100 square feet of surface in less than one hour. If the concrete has become too hard before washing, a comparatively smooth finish is obtained in a similar manner or by vigorously rubbing the surface with a rough brick. A green surface may be treated with a common scrubbing brush and water.

A fine sandy finish may be obtained after concrete has set by rubbing with a block of carborundum about 3 by 4 by $1\frac{1}{2}$ inches.

Another plan for removing the skin of cement is the acid process.†

Mr. H. P. Gillette‡ mentions a method employed in one case on the New York Central R. R. of chiseling sloping grooves, about $\frac{3}{4}$ inch deep and 2 inches apart, upon an old discolored concrete surface.

For a very smooth mortar surface, such as may be required for moldings, curved surfaces or carving, the interior surface of the mold may be plastered about $\frac{3}{4}$-inch thick, by hand or trowel, just in advance of the laying of the concrete, so that the concrete and mortar set up as one mass.

The advocates of dry mixed concrete often require a piece of board, corresponding in width to the thickness of the layer of concrete, to be placed on edge close to the form, the concrete rammed against it, and then the board removed and the space filled with mortar mixed in proportions 1 : 2 or 1 : 3. Another method, which can be used with mortar of a wetter consistency, is to place a thin board or a strip of sheet iron at the required distance from the form, usually about 2 inches, then to fill in the mortar between it and the mold, and the concrete on the other side of it, when it may be removed. In the best modern practice, facing mortar is omitted altogether, and the concrete is made wet enough to present a good surface.§

Marking the surface to resemble masonry is considered unnecessary from an architectural point of view, for the work is actually a monolith and

* Personal correspondence. See also *Engineering News*, Dec. 20, 1906, p. 656.

† See paper by Linn White, *Engineering Record*, Feb. 2, 1907, p. 126.

‡ *Engineering News*, July 24, 1902, p. 66.

§ Other methods of facing are described in the Report of the Association of Railway Superintendents of Bridges and Buildings, 1900.

should have that appearance, but if it is desired, triangular pieces may be nailed to the forms, or if tongued-and-grooved plank are used, the horizontal molding may be formed by a strip of wood gotten out to the preferred shape, and planed with a tongue and groove so as to fit between two planks as shown in Chapter XXIV.

The size of molding depends upon the class of masonry which is to be imitated. Mr. Edwin Thacher* specifies triangular moldings 2 inches wide by 1 inch deep.

To give a uniform color, in England† it is customary to use a rather stiff mortar in proportions 1 : 3 applied with a plasterer's hand float and worked in so thoroughly as to leave no body on the surface. In the United States a 1 : 2 grout is sometimes put on with a whitewash brush or small whisk broom. This, however, is liable to check.

A pumice stone paint used by Mr. H. I. Moyer has given satisfaction in practice. It consists of ground pumice-stone and Portland cement mixed in equal parts to the consistency of thick paint. After removing the board-marks with a block of carborundum, the surface is wet and the paint applied with a brush. When this first coat is hard, it is wet and the second coat applied.

Plastering. When plastering on external surfaces must be resorted to, special means must be taken to make it adhere and to prevent its checking. The forms must be wet instead of oiled; irregularities must be removed by chipping or rubbing; the entire surface should be roughened; and the coat of plaster should be as thin as possible, preferably not over $\frac{1}{16}$ or $\frac{1}{8}$-inch.

By throwing on plaster with considerable force, it bonds better than by spreading it. If the first coat is thrown on the second is more apt to adhere. A spatter-dash or a pebble dash finish is made by throwing on a mortar to leave it regular but rough.

Lafarge cement finish has been satisfactorily used for house walls by Mr. Benjamin A. Howes. The process is illustrated in a photograph shown in Fig. 107, page 293. The surface, which must be very true, is wet, and a neat solution of Lafarge cement is spread on with a whitewash brush. Before this has dried, a second coat in proportions about 1 : 3 of Lafarge cement and fine sea sand is spread with a steel trowel, floated with a wood float, then immediately wet down with a whitewash brush. The total thickness of the plaster should not be over $\frac{1}{16}$ inch.

If a thick plaster is necessary, the surface must be carefully roughened, wet, and coated with a neat cement grout, preferably spread on very thin with a wire brush, and then plastered immediately before it hardens. A plaster which has been found satisfactory is made using one-sixth to one-

* *Cement*, May, 1903, p. 107.
† Sutcliffe's *Concrete*, 1893, p. 324.

third part of lime putty to one part of cement by bulk, with enough sand to make it work sandy. For 3-coat work the second coat may have about as much hair as is used in brown coat work in interior plastering.

FORMS FOR MASS CONCRETE*

The forms for structures, such as buildings and sewers, are illustrated in the chapters treating upon these subjects.

The best lumber for forms or molds for concrete is white pine because it is easily worked and retains its shape after exposure to the weather. Except, however, where a very fine face is required, motives of economy

FIG. 107. Surfacing Wall with Mortar. (*See p.* 292.)

usually prompt the use of cheaper material, such as spruce or fir, or, for very rough work, even hemlock. Green lumber is preferable to dry because it is less affected by the water in the concrete.

If the planks or boards are thoroughly oiled and are not exposed too long a time to the hot sun and dry air, which tend to warp them, they may be used over and over again. Long exposure, however, will throw the surface out of true, and open up the joints. In some instances the same lumber can be employed in different places. For example, in the con-

* See also paper by Sanford E. Thompson on "Forms for Concrete Construction," Transactions National Association Cement Users, 1907, reprinted as Bulletin No. 13, Association of American Portland Cement Manufacturers.

struction of a factory building, Mr. Thompson specified 2-inch tongued-and-grooved roof plank of green spruce for the forms, and after using at least four times, no difficulty was found in laying it on the roof. The planks were merely slightly gritty and discolored by the oil employed to prevent adhesion of cement.

Lumber which is planed one side is essential to a smooth face, and where the forms must be removed within 24 or 48 hours it is sometimes advantageously employed for rough work because the concrete adheres less to planed lumber and that which does stick is easily scraped off, thus effecting a saving of labor which more than balances the cost of planing. Many concrete experts advise the use of beveled edge stuff in preference to tongued and grooved. The edges crush as the board or plank swells, and this prevents buckling.

Square corners and thin projections should be avoided when possible A beveled strip in an external corner will give it a finished appearance.

Either 1-inch boards or 2-inch plank are suitable for forms. The spacing of the studs depends in part upon the consistency of the concrete and the thickness of the walls. If the concrete is laid quite wet and the mass is large, there may be considerable pressure exerted before the cement sets. On the other hand, there is less liability of the boards being forced out of place by ramming than when a drier mixture is used. The authors have found that in comparatively thin walls laid with a wet mixture the stringers may be spaced 5 feet apart for 2-inch plank and 2 feet apart for 1-inch boards. This represents about the limit if an absolutely straight face is desired, and even with this spacing the lumber will spring slightly in places where very short lengths of it are used.

The size of the studding depends upon the height of the wall and the amount of bracing which it is convenient to use. For a low form of 1-inch stuff 2 by 4 inch studs may be satisfactory. If this size is used for a higher wall, horizontal timbers must be placed and carefully braced at distances about 5 feet apart to prevent the studs from springing. For 2-inch plank, as the studding is spaced farther apart, it must be heavier. Common sizes are 4 by 6 inches, 2 by 10 inches, and 4 by 10 inches, depending upon the character of the work and the material at hand. The toes of the diagonal braces which run from the studding down to the ground must rest securely against stout posts or other immovable supports. The use of these diagonals may be avoided in many cases or their number reduced by connecting opposite studs with through bolts or wire. An inexpensive method of connection is shown in Fig. 108, page 295. The wires are wound around opposite studs and then twisted with a stick, as a turn-buckle,

until the studs are the proper distance apart. To remove the forms the wires are cut and then trimmed off close to the concrete.

If in placing the concrete the forms commence to buckle, they must remain in their warped position unless trueness of face is of sufficient importance to warrant tearing down the concrete and replacing it. A carpenter is so accustomed to truing up his lumber after it is in place that it is

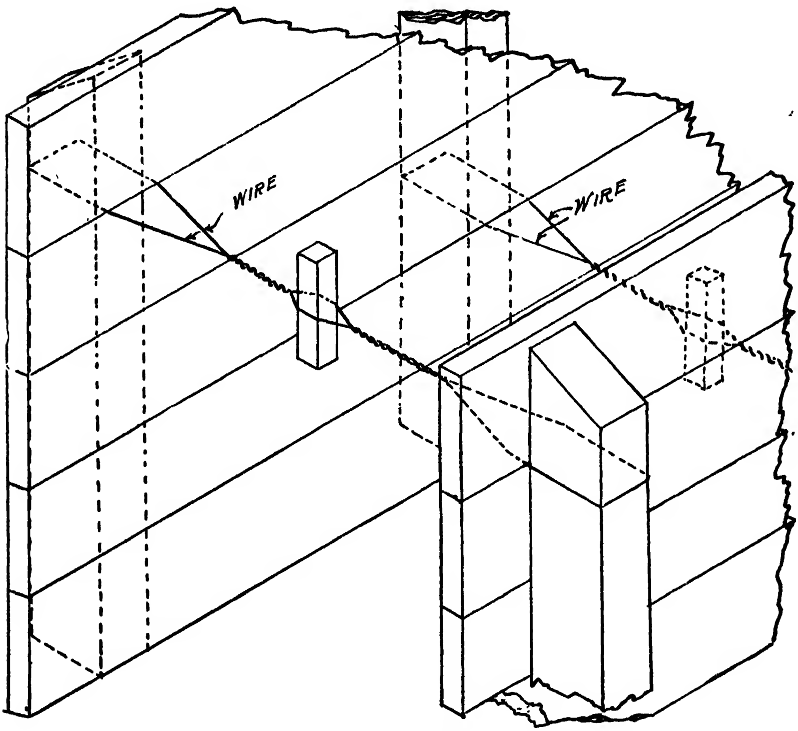


FIG. 108.—Method of Connecting Forms. (See p. 294.)

difficult for him to realize that a thin wall of concrete cannot be straightened in the same way. The fact that a crack once made in concrete which is set is almost impossible to repair cannot be too strongly impressed upon the woodworkers.

Concrete forms should be nearly water-tight but need not be absolutely so. Cracks of noticeable width which cannot be closed by wetting and swelling the lumber may be battened, and vertical joints between the ends

of planks may be stopped in the same way. Hard soap has also been used for this purpose.*

In a large structure such as a dam, cement bags filled with sand† may be piled to form the temporary end of a layer or series of layers of concrete.

Greasing Forms. Crude oil is an excellent and inexpensive material for greasing forms. This is a petroleum product sufficiently liquid to be readily applied with a large whitewash brush. The object is to fill the pores of the wood rather than to cover it with a film of grease. The oil must be applied every time the forms are set. Thin soft soap or a paste made from soap and water is also occasionally used. On an important job in England‡ the centering boards of arches were covered with strong packing paper soaked with linseed oil. Paper however is apt to wrinkle.

If the concrete is to set for several weeks before removing the forms, the cohesion of the concrete will be greater than its adhesion to the lumber, and no oil or grease will be necessary, although it is well to thoroughly wet the plank before laying the concrete against it. Always oil metal forms.

Removing Forms. The length of time which concrete must set before removing the forms depends upon the weather, the strain which is to come upon the work, and the consistency employed in mixing.

A good rule to follow when laying wet concrete upon which no pressure is to come immediately is to determine whether it is sufficiently hard by pressing upon it with the broad part of the thumb. If indented, the concrete is too soft to permit of removing the forms. It is sometimes possible in good drying weather, even if slow-setting Portland cement is used, to raise the forms within from 10 to 24 hours after placing the concrete, but care must be exercised that no blow or jar comes upon the fresh work. If the wall is very thin and is to be subjected immediately to earth or water pressure, it may be advisable to allow the forms to remain for several weeks. The setting of concrete is retarded by cold or by wet weather. When mixed very wet, it sets and attains its strength more slowly than when mixed with a small amount of water.

RUBBLE CONCRETE

Rubble concrete includes all classes of concrete in which large stones are placed by hand or by machinery. The term concrete rubble has been applied when the mass consists essentially of large stone laid in joints of concrete instead of mortar.

*George W. Lee, *Engineering News*, Mar. 19, 1903, p. 246.

†*Engineering News*, Aug. 27, 1903, p. 185.

‡K. Leibbrand in Proceedings Institution of Civil Engineers, Vol. CXIX, p. 227.

Rubble concrete comes in competition with pure concrete on the one hand, and with rubble masonry, — that is, stonework laid in cement mortar, — on the other hand. Its cost in large masses is usually less than that of pure concrete, because the expense of crushing the stones used as rubble is saved, and each large stone replaces a mass of mixed cement and aggregate, thereby saving a portion of the cement. As stone is always heavier than concrete made from the crushed material, because of the pores in the concrete, the replacing of portions of the latter by large stone increases its weight, and therefore its value for certain classes of construction. Large masses of rubble concrete can usually be laid cheaper than ordinary concrete, but where the mass is small and separate machinery or apparatus will be required for handling the large stones, its use may not be advantageous. It is especially suitable where the concrete materials are handled with derricks, because these may be employed to hook the stone or transport it in trays.

In comparison with large masses of rubble masonry laid in cement mortar, rubble concrete of similar quality is almost invariably found to be cheaper because scarcely any skilled labor is required. In a thin wall, not more than 3 feet thick, the rubble masonry may be cheaper because no forms are required. In estimating comparative costs of rubble masonry laid in Natural cement mortar and rubble concrete made with Portland cement, the fact must be considered that a wall of Portland cement rubble concrete may be made thinner than one of Natural cement masonry because it is stronger. The difference in strength is not merely due to the class of cement employed, but to the fact that in rubble concrete the stones are perfectly imbedded instead of being set up on small spawls in the manner customarily employed by stone masons.

The amount of cement used in rubble concrete varies not only with the proportions of the concrete mixture, but with the percentage of rubble introduced. Very much less cement is required in concrete than in a similar quantity of mortar of like strength, but concrete joints must be thicker than mortar joints, so that the result is often more cement is required per cubic yard for concrete than for rubble masonry. However, by employing a large percentage of stone, as was done at Boonton,* the quantity of cement may be brought below that for rubble masonry.

The strength of rubble concrete can be compared only theoretically to that of concrete or rubble masonry, because there are no testing machines in existence of sufficient capacity to break a mass of Portland cement masonry containing large stones. It is generally considered less than that

*See description, page 300.

of plain concrete, but, the authors believe, with insufficient ground. Less cement is contained in a cubic yard, which tends to lessen the strength, but, on the other hand, as stated above, the large stones add density which is a source of strength.

In concrete subjected to tension or bending the introduction of large stones might possibly be a source of weakness by forming planes of adhesion. On the other hand, the stones tooth into the mass and into each other, forming an irregularity of breaking surface which would tend to increase the strength. On long-time tests, too, the strength of the large pieces of stone, which is naturally greater per square inch than the strength of small pieces of broken stone, would naturally come into play. In compression this extra strength of the large stones, especially in their resistance to shearing, has a still greater influence upon the strength of the mass, and besides this they must necessarily bond and wedge with each other.

COMPARATIVE QUANTITIES OF MATERIALS FOR PLAIN AND RUBBLE CONCRETE

The cement and aggregate are often expressed as percentages of the total mass of plain concrete or of rubble concrete. This is confusing because there are various ways of expressing percentages, and, as suggested below, it is therefore clearer in ordinary cases to employ, instead, commercial measurements, such as cubic feet, cubic yards, or pounds.

Before the concrete is mixed, the volumes of materials may be compared by percentages, thus, proportions 1:3:6 have 10% cement, 30% sand, and 60% broken stone; but this is apt to be misleading, since loose volumes, — because of the different voids, — and weights, — because of different specific gravities, — do not exactly correspond to absolute or solid volumes in the finished concrete. By absolute volumes,* for example, a cubic foot of 1:3:6 concrete† may contain 0.079 cu. ft. of solid cement grain, 0.278 cu. ft. of solid sand grains, and 0.491 cu. ft. of solid stone particles, and may be said to have 7.9% cement, 27.8% sand and 49.1% stone. This is an exact method, but such percentages cannot be determined without very complete data.

For comparing costs of different concrete it is therefore best to discard the term percentages, and instead to express the quantity of each material as weights or loose volumes required for a unit volume, — say a cubic yard, — of compacted concrete. By this method a cubic yard of average 1:3:6 concrete (from the table on page 231) contains 1.11 bbl. cement.

*See example, p. 139.

†See item (23), p. 377.

0.47 cu. yd. loose sand, and 0.94 cu. yd. loose broken stone. If, now, rubble concrete is used and if on the average every cubic yard of this rubble concrete after being laid contains large rubble stone to the amount of 0.3 cubic yards (measured net, as solid stone), we may say that the rubble concrete contains 30% rubble, and each of the other materials are reduced by 30%, thus giving $1.11 \times 0.70 = 0.78$ bbl. cement, $0.47 \times 0.70 = 0.33$ cu. yd. sand, and $0.94 \times 0.70 = 0.66$ cu. yd. broken stone per cubic yard of concrete. From such data, the relative costs of materials for plain and rubble concrete may be readily compared.†

Proportion of Rubble in the Mass. The proportion of large stones which can be placed depends upon the size of these stones and upon their distance apart. In a heavy wall or dam the size may be limited simply by the strength of the machinery employed to handle them, whereas in a comparatively thin wall subjected to water pressure, it is evident that the stones should not be large enough to run nearly through the wall and might be limited to one-half or one-third of its width. Larger stones can be used with a wet than with a dry mixture since they bed more readily.

The distance between the stones varies in different specifications from 3 to 18 inches. If the concrete is mixed of dry consistency there must be space enough between the stones to ram the concrete thoroughly and force it into all the recesses, while with a wet mixture the spaces need be regulated merely by the dimensions of the stones in the concrete aggregate, care being exercised that they do not bridge or arch across between the large stones.

The quantity of rubble is usually expressed as a percentage of the total mass of the finished concrete. The percentage may vary from 20% to 64%, both of these quantities being mentioned by Mr. John W. Steven* as used in different places in Scotland. Nearly as much space must be left between two small stones as between two large ones, so that the percentage increases with the size. Into one of the Boonton dikes (4 feet 8 inches thick) of the Jersey City Water Supply Company, — where the stones were hoisted in derrick trays and unloaded by one or two men, — 20% of stone was introduced, and this may be taken as a fair average quantity for concrete containing "one-man" or "two-men" stone. In another Boonton dike, of the same thickness and similar in other respects, the stones were large enough to handle by derricks, and the quantity was increased to 33%, while in the large dam described below, 55% was the average quantity.

*Proceedings Institution of Civil Engineers, Vol. CXIII.

† See tables of Quantities of Materials, pp. 236, 237.

The amount of rubble may sometimes be most conveniently and accurately measured by weighing it in cart or car loads.

Methods of Laying Rubble Concrete. The forms for rubble concrete may be built as for ordinary concrete, or the faces of the work may be of cut stone or ashlar masonry.

Ordinarily, derrick buckets are the most suitable apparatus for placing the concrete, because the derrick can also be conveniently used for handling the stone.

One of the best examples of rubble concrete work which has come within the observation of the authors is the dam of the Jersey City Water Supply Company at Boonton, N. J.,* built in 1902–4 under the direction of Mr. William B. Fuller, Resident Engineer. The dam proper contains about 240 000 cubic yards of "cyclopean" or concrete rubble masonry, and the contract price at which this was let, which covered all labor and all materials excepting the cement, was $1.98 per cubic yard. Other bids ranged from $2.20 to $3.60 The rubble stones, which actually averaged in size from 1 to 2½ cubic yards each, were brought from the quarry about three miles distant over a standard gage track built for the purpose, and the stone for the concrete aggregate was also broken at the quarry, although it was not touched by hand from the time it entered the crusher until it was deposited in concrete. One of the distinctive features of the construction was the consistency of the concrete, which was mixed extremely wet, in fact, about like pea soup, so that when dumped it spread out, forming a level bed for the stone. As soon as a bucket of concrete was dumped, a large stone, which had come from the quarry on flat cars, was picked up by one of the stiff-legged derricks ranged on trestles along each face of the dam, and dropped, — with force, not gently lowered, — usually with its smoothest face down, into the mushy mass. Settling into place, it bedded itself in the concrete, and laborers joggled it with crowbars so as to bring it to a firm bearing and drive out all air bubbles. A tone lifted after placing left a bed conforming to the irregularities of the stone, and having the appearance of mortar, no stones being visible. Scraping this mortar in places showed that the stones of the concrete were covered with an exceedingly thin film of mortar.

The labor of actually placing the concrete and stone after bringing them to the dam may be estimated from the fact that each stiff-legged derrick supplied a gang of three or four laborers dumping concrete and joggling the stone, with one foreman mason, who not only looked after the depositing

* See drawing, Chap. XXVI. See also *Engineering Record*, Aug. 8, 1903, p. 152.

of the stone in the concrete, but also spent some of his time on the face stone masonry. In addition to these, there were the men mixing concrete and handling the cars of stone. Mr. Fuller stated that seven derrick gangs averaged about 700 cubic yards of concrete rubble masonry in ten hours, or about 100 cubic yards to a derrick. A maximum day's work for a derrick was about 125 cubic yards.

The concrete was proportioned 1 part Portland cement, $2\frac{3}{4}$ parts sand, $6\frac{3}{4}$ parts broken stone, the latter ranging in size from fine particles up to 3 inches in diameter. The masonry contains about 55% of rubble, the large stones being kept at least far enough apart so that the fist could be thrust between them. About 0.6 barrels of cement were used per cubic yard of concrete rubble masonry. This quantity is less than is generally used in a rubble wall built of fairly well dressed stones laid in 1:2 cement mortar; and where water-tight rubble is required and the stones are accordingly left as rough as possible, the quantity of cement is apt to average slightly more than one barrel per cubic yard.

In a dam built in eastern Connecticut in 1899 to 1901,* where methods somewhat similar to those jus described were employed, the quantity of cement averaged about two-thirds barrels per cubic yard of masonry.

The masonry dry dock at the Charlestown Navy Yard, which was begun in 1900, furnishes an example of rubble laid in dry mixed concrete. The stones, which were placed about 18 inches apart in all directions, averaged about $\frac{1}{2}$ cubic yard in volume, and had comparatively square faces and level beds. They occupied less than one-third of the total volume of the concrete. The concrete, mixed in proportions about 1 part Portland cement to 2 parts sand to 5 parts gravel, was deposited from buckets, and thoroughly rammed, and the stones, after washing with a hose, were placed by derrick. If a stone did not bed itself properly, the derrick picked up a heavy weight and allowed it to drop several times upon the stone to ram it into place.

DEPOSITING CONCRETE UNDER WATER

Although some engineers still specify that no concrete shall be laid under water, the many important structures which have been built of late years upon foundations of concrete deposited loose, to set and harden under water, prove that excellent work can be performed with proper selection of materials and care in laying. It is absolutely necessary, however, to lay the concrete by some means which will prevent the separation of the ingredients as they pass through the water. This has been accom-

*Described by Herbert M. Knight, *Engineering News*, June 12, 1902, p. 470.

plished, as discussed in the succeeding pages, by the following methods: (1) passing the concrete through a tube in a continuous flow, (2) lowering it in large buckets from which the concrete may be dropped in large masses, (3) confining it in bags, (4) forming the concrete into blocks on land, and after setting placing them by machinery or by floats, and (5) allowing the concrete to partially set in air and then depositing it in a "plastic" condition.

For sea water construction, the cement should be carefully tested to see that it is of standard quality.* Occasionally the water of a stream or pond may be impregnated with by-products, such as sulphuric acid from industrial plants, or with mineral impurities which prevent the concrete from setting properly.

Cofferdams, which need not be water-tight, are almost always necessary to prevent the concrete from spreading and the cement from washing away.

Laitance. "Laitance" is a French word, quite generally adopted in the United States and England for the light-colored powdery substance which is held in suspension by the water when cement or concrete is deposited below the surface. On land the same substance forms on the surface of concrete which has been mixed very wet.

The analysis of a sample of laitance† showed its composition to be as follows:

Silica (SiO_2)	16.00%
Alumina and Iron (Al_2O_3, Fe_2O_3)	8.66 "
Lime (CaO)	47.40 "
Magnesia Oxide (MgO)	2.40 "
Ignition loss	23.60 "

If calculated to a water and carbonic acid free basis the analysis becomes:

Silica (SiO_2)	20.94%
Alumina and Iron (Al_2O_3, Fe_2O_3)	11.30 "
Lime (CaO)	62.04 "
Magnesia Oxide (MgO)	3.14 "

Mr. Richardson notes that this composition corresponds with that of a normal Portland cement except that it is unusually high in alumina and iron, a fact which may be explained by the large amount of magma detected in the thin section examined. He further states:

I have had a thin section ground, but find that it shows no structure which is characteristic. The section consists largely of amorphous material of an isotropic nature, that is to say, it does not affect polarized light. It reveals a considerable amount of a yellow substance which seems to be the

*Also see Chapter XV, and page 308.
†Analyzed for the authors by Mr. Clifford Richardson.

undecomposed magma contained in the original cement. I have formed a material very similar to the "laitance" by shaking Portland cement with water, decanting the finer portion and allowing it to settle out and harden. This material, like your "laitance," is rather soft, and this is due to the fact that the Portland cement is much more thoroughly decomposed under these conditions than under ordinary ones, and this accounts for its character.

It is evident from these facts that the milky laitance which appears on concrete laid under water represents an actual loss of cement, which should be prevented by confining the mass until it reaches its position.

Depositing Concrete through Chutes. In his Treatise On Limes, Hydraulic Cements and Mortars,* Mr. Gillmore refers to a "trémie" used in laying concrete under water in Chesapeake Bay. This consisted essentially of a tube of boiler iron about 2 feet in diameter, and long enough to reach the place where the concrete is to be deposited. Similar apparatus is still employed for forming layers of concrete under water.

When building the piers of the Charlestown Bridge, Boston, a cofferdam was first constructed, and then a tube, about 14 inches in diameter at the bottom and 11 inches at the neck, with flaring top, was suspended by a differential hoist from a moving platform, as shown in Fig. 92, page 270. The tube was made in removable sections bolted together by outside flanges so that its length could be readily varied. Mr. William Jackson, Chief Engineer for the bridge, describes† the method of operation as follows:

The foot of the chute was allowed to rest on the bottom, and was filled with concrete dumped from wheelbarrows. The chute was then raised slowly from the bottom, allowing a part of the concrete to run out in a conical heap at the foot, while the loss was made good by dumping in more concrete at the top. The truck bearing the chute was then moved from side to side of the dam, so as to leave a ridge or bank of concrete crosswise of the pier, the chute being kept always filled or nearly filled by dumping more concrete into the hopper. The height of the ridge of concrete was regulated by the height to which the foot of the chute had been raised from the bottom. When the ridge was completed across the dam, the traveller supporting the truck was moved a short distance lengthwise of the pier, and the truck was moved back again across the dam, parallel to its former course, allowing the concrete to run out over the edge of the bank first deposited, widening it on the side to which the traveler had been moved, and this process was continued until the whole area of the foundation was covered with a layer of concrete, upon which, when it was sufficiently hardened, another similar layer or course could be deposited.

*Page 236.

†Third Annual Report, Boston Transit Commission, 1897, p. 74.

The thickness of each course depended upon the height to which the foot of the chute was raised above the top of the preceding course. Courses were laid up to 6 feet in thickness, but it is thought that the best results were attained with a thickness of 2 or 2½ feet.

If the bank is made too high, or if the bottom (or the top of the preceding course) is very uneven, or if the piles interfere with the motion of the chute, or if the chute is moved along or raised too rapidly, the concrete is likely to run out so fast as to empty the chute entirely before the flow can be checked. In this event the "charge" is said to be "lost," and the chute must be lowered again to the bottom and refilled. When the charge is lost the water rises inside the chute to the same level as that outside, and into this water the concrete must be dumped until the water is wholly displaced or absorbed by the concrete. This has a tendency to wash the concrete, and to separate the cement from the sand and gravel, and as it generally takes a cubic yard or more of concrete to displace all the water in the chute, there is danger that a rather large body of badly washed concrete will be deposited whenever the charge is lost. This danger threatens not only when the charge is accidentally lost, but whenever work is begun in the morning or after the mid-day intermission; for whenever the work stops the charge must be allowed to run out lest it set in the chute.

To obviate partially the evil of washed concrete, the contractor was directed, whenever work was begun after an intermission, or whenever the charge was lost or water leaked into the chute, to throw into it, before each wheelbarrow-load of concrete, until the water was displaced, a quantity of dry cement. He was also directed to begin work after an intermission with the chute near the center line of the pier, so that any body of washed concrete resulting would be completely surrounded by sound concrete.

After the workmen and the inspector had gained experience with the chute, the accidental loss of the charge was not a frequent occurrence, and the danger of an occasional body of partly washed concrete, surrounded as it must be by good concrete, was not looked upon as a very serious matter.

A difficulty sometimes met with in using the chute is that when a sudden rush of concrete takes place, even if the charge is not entirely lost, the concrete within the chute often falls far below the level of the water outside. The outside water then, especially if there is a deficiency of sand in the concrete, is likely to force its way through the concrete remaining in the bottom of the chute, tending to separate the cement from the sand and gravel, and making the concrete too wet, and so threatening a complete loss of charge. If there are any leaks in the joints of the chute, water comes in and tends to cause loss of charge, and this leakage is especially troublesome when the concrete in the chute falls below the level of the water outside.

The chute seems to work best when the concrete is mixed not quite moist enough to be plastic. If it is mixed too wet the charge is likely to be lost; if very dry there is a tendency to choking of the chute. The working of the chute is affected also by variations in the proportions of sand and

gravel. With gravel in excess the outside water too readily forces its way in at the bottom. With an excess of sand the concrete tends to clog in the chute.

Sometimes when the concrete becomes clogged in the upper part of the chute, the concrete below the clogged place continues to flow out, leaving a vacant space into which water forces itself through the concrete remaining in the bottom of the chute. When the clogged concrete above is loosened, it falls into this body of water, which, unable to find exit by the way through which it entered, is displaced by the falling concrete, and rises into the hopper, sometimes to a level considerably above that of the water outside.

In the construction of the foundations for the piers for the Cambridge, (Mass.) Bridge,* a tube was used in much the same way as that employed for the Charlestown Bridge. The concrete was dumped from derrick buckets into a hopper, below which was a tube 16 inches in diameter at the top and 22 inches in diameter at the bottom, built in 4-foot cylindrical sections, which telescoped one another, so that a length varying from 4 to 40 feet could be obtained. Each layer of concrete was 1 to 2 feet thick. The tube was suspended from a traveler running upon a pair of traveling trusses which rested at each end upon tracks laid on top of the cofferdam, so that concrete could be deposited at any point within the rectangle.

Depositing Concrete from Buckets. The opinion of engineers is divided as to whether the best method of depositing concrete under water is by a chute, as has just been described, or from a bucket. The objection to the former is the difficulty in always maintaining a continuous flow, while with the latter it is not so easy to place the layers uniformly and to prevent the formation of mounds which are more or less washed by the water. With careful superintendence, however, either of these methods is satisfactory.

The best results can be attained with buckets so constructed that the material flows out through the bottom. A mass of concrete deposited under water must be disturbed as little as possible, and in tipping a bucket the material is apt to be stirred. Various buckets with bottom doors have been devised for opening automatically when the place for depositing is reached. In one type, used in 1900 at the Charlestown Navy Yard, the slackening of the rope released latches which fastened the trap doors so that they opened as soon as the bucket commenced to ascend. Another style, designed by Mr. John F. O'Rourke, is shown in Fig. 109. The photograph shows the bucket closed. When it reaches the bottom the

*See article by Sanford E. Thompson, *Engineering News*, Oct. 17, 1901, p. 282.

handle slides down, allowing the doors to swing open and the concrete to drop out in a single mass. The bail catches when it has dropped to the bottom, so that when hoisting the bucket the doors remain open. Covers

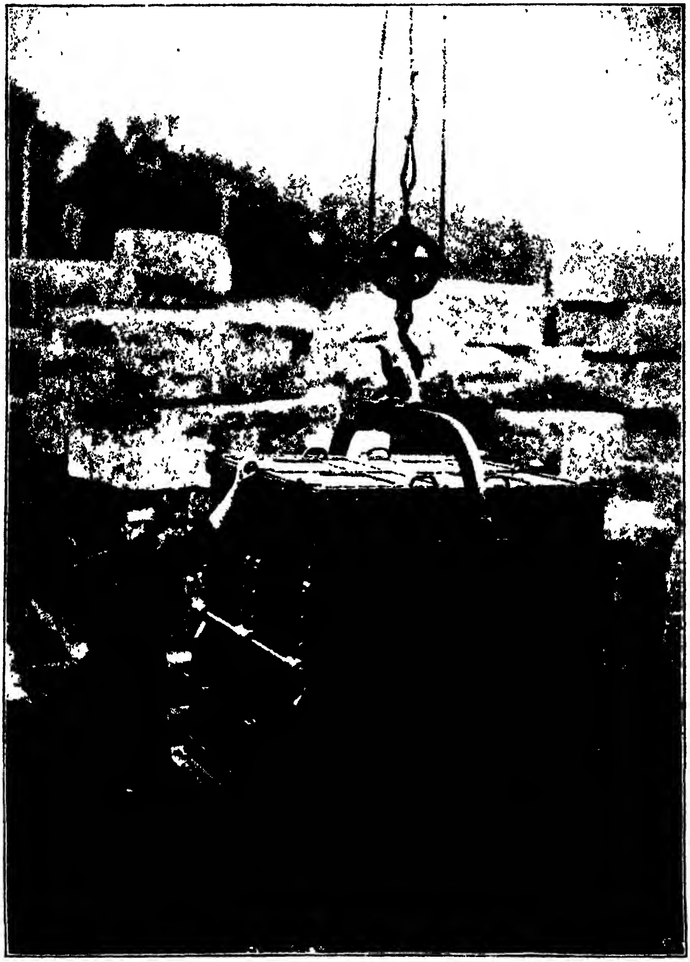


FIG. 109.—Bucket for Depositing Concrete. (*See p.* 305.)

prevent the water from rushing in at the top as the bucket is being lowered, and the V-shaped bottom lessens the disturbance of the water.

Depositing Concrete in Bags. Bags, varying from small paper or mus-

lin bags to jute sacks containing 100 tons,* have been employed in the past for holding concrete together as it passed through the water. In some cases the concrete has been placed in the bags dry †

Mr. William Dyce Cay in building the breakwater at Aberdeen Harbor Eng., employed bags holding from 28 to 50 tons of concrete. A bag was placed in the hopper bottom of a barge filled with concrete, and sewed up as the barge was being warped to place. When the doors of the hopper were released it fell into place.

John C Goodridge's method§ of laying concrete under water, employed in 1887, consisted in enclosing the concrete "in paper bags or other soluble envelopes, and then lodging the bags or envelopes so filled in the desired position under water, in such a manner that the bag or envelope shall not be ruptured until after or at the time it and its contents are in place."

Molded Blocks. Under some conditions, especially where it is difficult to construct a cofferdam and monolithic work is not required, blocks of concrete of any desired shapes are molded on land and placed after setting

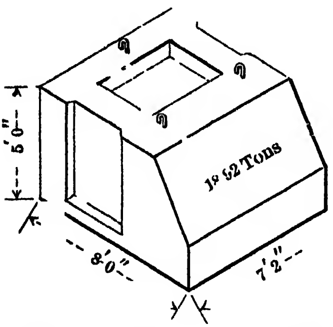


FIG. 110—Face Blocks of Buffalo Breakwater. (See p 307)

On the Buffalo breakwater,|| blocks weighing from 15 to 20 tons, one style of which is illustrated in Fig 110, were employed in parts of the structure For handling them, three iron bolts having legs bent to an angle at the ends and of unequal length, —one 24 inches long and the other 12 inches long, — so that the strain would occur in two separate planes, were sunk into the top face of each block. After placing them in position, grooves molded into their adjacent faces were filled with concrete so as to dowel them together.

In the harbor of the Welland Canal, Ontario,¶ blocks of somewhat smaller size were used just at the water level, with mass concrete placed on top of them. For handling these blocks four vertical channels, two on each side, were molded into each block, with recesses just below the central points to catch the four hooks used for moving it. As the hooks passed

* Proceedings Institution of Civil Engineers, Vol XXXIX, p 126, and LXXXVII, pp 101 and 126

† Lt Col J A Smith, *Engineering Record* March 23, 1895

§ U S. Patent, No 358 853

|| See article by Major T W Symonds, *Engineering News*, May 29, 1902, p. 420.

¶ *Engineering News*, May 15, 1902, p. 382

down in the channels, they projected so slightly that a block could be set close to the last one placed, and the hook removed without disturbing it.

As early as 1873, concrete blocks ranging in size from 13 to 60 tons in weight were used by the Department of Docks in New York City,* and in 1900 this method of construction was still in operation in that city.

In Belgium in 1899, for breakwater construction,† blocks about 25 feet square and 82 feet long, weighing 3 000 tons, were formed by building on the shore metal caissons of the required size, lining them with concrete, then floating to place, and removing plugs in the bottom so as to allow them to sink. The remainder of the concrete to fill the caisson was deposited in the interior.

Depositing Dry Concrete under Water. By dry concrete is meant in this case a mixture of aggregates and cement without water. This method, although occasionally practised, is undoubtedly one of the worst to employ in laying concrete under water. No matter how carefully the concrete is placed, more or less of the cement is carried off by the water. Experiments by Mr. B. B. Stoney‡ show, as one would expect, that a wall laid in this way is honeycombed, and is not nearly so dense as that formed of concrete mixed with water in the usual way before placing.

Plastic Concrete. Plastic or, as it is termed by Mr. Faija, "reset" concrete was once employed in England.§ The concrete was mixed on land with the smallest possible quantity of water, and allowed to set there about three to five hours, or until it attained the consistency of wet clay, before being deposited in the water. Mr. Kinniple claimed that setting eight hours on land before placing did not reduce the ultimate strength of the concrete, and that less of the cement was washed away.

Concrete in Sea Water. In the United States several instances have been noted where concrete has been disintegrated to the depth of 2 or 3 inches and sometimes more. The injury in all cases is limited to the space between high and low water mark, and frequently appears to be caused in part by frost action. Since other concrete close by is often intact, the chief cause for the defects seems to be in the character of the concrete. From the many cases of structures in good condition after many years, notably the docks in New York Harbor, the conclusion is drawn that concrete can be used with confidence in sea-water construction provided it is proportioned and laid with the best materials so as to form a dense impervious concrete. A still further precaution is to keep the concrete from immediate contact with sea-water by leaving the forms in place for several weeks.

*"Fabrication of Beton Blocks by Manual Labor," by Schuyler Hamilton, Transactions American Society of Civil Engineers, Vol. IV, p. 93.

†See paper by L. Vernon Harcourt in Proceedings Institution of Civil Engineers, Vol. CXII, p. 2.

‡Proceedings Institution of Civil Engineers, Vol. LXXXVII, p. 230.

§W. R. Kinniple, Proceedings Institution of Civil Engineers. Vol. LXXXVII, p. 65.

CHAPTER XVI

EFFECT OF SEA WATER UPON CONCRETE AND MORTAR*

By R. Feret,
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The principal conclusions which have been reached by the author of this chapter, as discussed in the following pages, may be summarized as follows:

(1) No cement or other hydraulic product has yet been found which presents absolute security against the decomposing action of sea water. (See p. 309.)

(2) The most injurious compound of sea water is the acid of the dissolved sulphates, sulphuric acid being the principal agent in the decomposition of cement. (See p. 310.)

(3) Portland cement for sea water should be low in aluminum (see p. 312), and as low as possible in lime. (See p. 311.)

(4) Puzzolanic material is a valuable addition to cement for sea water construction. (See p. 313.)

(5) As little gypsum as possible should be added, for regulating the time of setting, to cements which are to be used in sea water. (See p. 310.)

(6) *Sand containing a large proportion of fine grains must never be used in concrete or mortar for sea water construction.* (See p. 316.)

(7) The proportions of the cement and aggregate for sea water construction must be such as will produce a dense and impervious concrete. (See p. 316.)

EXTERNAL PHENOMENA

At present there is no hydraulic product which is known to be capable of resisting absolutely the decomposing influence of sea water. It is true that some concrete masonry has remained intact for a very long time in salt water, but with our present knowledge it is impossible to say why these structures have resisted so well, and there is little doubt that the cements from which they were made might have decomposed rapidly if they had been used under different conditions. In some cases, on the other hand, similar large structures subject to the action of sea water were

*The authors are indebted to Mr. Feret for this chapter, which has been especially prepared by him for this Treatise.

ruined in a few years and were torn down and completely rebuilt. Notable instances of this kind are the failures which occurred in the ports of Aberdeen,* Dunkerque, and Ymuiden.

Such occurrences have aroused great interest in the subject of the action of sea water upon mortars, and but few questions have received more careful study. In spite of this, however, it cannot be said that any sure means of preventing these failures have been found.

The decomposition manifests itself in various ways: sometimes the mortar softens, and little by little becomes disintegrated; sometimes the mortar becomes covered with a crust which finally cracks off; more often fine white veins develop on the surface of the mortar, these gradually grow large and open, the mortar swells, cracks, and falls off in small pieces or collapses in a pulp-like mass. Almost always the interior of the decomposed mortar is found to contain a soft white material which may be easily separated from it. The chemical composition of this substance is not, however, constant.† Generally, the more advanced the state of decomposition, the more readily the white material can be extracted from the mortar and the richer it is in magnesia. The proportion of sulphuric acid in it also increases with the degree of decomposition, though less uniformly.

ACTION OF SULPHATE WATERS

For several years the injurious action of sea water upon hydraulic compounds was attributed chiefly to the magnesia in the water. It is noteworthy, however, that chloride of magnesia is almost without action, while sulphate of magnesia acts very energetically upon cement, and it has now been ascertained that magnesia plays only a secondary part, while in fact it is the sulphuric acid combined as a soluble sulphate which is the real cause of the decomposition.

This has been confirmed in practise by the destruction of masonry washed by water which has traversed earth containing gypsum, or built from mortar made with sand which has been extracted from strata containing sulphate of lime.‡ A consideration of this fact makes it apparent how dangerous it is to use, in concrete or masonry subject to the action of sea water, cements to which the gypsum has been added for the purpose of regulating the rate of their setting or of increasing their initial strength.§

There are numerous instances in which brick masonry has rapidly de-

*Smith, Proceedings Institution Civil Engineers, Vol. CVII, 1891-92.
†Feret, Annales des Ponts et Chaussées, 1892, II, p. 93.
‡Bied, Annales des Ponts et Chaussées, 1902, III, p. 95.
§Feret, Annales des Ponts et Chaussées, 1890, I, p. 375.

composed because the bricks, burned with coal, contained alkaline sulphates which when drawn out by water attacked the mortar of the joints.*

These practical observations combined with certain laboratory experiments intelligently conducted have demonstrated that sulphuric acid is the principal agent in causing decomposition.

CHEMICAL PROCESSES OF DECOMPOSITION

Messrs. Candlot,† Michaelis,‡ and Deval§ have discovered successively by different methods that aluminate of lime $Al_2 O_3$ 3 CaO, which exists in cements in company with other calcareous salts, such as silicates, possesses the property of combining with sulphate of lime so as to give a double salt $Al_2 O_3$ 3 CaO, 3 (SO_3 CaO) combined with a large quantity of water with great increase in volume. This substance, moreover, has no firm coherence. It is soluble in pure water, but insoluble in lime water, a fact that explains its existence in a solid state in mortars.

On the other hand, even if the cements do not contain free lime when they are anhydrous, their setting under the action of water frees a part of the lime which was combined with the acid elements, principally with silica. If a soluble sulphate other than sulphate of lime is placed in contact with a hydraulic binding material during hardening or after having set, it produces, with the freed lime, sulphate of lime, which in turn combines with the aluminate, giving "sulpho-aluminate," and produces the swelling which causes the disintegration of the mortar. The same reactions would be produced, moreover, without the intervention of free lime as a result of the reaction of the sulphuric acid of the salt dissolved by the water upon a part of the lime of the binding material.

Although the formation of the sulpho-aluminate of lime seems to be the principal cause of the decomposition of cement by sea water and sulphate waters, it may not be the only one: the setting and the hardening of the cement in contact with water result in the separation of compounds rich in lime, in salts less calcareous, and in free lime. According to the nature of the medium and the conditions affecting its preservation, this reaction may be modified or counteracted in such manner that the hardening cannot follow its regular course; likewise, the lime set at liberty may be dissolved little by little in the water which penetrates the mortars, and may disappear by exosmose, giving place to other more or less injurious compounds.

*Zamboni, Industria, October 15, 1899.

†Ciments et Chaux Hydrauliques, Paris, 1891, p. 257.

‡Der Cement-Bacillus, Berlin, 1892.

§Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1900, I, p. 49.

These various phenomena are yet far from being satisfactorily explained; nevertheless, it appears that those cements which are richest in lime are the most quickly decomposed.

SEARCH FOR BINDING MATERIALS CAPABLE OF RESISTING THE ACTION OF SEA WATER

For a long time the efforts of experimenters have been directed toward finding a cement of such composition that it cannot be decomposed by sea water. Thinking at first that the destructive action of the water resulted from the substitution of the magnesia which it contained, for the lime of the cement, the idea was conceived of making cement by burning dolomitic limestone which consequently was composed largely of salts of magnesia. But it was found that the magnesia which this contained, since it was burned necessarily at a very high temperature, was slaked with great difficulty, and by its tardy hydration caused the mortar to swell. Cements were also made experimentally of baryta, a laboratory product whose high price does not permit its introduction into regular practice.*

After the discovery of the sulpho-aluminate of lime, the question changed its aspect, and alumina was considered a dangerous element in cement, the proportion of which ought to be reduced as much as possible. At present the specifications adopted by the Administration of Public Works in France limit to 8% the maximum amount of alumina allowed in cement intended for use in sea water, and this limit would be placed much lower were it not for the fact that in many localities it would be very difficult to obtain products containing less alumina. On the other hand, the percentage of alumina cannot be greatly reduced without at the same time rendering more difficult the burning of the cement, in which operation this element acts as a flux. Accordingly, it was suggested that the alumina be replaced by iron oxide. Cements have been made in the laboratory which were absolutely free from alumina and rich in iron, and these resisted sea water very well.† The various hydraulic cements and limes produced by the works of Teil, whose reputation is world-wide, contain not more than 2% of alumina, and some of them usually last much better in sea water than most of the Portland cements which contain between 7% and 8% of alumina. These too, however, become decomposed under certain conditions, but with this peculiarity — that their disintegration is not usually accompanied by any increase of volume.

*Le Chatelier, Annales des Mines, May and June, 1887.

†Le Chatelier, Congrès International des Matériaux de Construction, held at Paris in 1900, Vol II, Part 2, p. [illegible]

It has been noted that the cements which are the richest in lime decompose the most quickly in sea water. Based upon this observation, the experiment was also tried of making cements for marine use by burning mixtures less rich in carbonate of lime than the ordinary Portland cements. This diminished the strength of the cement, but the falling off in strength was only of secondary importance. The principal difficulty lay in the process of manufacture. In burning cements of this class there was produced in the kilns a considerable quantity of powder possessing only a comparatively feeble hydraulic power, which obstructed the draught. This difficulty was lessened by mixing ferruginous materials (ore, etc.), or even sulphate of lime,* with the raw materials before burning. Also, the use of rotary furnaces prevents the choking of the draught. As has just been said, cements low in lime do not attain as great strength as the ordinary Portland cements, but they generally resist the decomposing action of sea water better.

When the proportion of limestone is small, the burning can be done only at a very low temperature, and the cement obtained sets very quickly. Some of these low lime cements appear to resist chemical decomposition satisfactorily, while others resist no better than most of the Portland cements, a difference which has not yet been explained. In any case, on account of the rapidity of set, this class of cements cannot readily be used on large work, and, in fact, their use is mainly limited to special cases.

Another means of neutralizing the bad effects of the excess of lime liberated by the setting of Portland cement consists in mixing with the latter, before using, materials capable of combining with this lime so as to produce insoluble compounds. Puzzolans have been found to be the most useful material for this purpose. Laboratory tests, verified by experiments on a larger scale,† have shown that mortars made in this way generally resist sea water better than if they had been made from similar cements without puzzolanic material. Sometimes, too, their strength is increased by this mixture. It is conceivable, however, that the substances which in the Puzzolans appear as acids are less energetic in their action upon the lime of the cement than the sulphuric acids of sea water or of water containing gypsum, and that therefore in the end they will be displaced by the latter with the consequent decomposition of the mortar. This method cannot then be looked upon as giving absolute security against deterioration although it has been proved to be useful.

*Candlot, paper delivered at the meeting of the French and Belgian members of the International Association of the Materials of Construction, on April 25, 1903.

†Feret, Annales des Ponts et Chaussées, 1901, IV, p. 191.

METHOD OF DETERMINING THE ABILITY OF A BINDING MATERIAL TO RESIST THE CHEMICAL ACTION OF SULPHATE WATERS

One method is to gage the cement to be tested with sufficient water to obtain a plastic paste, spread this paste on glass plates so as to form cakes or pats with thin edges, immerse the pats in sea water, and observe them from time to time. But with this method the amount of deformation in the pats depends to a large extent upon the hardness of the paste at the time of immersion, so that a cement which cracks when immersed before setting may stand a long time without showing any trace or alteration if the pat is not placed in contact with the water until twenty-four hours after gaging. Further, the surface of the pat is quickly covered by a crust more or less thick resulting from the partial carbonization of the freed lime, so that the substitution of magnesia for a part of this lime and the presence of this crust may influence the decomposition of the underlying cement.

Another and more exact method consists in molding a block of cement or of mortar of a sufficient thickness; for example, a briquette such as is used for a tensile test. Allow this to harden in the usual way, say for twenty-eight days, then cut out from the center of this block a small solid disc with sharp edges, and immerse it in sea water or in a sulphate solution (saturated gypsum, sulphate of magnesia, etc.). In order to prevent all new superficial carbonization of the specimen, carbonic acid should not be allowed to come in contact with or be present in this liquid. When decomposition occurs in the cement it is indicated by cracks which appear at the edge of the disc after a lapse of a variable time.

As a third test, sea water under pressure can be made to filter continuously through mortars made with fine sand. The author of the present chapter uses for this test mortars containing from 250 to 450 kilograms (551 to 991 lb.) of cement per cubic meter (35.3 cu. ft.) of sand (corresponding approximately to proportions 1:6 to 1:3 by weight) which he gages to a plastic consistency and molds into cubes 50 square centimeters (7.74 sq. in.) on a face, with a tube of brass penetrating to the center of the block. After a few days the brass tubes are attached with India rubber tubes to a vessel containing sea water under a head of 2 meters (6.52 ft.). The amount of water which flows through each cube in a given time is accurately measured from time to time, the cube being immersed in sea water in a glass receptacle, where the state of preservation of the mortar can be closely observed.

Finally, the following quite rapid method is used in the laboratory at Boulogne. A mixture is made consisting of 100 parts of cement to be

tested and 300 parts marble ground to a fine powder. To this is added gypsum in the form of a very fine powder, varying progressively from 0% to 20% of the weight of the cement. Plastic mortars are then made from each of these mixtures, which are molded into prisms 2 by 2 by 12.5 centimeters (0.8 by 0.8 by 4.9 in.), allowed to harden for seven days in moist air, and then immersed in fresh water after the length of each has been exactly measured. The water is frequently renewed and at stated periods the lengths of the prisms are again measured, at which time their state of preservation is also examined.

The ability of the cement to resist decomposition by sulphates is indicated by the time taken for the prisms to expand abnormally and to develop cracks, and also by the quantity of gypsum which the binding material is able to bear for a given time without deterioration.

As a result of a long series of experiments, especially of those made by the last two methods, the conclusion has been reached that no binding material has as yet been found which will not be decomposed sooner or later when subjected to these tests, so that at present no cement can be looked upon as absolutely safe from the action of sea water.

MECHANICAL PROCESSES OF DISINTEGRATION

It seems possible to divide the phenomena of disintegration into two classes according as the destruction of the mortar is produced by a sort of progressive dissolution of its elements without appreciable change in volume, or as the products of decomposition, collecting in the pores, enlarge them and produce a scaling off and a weakening of the mortar. This second class of phenomena is much the more frequent and serious.

In both cases decomposition may be produced when the mortar is simply immersed, because of the penetration of the water into its pores and its renewal by the double phenomenon of endosmose and exosmose. But when the masonry is subjected to different degrees of pressure upon its opposite faces, as is usually the case, this tends to establish a current of water through it and the replacement of the dissolving elements goes on more actively. However, disintegration may, under these conditions, proceed more slowly if the current of water is strong enough to carry away the solid products of decomposition as they are formed. The writer has cited in a former paper* experiments which plainly show the difference between these two methods of decomposition: if lean mortars, made with the same cement and sands of different granulometric compositions, are kept in absolutely quiet sea water, those which disintegrate most rapidly are the ones

*Annales des Ponts et Chaussées, 1892, II, pp. 106 to 116.

into whose composition there enters no fine sand, but only medium sand or, and above all, coarse sand. These latter are the mortars that contain the voids of largest size. On the contrary, if a series of similar mortars are subjected to a continuous filtration of sea water, those made from coarse sand remain intact, while decomposition is more and more active for mortars containing more and more fine sand. *In practise this latter is the most frequent case, and, in fact, it has been verified that the destruction of concrete or mortar by sea water has in most cases been due to the use of too fine sands.*

This is a point which cannot be too strongly insisted upon, and experiments show that a rather lean mortar of coarse sand is much preferable to a mortar of fine sand, even when a very large quantity of cement is introduced into the latter. Fine sands ought to be banished relentlessly from sea water construction even when the cost of coarse sand is very high.* When stone is at hand, an excellent sand can be obtained economically by crushing it.

PROPORTIONS FOR MORTARS AND CONCRETES

From the preceding it is evident that the best means of fighting against sea water is to prevent as far as possible its penetration into the mortars and concretes, and accordingly to make those of great density. The authors of this volume have suggested in a preceding chapter (Chapter IX) with what size of sand and what quantity of cement this result can best be attained in mortars: the maximum density is obtained with a mortar containing sand composed of material having about two parts of very coarse grains to one of fine grains, including cement. Usually, natural sands, even the coarsest, contain a proportion of relatively fine sand sufficient to make it useless to add more with the cement. If a sand is used from which the fine grains have been screened, and this is mixed with about one-half of its weight of cement, a mortar is obtained at once very dense and of great strength, but whose use would often be too costly. In such cases the cement can be replaced by a mixture of sand and cement prepared in advance, such as the product known as "sand-cement," for the making of which a few factories have been built in Europe and also in America. It must be borne in mind, however, that this solution, excellent for mortars destined to remain in the air or to come in contact only with fresh water, would be poor to use in sea water, for very fine sand intimately mixed with cement separates its grains and increases the surface of attack, and various experiments have shown that this kind of mortar suffers severely in sea water.

*See also, Feret, Baumaterialienkunde, 1896, p. 139, and "Le Ciment," 1896, p. 212.

For use in sea water, on the contrary, if a good puzzolanic material can be procured on favorable terms, it is advantageous to grind this with the cement to take the place of the fine sand, so that in the mortar it may play both a mechanical and a chemical role, assuring to it a great density, and at the same time forming, with the lime freed by the setting, compounds which tend to harden the mortar and render it impermeable.

For concretes the law of greatest density is not the same as for mortars, and it has not yet been possible to express a general law. It is necessary to see that the concrete does not contain voids, and above all that the cement is not diluted by an excess of fine sand, which must always be considered as the greatest enemy of masonry in sea water.

In every case the sea water should be prevented from coming in contact with the work for as long a time as possible, so that the setting of the cement may be already considerably advanced. Yet it must not be forgotten that when the mortar contains a puzzolanic material its hardening can be properly effected only in the presence of moisture.

MIXTURES OF PUZZOLAN AND SLAG WITH CEMENTS

Tests by M. Vetillart and the writer, described in detail in a paper published in Annales des Ponts et Chaussées, 1908, I, page 121, indicate that Puzzolanic material may be of great value when mixed with Portland cement for concrete construction in sea-water, materially increasing the durability of the concrete without increasing its cost.

The conclusions reached in these tests are as follows:

The use of Puzzolan in hydraulic mortars in combination with the cement increases the strength, and in a great many cases appreciably retards disintegration by sea-water. It should be employed then, at least experimentally, in accordance with the following recommendations:

Grind the Puzzolan to the fineness of Portland cement.

Mix it mechanically with the cement so as to obtain an absolutely thorough mixture.

For Portland cement and a good natural Puzzolan, take two parts by weight of cement to one part of Puzzolan.

Select only Puzzolan of known good quality; the use of gaize slightly roasted is especially recommended.

If other kinds of cement or limes are used with Puzzolan, or if the Puzzolan is of doubtful quality,—especially if it is obtained from granulated slag or a similar industrial by-product,—determine the proportions of the mixture by means of preliminary trials based on tests of strength.

Add to the sand the mixture of cement and Puzzolan as pure cement would be added, and in the same proportions; mix and place the mortar in the usual manner.

Always use for comparison with the Puzzolan mortar, specimens of mortar, of the same proportions and made under identical conditions, in

which the mixture of cement and Puzzolan is replaced by the same weight of pure cement.

Allow the Puzzolan mortar to harden in the presence of moisture.

It is as yet impossible to suggest detail rules for the acceptance and control of Puzzolan cements. The recommendation is made, however, that their ability to resist the decomposing action of the salts in sea-water be compared to the resistance of pure cements by means of the test with sulphate magnesia already referred to.*

VARIOUS PLASTERS AND COATINGS

Various methods have been tried to prevent sea water from wetting masonry too soon, either by coating the work with materials designed to obstruct the pores, or by covering it with a layer more or less thick and more or less impermeable, consisting usually of a rich mortar, clay, bituminous materials, etc.

This method of protecting the work is generally rather costly and is not applicable to all kinds of construction. Besides, it presents this disadvantage, that if by accident there is any break in the continuity of the covering, the sea water finds a passage towards the heart of the masonry and creeps in from one place to another, so that often the coating offers only an illusory security.

In certain cases, a coating is formed spontaneously by the carbonization of the lime in the parts of the mortar near the free surface, and this action is aided by the development of sea organisms such as sea-weed and shellfish. This cause, together with the differences in the saltness and the temperature of the water, and the course of the ocean currents, is the one which is most often called upon to explain why mortars decompose more quickly in some regions than in others.

* See also Annales des Ponts et Chaussées, 1908, I, p 107

CHAPTER XVII

LAYING CONCRETE AND MORTAR IN FREEZING WEATHER

The results of practise and experiment with cements exposed to frost, which are discussed more in detail in the following pages, may be summarized as follows:

(1) Most Natural cements are completely ruined by freezing. (See p. 320.)

(2) The setting and hardening of Portland cement in concrete or mortar is retarded, and the strength at short periods is lowered, by freezing, but the ultimate strength appears to be but slightly, if at all, affected. (See p. 321.)

(3) A thin scale is apt to crack from the surface of concrete walks or walls which have been frozen before the cement in them has hardened. (See p. 320.)

(4) Frost expands Natural cement masonry and settlement results with the thawing. (See p. 320.)

(5) Heating the materials hastens setting and retards the action of frost. (See p. 323.)

(6) Salt lowers the freezing point of water, and in quantities up to 10% of the weight of the water does not appear to affect the ultimate strength of the concrete or mortar. (See p. 324.)

(7) In practise concrete work should be avoided if possible in freezing weather, because of the difficulty and expense of attaining perfect results. (See p. 320.)

EFFECT OF FREEZING

Numerous experimental tests have been made, chiefly in the United States, where the effect of frost is a more serious question than in England, France, or Germany, to determine the effect of freezing temperatures upon hydraulic cements. Although the conclusions of different experimenters are not in perfect accord, it is the generally accepted belief, corroborated by tests under the most practical conditions and by the appearance of concrete and mortar in masonry construction, that the ultimate effect of freezing upon Portland cement concrete and mortar is to produce only surface injury.

In their practise and research the authors have never discovered a case,

either in laboratory work or in practical construction, where Portland cement concrete or mortar laid with proper care has suffered more than surface disintegration from the action of frost. They do not wish to imply, however, that it is always expedient to lay Portland cement masonry in freezing weather, for the expense of laying is increased, and it is much more difficult to satisfactorily mix and place the materials. Mortar for brick and stone masonry freezes in the tubs and in the joints, while in laying concrete the surface freezes unless measures are taken to prevent it, and any dirt or "laitance" which rises to the surface of wet mixtures is hard to remove. It is a well-known fact that a thin crust about $\frac{1}{16}$ inch thick is apt to scale off from granolithic or concrete pavements which have frozen, leaving a rough instead of a troweled wearing surface, and the effect upon concrete walls is often similar. It may be stated as a general rule that concrete work should, if possible, be avoided in freezing weather, although if circumstances warrant the added expense, with proper precaution and careful inspection mass concrete may be laid with Portland cement at almost any temperature.

Most Natural cements, on the contrary, are seriously injured by frost especially by alternate freezing and thawing, and while occasional cases are on record, especially in heavy stone masonry in which the weighted joints have thawed slowly, where Natural cement mortar has been laid in freezing weather without serious results, numerous examples might be cited where even after several years the concrete or mortar was but slightly better than sand and gravel. Mr. Thompson has observed this result in Natural cement mortar laid during the comparatively warm winter of North Carolina on days when the temperature was considerably above freezing at the time of laying, and also in the cold climate of Maine where the mortar froze as it left the trowel and did not thaw until spring.

The settlement of the masonry when thawing is often a serious characteristic of Natural cements. Stone masonry walls laid in freezing weather in Natural cement mortar may settle as much as $\frac{1}{2}$ inch in the height of a window jamb.

Experiments upon Natural cement mortars have not positively confirmed the judgment reached by nearly all engineers experienced in construction in freezing weather. Occasional tests are recorded in which such mortars, especially when subjected to a uniformly cold temperature and then suddenly thawed, have attained full strength, but these are insufficient to warrant the use of any except Portland cements when frost is likely to occur before the mortar is thoroughly dry.

The prevention of injury from frost in certain cements may be due, at

least in part, to the internal heat produced when setting. In the interior of a large mass, some cements, especially high grade Portlands, attain a high temperature. (See p. 130.)

Freezing Experiments. An extensive series of experiments upon frozen mortars has been conducted by Mr. Thomas F. Richardson, at the Wachusett Dam in Massachusetts. The results of tests extending up to one year showed that although briquettes mixed 1 part cement to 3 parts sand had less strength at the end of seven days than those which had not been frozen, the frozen specimens after longer periods, especially at the end of one year, gave as high and often higher strength than those which were kept at ordinary temperatures. The conclusion was reached, therefore, that Portland cement mortar is not permanently injured by freezing.

Mr. Richardson's experiments were conducted in the middle of the winter of 1902. He gives the following description* of the tests:

Two bags of Portland cement were thoroughly mixed together and all the briquettes were made from cement from these bags. Masonry work on the Wachusett Dam was in progress during the period, and briquettes were made each week and submitted to the same conditions as the masonry, the molds being filled with mortar and placed out doors in the air, not in water, immediately after filling.

Briquettes were made at the same time as the ones exposed to the weather, and kept in the laboratory, either in the air or in water, those in the air approximating more closely the conditions which obtained on the masonry construction at the dam. About ⅓ of the briquettes out doors were exposed to temperatures as low as 9° above zero in the first 24 hours, and some of them to temperatures as low as 12° below zero in the first week. Salt was used in most of the experiments, the quantity ranging from 4 to 16 pounds per barrel of cement, the average being about 6 pounds or about 3% by weight of water. Our experiments indicate that 8 pounds of salt per barrel of cement is sufficient, even in the coldest weather, and the results from 4 pounds are very nearly as good; 16 pounds do not seem to give quite as good results.

The following table gives the average results of the experiments:

Effect of Frost upon Tensile Strength of 1:3 Mortar. (*See p.* 321.)
BY THOMAS F. RICHARDSON.

Briquettes Kept	No. of Briquettes	Tensile Strength, lb. per sq. in.				
		7 d.	28 d.	3 mo.	6 mo.	1 yr.
Water in laboratory	20	268	304	359	370	401
Air in laboratory	20	298	352	364	392	517
Out doors, below freezing	80	139	238	344	435	627

*Kindly furnished by Mr. Richardson for this Treatise.

The briquettes were made in sets of 5, consequently 4 experiments are shown for water and air in laboratory, and 16 for out doors.

In France similar results have been reached by Mr. P. Alexandre* as to the effect of temperatures slightly above freezing.

Mr. Charles S. Gowen† also has concluded from his tests that "there is no indication that freezing reduces the ultimate strength of the mortar, although it delays the action of setting."

The effect of different uniform temperatures upon neat cements and mortars is illustrated in Fig. 111, which is selected and adapted by the authors from a series of experiments by Mr. J. E. Howard‡ at the Watertown Arsenal. The results with both neat cements and mortars show but

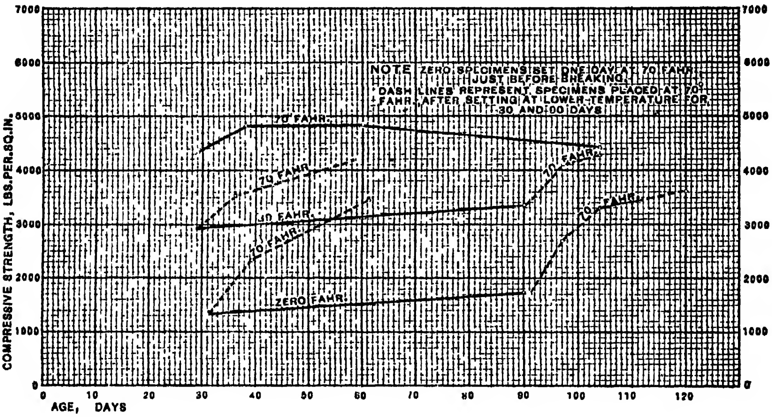


FIG. 111.—Strength of Neat Portland Cement Mortar, 2-inch Cubes, Set in Air at Different Temperatures. (*See p.* 322.)

slight increase in strength while the specimens are maintained at 0° Fahr. (—18° Cent.), but a decided increase in strength as soon as they are subjected to a higher temperature. The zero cubes were removed from the freezer and allowed to set one day at 70° Fahr. (21° Cent.) before breaking.

Cold retards setting. Prof. Tetmajer§ found, for example, that 1:3 Portland cement mortar which attains its initial set at 2¾ hours and its final set at 8½ hours when mixed at 65° Fahr. (18° Cent.), at a temperature of freezing reaches its initial and final set at 21 and 38 hours respectively.

*Annales des Ponts et Chaussées, 1890, II, pp. 302 and 422.

†Proceedings American Society for Testing Materials, 1903, p. 393.

‡Tests of Metals, U. S. A., 1901, p. 530.

§Johnson's Materials of Construction, 1903, p. 616.

METHODS OF CONSTRUCTION IN FREEZING WEATHER

Certain classes of concrete construction, such as foundations or heavy walls, whose face appearance is of no consequence and which will have opportunity to thaw and then thoroughly harden before loading, may be laid in freezing weather with first-class Portland cement, but it is absolutely necessary to thoroughly remove all dirt and frozen "laitance" (see p. 393) before placing fresh concrete. This is a much more difficult matter than would appear, because frozen dirt has the same appearance as set concrete.

In the case of structures which must not be permitted to freeze, work may often be conducted by maintaining the atmosphere artificially above the freezing point. In temperatures only a few degrees below freezing, it is a common practise to heat the materials, the heat tending both to accelerate the setting of the cement and to lengthen the time before the mixture becomes cold enough to freeze. The addition of salt lowers the freezing point of the water, and therefore of the concrete or mortar.

Protection from Frost. The method of maintaining masonry above the freezing point depends upon the character of the structure.

In building construction, the reinforced concrete must be kept from freezing and maintained at a fairly high temperature to permit proper hardening. A common plan is to cover a floor as soon as laid with clean straw, free from manure, to a depth of about 12 inches, and then protect the columns and girders underneath by temporary canvas walls surrounding the entire building, heating the enclosed space with stoves.†

A dam was constructed at Chaudiere Falls, P. Q.* when the temperature was 20° below zero. A house 100 feet long by 24 feet wide was built over a portion of the dam in sections about 10 feet square, bolted together, and heated by sheet-iron stoves about 18 inches in diameter by 24 inches high, burning coke. The concrete was mixed and laid in this house, which, when one portion of the dam was completed, was taken down and erected in another place.

Heating the Materials. Where hand-mixing is employed, an arrangement used on the Newton, Mass., sewers is useful. Sand for one or more batches is placed in a bottomless box containing a coil of steam pipe, the exhaust end of which is then extended to the mixing platform and arranged to discharge through the bottom of the platform into the bottomless box employed for measuring the stone, so that the latter is heated by the exhaust steam. The cement is warmed by piling the bags on top of the sand box.

† Transactions American Society Civil Engineers, Vol. LX, 1908, p. 453.
* *Engineering News*, May 7, 1903, p. 402.

An ordinary sand heater, such as is used for asphalt materials, may also be employed, and the stone heated by steam from a hose. A modification of the sand heater,* arranged to form the combined water, sand, and stone heater illustrated in Fig. 112, has been used on the New York Central Railroad.

Experiments by Mr. Thomas F. Richardson† tend to show that heating the materials of mortar has but little, if any, permanent effect upon its strength.

Addition of Salt. Because of its cheapness salt is most commonly employed to lower the freezing point of water. Other materials, such as

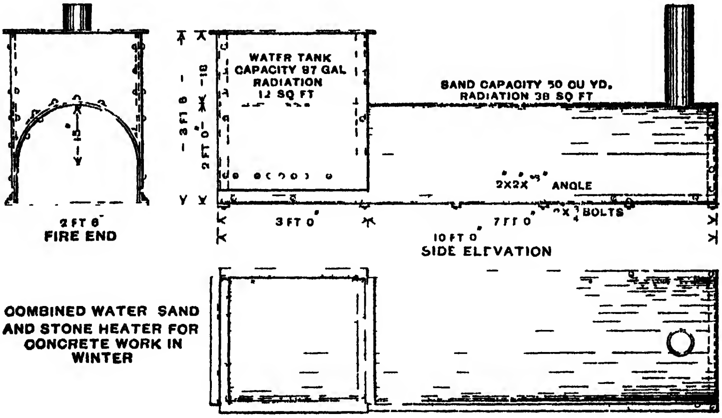


FIG. 112.—Combined Water, Sand, and Stone Heater for Concrete Work in Winter (*See p* 324)

glycerine, alcohol, and sugar, have been experimentally employed, but these appear to have a tendency to lower the strength of the mortar

Salt has been more extensively employed in mortars than in concretes. Rules have been formulated for varying the percentage of salt with the temperature of the atmosphere Prof Tetmajer's‡ rule, for example, reduced to Fahrenheit units, requires 1% by weight of salt to the weight of the water for each degree Fahrenheit below freezing

A rule frequently cited in print, which practical tests by the authors have proved to be entirely inadequate, is to require one pound of salt to 18 gallons of water for a temperature of 32° Fahr. and an increase of one

*George W. Lee in *Engineering News*, March 19, 1903, p 246.
†Report Metropolitan Water and Sewerage Board, 1904, p. 110.
‡Johnson's Materials of Construction, 1903, p 615

ounce for each degree of lower temperature. For 16° Fahr. this corresponds to but slightly more than 1% of the weight of the water, an amount too small to be effective. Since the temperature of the air usually cannot be determined in advance, an arbitrary quantity is as suitable as a variable one. In the New York Subway work in 1903, 9% of salt to the weight of the water was adopted. On the Wachusett Dam, during the winter of 1902, 4 pounds of salt were used to each barrel of cement. For 1:3 mortar this corresponded to about 2% of the weight of the water.

Experiments show that ordinary "quaking" concrete in proportions 1:2½:5 requires about 130 pounds of water per barrel of Portland cement, hence 10% of salt in average concrete is equivalent to 13 pounds per barrel of Portland cement. Ordinary 1:2½ mortar requires about 120 pounds of water per barrel of Portland cement, hence 10% of salt in average mortar is equivalent to about 12 pounds salt per barrel of Portland cement. Salt is sometimes added in sufficient quantity to "float a potato" or an egg. According to tests of the authors, about 15% of salt to the weight of the water is required to float a potato, and about 11% to float an egg.

Recent experiments, by Mr. Gowen* and Mr. Richardson,† extending up to a period of one year, tend to show that salt in a quantity corresponding to at least 10% of the weight of the water does not lower the ultimate strength of ordinary mortar. The time of setting, however, is considerably increased and the strength at short periods is lowered. The effect, at laboratory temperature, of 10% salt with 1:3 Portland cement mortar is illustrated in the following table:

Tensile Strength of 1:3 Mortars made with Fresh and Salted Water.

BY CHARLES S. GOWEN.

	1 week.	mo.	3 mos.	6 mos.	9 mos.	12 mos
Fresh water used.........	112	183	268	335	351	458
Salted water used.........	68	131	215	266	301	413

In Mr. Richardson's experiments‡ smaller percentages of salt proved beneficial. Portland cement mortar in proportions 1:3, mixed with 4 and 8 pounds of salt per barrel cement (corresponding respectively to about 2% and 4% of the weight of the water), gave slightly higher tensile strength than the unsalted mortar at all periods from 7 days to one year.

Experiments by Mr. E. S. Wheeler§ indicate that the use of 10% of salt tends to prevent the swelling of briquettes in the molds, even if the specimens freeze.

*Proceedings American Society for Testing Materials, 1903, p. 393.

†Report Metropolitan Water and Sewerage Board, 1903, p. 112.

‡See page 321.

§Report Chief of Engineers, U. S. A., 1895, pp. 2963 to 2971.

Practical Proportion of Salt. Since in practice it is impossible to tell how low the temperature will fall before the concrete sets, Mr. Thompson has adopted the arbitrary rule of 2 pounds of salt to each bag of cement to be used when the temperature is expected to fall several degrees below freezing, and if experience shows that this is not quite sufficient to prevent the frost catching the surfaces, 3 pounds of salt per bag of cement are to be used instead.

The salt can be added most conveniently by putting it into the mixing water. To determine the amount of salt per barrel or per tankful of water, the quantity of water used per bag of cement must be noted and from this the amount can be readily figured.

Calcium Chloride. Experiments indicate that calcium chloride added in quantities not exceeding 2% of the weight of the cement is an effective agent for lowering the freezing point of the concrete. It should be used with caution, however, since a larger quantity than this is likely to so hasten the set as to make the concrete difficult to handle.

CHAPTER XVIII

FIRE AND RUST PROTECTION

Observations of steel imbedded in concrete which has been exposed to fire or to corrosive action, and experimental tests prove conclusively that 1½ to 2 inches of dense Portland cement concrete, made in ordinary proportions, with broken stone, gravel, or cinders, of good quality, and mixed wet, will effectually resist the most severe fire liable to occur in buildings, and will prevent the corrosion of steel even under extraordinary conditions. In members of inferior importance or which are only liable to fire of comparatively low temperature, a less thickness of concrete, in many cases ¾-inch or even ½-inch, will prove effective. (See p. 333.)

In buildings concrete has been found a more effective fire-resisting material than terra-cotta (see p. 333) and fully equal to first-class brickwork. Brickwork cannot exist in a structure except in combination with some other material like steel or wood, which is seriously affected by fire, whereas concrete reinforced with steel may replace not only the brickwork, but also the steel or wood columns and beams.

PROTECTION OF STEEL BY CONCRETE

Tests by Prof. Charles L. Norton

Extended practical tests have been conducted by Prof. Charles L. Norton for the Insurance Engineering Station in Boston. As a result of experiments made in 1902 upon several hundred specimens, he concludes:*

(1) Neat Portland cement, even in thin layers, is an effective preventive of rusting.

(2) Concretes, to be effective in preventing rust, must be dense and without voids or cracks. They should be mixed quite wet where applied to the metal.

(3) The corrosion found in cinder concrete is mainly due to the iron oxide, or rust, in the cinders, and not to the sulphur.

(4) Cinder concrete, if free from voids and well rammed when wet, is about as effective as stone concrete in protecting steel.

In his first series of experiments, round rods of mild steel, soft shee steel, and expanded metal were each imbedded in the center of blocks o

**Engineering News*, October, 1902, p. 334.

concrete, 3 by 3 by 8 inches. Neat cement, 1:3 mortar, and concrete in proportions 1 cement : 5 broken stone; 1 cement : 7 cinders; 1 cement : 2 sand : 5 broken stone; and 1 cement : 2 sand : 5 cinders, were employed for imbedding the steel. The stone was chiefly of trap rock. These specimens, after setting, were subjected continuously to the action of steam, air, and carbon dioxide. Unprotected pieces of steel were also exposed to the same test.

At the end of three weeks the unprotected pieces of steel "were found to consist of rather more rust than steel." The protection of the steel incased in neat cement was perfect. The remaining specimens, in mortar and concrete, were seriously corroded in spots, but it was observed that the "rust spot was invariably coincident with either a void in the concrete or a badly rusted cinder. In the more porous mixtures, the steel was spotted with alternate bright and badly rusted areas, each clearly defined." One point is exceedingly instructive:

> In both the solid and the porous cinder concretes, many rust spots were found, *except where the concrete had been mixed very wet, in which case the watery cement had coated nearly the whole of the steel, like a paint, and protected it.*

Protection of Rusty Steel. In 1903, Prof. Norton made tests to determine the protection afforded ordinary rusty or dirty steel. He found that while unprotected steel "vanished into a streak of rust," if protected by an inch or more of sound concrete, not only the sound steel but ordinary structural steel of any degree of cleanliness likely to be in use in a building is unaffected by such extreme treatment as was accorded it in the tests. The conditions of these later experiments were similar to those of the previous year. Each piece of steel was stamped, and this removed loose scale. Dirt was removed by a soft wire brush. The steel was imbedded to a depth of 1½ inches in all directions in broken stone concrete of proportions 1:2½:5 and in cinder concrete of proportions 1:3:6. The treatment of the specimens was similar to that of the previous ones.

A portion of Prof. Norton's conclusions* are given in the following paragraphs:

> **Condition of Specimens.** After varying lapses of time from one to three months for the specimens in the "corroders," and from one to nine months for the others, the specimens were broken out of the briquettes cleaned by brushing, and weighed and calipered. Not one specimen had

*Engineering News. January, 1904, p. 30.

shown any sensible change in weight or dimension, except where the concrete had been poorly applied. Some specimens were purposely bedded in very dry concrete, and some in concrete partly set, and many of these were not well covered and the steel was seriously attacked where there were voids or cracks. Of the hundreds of specimens of rusty steel examined, not one which had a continuous unbroken coating of concrete gained or lost anything in volume or weight by treatment which caused the practical destruction of some of the unprotected specimens. If loss by corrosion as great as 1-1000 of the loss occurring with the unprotected specimens had been experienced in the case of the protected pieces it would have readily been noted.

Conclusions. It would therefore seem that if we admit that from a severe trial of a short duration, we may judge relatively of the effects of the less severe but longer test of time, it can not be questioned that structural steel is safe from corrosion if incased in a sound sheet of good concrete, at least for a period of years so long as to make the subject of more interest to our great-grandchildren's children than to us. We know that bare steel does not rust and fall down over night, and that much of the steel standing has been bare of everything that could protect it, for long years, and it seems to me beyond question that steel properly covered in concrete may well be expected to last far longer than the changes in our cities will allow any building to remain.

Protection by Cinder Concrete. There is one limitation to the whole question, that is the possibility of getting the steel properly incased in concrete. Many engineers will have nothing to do with concrete because of the difficulty in getting "sound" work. This is especially true of cinder concrete, where the porous nature of the cinders has led to much dry concrete and many voids, and much corrosion. I feel that nothing in this whole subject has been more misunderstood than the action of cinder concrete. We usually hear that it contains much sulphur and this causes corrosion. Sulphur might, if present, were it not for the presence of the strongly alkaline cement; but with that present the corrosion of steel by the sulphur of cinders in a sound Portland concrete is the veriest myth, and as a matter of fact the ordinary cinders, classed as steam cinders, contain only a very small amount of sulphur. There can be no question that cinder concrete has rusted great quantities of steel, but not because of its sulphur, but because it was mixed too dry, through the action of the cinders in absorbing moisture, and that it contained, therefore, voids; and secondly, because in addition the cinders often contain oxide of iron which, when not coated over with the cement by thorough wet mixing, causes the rusting of any steel which it touches.

Mix Wet. There is one cure and only one, *mix wet* and mix well.* With this precaution I would trust cinder concrete quite as quickly as stone concrete in the matter of corrosion.

Rust no Protection for Steel. It has been suggested that steel which has been rusted to a slight depth becomes protected by this coating from further rusting. Nothing could be further from the truth. A large num-

*See page 280 for the authors' definition of a very wet mixture.

ber of specimens were rusted by repeated alternate wetting and drying to see if they finally reached a constant condition. Instead of doing this, they all showed an irregular but persistent loss in weight, on further rusting, until some had practically been washed away.

Small Rods. The increasing use of steel of small dimensions in floors and roofs, twisted rods, expanded metal, etc., has caused some question as to the advisability of their use in view of the possible great effects of corrosion, as compared with the effects of corrosion on larger members, but with sound concrete of a thickness of about 1½ in. between the steel and the weather I do not question the durability of these lighter members.

CHEMICAL UNION OF STEEL AND CEMENT

Experiments of Mr. Breuillé* indicate that clean steel may form with cement a chemical combination which is soluble in water. This presents an additional reason for making concrete in which steel is imbedded as impervious as possible, to avoid the penetration of moisture which will wash away this chemical compound, if such is found to exist in actual structures. Large I-beams imbedded in concrete would be especially subject to deterioration from this cause, but as rust rarely forms between two plates of steel which are riveted together in a bridge, even although the rest of the structure is badly corroded, the danger is probably insignificant.

Cement Paint for Protecting Steel. The property of neat cement which prevents steel from corrosion is taken advantage of in different forms of cement coating. Mr. Maximillian Toch in 1903† made a series of experiments upon metal covered with various preparations of cement, and drew the following conclusions:

(1) A proper cement paint can be applied to a surface that has begun to oxidize, and further oxidation will be arrested.

(2) If the cement be absolutely fine and free from iron, calcium sulphate and sulphites, and of low specific gravity, it will set on the surface within a very short time, and eventually become an integral part of the metal.

For exposed iron work Mr. Toch recommends a protective coat of cement paint followed by a coat of linseed oil paint. To protect from the fumes of a factory, he states that after applying three coats of cement paint, an alkali-proof, adherent paint may be spread, and an absolute protection afforded to the iron.

Mr. J. W. Schaub‡ refers to the use of cement mortar in Europe and in

*J. W. Schaub in Transactions American Society of Civil Engineers, Vol. LI, p. 124.

†Lecture on the Permanent Protection of Iron and Steel, delivered before the New York Section of the American Chemical Society, March 6, 1903.

‡*Engineering News*, June 16, 1904, p. 561.

the United States for coating iron exposed to destructive agencies. He says:

> The mortar is usually a mixture of 1 cement and 2 sand, applied with a brush as a wash. Five or six coats are applied in this way to give the metal a proper coating. This is especially applicable in the case of the iron work exposed in roundhouses, where the gases from locomotives are so destructive, and where paint is so inefficient.

FIRE PROTECTION

Numerous experimental tests* have been made showing the value of concrete as a fire-resisting material, but the best proof of its ability to resist the heat of a severe fire — such as is liable to occur in an office or factory building — lies in the fact that concrete has actually withstood very severe fires more successfully than have terra-cotta and various other so-called fireproof materials.

The reinforced concrete factory of the Pacific Coast Borax Co. at Bayonne, N. J., passed through a severe fire in 1902. Still more recently, in 1904, occurred the conflagration at Baltimore in which many building materials utterly failed.

Such practical tests, further confirmed by numerous experiments with test buildings of reinforced concrete, have proved that while in a severe fire, where the temperature ranges from 1600° to 2000° Fahr., the surface of the concrete may be injured to a depth of from $\frac{1}{2}$ to $\frac{3}{4}$ inch, the body of the concrete is unaffected, so that the only repairs required consist of a coating of plaster, and even this only in rare instances.

Tests upon small briquettes of cement placed in a furnace indicate that the strength of cement is destroyed by a heat reaching a dull, red color,† but as stated below, in an actual fire, the injured material protects the rest of the concrete so that the danger is theoretical rather than real.

Fire in Borax Factory. The fire in the 4-story reinforced concrete factory of the Pacific Coast Borax Company, built entirely of concrete except the roof, utterly destroyed the contents of the building, the roof, and the interior framework, but the walls and floors remained intact except in one place where an 18-ton tank fell through the plank roof and cracked some of the floor beams, and in one place on the outside of the wall where the surface of the concrete was slightly affected. The fire was so hot that brass and iron castings were melted to junk. A small annex,

*See References, Chapter XXIX.

†Digest of Physical Tests, Vol. I, p. 217.

built of steel posts and girders, was completely wrecked, and the metal bent and twisted into a tangled mass.

Baltimore Fire. The effect of the fire upon the concrete in various buildings located in the center of the burned districts of Baltimore is best appreciated by an examination of the reports of experts upon the fire. Capt. John S. Sewell, in his report to the Chief of Engineers, U. S. A.,* in referring to the fire in one of the buildings built with reinforced concrete columns, beams, and arches, writes:

> It was surrounded by non-fireproof buildings, and was subjected to an extremely severe test, probably involving as high temperature as any that existed anywhere. The concrete was made with broken granite as an aggregate. The arches of the roof and the ceiling of the upper story were cracked along the crown, but in my judgment very slight repairs would have restored any strength lost here. Cutting out a small section — say an inch wide — and caulking it full of good strong cement mortar would have sufficed. The exposed corners of columns and girders were cracked and spalled, showing a tendency to round off to a curve of about 3 in. radius. In the upper stories, where the heat was intense, the concrete was calcined to a depth of from ¼ to ¾ inch, but it showed no tendency to spall, except at exposed corners. On wide, flat surfaces, the calcined material was not more than ¼-inch thick, and showed no disposition to come off. In the lower stories, the concrete was absolutely unimpaired, though the contents of the building were all burned out. In my judgment, the entire concrete structure could have been repaired for not over 20% to 25% of its original cost. On March 10, I witnessed a loading test of this structure. One bay of the second floor, with a beam in the center, was loaded with nearly 300 pounds per sq. ft. superimposed, without a sign of distress, and with a deflection not exceeding ⅛-inch. The floor was designed for a total working load of 150 pounds per sq. ft. The sections next to the front and rear walls were cantilevers, and one of these was loaded with 150 pounds per sq. ft. superimposed, without any sign of distress, or undue deflection.

Captain Sewell concludes as a result of the examination of this and other buildings containing reinforced concrete construction:

> As the material is calcined and damaged to some extent by heat, enough surplus material should be provided to permit of a loss of say ¾-inch all over exposed surfaces, if the structure is to be exposed to fire; moreover, all exposed corners should be rounded to a radius of about 3 inches. This latter precaution would add much to the resistance of all materials used in masonry — whether bricks, stone, concrete or terra-cotta — if they are to be exposed to fire.

**Engineering News*, March 24, 1904, p. 276.

Concrete Versus Terra-Cotta. Prof. Norton, in his report on the Baltimore fire to the Insurance Engineering Experiment Station,* says:

Where concrete floor arches and concrete-steel construction received the full force of the fire it appears to have stood well, distinctly better than the terra-cotta. The reasons I believe are these: First, because the concrete and steel expand at sensibly the same rate, and hence when heated do not subject one another to stress, but terra-cotta usually expands about twice as fast with increase in temperature as steel, and hence the partitions and floor arches soon become too large to be contained by the steel members which under ordinary temperature properly enclose them. Under this condition the partition must buckle and the segmental arches must lift and break the bonds, crushing at the same time the lower surface member of the tiles.

When brick or terra-cotta are heated no chemical action occurs, but when concrete is carried up to about 1 000° Fahr. its surface becomes decomposed, dehydration occurs, and water is driven off. This process takes a relatively great amount of heat. It would take about as much heat to drive the water out of this outer quarter-inch of the concrete partition as it would to raise that quarter inch to 1 000° Fahr. Now a second action begins. After dehydration the concrete is much improved as a non-conductor, and yet through this layer of non conducting material must pass all the heat to dehydrate and raise the temperature of the layers below, a process which cannot proceed with great speed.

Cinder Versus Stone Concrete. Prof. Norton compares the action of stone and cinder concrete in the Baltimore fire as follows:

Little difference in the action of the fire on stone concrete and cinder concrete could be noted, and as I have earlier pointed out, the burning of the bits of coal in poor cinder concrete is often balanced by the splitting of the stones in the stone concrete. I have never been able to see that in the long run either stood fire better or worse than the other. However, owing to its density the stone concrete takes longer to heat through.

Further experiments are required to determine the relative durability under extreme heat of concrete made with different kinds of broken stone. It seems probable, from the composition of the rock, that hard trap or gravel may be preferable to limestone, slate, or conglomerate as fire-resisting material.

Thickness of Concrete Required to Protect Metal from Fire. The conclusion reached by Prof. Norton† from tests upon concrete arches is that two inches of good concrete gives perfect assurance of safety in case of fire, even if the steel to be protected is in the form of I-beams. Rods of

Engineering News, June 2, 1904, p. 529.

†*Insurance Engineering*, Dec., 1901, p. 483.

small dimensions can be more effectively coated, and it appears evident from the various tests and from practical experience in severe fires that 1½ inches of concrete around steel rods is sufficient protection. The Pacific Borax Company's fire and other similar tests indicate that in slabs of reinforced concrete, ½ inch to ¾ inch affords ample protection. Secondary members, such as cross girders, or slabs of long span, should have a thickness of concrete outside of the steel varying from ¾ inch to 1½ inch. Although in slabs protected by only ½ inch of concrete, the latter may be softened by an extreme fire, and the metal exposed when it is struck by the stream from a hose, the metal in the majority of cases would still remain practically uninjured, and it is questionable economy to put an excess of material where there is so little probability of its being needed, and where a failure would merely produce local damage.

THEORY OF FIRE PROTECTION

Mr. Spencer B. Newberry, in an address delivered before the Associated Expanded Metal Companies, Feb. 20, 1902,* gives the following explanation of the fire-proof qualities of Portland cement concrete:

The two principal sources from which cement concrete derives its capacity to resist fire and prevent its transference to steel are its *combined water and porosity*. Portland cement takes up in hardening a variable amount of water, depending on surrounding conditions. In a dense briquette of neat cement the combined water may reach 12%. A mixture of cement with three parts sand will take up water to the amount of about 18% of the cement contained. This water is chemically combined, and not given off at the boiling point. On heating, a part of the water goes off at about 500° Fahr., but the dehydration is not complete until 900° Fahr. is reached. This vaporization of water absorbs heat, and keeps the mass for a long time at comparatively low temperature. A steel beam or column embedded in concrete is thus cooled by the volatilization of water in the surrounding cement. The principle is the same as in the use of crystallized alum in the casings of fireproof safes; natural hydraulic cement is largely used in safes for the same purpose.

The porosity of concrete also offers great resistance to the passage of heat. Air is a poor conductor, and it is well known that an air space is a most efficient protection against conduction. Porous substances, such as asbestos, mineral wool, etc., are always used as heat-insulating material. For the same reason cinder concrete, being highly porous, is a much better non-conductor than a dense concrete made of sand and gravel or stone, and has the added advantage of lightness. In a fire the outside of the concrete may reach a high temperature, but the heat only slowly and imperfectly penetrates the mass, and reaches the steel so gradually that it is carried off by the metal as fast as it is supplied.

*Cement, May, 1902, p. 95.

TESTS OF FIRE RESISTANCE

Prof. Ira H. Woolson of Columbia University has made several series of tests* to determine the effect of heat upon the strength and elastic properties of the concrete and upon the thermal conductivity of the concrete and the imbedded steel.

Effect Upon Strength. Tests to determine the effect of heat treatment upon the strength and elastic properties of different mixtures showed that the trap concrete was least affected. Concrete two months old, in proportions 1:2:4, the crushing strength of which before heating was about 2500 pounds per square inch tested in 7-inch cubes, after being subjected to a heat of 1500° Fahr. for two hours gave a strength of about 1000 pounds per square inch. However, since this reduction in strength was due at least in part to the reduction in the effective area because of the surface deterioration (if the surface was injured to a depth of 1¼ inches the effective area would be reduced from 49 sq. in. to 20 sq. in.), it is probable that the interior of the blocks was affected very little. The concrete made with gravel, which in these tests was nearly pure quartz having a high coefficient of expansion, was affected to a much greater extent. Cinder concrete, which showed a normal crushing strength of about one-half that of the trap, after heat treatment gave a corresponding weakening.

The modulus of elasticity of the concrete was always greatly reduced by heat treatment.

CONDUCTIVITY OF CONCRETE AND IMBEDDED STEEL

As a result of the conductivity tests, which were made upon specimens of trap, gravel and cinder concrete having thermo-couples for measuring heat transmission imbedded so as to indicate the temperature at points varying from ½ inch to 6 inches from the heated face, Prof. Woolson drew the following conclusions:†

All concretes have a very low thermal conductivity, and herein lies their ability to resist fire.

When the surface of a mass of concrete is exposed for hours to a high heat, the temperature of the concrete one inch or less beneath the surface will be several hundred degrees below the outside.

A point 2 inches beneath the surface would stand an outside temperature of 1500° Fahrenheit for two hours, with a rise of only 500° to 700°, and points with three or more inches of protection would scarcely be heated above the boiling point of water.

* Proceedings of American Society for Testing Materials, Vol. V, 1905, p. 335; VI, 1906, p. 433; VII, 1907, p. 404.

† Proceedings American Society for Testing Materials, Vol. VII, 1907. p. 408.

The fact that cinder concrete showed a higher thermal conductivity than the stone concrete would indicate that its well-known fire-resistive qualities are due, in part at least, to the incombustible quality of the cinder itself.

The thermal conductivity of the gravel concrete* was fully as low as that of the trap, but the specimens of gravel concrete cracked and crumbled in many cases when the trap and cinder specimens under similar treatment remained firm and compact.

In the tests on the conductivity of imbedded steel with the end projecting from concrete, Prof. Woolson found practically the same results with concrete from all three aggregates. With the temperature of the end surface of the concrete and the projecting end of the bar 1700° Fahrenheit, a point in the bar only 2 inches from the heated face of the concrete developed a temperature of only 1000° Fahrenheit, while at a point 5 inches in the concrete the temperature was only 400° to 500°, and at 8 inches the temperature reached only the heat of boiling water.

From these results Prof. Woolson concludes that "where reinforcing metal is exposed in the progress of a fire, only so much of the metal as is actually bare to the fire is seriously affected by it."

Tests by the National Fire Protection Association† in 1905 upon beams 8 inches by 11¼ inches by 6 feet long, of different kinds of concrete, showed that the strength of rods imbedded 1 inch from the lower surface was reduced about 25 per cent after heating to a temperature of 2000° Fahrenheit for one hour. With rods imbedded 2 inches a similar reduction in strength occurred after 2 hours and 20 minutes heating, and the strength of the concrete was appreciably reduced to a depth of 4 inches from the sides and bottom.

The hardest and densest mixtures were usually the poorest conductors of heat; the cinder concrete gave, however, a slower rise of temperature than the others.

INFLUENCE OF CRACKS IN REINFORCED CONCRETE UPON THE CORROSION OF STEEL

It has been seriously questioned whether the minute cracks which open in a concrete beam and slab even under loads which are absolutely safe do not permit corrosion of the steel reinforcement. Tests by E. Probst‡ in

*As stated in connection with the tests on preceding page, this gravel was nearly pure quartz. In other tests, concrete with gravel containing a larger percent of slate or other similar material has given much better results.

†*Cement*, January, 1906, p. 273.

‡Report of the Royal Department of Testing Materials in Gross Lichtenfelde, West Prussia.

Germany, in 1907, indicate very conclusively that steel in reinforced beams, laid in ordinary wet concrete used in practical construction, is in no danger of rusting through the cracks formed in the concrete under tension, until nearly the breaking point of the steel. The specimens, 34 beams, which contained both plain and deformed bars and rusted and unrusted steel, were subjected in loading to the action of a mixture of oxygen, carbon dioxide and steam, for a period of from 3 to 12 days. Unprotected steel subjected to this mixture was badly rusted in two hours. After breaking up the specimens of concrete no rust was found even on steel stressed to its elastic limit, although some was discovered on steel stressed nearly to its breaking point, which could be attributed to large cracks extending to the metal and uncovering it.

PROTECTING STRUCTURAL STEEL

In San Francisco at the time of the earthquake and fire, April, 1906, there were few concrete structures, but these stood the test of fire and shock on the whole better than any other material.*

Observations after the fire indicate that concrete is also an effective protection for steel frame construction, but that it preferably should be enclosed in a metal basket.

Captain John S. Sewell, Engineer Corps, U. S. A., in his report to the U. S. Government† suggests that when such a basket is used the total thickness of concrete upon the exposed flanges of girders and floor beams should be 2 to 3 inches according to circumstances. For columns incased in a metal basket or cage, a thickness of 3 to 4 inches was recommended.

The structural steel in the Boston subway,‡ imbedded for twelve years in concrete or protected by the cement mortar joints of brick arches, was found upon examination during changes in the structure to be free from rust. The only exception to this was under the rather large base plates (21 by 24 inches) of columns, where a thin layer of rust frequently was found, having tubercles sometimes ¼ inch thick. This was evidently due to the settling of the finer parts of the concrete under the plates. The small base-plates were practically free from rust.

* Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 208.
†U. S. Geological Survey, Bulletin 324, 1907.
‡Personal correspondence with Mr. Howard A. Carson, Chief Engineer.

CHAPTER XIX

WATER-TIGHTNESS

A wall of concrete may be rendered water-tight in several ways:

(1) By accurately grading and proportioning the aggregates and the cement. (See p. 339.)

(2) By special treatment of the surface of the concrete. (See p. 341.)

(3) By the introduction of foreign ingredients into the mixture. (See p. 342.)

(4) By the application of layers of waterproof material, such as asphalt and felt. (See p. 343.)

It is often advisable to combine two or more of these methods.

In the succeeding pages directions are given for practically applying these methods, and experimental investigation is cited.

LAYING CONCRETE FOR WATER-TIGHT WORK

The manner of laying the concrete in walls or floors which are to withstand water pressure is as important as the proportioning of its ingredients. Approved methods of placing are fully described in Chapter XV.

The chief points applicable to water-tight work are briefly recapitulated as follows:

(*a*) Mix concrete of quaking or of wet consistency. (See p. 338.)

(*b*) Place concrete carefully so as to leave no visible stone pockets.

(*c*) Lay the entire structure, if possible, in one continuous operation, working night and day when necessary.

(*d*) If joints are unavoidable, clean and roughen the old surface, then wet it and coat with a layer of cement or mortar. (See p. 284.)

(*e*) Make suitable provision for contraction by special joints, or by steel reinforcement without joints. (See p. 285, also chapter xxi.)

Effect of Consistency. A series of experiments, conducted by the authors, upon several blocks of mortar mixed in the same proportions of cement, sand, and stone, but with different proportions of water, indicates that the best consistency for concrete designed to withstand water pressure is intermediate between a *quaking* and a *mushy* mixture, as defined on page 280.

Also, the general conclusion was reached that with the same dry materials the consistency producing the greatest density after setting gives the most

impermeable mortar or concrete up to the point of a very wet consistency, when the excess of water affects the chemical composition of the cement, forming "laitance" (see p. 302), and thus reduces both the strength and the water-tightness of the specimen. After setting, the very wet specimens were found to have about the same density as the medium and mushy mixtures, because the cement, sand, and stone settled into place and expelled the surplus water.

PROPORTIONING WATER-TIGHT CONCRETE

The proportions* employed to resist the percolation of water usually range from 1 : 1 : 2 to 1 : 2½ : 4½, the most common mixtures being 1 : 2 : 4 or 1 : 2½ : 4½. However, with accurate grading by scientific methods, such as are described in Chapter XI, water-tight work may be obtained with proportions as lean as 1 : 3 : 7. (See p. 183.) Permeability, the quality of allowing water to pass through, and porosity, the property of containing pores or voids, are not synonymous terms, and the most porous material is not necessarily the most permeable, because the dimensions of the voids as well as their volume affect by capillarity the passage of water.

For maximum water-tightness a mortar or concrete may require a slightly larger proportion of fine grains in the sand than for maximum density or strength, but otherwise the general principles discussed on page 172 are applicable. A mixed aggregate (such as is shown in Fig. 61, p. 173) evidently has fewer channels through which the water can pass than an aggregate consisting of coarse stone and sand (such as is shown in Fig. 59, p. 172), provided the character and relative proportioning of the finest particles are the same in both cases. Recent tests indicate that gravel produces more water-tight concrete than broken stone under similar conditions.

Porosity of Concrete. The total voids, air plus water, in first-class concrete and mortar of various proportions are shown in column (20) of the table of Mr. William B. Fuller's experiments on pages 376 and 377. The percentage of total voids in the mortars averages about 26%, while in the concretes, of proportions commonly employed in practice, the voids range from 13% to 17%.

In neither the concrete nor the mortar do these percentages ever represent air alone. A portion of the water, an amount estimated at 8% of the weight of the cement,† corresponding to about 2½% of the volume of

*Proportions are based on an assumed unit of 100 lb. cement per cu. ft. or the equivalent of 3.8 cu. ft. to the barrel. (See p. 217.)

†Allen Hazen in Transactions American Society of Civil Engineers, Vol. XLII, p. 128.

ordinary concrete, combines with the cement, and a still larger portion of the water remains in the pores unless dried by artificial heat.

The porosity of mortars is discussed on page 127.

Size of Stone. Authorities disagree as to the relative advantages of small stone ranging between ½ and one inch, and coarse stone, ranging from ½ inch up to, say, 2½ inches. The latter is theoretically the better, but it is sometimes claimed that the fine material can be placed more satisfactorily. This depends upon the workmanship. With proper selection of materials and care in laying, the concrete containing the coarse stone produces excellent work, as is illustrated by the constructions at Little Falls, N. J. (see Chapter XXVIII), and Boonton, N. J. (see Chapter XXVI), where carefully graded stone up to 2½ or 3-inch diameter was used.

If very fine stone, under ½-inch, and containing dust, is used for the coarser aggregate, the addition of sand may increase the porosity and the permeability, because concrete with such small stone is practically a mortar, and the finer particles of stone are really sand. A concrete in proportions 1 part cement : 2 parts sand : 4 parts unscreened stone less than ½-inch diameter, makes a porous concrete, while a mixture 1 part cement : 2 parts sand : 4 parts stone ¼-inch to 1½-inch diameter, makes a dense one. With the small stone, proportions 1 : 1 : 2 would be the leanest advisable mixture.

The method of proportioning by mechanical analysis, as described by Mr. Fuller in Chapter XI, has been found in practice to produce impermeable concrete.

THICKNESS OF CONCRETE FOR WATER-TIGHT WORK

It is impossible to specify definite thicknesses of concrete to prevent percolation under different heads of water, because of variations in proportions and methods of laying. We have known rain water under a head of 2 or 3 inches to percolate through a 4-foot wall of excellent concrete of dry consistency. On the other hand, had the same materials been mixed to a wetter consistency and placed with no joints between successive layers, concrete but a few inches thick would have withstood a high head.

The best criterions for thicknesses of walls of first-class concrete are obtained from actual examples. Instances are cited in Chapters XXVI and XXXVIII of water-tight concrete 4 inches thick sustaining a head of 4 feet, concrete 15 inches thick sustaining a head of 40 feet, and concrete 5.5 feet thick sustaining a head of 100 feet.

SPECIAL TREATMENT OF SURFACE

Various methods of treating the surface of concrete have been employed to increase the water-tightness.

Plastering. Plastering the surface of concrete with rich Portland cement mortar in proportions 1 : 1 or 1 : 1½ is the method which first occurs to one, but in temperate or cold climates it is only useful for walls below the surface of the ground and therefore not subject to atmospheric changes. In such cases it can sometimes be used as a substitute for, or in connection with, paper and asphalt.

In certain sections of the Boston Subway*, a 6 inch wall of concrete was laid up next to the bank of earth and plastered with a layer of 1 : 1 mortar about ½ inch thick. After spreading the mortar with a plasterer's ordinary metal float (see Chapter XXIII.) the surface was run over with a toothed roller about 12 inches long by 4 inches in diameter, which pressed the plaster into any crevices, and left a rough surface. The main wall of concrete forming the lining of the Subway was then laid up against this plastered surface.

On the arch of the approaches to the East Boston tunnel, a layer of plaster, like that on the walls, was spread before laying the final 6-inch thickness of concrete, thus forming a water-tight joint in the interior of the arch ring.

Granolithic Finish. On horizontal or inclined surfaces, a granolithic surface of rich mortar of Portland cement and sand, or Portland cement and screenings in proportions about 1 : 1 may be laid and troweled, as in sidewalk construction. (See Chapter XXIII.) The surface finish must be placed at the same time as the base, and with the same, that is, Portland cement.

Troweling Surface. The water-tightness of horizontal or inclined layers of concrete can be greatly increased by troweling the concrete in the same manner that granolithic work is troweled. (See Chapter XXIII.) This brings the cement to the surface, and produces a dense, hard surface which is nearly equal to a surfacing of rich mortar. This is very effective for surfacing a structure like the inclined face of the dam shown in Chapter XXVI.

In experimenting upon the permeability of different concretes, the authors have noticed that even the very light joggling which is necessary to compact a wet concrete, and also the ramming of a stiffer mixture, increases the impermeability of the concrete. Even after chipping off the top of the specimen for a depth of ⅛ or ¼ of an inch, the flow will be several times less than when the pressure is directed upon its under surface.

* In Subway construction since 1902 and in the tunnel built in 1907–9, the trench frequently was shored with 2½-inch reinforced concrete sheeting (*See* Chap. XXV), the surface evened with plaster, if necessary, and water-proofing applied.

Grout. Portland cement grout is preferable to plaster for coating the soffits of arches or for wall surfaces. It is also valuable for coating the interior of cisterns or tanks.* The grout should of course be applied against the surface which is to come in contact with the water, and if the wall is to be made impervious in both directions, both sides should be washed.

A specially prepared cement wash has been found effective in preventing dampness in masonry.†

Alum and Lye Waterproof Wash. United States Army Engineers‡ have satisfactorily employed a wash of alum and concentrated lye mixed in proportions one pound lye, 2 to 5 pounds alum, and 2 gallons of water, which has been used with good success in several instances.

Special Coatings. A few patented compounds which have proved successful are on the market. These are generally used with neat cement or mortar. In many cases it has been found possible to waterproof the face of the wall instead of the back upon which the water presses.

INTRODUCTION OF FOREIGN INGREDIENTS

The principal advantage of introducing foreign ingredients into a mortar or concrete is to permit the use of a lean mixture, the fine particles of hydrated lime, or whatever may be used, tending to reduce the volume and the dimensions of the voids. Every case must be studied by itself, since it is frequently cheaper to obtain the required water-tightness by adding cement than by admixtures..

Lime and Puzzolan Cement. The effect of the addition of lime in small quantities is chiefly mechanical, and the quantity which should be employed depends, therefore, upon the fineness of the sand and the proportions of the mixture.

Although it is impossible to replace the water which separates the grains in neat cement paste or rich mortar with a material like lime, a series of tests§ made by one of the authors in 1908 indicates that the introduction of a small percentage of hydrated lime into the concrete for small structures like tanks will render them more watertight, especially at early periods, and also that for large masses of concrete the addition of hydrated lime may permit the use of leaner proportions. The percentage of hydrated lime to use varies with the proportions of concrete and the character of the materials,

* J. W. Schaub, Transactions American Society of Civil Engineers, Vol. LI, p. 123.
† Oscar Lowinson, Transactions American Society of Civil Engineers, Vol. LI, p. 125.
‡ C. B. Hegardt in Report Chief of Engineers, U. S. A., 1902, p. 2482.
§ "Permeability Tests of Concrete with Addition of Hydrated Lime," by Sanford E. Thompson, American Society for Testing Materials, Vol. VIII, p. 500.

permissible quantities in practice ranging from 5 to 15 per cent of the weight of the cement. Results of tests with different proportions are given in the paper mentioned.*

Lime paste made from a given weight of hydrated lime occupies about 2½ times the bulk of paste made from the same weight of Portland cement and is therefore very efficient in void filling.

The strength of concrete has been found in some cases to be slightly reduced by the addition of hydrated lime, but not in a sufficient degree to influence its use in a water-tight wall, where the strength is seldom a determining quality.

The effect of the addition of lime upon the strength and density of mortar is discussed on page 154d.

Unslaked lime must not be used under any circumstances. (See p. 156.)

Puzzolan cement, unlike lime, tends to increase the strength even of neat cement and rich mortars,† in many cases 20% by weight of total dry materials being beneficial if the Puzzolan cement is ground with the Portland. Undoubtedly the impermeability is similarly increased, since mixtures of Portland and Puzzolan cements have been found to well resist the action of sea-water.‡

In Japan in the Nagasaki Dock,§ concrete blocks were made in proportions 0.25 lime; 1 Puzzolana; 1 Portland cement; 4 sand; 8 gravel:

Clay. Pure clay, finely powdered and free from any trace of vegetable matter, has been found to appreciably increase the water-tightness of concrete,‖ especially of lean mixtures. In certain cases 5 per cent of clay to the weight of the sand has been found effective. The proportions should vary with the character of the aggregates.

Clay acting as a colloid in combination with an electrolyte such as alum sulphate has been suggested by Mr. Richard H Gaines¶ for increasing water-tightness. Tests by him show a marked decrease in the flow of water due to these materials either added alone or in combination.

Pulverized Rock. Mortars 1 : 3 and leaner, and concrete made with these proportions of cement and sand to the stone, are increased in strength,† and probably in impermeability, by the addition of rock pulverized as finely as the cement and equal to it in weight, although if the natural sand is very

* "Permeability Tests of Concrete with Addition of Hydrated Lime," by Sanford E. Thompson, American Society for Testing Materials, Vol. VIII, 1908, p. 500.

† Feret's *Chimie Appliqéue*, 1897, pp. 477 and 493.

‡ See R. Feret, Chapter XVI.

§ N. Shirrishi, Transactions American Society of Civil Engineers, Vol. LVI, 1906, p. 76.

‖ See paper on "Waterproofing Cement Structures," by James L. Davis, Proceedings National Association of Cement Users, Vol. IV, 1908, p. 328.

¶ Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 159.

fine or contains dust, the addition of fine material is not beneficial.

Alum and Soap. A soap and alum mixture in various proportions sometimes is used to make what is called "waterproof mortar." The Sylvester Process mixture employed in New York Harbor by Major W. L. Marshall‡ was made by "taking one part cement and 2½ parts sand and adding thereto ¾ of a pound of pulverized alum (dry) to each cubic foot of sand, all of which was first mixed dry, then the proper amount of water—in which had been dissolved about ¾ of a pound of soft soap to the gallon of water—was added, and the mixing thoroughly completed. The mixture is little inferior in strength to ordinary mortar of the same proportions and is impervious to water, and is also useful in preventing efflorescence."

The effect of alum and soap in diminishing the permeability has been experimented upon by Mr. Edward Cunningham§ and Prof. W. K. Hatt,§ and found useful for small structures.

LAYERS OF WATERPROOF MATERIAL

The use of cement plaster has already been described on page 419.

Layers of waterproof paper or felt cemented together with asphalt or bitumen or tar are extensively used,—and sometimes asphalt alone,—to form an impervious layer. A mixture of alum and lye has also been tried.

Paper or Felt Waterproofing. Layers of paper or felt with tar or asphalt between them are employed for a waterproof course in concrete floors, roofs, and walls of underground structures of large or long area, like tunnels and subways, which require special protection from infiltration of water. The materials range from ordinary tarred paper, laid with coal tar pitch, to asbestos or asphalted felt, laid in asphalt. Coal tar products appear to be satisfactory when made to contain a large percentage of carbon, and are being used by many in preference to asphalt.

In the New York Subway, portions of which are built below tide-water, much of the waterproofing consists of layers of felt laid in asphalt. The specifications,** approved by Mr. William Barclay Parsons, Chief Engineer, contain the following requirements for the materials:

The asphalt used shall be the best grade of Bermudez, Alcatraz, or lake asphalt, of equal quality, and shall comply with the following requirements: The asphalt shall be a natural asphalt or a mixture of natural asphalts, con-

‡ Report Chief of Engineers, U. S. A., 1901, p. 918.

§ Transactions American Society of Civil Engineers, Vol. LI, pp. 127 and 128.

** Contract No. 2, June, 1902, p. 107.

taining in its refined state not less than ninety-five (95) per cent of natural bitumen soluble in rectified carbon bisulphide or in chloroform. The remaining ingredients shall be such as not to exert an injurious effect on the work. Not less than two-thirds (⅔) of the total bitumen shall be soluble in petroleum naphtha of seventy (70) degrees Baumé or in Acetone. The asphalt shall not lose more than four (4) per cent of its weight when maintained for ten (10) hours at a temperature of three hundred (300) degrees Fahrenheit.

The use of coal tar, so-called artificial asphalts, or other products susceptible to injury from the action of water, will not be permitted on any portion of the work, or in any mixtures to be used.

The felt used for waterproofing shall be dipped in asphalt and weigh not less than fifteen (15) pounds to the square of one hundred (100) feet. All felt shall be subject to the inspection and approval of the engineer.

With reference to the laying of the water-proofing the contract required:*

Each layer of asphalt fluxed as directed by the engineer must completely and entirely cover the surface on which it is spread without cracks or blowholes.

The felt must be rolled out into the asphalt while the latter is still hot, and pressed against it so as to insure its being completely stuck to the asphalt over its entire surface, great care being taken that all joints in the felt are well broken, and that the ends of the rolls of the bottom layer are carried up on the inside of the layers on the sides, and those of the roof down on the outside of the layers on the sides so as to secure a full lap of at least one (1) foot. Especial care must be taken with this detail.

None but competent men, especially skilled in work of this kind, shall be employed to lay asphalt and felt.

When the finishing layer of concrete is laid over or next to the waterproofing material, care must be taken not to break, tear, or injure in any way the outer surface of the asphalt.

Any masonry that is found to leak at any time prior to the completion of this work shall be cut out and the leak stopped.

Method of Laying Paper or Felt. The waterproof layer of a floor may be laid directly upon the ground if the soil is fairly dry and firm, but is usually spread upon a layer of concrete from 4 to 8 inches thick. In the former case† the first layer consists of strips with a 2 to 6-inch lap cemented with asphalt, and the remaining layers are mopped on. Upon a concrete base it is customary to first spread a layer of asphalt upon the concrete, although, if the concrete is damp, the bottom layer of paper or felt may be placed dry, as described above.

The "ply" in waterproofing,—that is, the number of layers which cover all parts of the surface,—varies from 2-ply to 10-ply. It is considered better practice to "shingle" the strips than to place each ply or layer independently.

* Contract No. 2, June, 1902, p. 107.

† This method was followed in portions of the floor in the approaches to the East Boston Tunnel.

If the surface to be waterproofed is rough it may be leveled with cement mortar. It must be dry before applying the tar or asphalt. The asphalt is heated and brought, generally in buckets, to the work. Several rolls of paper are started consecutively. Ahead of each roll, as it is unrolled, the liquid asphalt is swabbed upon the concrete with a mop, so that the paper or felt is spread directly upon the fresh hot stuff. As soon as the first roll is started the second is placed to overlap the first, a width depending upon the number of ply to be laid. For example, if the felt is 32 inches wide and is laid 3-ply, the second roll is lapped upon the first about 22 inches. As this is unrolled (in the same general direction as the first roll) the surface ahead of it is mopped with asphalt, as described above. A third roll is immediately started, lapping both of the two others, and so on for the entire width of the surface to be covered.

A waterproof course of this character always forms a distinct joint in the mass, thus destroying its cohesion upon that plane, and the strength of the concrete in bending on the two sides of the layer must be considered independently.

Asphalt Waterproofing. Asphalt is sometimes laid as a waterproof course in one or more continuous sheets, and is also used for filling contraction joints in concrete.

In the sedimentation basin for the Albany (N. Y.) Filtration Plant* 16 inches of clay and gravel puddle were covered with 6 inches of concrete laid in blocks 7 feet square, with ½-inch asphalt joints 3 inches deep, that is, extending halfway through the concrete. This proved to be a successful treatment.

In the Astoria (Ore.) Water Works† the bottom of the reservoir consisted of 6 inches of concrete in approximate proportions, one packed cement : 0.7 sand : 3.5 fine gravel : 6.5 broken stone, covered with a ⅜-inch finishing coat of 1 : 1 mortar, and upon this two layers of Alcatraz brand asphalt. The first layer was of natural liquid asphalt, and the second was the product of refining natural rock asphalt with about 20% of the liquid as a flux. Mr. Adams made the rule that no asphalt should be placed until after the concrete had set at least two weeks, and was well dried out. All dust was carefully removed from the concrete, and the asphalt was applied with twine mops. The slopes of the reservoir were lined with brick laid in asphalt upon 6 inches of concrete. Under ordinary conditions such complete measures are unnecessary.

In the construction of government fortifications by the United States

* Allen Hazen in Transactions American Society of Civil Engineers, Vol. XLIII, p. 258.

† Arthur L. Adams in Transactions American Society of Civil Engineers, Vol. XXXVI, p. 29.

Army Engineers, numerous methods of waterproofing have been used,* in some cases an asphalt course being placed between two layers of concrete. Asphalt paint has been used for a protective coating where earth is to be deposited above or against it.†

A ¼-inch coating of asphalt applied hot with a mop upon a surface already covered with grout (see p. 339) has been satisfactorily used by Mr. J. W. Schaub‡ for coating the interior of tanks where the head is greater than 10 feet. He considers this sufficient to withstand a water pressure of 60 feet.

Mr. Schaub‡ also suggests the method of building the wall in two parts and filling the core or hollow space between with asphalt.

CONSTRUCTION WITHOUT WATERPROOFING

New York Subway Practice. Formerly asphalt waterproofing was required on the floors, walls and roof of the New York Subway, varying in thickness from 3 to 6-ply or else using two layers of waterproofing with one or more layers of brick dipped in asphalt. It was found, however, that the sections of subway waterproofed in this way were not so cool as other sections because the waterproofing prevented radiation of heat. Consequently, it was proposed to use the waterproofing below high water level but extending only 2 feet above it, except in special localities. The concrete was to be reinforced longitudinally, as well as laterally, using a rich mixture, well spaded. This was further protected by a blind drain composed of broken stone 6 inches thick on the top of the subway and hollow tile built against the walls.§

Philadelphia Subway Practice In all of the subway work it is the practice to rely on the proper placing of the concrete for waterproofing except that on the roof a layer of asphalt 1-inch thick is used. Longitudinal reinforcement, generally to the amount of 0.3 per cent., is introduced to prevent cracking of the walls.‖

METHODS OF TESTING PERMEABILITY

Permeability tests are somewhat difficult to make because of the many variables which must be provided for. In all cases it is advisable to measure the water which has passed through the specimen and not the water

* Report Chief of Engineers, U. S. A., 1901, pp. 911 to 925, and 1902, pp. 2451 to 2484.

† Report Chief of Engineers, U. S. A., 1902, p. 2473.

‡ Transactions American Society of Civil Engineers, Vol. LI, p. 123.

§ Personal correspondence with Henry B. Seaman, Chief Engineer, 1909.

‖ Personal correspondence with Charles M. Mills, Principal Assistant Engineer, 1909.

flowing into it. Results of permeability tests are comparable only among the specimens of each individual series. The methods which have been successfully employed may be outlined as follows:

Cementing a pipe upon the top of a block of concrete similar to the plan employed by the French Commission for mortar.*

Incasing a block on all sides except the top and bottom and forcing the water through.

Making thin discs and confining the water pressure to the center by means of gaskets.

These three methods as they have been developed are illustrated in Figs. 113, 114 and 115.

In Fig. 113, an apparatus designed by one of the authors,† the pipe is enlarged to 4 inches diameter to give a good surface of concrete and permit

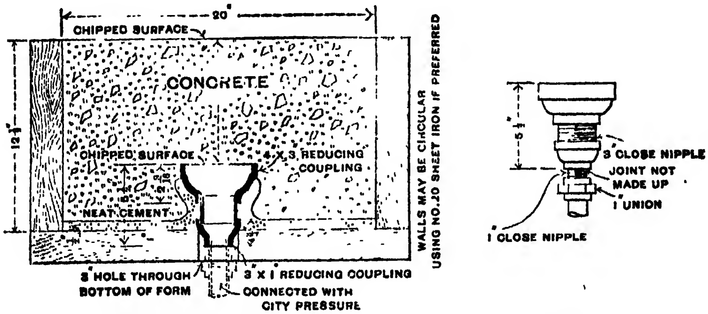


FIG. 113. Detail of Specimen for Testing Permeability.* (*See p.* 348.)

thoroughly chipping it, while at the same time the external pipe connections are small, so that tight joints can be made readily. The walls of the mold may be coated with neat cement as well as the bottom, if desired, the concrete being placed in any case before the neat cement has begun to stiffen.‡

The apparatus used at Jerome Park§ is a still better although somewhat more expensive design which is capable of modification to suit the size of the specimen. The concrete specimens, which are described at length in the paper referred to, were made first and afterward coated with neat

* See p. 128.

† "Permeability Tests of Concrete with Addition of Hydrated Lime," by Sanford E. Thompson, American Society for Testing Materials, Vol. VIII, p. 506.

‡ For example of the method adopted in earlier experiments, see "Consistency of Concrete," by Sanford E. Thompson, Proceedings American Society for Testing Materials, Vol. VI, 1907, p. 374

§ See "Laws of Proportioning Concrete," by William B. Fuller and Sanford E. Thompson, Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 67.

cement by placing in a mold after thoroughly roughening and wetting the surfaces. (See Fig. 114.)

Molding concrete in iron pipe is not satisfactory because the concrete shrinks in setting and there is consequently danger of leakage.

The method used in the St. Louis Structural Materials Laboratory* is illustrated in Fig. 115. This plan requires expensive castings and great care to make a water-tight joint at the rubber washers.

In tests of permeability the apparatus must be designed so as to make all the water pass through the concrete; the surface of the specimen must be cut down to the pure interior concrete to prevent surface effects; the mix must be very uniform, the size of the specimen being proportioned to the maximum size of the aggregate; sufficient water must be used to produce uniformity, the consistency depending upon the purpose of the tests; a slight excess of sand rather than a deficiency must be used to prevent large voids; if neat cement is used as a coating, it must be molded with the concrete or else the surface of the concrete must be chipped rough and soaked with water before applying the cement paste, and it must be kept wet for some time; the specimen should be soaked for 24 hours before testing.

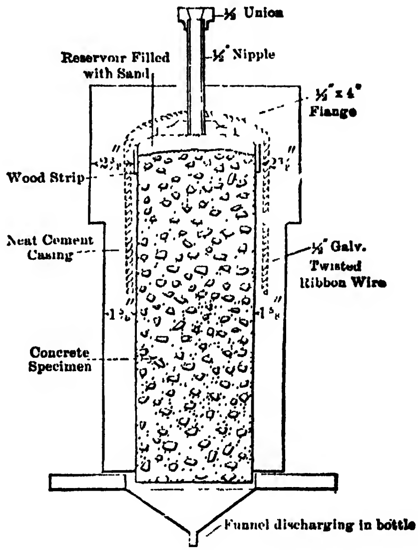


FIG. 114.
Permeability Specimen used at Jerome Park. (*See p.* 348.)

LAWS OF PERMEABILITY

The following conclusions have been reached with reference to the permeability of concrete and mortar:

(1) The permeability or flow of water through concrete is less as the percentage of cement is increased, and in very much larger inverse ratio.†

(2) The permeability is less as the maximum size of the stone is greater. Concrete with maximum size stone of 2½-inch diameter is, in general, less

* Bulletin No. 329, U. S. Geological Survey, 1908, by Richard L. Humphrey.

† See foot-note p. 350.

permeable than that with 1-inch maximum diameter stone, and this is less permeable than that with ½-inch stone.*

(3) Concrete of cement, sand and gravel, is less permeable than concrete of cement, screenings and broken stone; that is, for equal permeability, a slightly smaller quantity of cement is required with rounded aggregates like gravel than with sharp aggregates like broken stone.*

(4) Concrete of mixed broken stone, sand and cement, is more permeable than concrete of gravel, sand and cement, and less permeable than

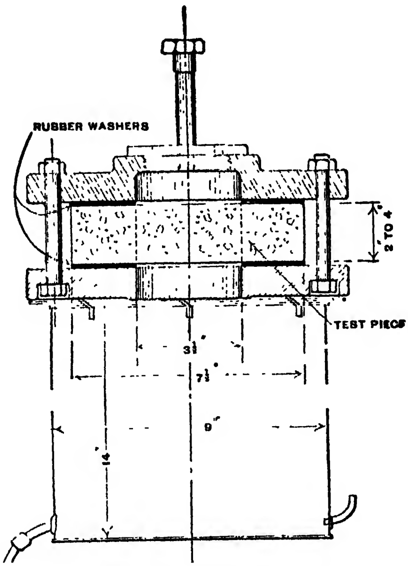


FIG. 115. Permeability Specimen used at St. Louis. (*See p.* 349.)

similar concrete of broken stone, screenings and cement; that is, for watertightness, less cement is required with rounded sand and gravel than with broken stone and screenings.*

(5) Permeability decreases materially with age.*

(6) Permeability increases nearly uniformly with the increase in pressure.*

(7) Permeability increases as the thickness of the concrete decreases, but in a much larger inverse ratio.*

(8) Of mortars containing the same percentage of cement but of variable granulometric composition, the most impermeable are those containing

* "Laws of Proportioning Concrete," by Fuller and Thompson, Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 72.

equal parts of coarse grains, G, and fine grains, F (see p. 142), the latter including the cement.*

(9) Decomposition by the passage of sea-water through mortars mixed in equal proportions by weight increases as the sand contains more fine grains.*

(10) Medium and fairly wet consistencies produce concrete much more water-tight than dry consistencies, and slightly more water-tight than very wet consistencies.†

(11) The surface of concrete as molded is much more water-tight than the bottom of a specimen, because of the fine material which rises to the top.†

RESULTS OF TESTS OF PERMEABILITY

The table which follows gives the comparative permeability of concrete specimens 18 inches in length and 6 inches square, made up as shown in Fig. 114. The various qualities are referred to in paragraphs which follow:

Effect of Shape of Stone Upon Permeability. In the table it is noticeable that the most permeable concrete is that composed of broken stone and screenings; the next, that containing broken stone and natural sand; and the most water-tight of all (comparing similar percentages of cement), the concrete of gravel and sand. The rounded gravel stone and sand evidently flow better and make a more homogeneous mix. It is noticeable also in the Jerome Park permeability tests that the results from the sand and gravel specimens were the most uniform.

Effect of Percentage of Cement Upon Permeability. The table on the following page illustrates the very great increase in water-tightness with the richness of the mixture. The most extreme differences are noticed in the specimens with broken stone and screenings.

Increase of Permeability With Pressure. A comparison of the columns in the table shows that the rate of flow increases nearly uniformly with the increase of pressure.

Effect of Thickness of Concrete Upon Permeability. Other experiments, not here recorded, indicate that the rate of flow increases as the thickness of the concrete decreases, but in a much larger inverse ratio. Speci-

* R. Feret in Annales de Ponts et Chaussées, 1892, II, p. 109.

† "The Consistency of Concrete," by Sanford E. Thompson, Proceedings American Society for Testing Materials, Vol. VI, 1906, p. 358.

mens 17 inches in length in proportions 1 : 6.5 by weight were practically water-tight, whereas specimens of half that length passed considerable water.

Effect on Permeability of Percentage of Cement, Character of Aggregate and Pressure,

By Fuller and Thompson* (See p. 351)

Thickness of Specimens 18 inches. Area of contract 36 square inches.
Maximum diameter of stone 2½ inches.

Proportions by weight	Percentage of cement to total dry materials	Kind of material: Stone	Kind of material: Sand	Time in which water appears after starting pressure	Rate of flow of water in grams per minute, at the following pressures, per square inch: 20 lb.	40 lb.	60 lb.	80 lb.
	%			min.				
1 : 11.5	8.0	Crushed stone	Screenings	7	25	161	237	273
1 : 9	10.0	"	"	3	11	24	37	49
1 : 7	12.5	"	"	3	15	22	30	38
1 : 5.8	15.0	"	"	5.5	5	8	10	12
1 : 8.8	10.2	Crushed stone	Sand	9	4	11	17	22
1 : 6.9	12.7	"	"	10	2	2	3	3
1 : 5.5	15.6	"	"		0	0	0.7	1.4
1 : 10.8	8.5	Gravel	Sand	3	15	25	38	43
1 : 8.4	10.6	"	"	17	1	3	5	6
1 : 6.5	13.0	"	"	100	0	0	0	0.5
1 : 5.3	15.9	"	"	98	0	0	0	1.4

Rate of Flow. The Jerome Park tests indicate that if the surface of the concrete is clean and the water pure, the flow is very nearly constant for a considerable period. During a four hours' test there was no appreciable differences in the rate of flow. This result is somewhat contrary to other tests, but it is probable that in many cases the apparent plugging up of the pores is due to impurities in the water or to the early age of the concrete.

Effect of Size of Stone Upon Permeability. The following table gives the comparative permeability of concrete in the same proportions mixed with stone of different maximum size. The difference in this case is evidently due to the greater density of the concrete composed of the large stone.

* Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 132.

Effect of Size of Stone on Permeability

BY FULLER AND THOMPSON* (*See p.* 352.)

Thickness of Specimens 18 *inches.* *Area of contact* 36 *square inches.*

Aggregates, crushed stone and natural sand.

PROPORTIONS BY WEIGHT	PERCENTAGE OF CEMENT TO TOTAL DRY MATERIAL	MAXIMUM SIZE OF STONE	TIME IN WHICH WATER APPEARS	RATE OF FLOW OF WATER IN GRAMS PER MINUTE AT THE FOLLOWING PRESSURES PER SQ. IN.			
	%	in.	min.	20 lb.	40 lb.	60 lb.	80 lb.
1 : 2.9 : 5.7	10.2	2¼	7	1	4	8	12
1 : 2.9 : 5.7	10.2	1	26	0	5	10	15
1 : 2.9 : 5.7	10.2	½	29	0	10	17	20

Effect of Coarseness of Sand Upon Permeability. As stated, tests by Mr. Feret have indicated that for maximum watertightness more fine sand is required than for maximum strength. This is borne out by tests by one of the authors, the results of which are given in the following table. The tests were made in connection with the preparation of specifications for the Waltham Reservoir.†

Tests to determine Relative Permeability of Concrete with Coarse and Fine Bank Sand

BY SANFORD E. THOMPSON. (*See p.* 353.)

Proportions 1 : 3 : 6 *by Volume or* 1 : 2.8 . 5.7 *by Weight.* *Age* 32 *days*

CHARACTER OF SAND	DENSITY $c + s + g$	WATER PASSING IN GRAMS PER MINUTE
(1) All coarse....................	0.853	145.1
(2) ⅝ coarse, ⅜ fine..................	0.846	10.4
(3) ⅔ coarse, ⅓ fine..................	0.843	43.0
(4) All fine..........................	0.813	30.2

Analyses of Natural Bank Sand and Screened Gravel used in Tests

SIEVE	TOTAL PER CENT PASSING SIEVES		
	Coarse Sand	Fine Sand	Screened Gravel
	%	%	%
1 inch...........			100
½ inch...........			50
¼ inch...........	100		0
No. 5............	88		
No. 12...........	77	100	
No. 40...........	32	96	
No. 200..........	3	27	

*Transactions American Society of Civil Engineers, Vol. LIX, 1907, p. 136.

†See p. 702

CHAPTER XX

STRENGTH OF PLAIN CONCRETE

The strength of plain concrete, that is, of concrete without steel reinforcement, is governed primarily by

(1) The quality of the cement.

(2) The texture of the aggregate.*

(3) The quantity of cement in a unit volume of concrete.

(4) The density† of the concrete.

The percentage of cement and the density of the concrete, which are of special importance to the user in determining the proportions of materials, may be expressed more explicitly as follows:

(1) With the same aggregate the strongest concrete is that containing the largest percentage of cement in a given volume of concrete, the strength varying nearly in proportion to this percentage.

(2) With the same percentage of cement but different arrangement of the aggregates, the strongest concrete is usually that in which the aggregate is proportioned so as to give a concrete of the greatest density, that is with the smallest percentage of voids. In many cases relative densities nearly correspond to relative weights.

Although these laws have been long recognized in a general way, having been partially proved by experiments of Mr. John Grant as early as 1871, but few attempts have been made to apply them practically in the comparison of strengths of different mixtures of concrete.

The authors have evolved a formula (see p. 356) from which, knowing the exact quantities of the raw materials entering into a concrete of a certain strength, it is possible to estimate the approximate strength of any other concrete mixed in different proportions of the same materials, under similar conditions of manufacture, storage, age, and methods of testing.

The compressive fiber strength of concrete, which is an essential factor in the design of reinforced concrete, is proportional to the strength of concrete in direct compression.

The table of tests of beams on page 376 covers so wide a range of proportions that it may be employed for comparing the transverse strength of different mixtures.

*The word aggregate is defined on page 1.

†The meaning of density is illustrated on pages 172 and 173.

Further information relating to the strength of concrete made from different materials and under various conditions is presented under separate headings in this chapter. The methods of making concrete specimens for testing are outlined on page 395.

COMPRESSIVE STRENGTH OF CONCRETE

The actual strength of concrete in compression, because of the limited capacity of testing machines, can be determined only by experiments upon comparatively small specimens from 4 to 12 inches square. The results from tests of such specimens are probably slightly lower than the actual strength of concrete in practice, carefully mixed and laid, because of the difficulty in obtaining homogeneous specimens. Experiments by the authors show that the strength of the same mixture tends to increase with the size of the specimen even if the relative dimensions remain constant. Of course carelessness or inexperience will produce irregular work in either actual or experimental construction.

The experimental strength of concrete is not always a criterion for fixing the proportions of mixture, in fact most concrete must be made stronger than the theoretical loading would require. A lean concrete, for example, although it may gain sufficient strength before the load is applied, may not be sufficiently strong at a short period to permit the removal of the molds or the ordinary wear during building, or for many purposes the lean concrete may be too porous. Often a lean Portland cement concrete may thus present no special advantage over a richer natural cement concrete. (See Chapter IV.)

Comparative Strength of Concretes of Different Proportions. The formula for strength of mortar derived by Mr. R. Feret and presented on page 141, as Mr. Feret himself states,* is not applicable to concrete. Our formula for concrete mixtures is therefore presented as a practical working formula of sufficient accuracy to compare the compressive strength of mixtures of the same materials in different proportions. Starting with the principles laid down in the two fundamental laws stated at the commencement of the chapter, it is evolved by trial by the method given on page 357, to fit the average results of a large number of tests made in this country and Europe.

Let

P = unit compressive strength of concrete.

c = absolute volume† of cement in a unit volume of concrete.

*Chimie Appliquée, p. 522.

†Method of determining densities and absolute volumes are described on page 135.

s = absolute volume of sand in a unit volume of concrete.
g = absolute volume of stone in a unit volume of concrete.
M = a coefficient, constant for all proportions of the same material mixed and stored under similar conditions, but varying with the texture of the coarse aggregate and the age of the specimen.

Then

$$P = M\left(\frac{c}{1 + c - (s + g)} - 0.1\right) \qquad (1)$$

The absolute volumes, as indicated on page 138, are really ratios of the actual volume of the concrete, representing the actual mass or total volume of solid particles in a unit volume of concrete. Since ratios are independent of the unit selected, the absolute units are the same for any system of measurement, and by changing the value of M the formula is adapted to English or Metric System. For example, if P expressed in terms of kilos grams per square centimeter requires a value of $M = 880$, P in pounds per square inch will require a value of $M = 880 \times 14.2^* = 12\ 500$. It follows that knowing for a given age the value of M and the strength of a concrete composed of known percentages of materials, it is possible to estimate the compressive strength at the same age of any other concrete of exactly known composition made under like conditions from similar materials, but differently proportioned.

A very slight variation in the values of the terms will so largely influence the result that the formula is only useful, on the one hand, where the specific gravities of the materials and the weights entering into a unit volume of concrete are determined so accurately that the absolute volumes can be calculated, and, on the other hand, for comparison of the strength of different mixtures of concrete under assumed average conditions. For the latter purpose the specific gravity of cement may be taken at 3.1 and of sand at 2.65, the weight of a barrel of cement as 376 pounds, the weight of the dry sand contained in a cubic foot of moist sand as 89 pounds and the percentage of voids in the stone as 46%. In computations, values of absolute volumes must be carried to three places of decimals.

Now let

P' = compressive strength in pounds per square inch.
c_b = barrels of cement contained in a cubic yard of the concrete.
s_c = cubic yards of sand contained in a cubic yard of concrete.
g_c = cubic yards of stone contained in a cubic yard of concrete.
M' = a coefficient adapted to pounds per square inch.

Then assuming solid cement with no voids to weigh 193 lb. per cu. ft. and the solid particles of sand 165 lb. per cu. ft. formula (1) becomes,

$$P' = M'\left\{\frac{c_b \dfrac{376}{193}}{27 + \dfrac{376}{193}c_b - 27\left(\dfrac{89}{165}s_c + 0\ 54g_c\right)} - 0.1\right\}$$

$$P' = M'\left\{\frac{c_b}{13.85 + c_b - 7.48\,(s_c + g_c)} - 0.1\right\} \qquad (2)$$

This formula, as stated above, is only adapted for average comparative determinations, or where the conditions exactly correspond to those assumed. It may be adapted to other sand and stone by altering the coefficients of s_c and g_c. The table on page 360 is based upon these formulas (1) and (2).

Formula (1) on page 356 is based upon the actual strength of concrete, as determined by tests of Mr. E. Candlot in France and those of several other authorities at the Watertown Arsenal, U. S. A. To illustrate its

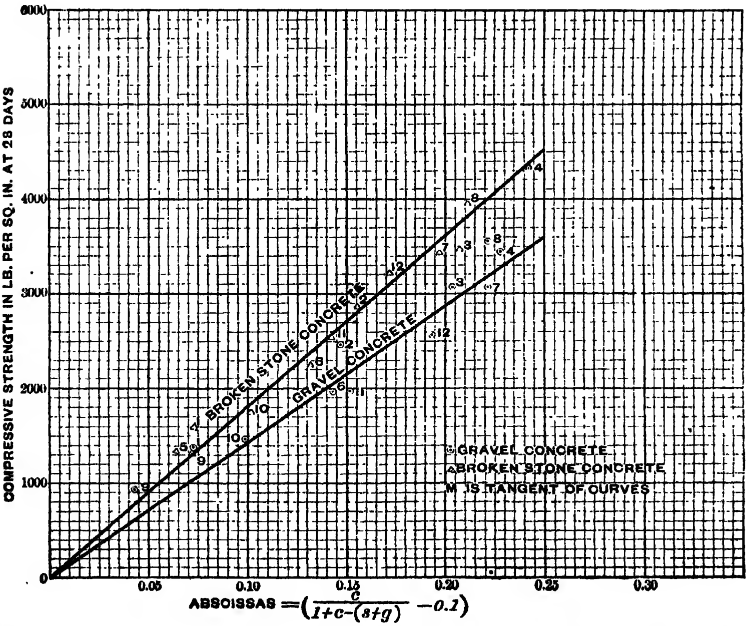


FIG. 116.—Comparison of Authors' Formula with Tests of E. Candlot. (See p. 358.)

agreement with actual experiments, tests of Mr. Candlot upon broken stone and gravel concrete 28 days old, quoted in full on page 367, are plotted on the diagram, Fig. 116, page 357, and Mr. George A. Kimball's tests made at the Watertown Arsenal on specimens 6 months old in Fig. 117.

The accuracy of the formula is shown by the nearness of the points on

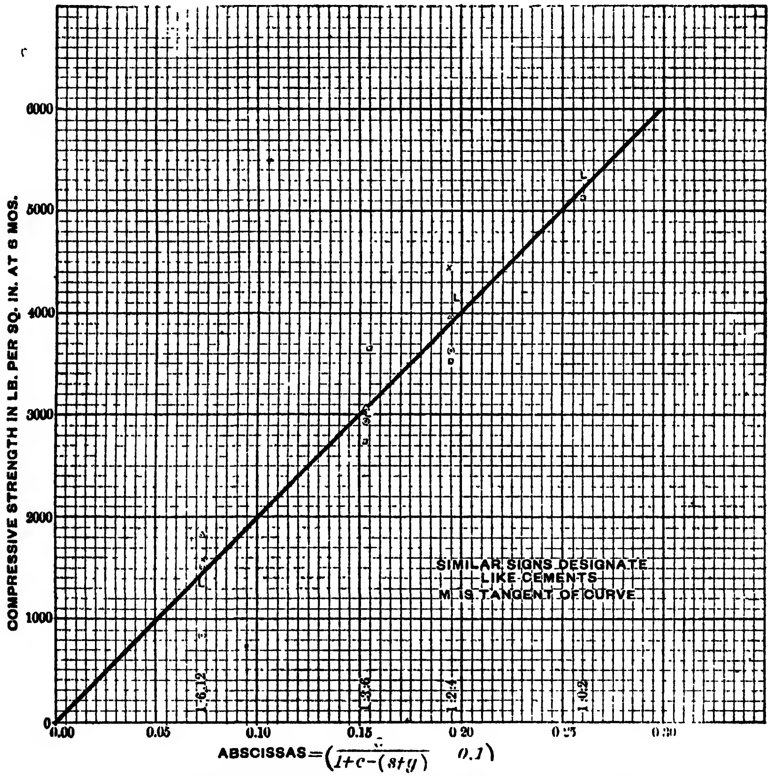


FIG. 117.—Comparison of Authors' Formula with Tests of George A. Kimball. (*See p.* 358.)

each diagram to straight lines starting from the origin. The abscissa of each point is determined by calculation of the term in brackets in formula (1), and the ordinate is the actual breaking strength of the specimen at the given period. The value of M in each case is the tangent of the straight line drawn through the points. If Mr. Candlot's tests are plotted on cross-section paper and smooth curves of growth in strength drawn through

them, it will be found that the new values taken from such curves, which partially eliminate inequalities in the breaking, approach even more nearly to the straight lines.

After a study of the strength of concrete at different periods, the authors suggest the following values for M at different ages. The values for broken stone concrete are based upon stone ranging in size from 2 to 2½ inch down to ¼ to ½ inch. For broken stone of finer size the values will be slightly lower. The composition of the concrete does not affect the value of M, since the term of the formula in large brackets is itself dependent upon the proportions of the mixture and the density of the concrete. The values of M are directly proportional to relative strengths at different ages.

Value of Coefficient M for Compressive Strength in Pounds per Square Inch.

Age.	Coefficient M for broken stone concrete	Ratio of growth based on age at one month
7 days	9 500	0.76
1 month	12 500	1.00
3 months	15 600	1.25
6 months	16 900	1.35
1 year	18 000	1.44

The ratios, which are taken from the curve on page 375, are based on the assumption that growth in strength of concrete, mixed under similar conditions and of similar consistency, is the same for all proportions of like materials. This, as stated on page 374, is not strictly true, but is sufficiently accurate for practical purposes.

Table of Compressive Strength. The strength of concrete mixed in various proportions, given in the table on page 360, is based upon a strength with proportions 1: 3: 6, that is, one barrel cement to 11.4 cubic feet sand to 22.8 cubic feet stone, of 1950 lb. per square inch at the age of one month, this value being selected as the average of tests by different experimenters. It corresponds to a value of M of 12 500. Using 1950 lb. per square inch for 1: 3: 6 as the starting point, the strengths for other mixtures are calculated from formula (1) page 356, the absolute units for the different proportions being deduced from the average quantities of cement, sand, and stone, contained in a unit volume of concrete. The values employed are similar to those in the table on page 231, except that it was necessary to carry them to three places of decimals. The strength at the age of six months is based on the growth in strength given on the curve on page 375. The assumption, which corresponds to average conditions, is made that a cubic foot of moist bank sand contains 89 lb. of

dry grains having a specific gravity of 2.65, and that the specific gravity of the cement is 3.1. The stone is assumed equal in quality to sound, hard limestone, ranging in size from ¼ inch to 2 inches. Stone of ½ inch maximum size may give strength about 20% lower. Specimens mixed of very wet consistency show lower strength especially at early periods. Cold weather retards strength. Prisms test lower than cubes.

The values in the table may be readily transformed to safe working strength by dividing by the proper factor of safety.

Relative Compressive Strength of Portland Cement Concrete of Different Proportions.

Based on Cube Specimens and Medium Consistency.

(*See important foot-notes, also* *p.* 359.)

Proportions.			Age, one month. Voids in Broken Stone or Gravel.					Age, six months. Voids in Broken Stone or Gravel.				
Cement.	Sand.	Stone.	*50% lb. per sq. in.	†45% lb. per sq. in.	‡40% lb. per sq. in.	§30% lb. per sq. in.	§20% lb. per sq. in.	*50% lb. per sq. in.	†45% lb. per sq. in.	‡40% lb. per sq. in.	§30% lb. per sq. in.	§20% lb. per sq. in
1	1½	2	2880	2860	2840	2800	2760	3890	3870	3840	3780	3730
1	1½	3	2780	2750	2720	2670	2610	3750	3710	3680	3600	3530
1	1½	4	2680	2650	2610	2540	2460	3620	3570	3520	3430	3330
1	2	3	2560	2540	2510	2460	2410	3460	3420	3390	3320	3250
1	2	4	2480	2440	2410	2350	2290	3340	3300	3250	3170	3090
1	2	5	2400	2350	2310	2230	2170	3230	3180	3120	3010	2930
1	2	6	2320	2260	2230	2140	2060	3130	3060	3010	2890	2780
1	2½	3	2370	2340	2320	2270	2230	3200	3160	3130	3070	3020
1	2½	4	2290	2260	2230	2180	2110	3090	3050	3010	2940	2850
1	2½	5	2210	2180	2130	2070	2000	2980	2940	2880	2790	2700
1	2½	6	2140	2100	2060	1980	1910	2890	2830	2780	2670	2570
1	3	4	2120	2090	2060	2020	1970	2860	2830	2780	2720	2660
1	3	5	2060	2030	1990	1930	1870	2780	2740	2690	2610	2530
1	3	6	1990	1950	1910	1840	1770	2680	2630	2580	2480	2390
1	3	8	1860	1810	1770	1680	1600	2510	2440	2390	2280	2160
1	4	6	1710	1680	1650	1590	1530	2310	2270	2220	2140	2070
1	4	7	1660	1620	1590	1530	1460	2240	2190	2150	2060	1980
1	4	8	1610	1570	1530	1460	1400	2170	2120	2070	1970	1880
1	4	10	1510	1460	1420	1340	1260	2040	1980	1920	1810	1700
1	5	10	1310	1270	1230	1160	1090	1770	1720	1660	1570	1470
1	6	12	1060	1020	980	910	840	1430	1380	1320	1230	1140

NOTE.—Proportions are based on a barrel of 3.8 cu. ft. Values are for average ultimate strength, which must be divided by a factor of safety for working loads. Quality of materials and methods of mixing may affect the strength by 25% in either direction, while the relative values for different proportions are not materially changed.

*Use 50% columns for broken stone screened to uniform size.
†Use 45% columns for average conditions and for broken stone with dust screened out.
‡Use 40% columns for gravel or mixed stone and gravel.
§Use these columns for graded mixtures.

In the table the stone with the smaller percentage of voids gives the lower strength. This is due to the proportioning by volume. To illustrate, a cubic foot of stone measured loose with 40% voids contains more solid material than stone with 50% voids, and hence makes a greater bulk of concrete with the same proportions by volume. This is further illustrated in the table on page 234. Consequently, there is less cement in a unit volume of the concrete when the stone has 40 per cent voids; and while the density is slightly greater, it is not enough greater to counterbalance the decrease in the percentage of cement. If the proportions had been altered so as to use less sand with the stone having 40 per cent voids, the concrete would have been stronger, with the same amount of cement per cubic yard of concrete, because of the greater density.

From this it must not be inferred that the aggregate with the largest percentage of voids is best to use. As indicated above, it requires more cement to a given volume of concrete, and the concrete is apt to be slightly less dense than with an aggregate having fewer voids, so that the latter is usually the more economical even although it is sometimes slightly inferior in strength. In the example in the preceding paragraph, with Portland cement at $2 per barrel, the concrete with stone having 50% voids would require 0.11 bbl. more cement per cubic yard than the concrete with stone having 40% voids, and would therefore cost 22 cents higher per cubic yard.

The following table is presented to indicate in round numbers the probable

Approximate Average Crushing Strength of Concrete

PROPORTIONS BY VOLUME.	MEDIUM CONSISTENCY.		WET CONSISTENCY			
	Cubes.		Cubes.		8 by 16 inch Cylinders	
	30 days. lb. per sq. in.	6 mos. lb. per sq. in.	30 days lb. per sq. in.	6 mos lb. per sq. in.	30 days. lb. per sq. in	6 mos. lb. per sq. in.
1 : 1½ : 3	2800	3700	2600	4100	2300	3600
1 : 2 : 4	2500	3300	1900	3100	1700	2700
1 : 2½ : 5	2200	2900	1700	2700	1500	2400
1 : 3 : 6	1900	2600	1500	2400	1300	2100
1 : 4 : 8	1500	2100	1000	1600	900	1400

Proportions are based on the unit measure of one barrel (4 bags) cement assumed as 3.8 cu. ft. The first column of strength values is taken from the table on the opposite page; the cylinders at one month are arranged as averages of a large number of tests in various laboratories made during the years 1904 to 1908; the ratio of strength of cubes to cylinders is based upon the St. Louis tests (p. 370) and the growth of strength of wet consistency upon tests by the authors (p. 384). The ultimate strength of long columns is probably from 90 to 95 per cent of the strength of cylinders (p. 370.)

strength of different mixtures of concrete under working conditions. As stated on the opposite page, so many conditions affect the strength that such data can be presented only as extremely rough approximations.

Variation in Weight of Concrete of Different Proportions. The weights of specimens of similar concrete are of interest in comparing the relative strength of different mixtures or of different specimens of the same mixture. Of twelve pairs of duplicate cubes which the authors had tested in 1903 at the Watertown Arsenal and the Massachusetts Institute of Technology, the heavier specimen, except in one case, was found to be the stronger.

The following table of tests selected from tests of concrete and mortar cubes made by Mr. James E. Howard* at the Watertown Arsenal illustrates the comparative variation in weight and strength of concrete mixed in varying proportions:

Weights of Portland Cement Concrete of Different Proportions.

Age four months. Watertown Arsenal. (*See p.* 362.)

Item	Proportions by volume: Cement	Sand	Broken Stone†	Weight per cu. ft. lb.	Compressive strength per sq. in. lb.	Item	Proportions by volume: Cement	Sand	Broken Stone†	Weight per cu. ft. lb	Compressive strength per sq. in. lb.
1	1	1	0	136.5	4370	11	1	5	10	140.2	797
2	1	2	0	134.2	2506	12	1	6	12	138.2	738
3	1	3	0	133.8	1812						
4	1	4	0	120.9	830	13	1	2	2	140.3	1768
5	1	5	0	119.3	532	14	1	2	3	145.2	1911
6	1	6	0	116.9	169	15	1	2	4	149.1	2147
7	1	7	0	111.5	118	16	1	2	5	150.9	2452
						17	1	2	6	151.2	2124
8	1	2	4	150.7	2178	18	1	2	7	146.4	1650
9	1	3	6	146.9	1815	19	1	2	8	142.4	1295
10	1	4	8	143.2	1135						

Compressive Tests of Plain Concrete. The tests on pages 363, 367, and 366 (Fig. 119), are selected from among the best series of concrete experiments on record in America and Europe, so that the reader may form a general idea of the results obtained by expert experimenters. For practical comparisons of strength of different mixtures, reference should be made to the more complete table on page 360. The variation in strength of concretes mixed in the same proportions is due not only to the difference in the materials, but also to the different methods of making the tests, and to the fact that in many cases the unit of measurement

*Tests of Metals, U. S. A., 1899, pp. 788–795.

†Items (8) to (12), 2½ inch screened broken trap, and items (13) to (19), 1½ inch screened broken trap.

Strength of Concrete in Compression from Various Authorities. Age, one month.

In pounds per square inch. (*See p.* 362)

AUTHORITY.	Size of Specimen.	1:0	1:1	1:2	1:3	1:4	1:0:2	1:0:5	1:1:3	1:1½:4	1:2:2	1:2:3	1:2:4	1:2:5	1:2:7	1:2½:5	1:3:2	1:3:3	1:3:4	1:3:5	1:3:6	1:3:7½	1:4:3	1:4:5	1:4:8	1:5:10	1:6:12
Portland Cement Concrete.																											
[1]Hawley & Krahl *a*	6″																				1795						
[2]Henby *b*	3″ x 3½″ x 11″												2800	2500							1900				1150		
[3]Humphrey *c*	6″																				1620						
[4]Luegers													2560								1990				1710	1350	1140
[5]Von Mauk	8″	4350	3560	3200	2560	1630					2060	1570					1570	1420	920				850				
[6]Dyckerhoff *d*	4″							1000				2800		2430						1660				1290			
[7]G. A. Kimball *e*	12″						3846						2399								2164						994
[8]Taylor & Thompson *f*	12″															3255											
[9]Woodb'y & Leight'n *g*	12″																			1623							
[10]Watertown Arsenal *h*	12″	5500	4400						4917											2042							
[11]Webster	6″																				1763						
Natural Cement Concrete.																											
[12]Clarke & Son	12″									1449					746							991					
Cinder Concrete with Portland Cement.																											
[13]Watertown Arsenal	12″								1466			1098	904	769													
[14]Henby *i*	3½″ x 3½″ x 11″												1000	750							550	450					

[1]Engineering News, June 7, 1900, p. 375.
[2]Journal Association Engineering Societies, Sept., 1900, p. 145.
[3]Transactions American Society of Civil Engineers, Vol. XLVIII, p. 561.
[4]Lueger's Lexicon, Vol. II, p. 295.
[5]Portland Zement, Berlin, p. 92.
[6]Portland Zement, Berlin, p. 90.
[7]Tests of Metals, U. S. A., 1899, p. 740.
[8]Tests at Watertown Arsenal, 1903.
[9]Tests at Watertown Arsenal, 1896.
[10]Tests of Metals, U. S. A., 1898, p. 650.
[11]Tests at Philadelphia, Penn., 1903.
[12]Tests of Metals, U. S. A., 1901, p. 611.
[13]Tests of Metals, U. S. A., 1898, p. 569.
[14]Journal Association of Engineering Societies, Sept., 1900, p. 145.

*a*Drift sand, blue limestone.
*b*½″ to 2″ limestone.
*c*Gravel concrete.
*d*Stone 0.2″ to 1.2″ diameter.
*e*Average of 5 cements. Conglomerate stone.
*f*Newburyport sand ¼″ to 1″ trap.
*g*Broken trap.
*h*1″ trap.
*i*Cinders unscreened.

used in proportioning is indefinite, and, as discussed on page 218, similar nominal proportions may apply to quite different actual mixtures. Notwithstanding these opportunities for variation, however, it is noticeable that the results reached by different parties really show less percentage

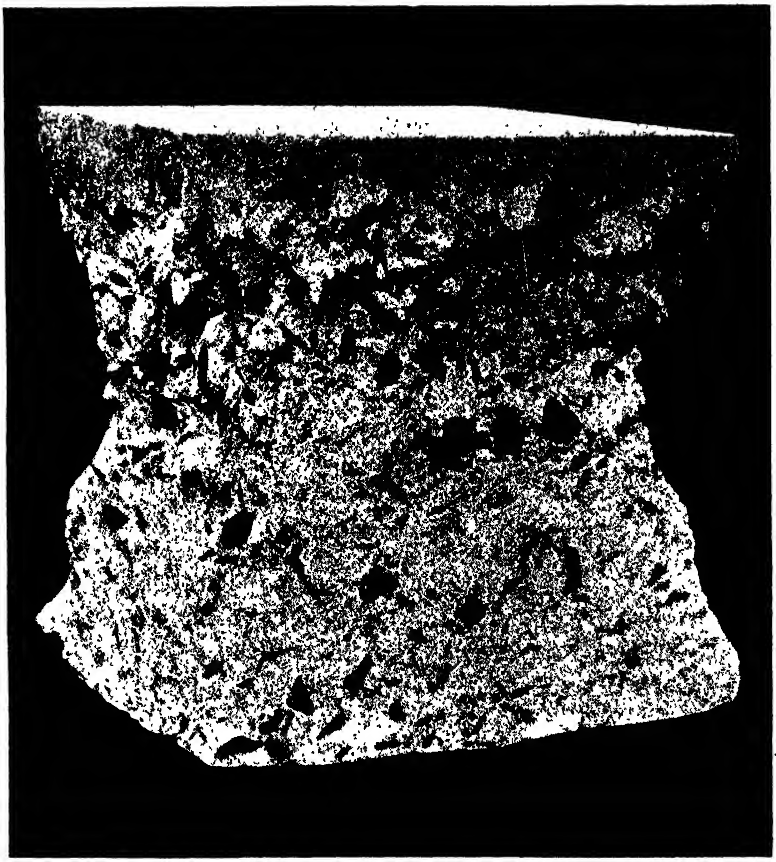


Fig. 118. Twelve-inch Concrete Cube after Crushing in Emery Testing Machine at Watertown Arsenal. (*See p.* 365.)

variation than is expected in the tensile tests of neat cements and sand mortars in different laboratories even with the same brand of cement.

In the table on page 363 of data from various authorities, only tests at the age of one month are recorded. Strength of the specimens at longer

and shorter periods may be estimated by referring to the curve in Fig. 122, page 375.

The appearance of a concrete cube after crushing, showing the manner in which the sides flake off, leaving a double pyramid, and the shearing of the particles of stone, is illustrated in Fig. 118. The specimen is one of a series tested for the authors at the Watertown Arsenal, U. S. A.

Kimball's Tests. A series of experiments upon 12-inch cubes made by Mr. George A. Kimball,* Chief Engineer of the Boston Elevated Railway Company, and tested at the Watertown Arsenal, although included in the above table, covers so wide a range in time and proportions that more complete values are worth quoting and are presented in the curves on page 366. Mr. Kimball also determined the elastic properties of these specimens, and tested some of the specimens with a concentrated load, as referred to on page 368. He states that the stone used was conglomerate from Roxbury, Mass., containing 49.5 per cent. voids. Its analysis was as follows:

Passing	2½-inch ring	100.0%
"	2-inch "	95.2%
"	1-inch "	18.5%
"	½-inch "	0.5%

The sand and cement were made into a mortar of about the consistency of damp sand, and then spread upon the stone, which previously had been drenched with water. After ramming with iron rammers and tamping bars, the water barely flushed to the surface of the 1: 0: 2 and 1: 2: 4 mixture, while the surface of the 1: 3: 6 and the 1: 6: 12 mixtures appeared merely moist, so that the concrete was what ordinarily would be termed dry. The average quantity of water used with the different mixtures in addition to the water for wetting the stone is expressed in percentages of the weight of the cement and of the cement plus sand as follows:

Percentages of Water Employed in Kimball's Tests.

		In terms of weight of cement.	In terms of weight of cement plus sand.†
Mixture	1: 0: 2	20.9%	20.0%
"	1: 2: 4	30.3%	10.7%
"	1: 3: 6	39.3%	10.5%
"	1: 6: 12	71.1%	8.6%

These percentages *do not* include the water used in wetting the stone.

The specimens were made in cold weather, and therefore set slowly.

*Tests of Metals, U. S. A., 1899, p. 717.

†Approximate.

They remained from two to seven days (most of them three to four days) in the molds, and were then placed, until tested, in wet ground. Mr. Kimball's remarks with reference to the leanest mixtures are of interest as illustrating the frequent necessity of using richer proportions than the actual loading requires.

The 1:6:12 blocks were in poor condition. This was due to the difficulty of getting so lean a mixture well rammed into the corners of molds so small as 12-inch, and to the fact that the concrete had not attained sufficient strength, even though handled with care, to hold together well in the process of removal from the molds. The cubes of this mixture should have had a longer time to set before taking them out of the forms. In our foundation work we have used this mixture only as a filling with which to replace soft ground and on which to build the foundations proper.

The diagram in Fig. 119 shows Mr. Kimball's resultant curves* for the

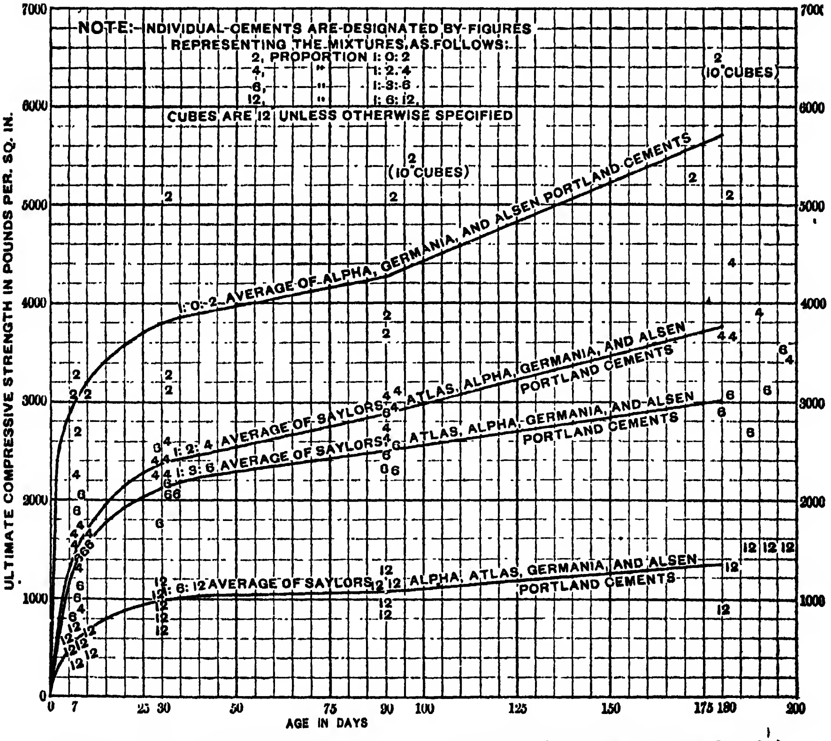


FIG. 119.—Tests on Concrete Cubes by Geo. A. Kimball (Watertown Arsenal, 1889). (*See p.* 365.)

*From data presented to the authors by Mr. Kimball.

different proportions based on an assumed weight of cement of 100 lb. per one cubic foot at the various ages. The results from individual brands of cements are shown by separate points.

Candlot's Tests. The table below, giving results of tests by Mr. E. Candlot,* of France, converted into English units, is of special value because of the accuracy in recording the data, the extreme variation in proportions and the number of periods at which specimens were

Tests of Strength of Concrete made with Different Proportions.

BY E. CANDLOT. (*See p.* 367.)

Proportions based on packed cement†	Volume of mortar in terms of percentage of volume of stone	Actual quantity of materials				Gravel concrete							Broken stone concrete						
									Ultimate compressive strength in lb. per sq. in.							Ultimate compressive strength in lb. per sq. in.			
		Cement	Sand	Stone	Water	Volume of Concrete	Cement in 1 cu. ft. of concrete	Weight per cu. ft. of concrete after setting	7 Days	28 Days	6 Months	1 Year	Volume of Concrete	Cement in 1 cu. ft. of concrete	Weight per cu. ft. of concrete after setting	7 Days	28 Days	6 Months	1 Year
	%	lb.	cu.ft.	cu.ft.	cu.ft.	cu.ft.	lb.	lb.					cu.ft.	lb.	lb.				
1:6.4: 8.2	67	551	35.3	45.0	6.36	58.3	9.5	144.8	1031	1387	1280	1292	54.8	10.1	142.3	1316	1600	1636	1943
1:3.6: 4.7	67	992	35.3	46.6	7.42	61.1	16.2	147.3	1458	2454	2583	3225	56.9	17.4	147.0	2340	2845	3319	3506
1:2.5: 3.6	67	1433	35.3	50.0	8.55	65.0	22.0	150.4	2312	3094	3485	4385	61.1	23.4	149.8	2845	3485	4883	5020
1:1.6: 2.8	67	2205	35.3	62.0	10.77	78.0	28.0	149.8	2632	3414	3579	5500	73.0	30.6	151.0	3985	4303	4623	5974
1:6.4:10.9	50	551	35.3	60.1	6.36	67.8	8.1	142.3	747	924	1031	1707	63.6	8.7	142.3	1316	1387	1494	1683
1:3.6: 6.3	50	992	35.3	62.2	7.42	70.6	14.0	145.4	1743	1091	2530	2064	67.1	14.7	146.6	2008	2241	2845	3201
1:2.5: 4.7	50	1433	35.3	67.8	8.55	73.8	19.4	149.1	2169	3058	3532	4505	70.6	20.3	148.5	2276	3414	3627	5262
1:1.6: 3.7	50	2205	35.3	82.6	10.77	91.1	24.2	150.4	2952	3592	4054	5050	86.2	25.5	151.0	3556	3982	4338	5572
1:6.4:13.6	40	551	35.3	75.0	6.36	79.5	6.9	141.0	676	924	1078	1375	70.6	7.8	143.5	1280	1316	1138	1778
1:3.6: 7.8	40	992	35.3	77.7	7.42	84.8	11.7	142.3	1031	1494	1518	2608	78.8	12.6	142.3	1494	1778	2347	2822
1:2.5: 5.0	40	1433	35.3	84.8	8.55	90.4	15.9	145.4	1245	1902	2654	3247	85.5	16.7	146.0	2205	2525	2963	3201
1:1.6: 4.7	40	2205	35.3	103.3	10.77	106.7	20.7	149.2	2454	2560	3319	4503	102.4	21.5	146.6	2560	3200	3532	3936

NOTE.—The gravel weighed 96.8 lb. per cu. ft. and contained 40% voids. The broken stone weighed 85.5 lb. per cu. ft. and contained 47.4% voids. Both the gravel and broken stone had been passed through a screen having meshes of $1\frac{1}{2}''$ diameter. The sand weighed 81.2 lb. per cu. ft., thus containing 50.4% voids, and had been passed through a No. 12 sieve. The cubes were 10 centimeters (4 in.) on an edge.

crushed. The application of these tests to the authors' formula for strength is discussed on page 357.

The Effect of Concentrated Loading. In concrete foundations for piers and in concrete footings it is customary to load an area smaller than that of the surface of the concrete. The question at once arises whether the stress shall be based upon the load divided by the total area of the concrete footing or by the area of contact. Experiments made upon concrete and other materials show that neither of these methods is correct, but that an intermediate area should be selected for computation.

*Candlot's Ciments et Chaux Hydrauliques, 1898, pp. 446, 447.

†Assuming 3.8 cu. ft. in 1 bbl of 376 lb.

In connection with the designing of concrete footings for the Boston Elevated Railway, 12-inch cubes were crushed by concentrating the load upon plates 10 by 10 inches and 8 by 8¼ inches.* At Lehigh University in 1908 a set of experiments was made upon the strength of 6 by 6 inch cubes of 1:2:4 proportions where the compressed area varied from the entire area of the specimen down to 1.21 square inches.

In the diagram, Fig. 120, both sets of values† are plotted. The two sets agree where they overlap, and also are similar in general direction, and, in fact, in actual values of the ordinates, to curves drawn by Prof. J. B. Johnson‡ illustrating Bauschinger's tests upon other materials than concrete.

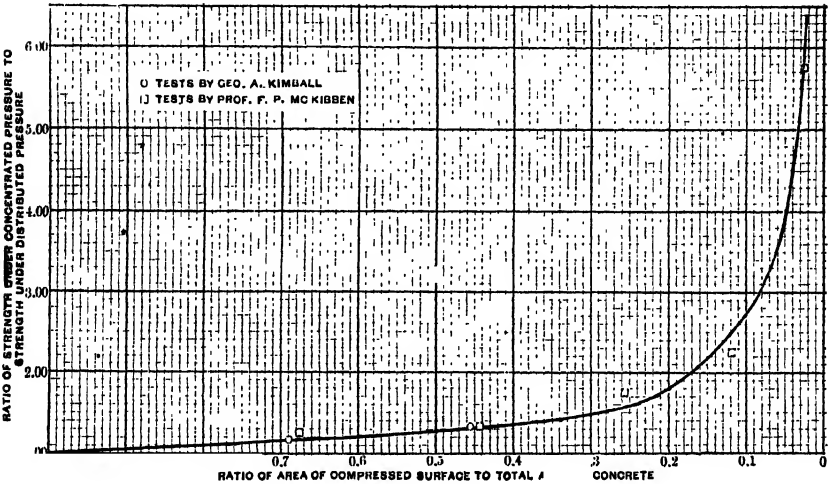


FIG. 120. Concentrated vs. Distributed Loading. (*See p.* 368.)

In considering the smaller areas, as indicated by the smaller ratios of area, the fact must be considered that the compressed surface deforms, that is, actually compresses under the load, and the amount of deformation, which may be approximately estimated from the modulus of elasticity, may sometimes be the limiting consideration. Also, in the small areas the possibility of punching through must be considered.

The method of using the curve shown in Fig. 120 is best illustrated in the following examples:

* Tests of Metals, U. S. A., 1899, p. 740.

† From data presented to the authors by Mr. George A. Kimball and by Prof. Frank P. McKibben.

‡ Johnson's Materials of Construction, p. 33.

Example 1.—What dimensions of pedestal would be required to safely support a load of 40 tons concentrated upon a plate 10 inches square, assuming an allowable distributed stress upon the concrete of 650 lb. per square inch?

Solution.—Forty tons or 80 000 pounds on 100 square inches represents 800 lb. per square inch, and the ratio of pressure required under the concentrated load to the allowable pressure is therefore $\frac{800}{650} = 1.23$; hence from the curve, the total area of concrete necessary is $\frac{100 \text{ sq. in.}}{0.55} = 182$ square inches.

Example 2.—The breaking strength of a 12-inch cube of 1 : 2 : 4 concrete having chamfered edges, so that the area of contact of the load is reduced to 9 by 9 inches, or 81 square inches, is 324 000 pounds. What may be considered as the ultimate strength of the concrete when loaded over its full area?

Solution.—The strength per square inch of the cube figured on its chamfered surface is $\frac{324\ 000}{81} = 4\ 000$ lb. per square inch. The ratio of the compressed surface to the total area is $\frac{81}{144} = 0.56$, and from the diagram we find the ratio of strength to be 1.22. Dividing 4 000 pounds, the unit strength on the concentrated surface by this gives as the probable ultimate of the concrete when loaded over its full area, 3 280 lb. per square inch.

The Strength of Short Prisms. The theoretical angle of rupture in crushing is about 60° with the horizontal, and, as a matter of fact, cubes or prisms of concrete will leave, after crushing, pyramids whose surfaces are at an angle of about 60° with the base. To develop simply the normal compressive strength, the height of a specimen should be at least $1\frac{1}{2}$ times, and preferably 5 times, its least lateral dimension.

The following formula evolved by Prof. Johnson* by plotting results of experiments by Prof. Bauschinger with sandstone prisms, and by Mr. Charles Bouton with cast-iron prisms, may be used for comparing approximately the strength of prisms and cubes. Prof. Johnson states that the law holds between ratios of height to breadth of 0.4 to 5.0, the limits of the observations.

$$\frac{\text{strength of prism}}{\text{strength of cube}} = 0.778 + 0.222\frac{b}{h} \quad \ldots\ldots (3)$$

where b = least lateral dimension of specimen,

and h = height of specimen.

* Materials of Construction, 1903, p. 31.

Although we have not sufficient data to prove that this formula is exactly applicable to concrete, a study by the authors of tests at the Watertown Arsenal* tends to show that, considering the variability of the material, it is probably sufficiently accurate for practical use. In the Arsenal experiments square prisms were employed, varying in cross-section from 4 by 4 inches to 12 by 12 inches and ranging in height from 1 to 2 inches up to that of a cube. In every case the shorter prisms gave much higher strength than the cubes.

Example.—If the compressive strength per square inch of a 12-inch cube is 4 000 lb., what strength may be expected from a prism 12 inches square and 18 inches high?

Solution.—Substituting in formula (3), we have

$$\frac{x}{4000} = 0.778 + 0.222\tfrac{12}{18}$$

$$x = 3704$$

Theoretically, specimens of the same shape, as, for example, all sizes of cubes, should have the same strength per unit of area. In practice, large concrete cubes are apt to show higher unit strength than smaller ones; experiments by the authors, for example, giving in every case higher unit strength for 12-inch than for similar 8-inch cubes. However, the average unit weight of the 8-inch cubes was much lower than that of the 12-inch cubes made from the same batches of materials, indicating the difference in strength to be due to the fact that the materials can be more compactly placed in a large than in a small mold.

The standard compression specimen adopted by the Joint Committee on Concrete and Reinforced Concrete is a cylinder 8 inches in diameter by 6 inches long.

Strength of Cubes vs. Cylinders vs. Columns. Computations from the United States Government tests at St. Louis† comparing the strength of 6 inch cubes and standard cylinders 8 inches diameter by 16 inches long gives a ratio of strength of cylinders to cubes at ages of thirteen and twenty-six weeks as 0.88. This coincides almost exactly with the above formula.

But few comparative tests of cylinders and columns are available, but these indicate that the above formula is fairly correct and on the safe side when comparing the probable strength of a column with the given strength of a cylinder.

* Quoted and tabulated by Committee on Compressive Strength of Cements of the American Society of Civil Engineers in Transactions, Vol. XVIII, p. 264.

† U. S. Geological Survey, Bulletin 344, 1908.

Plain Concrete Columns. There are few comparative records of the strength of concrete columns of different heights, but both theory and experiments tend to show that there is no appreciable difference in the compressive strength of columns of heights differing within ordinary limits, ranging, say, from a height of 3 to 14 times the least lateral dimension, provided the loading is exactly central. Prussian regulations,* 1904, require that computation shall be made for flexure, if the height exceeds 18 times the least diameter.

In 1897 tests were made at the Watertown Arsenal† on 12 by 12 inch columns of plain concrete, built by the Aberthaw Construction Company,

Compressive Strength of Mortar and Concrete Columns
Length of Columns 8 feet
Watertown Arsenal (See p. 371.)

Nominal size of column	Composition: Cement	Sand	Stone	Kind of stone	Age: Months	Days	Weight lb. per cu. ft.	Strength lb. per sq. in.	Date of test and reference to Tests of Metals
10″ Diameter	Neat	0	0	None	10	25	129	*a* 7000	1907 p. 186
10″ Diameter	1	1	0	None	6	11	152	4320	1906 p. 473
12″ × 12″	1	2	0	None	6	0	130	3070	1905 p. 379
12″ × 12″	1	1	1	¼″ to ½″ trap	7	10	142	3522	1907 p. 182
12″ × 12″	1	1	2	½″ to 1½″ trap	5	7	154	3000	1905 p. 331
10″ Diameter	1	1½	3	¾″ to 1½″ trap	10	25	152	3576	1907 p. 192
12″ × 12″	1	2	4	½″ to 1½″ trap	6	5	150	1990	1905 p. 334
12″ Diameter	1	3	6	½″ to 1½″ trap	5	5	146	*b* 1446	1906 p. 535
12″ Diameter	1	3	6	Cinders	5	0	101	698	1906 p. 537

a Maximum load applied, column not ruptured.
b A similar column failed at 750 lb. per sq. in. but the lower end of this column was less sound than the upper part because of leakage of the mold.

ranging from 2 to 14 feet in length. The results of these tests concur with the theory of columns in showing that up to at least 14 diameters there is but little decrease in strength as the length of the column increases.

The table presented above gives results selected from tests made by Mr.

* See *Engineering Record*, July 2, 1904, p. 25.
† Tests of Metals, U. S. A., 1897, p. 383.

Howard at the Watertown Arsenal* in 1905, 1906 and 1907, on concrete and mortar columns. Generally the first sign of failure in the columns appeared in the form of oblique and longitudinal cracks, occurring usually from 0 to 3 feet distant from one end, although sometimes extending the entire length.

A comparison of the strength of plain and reinforced columns is presented in the next chapter.

Strength of Machine vs. Hand Mixed Concrete. Mixing in a well designed machine produces a more homogeneous concrete than is possible by hand except with excessive labor. The relative strength of the concrete of course varies with the conditions, but tests indicate that ordinarily 10 to 20 per cent greater strength may be expected in a first-class, machine mixed concrete, properly handled. It is probable that this more thorough mixing at least balances the extra care given to laboratory specimens, so that in ordinary practice, strength as great, if not greater, than in the laboratory, may be expected

Eccentric Loading. The effect of eccentric loading, that is, of having the center of gravity of the load one side of the center of the column, is to lessen its compressive strength. A similar effect is produced by loading a column already bent, or by constructing it of unsymmetrical shape, as by bulging one side.

Most columns in actual structures are loaded more or less eccentrically, and this is especially the case with wall columns, which have all the floor loading upon one side. This must be allowed for in designing the columns.

The ordinary formula for the compressive fiber stress due to eccentric loading upon solid rectangular columns, as illustrated in Fig. 121, is as follows:

Let

P = total load.

A = area of columns.

e = eccentricity.

b = breadth of column.

f = average unit pressure.

f' = total unit pressure on outer fiber nearest to line of vertical pressure.

Then

$$f' = \frac{P}{A}\left(1 + \frac{6e}{b}\right) \tag{4}$$

The use of the formula is illustrated by the following example.

*Tests of Metals, U. S. A., 1905, 1906, 1907.

FIG. 121. Eccentric Column Loading. (See p. 372.)

Example. — What will be the increase in pressure in a column 2 feet square due to placing the loading 6 inches off center?

Solution. — With central loading the pressure is, $f = \dfrac{P}{A}$ hence

$$f' = f\left(1 + \frac{6e}{b}\right)$$

Substituting the values $e = 0.5$ and $b = 2$

$$f' = 2\tfrac{1}{2} f$$

that is, the pressure on outer fibre is increased $2\frac{1}{2}$ times.

Concrete vs. Brick Columns. The compressive strength of brick piers is of interest to the concrete engineer for comparing brick and concrete columns. Tests made at the Watertown Arsenal and quoted by the Committee of the American Society of Civil Engineers on the Compressive Strength of Cement* give the ultimate strength of common brick piers about eighteen months old as ranging from 800 to 2 400 pounds per square inch, the results for brick laid with lime mortar averaging nearer the lower figure, and those for 1 : 2 Portland cement mortar nearer the higher figure.

Prof. William H. Burr,† after discussing the strength of brick piers under various conditions, states that

> The results of all the experimental investigations available in connection with brick masonry and experiences in the best class of engineering work indicate that masonry laid up of good hard-burnt common brick may safely carry a working load of 15 to 20 tons per square foot or 210 to 280 pounds per square inch. In the construction of this class of masonry where the duties are to be severe it is of the utmost importance that the best class of Portland cement mortar be employed, as the carrying capacity of brick masonry depends largely, if not chiefly, upon the character of the mortar.

These working stresses are about one-half those recommended for good 1 : 2 : 4 concrete in the chapter which follows.

More recent tests by Professors Talbot and Abrams‡ indicate that the strength of the brick column varies with the quality of the brick, the quality of the mortar and the care in laying.

SAFE STRENGTH OF CONCRETE

The working strength to be used for concrete is fully discussed in the

* Transactions American Society of Civil Engineers, Vol. XV, p. 717, and Vol. XVIII, p. 264.
† Burr's Materials of Engineering, 1903, p. 428.
‡ University of Illinois, Bulletin No. 27, Sept. 1908.

chapter which follows. For proportions and conditions differing from those presented there, reference may be made to the relative strengths discussed in the preceding pages.

In many structures the actual strength of the concrete does not enter into the calculation. The dimensions of a concrete foundation, for example, are often determined by the area of the superimposed structure, or else, on the other hand, by the bearing power of the soil. In such cases it often would be theoretically possible to come nearer to the working strength of the concrete by using very lean proportions, were it not prohibited by the porosity of the mass or its low strength at short periods. However, by grading the materials so as to reduce the voids, a lean mixture is often economical.

The unit pressure to be selected depends not only upon the strength of the concrete as determined by its proportions, the character of the raw materials, and the methods of mixing, but also upon the character and importance of the structure, the nature of the pressure,—whether by direct compression or bending, whether from a live or dead load, or whether acting directly or through a cushion of inert material,—and the time of setting before placing the load.

GROWTH IN STRENGTH OF CONCRETE

Records from various tests made upon similar specimens of concrete at different periods are plotted in the diagram, Fig. 122. The curve illustrates the growth in strength which may be expected in ordinary average concrete made with first-class materials. The ordinates on the diagram represent ratios of the strength at various periods to the strength at the age of one month, in order that the curve may be of general application to various mixtures. If, for example, the strength of any concrete at one month is found to be 2 000 pounds per square inch, the strength of the same concrete at the age of six months may be assumed to be 2 000 multiplied by 1.35, the ordinate at six months, or 2 700 pounds per square inch.

The curve does not allow for the fact that the growth in strength varies to a certain extent with different materials, with different proportions, and with different percentages of water employed in mixing. As stated on page 386, with age, the strength of gravel concrete appears to gain on the strength of broken stone concrete. The growth, too, at periods beyond, say three months, is undoubtedly affected by the hardness or strength of the particles of the coarse aggregate, since a concrete of poor material will reach its ultimate strength earlier than one of good material. The tests of Mr. Kimball (see page 366) tend to show that the increase with age

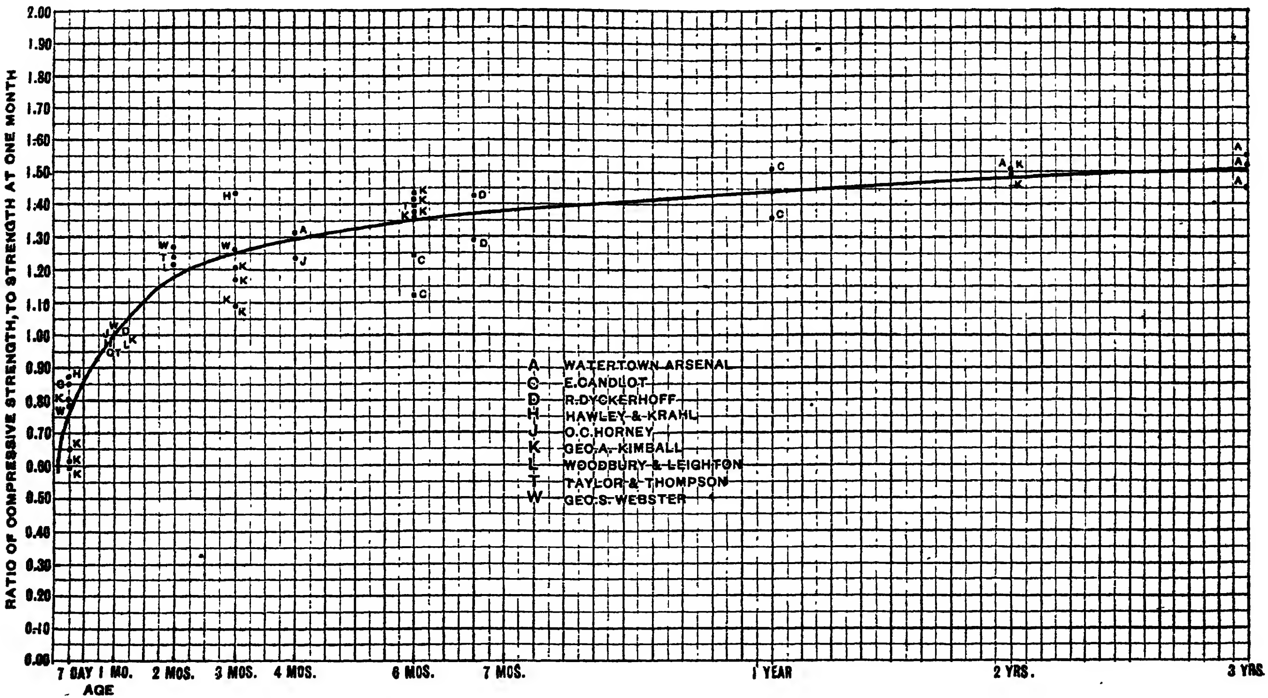


Fig 122 — Growth in Compressive Strength of Portland Cement Concrete. (See p. 374.)

Data Concerning Composition and Transverse Strength of Concrete Beams Tested at Little Falls, N. J., by Wm. B. Fuller, C. E.

During the year 1901. Beams, 6x6x72 inches. Spans, 30 and 60 inches. Atlas Portland Cement, River Silica Sand. Crusher Run Trap Rock, ¼ to 3 inches nominal diameter. (*See p.* 378.)

Item.	Proportions by weight. C.S G	Weight in Pounds of Material in one cu. ft. of Beam as Mixed. Cement.	Sand	Stone.	Total Dry Mix.	Water.	Totals. As Mixed.	Totals. Minimum.	Totals. Maximum	Calculated Volume, in cu. ft. of Material in one cu. ft. of Beam as mixed. Cement.	Sand.	Stone.	Total Sand and Stone.	Total Dry.	Water, 62.4.	Total.	Volume of Voids in one cu. ft. Cement.	Aggregate	Total.	Modulus of Rupture. Age. Days.	Number of Breaks.	Pounds per sq. in. Maximum.	Minimum.	Average.	Per cent. probable error of average.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
(1)	1:0:0	112.9			112.9	24.1	137.0	121.9	138.8	.585				.585	.386	.971	.415	.000	.415	32	6	968	856	906	1.0
(2)	1:0:1	69.1		69.1	138.2	15.4	153.6	143.7	155.2	.358		.370	.370	.728	.247	.975	.254	.018	.272	32	6	872	668	772	2.8
(3)	1:0:2	49.9		99.8	149.7	12.9	162.6	153.7	162.6	.259		.534	.534	.793	.207	1.000	.183	.024	.207	33	6	802	668	731	2.4
(4)	1:0:3	38.0		113.8	151.8	11.2	163.0	154.8	163.9	.197		.609	.609	.806	.180	.986	.140	.054	.194	34	6	724	580	622	2.4
(5)	1:0.4	27.4		109.4	136.8	9.7	146.5	139.0	154.0	.142		.585	.585	.725	.155	.880	.101	.174	.275	34	3	251	236	241	2.3
(6)	1:1:0	64.9	64.9		129.8	15.3	145.1	135.0	146.7	.336	.393		.393	.729	.245	.974	.238	.033	.271	34	6	866	628	734	3.8
(7)	1:1:1	47.0	47.0	47.1	141.1	12.3	153.4	144.9	154.8	.244	.285	.252	.537	.781	.197	.978	.172	.047	.219	34	6	744	649	708	1.6
(8)	1:1:2	37.2	37.2	74.4	148.8	10.3	159.1	151.8	160.3	.193	.225	.398	.623	.816	.165	.981	.136	.048	.184	33	6	798	646	710	3.0
(9)	1:1:3	30.1	30.2	90.4	150.7	10.8	161.5	153.1	161.8	.156	.183	.483	.666	.822	.173	.995	.110	.068	.178	34	6	732	573	655	2.3
(10)	1:1:4	25.9	25.9	103.6	155.4	9.7	165.1	157.5	165.1	.134	.157	.554	.711	.845	.155	1.000	.095	.060	.155	33	6	512	446	486	1.9
(11)	1:1:5	22.6	22.6	113.0	158.2	7.8	166.0	160.0	167.1	.117	.137	.604	.741	.858	.125	.983	.083	.059	.142	34	6	542	481	504	1.6
(12)	1:2:0	43.5	86.9		130.4	12.9	143.3	133.9	145.9	.225	.527		.527	.752	.207	.959	.160	.088	.248	33	6	640	592	616	0.9
(13)	1:2:1	34.1	68.3	34.1	136.5	12.9	149.4	139.2	150.7	.177	.414	.182	.596	.773	.207	.980	.125	.102	.227	33	6	572	459	523	2.6
(14)	1:2:2	28.6	57.1	57.1	142.8	11.7	154.5	145.1	155.3	.148	.346	.305	.651	.799	.188	.987	.105	.096	.201	33	6	552	485	528	1.8
(15)	1:2:3	25.3	50.6	76.0	151.9	7.4	159.3	153.9	161.6	.131	.307	.406	.713	.844	.119	.963	.093	.063	.156	33	1			471	∞
(16)	1:2:4	22.3	44.7	89.4	156.4	7.4	163.8	158.2	164.8	.116	.271	.478	.740	.865	.119	.984	.082	.053	.135	33	6	480	399	439	3.0
(17)	1:2:5	19.8	39.5	98.5	157.8	7.4	165.2	159.4	166.0	.103	.239	.527	.766	.869	.119	.988	.073	.058	.131	33	6	413	349	380	2.2
(18)	1:2:6	17.5	35.0	105.1	157.6	8.2	165.8	159.0	166.0	.091	.212	.562	.774	.865	.131	.996	.064	.071	.135	35	5	410	234	319	9.5

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
(19)	1:3:0	32.0	95.8		127.8	12.2	140.0	130.4	143.6	.166	.581		.581	.747	.196	.943	.117	.136	.253	35	6	432	392	418	1.2
(20)	1:3:1	27.4	82.3	27.4	137.1	10.9	148.0	139.3	150.4	.142	.499	.146	.645	.787	.175	.962	.101	.112	.213	35	5	392	274	360	3.3
(21)	1:3:3	21.0	63.2	63.1	147.3	8.6	155.9	149.0	158.0	.109	.383	.337	.720	.829	.138	.967	.077	.094	.171	33	6	369	338	355	1.2
(22)	1:3:5	16.6	49.9	83.2	149.7	8.0	157.7	151.0	160.1	.086	.302	.445	.747	.833	.128	.961	.061	.106	.167	33	3	308	262	285	3.3
(23)	1:3:6	15.3	45.9	91.9	153.1	8.5	161.6	154.3	162.6	.079	.278	.491	.769	.848	.136	.984	.056	.096	.152	33	5	246	213	226	2.1
(24)	1:3:7	14.3	42.8	100.0	157.1	8.2	165.3	158.2	165.3	.074	.259	.535	.794	.808	.132	1.020	.053	.079	.132	33	2	257	220	239	5.4
(25)	1:3:8	13.1	39.5	105.0	157.6	7.2	164.8	158.6	165.8	.068	.239	.561	.800	.868	.115	.983	.048	.084	.132	33	4	192	159	178	2.9
(26)	1:3:9	12.1	36.3	109.1	157.5	6.5	164.0	158.5	165.0	.063	.220	.583	.803	.866	.104	.970	.044	.090	.134	33	3	176	123	145	8.2
(27)	1:4:0	25.3	101.1		126.4	11.3	137.7	128.4	142.4	.131	.613		.613	.744	.181	.925	.093	.163	.256	33	6	294	262	279	1.8
(28)	1:4:2	18.9	75.7	37.0	132.5	12.8	145.3	134.0	147.5	.098	.450	.203	.662	.760	.205	.965	.069	.171	.240	34	4	235	198	210	3.1
(29)	1:4:4	15.8	63.0	63.1	141.9	10.7	152.6	143.2	154.3	.082	.382	.337	.719	.801	.171	.972	.058	.141	.199	34	3	219	202	209	1.8
(30)	1:4:6	13.6	54.2	81.4	149.2	10.0	159.2	150.3	159.0	.070	.328	.435	.763	.833	.160	.993	.050	.117	.167	34	2	184	114	149	16.7
(31)	1:4:7	12.7	50.6	88.6	151.9	9.5	161.4	152.9	161.4	.066	.307	.474	.781	.847	.153	1.000	.047	.106	.153	34	2	190	170	181	4.3
(32)	1:4:8	11.7	46.8	93.5	152.0	9.7	161.7	152.0	161.6	.061	.284	.500	.784	.845	.155	1.000	.043	.112	.155	34	2	158	156	157	0.4
(33)	1:4:9	11.1	44.3	90.8	155.2	8.7	163.9	156.1	163.0	.058	.268	.534	.802	.860	.140	1.000	.041	.099	.140	34	2	127	120	124	2.0
(34)	1:4:10	10.6	42.6	106.5	159.7	7.3	167.0	160.5	107.0	.055	.258	.570	.828	.883	.117	1.000	.039	.078	.117	34	2	133	130	132	0.8
(35)	1:5:0	20.9	104.7		125.6	13.0	138.6	127.3	141.6	.108	.635		.635	.743	.208	.951	.077	.180	.257	33	4	180	170	173	1.0
(36)	1:5:3	15.5	77.5	46.5	139.5	10.8	150.3	140.7	152.0	.080	.470	.249	.710	.799	.173	.972	.057	.144	.201	34	2	153	149	151	0.9
(37)	1:5:5	13.4	67.1	67.1	147.6	10.2	157.8	148.7	157.9	.060	.407	.359	.766	.835	.163	.998	.049	.116	.165	34	2	163	159	161	0.9
(38)	1:5:7	11.8	58.8	82.3	152.0	8.0	161.8	153.5	161.8	.061	.356	.440	.796	.857	.143	1.000	.043	.100	.143	34	2	134	123	129	3.0
(39)	1:5:9	10.6	53.3	95.0	159.8	6.8	166.6	160.6	166.6	.055	.323	.513	.836	.891	.109	1.000	.030	.070	.109	33	2	113	105	109	2.6
(40)	1:5:11	0.3	40.7	102.6	158.0	7.5	166.1	159.3	166.1	.048	.283	.549	.832	.880	.120	1.000	.034	.086	.120	33	2	120	113	116	2.1
(41)	1:6:0	17.4	104.3		121.7	13.8	135.5	123.1	130.0	.090	.632		.632	.722	.221	.943	.064	.214	.278	33	2	94	92	93	0.8
(42)	1:6:2	14.5	87.1	29.1	130.7	12.4	143.1	131.9	145.7	.076	.528	.156	.684	.760	.199	.950	.053	.187	.240	33	2	102	102	102	∞
(43)	1:6:4	13.1	78.4	52.3	143.8	10.8	154.6	144.8	154.8	.065	.475	.280	.755	.823	.173	.996	.048	.129	.177	33	2	115	111	113	1.2
(44)	1:6:6	11.2	67.5	67.5	140.2	10.0	150.2	147.1	150.0	.058	.400	.301	.770	.828	.160	.988	.041	.131	.172	33	1			78	∞
(45)	1:6:8	10.2	61.5	82.0	153.7	8.5	162.2	154.5	162.2	.053	.373	.438	.811	.864	.136	1.000	.037	.099	.136	33	1			84	∞
(46)	1:6:10	0.3	56.0	93.2	158.5	7.2	165.7	150.2	165.7	.048	.330	.408	.837	.885	.115	1.000	.034	.081	.115	33	2	91	87	89	1.6
(47)	1:7:0	15.6	100.2		124.8	12.8	137.6	126.0	140.8	[illegible]	.662		.662	.745	.205	.948	.057	.200	.257	33	1			95	∞
(48)	1:8:0	14.0	112.4		120.4	11.5	137.0	127.5	[illegible]	.275	.681		.681	.754	.184	.938	.051	.105	.246	33	1			41	∞

Volumes, cu. ft. per 100 lb., as mixed. — cement 1.00, sand 1.08, stone 1.02. Specific gravity cement paste, 1.81; cement, 3.09; sand, 2.64; stone, 2.00. Weights, pounds per cu. ft. as mixed,—cement, 100; sand, 93; stone, 98; water, 62.4. Tensile strength of cement, lb. per sq. in., neat; 7 days, 834; 28 days, 919. Mechanical Analyses.—Per cent by weight, of grains below diameter in inches; sand, 100%, 0.25; 90%, 0.10; 75%, 0.06; 50%, 0.028; 25%, 0.014; 0%, 0.003; stone, 100% 2.1; 0%, 0.17. Col. 9 = Col. 3 x 0.98 + Col. 6. Col. 10 = Col. 6 + Col. 20 x 62.4. Cols. 9 and 10 represent minimum and maximum weights per cubic foot.

is greater with rich than with lean concrete, but on the other hand, tests of specimens made at the Watertown Arsenal indicate the reverse. The difference is slight in both cases, however, and it may be assumed for practical purposes that the rate of growth is approximately the same whatever the proportions. A wet consistency of the concrete produces lower strength, especially at early periods, and a larger percentage of growth than is indicated in the diagram. (See page 383.)

The curve does not apply to concretes of Natural cement mortar. 12-inch cubes of concrete in various proportions made from Akron Star cement tested at the Watertown Arsenal for William Wirt Clarke & Son* show an average ratio of increase in strength between one month and one year of 1.96. With this series of specimens the average strength at the age of one year was no greater than at seven months, but this is probably an exceptional case, since, for instance, tests by Capt. William M. Black on Natural cement concrete show a slower and continual growth, with an equally large ultimate strength.

TRANSVERSE STRENGTH OF CONCRETE

The strength of a beam of plain concrete is limited by the tensile strength of the concrete at the place of greatest strain, which, with vertical loading, is its lowest surface. The value of this transverse "fiber" strength or modulus of rupture is of less importance than the crushing strength, because, on account of the brittleness of concrete in tension, that is, its liability to crack from shrinkage or sudden loading, it is seldom safe, and usually is not economical, to construct beams or girders without metal reinforcement. Most formulas for reinforced design disregard the tensile strength of the concrete. In certain computations, however, the tensile strength must be considered. Since concrete beams can be broken with less powerful and less expensive apparatus than crushing specimens, this form of specimen is often convenient for comparing the relative strength of different mixtures or different materials, and while the ratios thus obtained will not exactly coincide with those for crushing strength, they will be sufficiently close for many purposes.

Fuller's Beam Tests. The table† on page 376 gives the results of a comprehensive series of tests of 6 by 6 by 72-inch beams made by Mr. William B. Fuller at Little Falls, N. J. Although different materials than those used by Mr. Fuller will of course show slightly different strength, the table is sufficiently representative of average conditions to permit its use for comparisons of different proportions, and, with a proper

*Tests of Metals, U. S. A., 1901, p. 609.

†Especially prepared for this treatise by Mr. Fuller.

factor of safety, as a working guide to the safe transverse strength of concrete.

The proportions are given by weight but can be transformed to volume measure by referring to the footnote. The various columns present valuable data on weights and volumes and voids.

The curves in Fig. 123 are plotted from the results in the table, and illustrate also the proportions corresponding to maximum strength for a given per cent. of cement.

Tests by other authorities are mentioned under Strength of Beams in References, Chapter XXXI.

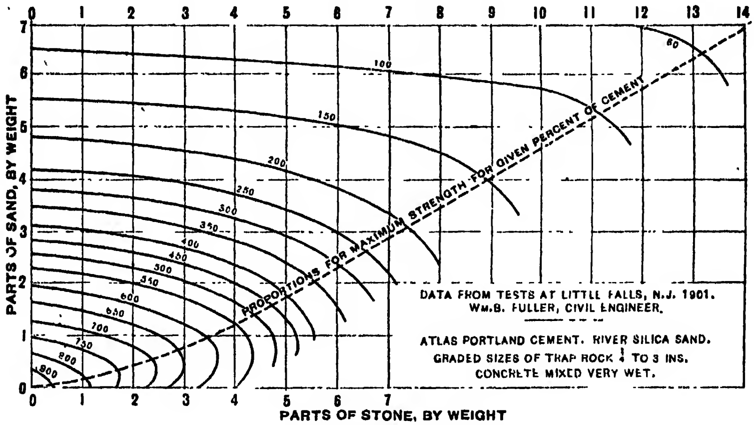


FIG. 123. Curves showing strength of beams in pounds per square inch for various proportions by weight of sand and stone to one part Portland cement. Age 34 days

Formula for Transverse or Bending Stress in Plain Concrete. The common formulas for representing the longitudinal forces of compression and tension upon a beam are usually expressed with the following notation:

Let

f = intensity of stress at any point in the beam.

M = bending moment.

I = moment of inertia about its neutral axis of section containing the point under consideration.

y = distance of the point from the neutral axis.

b = breadth of beam.

h = height of beam.

Then $f = \frac{My}{I}$ (5) also, $M = \frac{fI}{y}$ (6)

For rectangular sections, $I = \frac{bh^3}{12}$ and up to the elastic limit for beams of homogeneous material (but not for reinforced beams), $y = \frac{1}{2} h$.

Hence for rectangular beams of homogeneous material,

$$f = \frac{6M}{bh^2} \qquad (7) \qquad\qquad \text{also, } M = \frac{1}{6} f b h^2 \qquad (8)$$

In considering the strength of a beam, since the stress is greatest at one or the other of the surfaces, y is generally understood to represent the distance of the most strained fiber from the neutral axis, and f the intensity of stress upon this fiber.

The neutral axis — which is the line formed by the intersection of any cross section with the neutral plane, the plane upon which there is no longitudinal stress of either tension or compression — in a beam of homogeneous material passes through the center of gravity of the cross section. This is true for mortar and concrete which contain no reinforcement in the earlier stages of loading. Since, however, the neutral axis passes through the center of gravity of the beam only within the elastic limit,* the fiber stress, f, at the breaking point, as obtained by the common formula, does not represent the actual tensile stress upon the material. The comparative relations between different results, however, are unaffected by this limitation of the law, and the formula can therefore be used for comparing the strength of beams composed of similar material. For example, while the stresses at the instant of breaking, that is, the moduli of rupture, as figured by the formula, are not strictly correct either for 8 or 10 inch beams, they are nearly *proportional* to the actual stresses, so that the strength of plain concrete beams of different dimensions may be compared by means of the formula without appreciable error.

For convenience in designing, a table is given in Chapter XXI for bending moments caused by uniformly distributed loads and for loads concentrated at different points. Also, in the same chapter, the moments of inertia, I, for various sections are tabulated. These tables are applicable for the most part to both plain and reinforced beams.

* Although concrete and mortar have no true elastic limit the general principles apply to beams of these materials.

Relation of Transverse to Compressive Strength of Concrete. There is no fixed relation between the tensile fiber stress of concrete beams and the crushing strength of specimens made from the same material under identical conditions. The growth of strength is different in the two classes of tests, and although the general laws of increase in strength due to increasing the percentage of cement and the density appear to hold in both cases, the authors' formula given on page 356 for compressive strength is not applicable to transverse tests.

Experiments by the authors comparing 8-inch cubes and 8-inch beams of 1: 2½: 5 concrete give a ratio of crushing strength to modulus of rupture at one and two months of 6: 1.

Mr. A. Fairlie Bruce† states from his experiments on the strength of concrete bars and arch ribs that he found the ratio between the crushing strength of the arch and the modulus of rupture of the bars to be about 6: 1 for concrete two to four weeks old, then increasing to about 10: 1 at the age of six months.

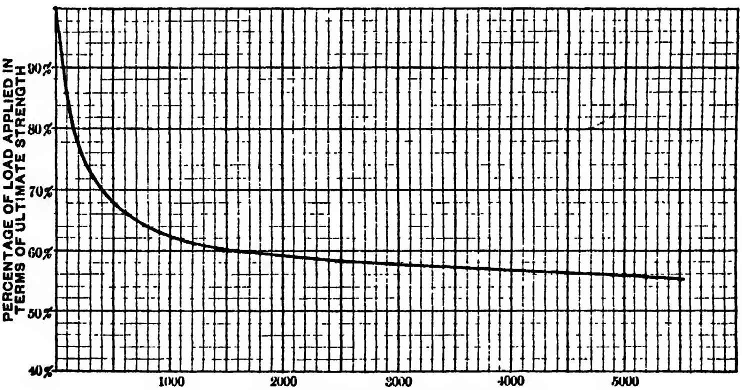


FIG. 124.—Fatigue of Neat Cement under Compression. *(See p. 381.)*

THE FATIGUE OF CEMENT

The action of cement under repeated stresses has been slightly investigated by Prof. J. L. Van Ornum* at Washington University. The experiments were made upon 2-inch neat Portland cement cubes four weeks old. The results of tests on 92 blocks are shown in the diagram in Fig. 124. The effect upon concrete of repeated applications of a load is discussed in the following chapter.

† *Engineering Record*, Oct.31, 1903, p. 533.
* Transactions American Society of Civil Engineers, Vol. LI, p. 443.

STRENGTH OF CONCRETE IN SHEAR

The actual strength of concrete* in direct shear is much greater than was formerly supposed because in many of the earlier tests this was confused with diagonal tension which, as indicated in the following chapter, may be dangerous in a beam even when the vertical shear is small. Owing to the difficulty in eliminating in experiments the effect of bearing action, diagonal tension and beam stresses in general, it is not easy to devise a form of test specimen and a manner of testing which will determine satisfactorily the resistance of concrete to direct shear. In tests made at the Massachusetts Institute of Technology under the direction of Prof. Charles M. Spofford in 1904 and 1905, the final failure of the specimens appeared to be by true shear. These tests gave a shearing strength ranging in general

Shearing Strength of Concrete

By Prof. Charles M. Spofford.

Massachusetts Institute of Technology. (*See p.* 382)

Age of Concrete 24 to 32 days

Mixture.	Method of Storing.	Shearing Strength lb. per sq. inch.			Av Compressive Strength in lb. per sq. inch.	Ratio of Compression to Shear.
		Maximum.	Minimum.	Average.		
1 : 2 : 4	Air	1630	960	1310	2070	0.63
1 : 2 : 4	Water	2090	1180	1650	2620	0.63
1 : 3 : 5	Air	1590	890	1240	1310	0.95
1 : 3 : 5	Water	1380	840	1120	1360	0.82
1 : 3 : 6	Air	1450	950	1180	950	1.24
1 : 3 : 6	Water	1200	1040	1120	1270	0.88
Average Ratio for 1 : 2 : 4 and 1 : 3 : 5 Concrete						0.76

from 60 to 80 per cent of the compressive strength of the concrete, which agrees substantially with experiments made by Prof. Arthur N. Talbot† in 1906.

This direct shear must not be confounded with shear in a beam involving diagonal tension where the concrete may break with a shearing stress 10% of the crushing strength.

At the Institute three grades of concrete were used, and the specimens were stored both in air and water. The test specimens were cylinders 5 inches in diameter by 18 inches long, and in testing, the end thirds of the

* Shearing tests of mortar, by Mr. Feret, are recorded on page 136.

† University of Illinois, Bulletin No. 8, 1906.

cylinders were held rigidly by cast iron yokes, the pressure being applied through a cast iron half cylinder bearing, fitting between the two yokes, so as to shear the concrete across two planes. To compare the compressive strength of the concrete with the shearing strength, six extra cylinders of the same dimensions were crushed. The following table gives the relation between the shearing and crushing tests.

From the experiments made at the University of Illinois, referred to, the conclusion was drawn that the resistance to shear is dependent upon the strength of the stone as well as upon the strength of the mortar, and for the richer mixture the strength of the stone probably exerts the greater influence.

EFFECT OF THE CONSISTENCY UPON THE STRENGTH

The general result of experiments and practice tends to show that the strongest concrete can be secured with a mixture containing only sufficient water to produce a film of mortar upon the surface after very hard ramming in thin layers, but with a wetter "quaking" mixture the ultimate strength will be nearly as high as with the dry mixture, and because of the greater ease in laying and obtaining a homogeneous mass, it is generally to be preferred. An excess of water injures the cement by decomposing parts of it before it has had opportunity to set. The actual strength of concrete is often of less importance than other considerations. If, as in many classes of structures, there is an excess of strength, cheapness in placing, the appearance of the surface, or the proper imbedding of reinforcing metal, may be of primary importance. In such cases the quantity of water must be suited to the attendant conditions.

The curves in Fig. 125 are plotted from experiments by the authors* upon the strength, density†, and permeability of the concrete mixed with different percentages of water. In the three curves the points of maximum density, strength and water-tightness all lie not far from the medium quaking consistency, although for maximum water-tightness a still softer consistency appears to be slightly more efficient.

These tests further indicate that (1) the consistency which will produce the densest concrete will result in the greatest ultimate strength provided an excess of water is not employed; (2) dry mixtures attain highest strength at short periods, but mixtures of quaking consistency approach the dryer specimens after longer setting; (3) very wet mixtures, especially of lean proportions, may be chemically injured, by the excess of water.

* Proceedings of American Society for Testing Materials, Vol. VI, 1906, p. 358.

† See p. 1 for definition and p. 138 for method of determining density.

Effect of "Laitance." Whenever concrete is laid under water, the water is likely to be clouded by what appear to be particles of cement floating up from the mass which is being laid. This whitish substance is generally termed "laitance." A similar formation occurs on the surface

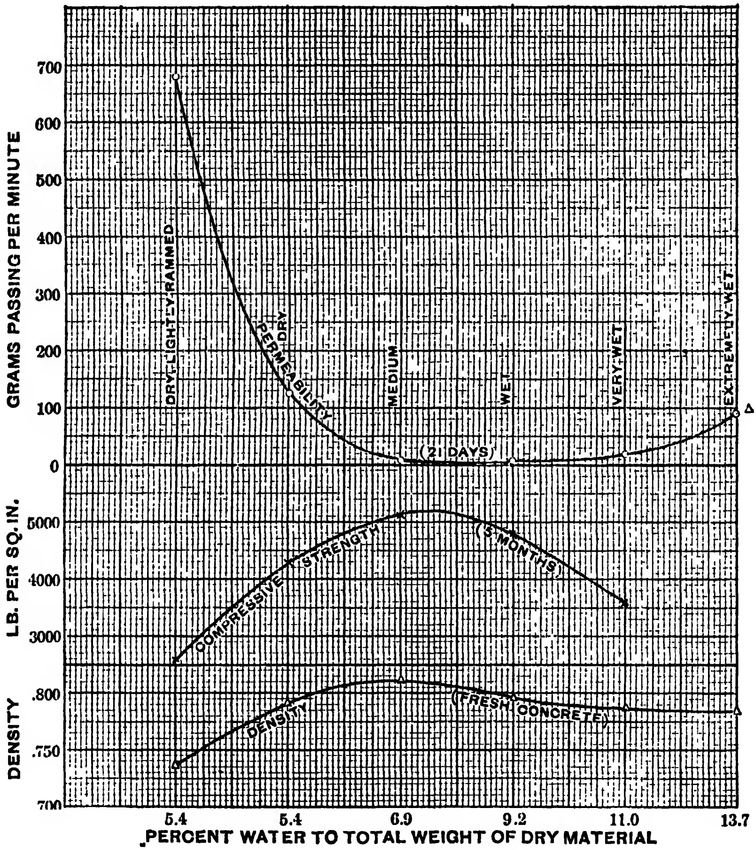


FIG. 125.—Comparative Permeability, Strength and Density of 1 : 2¼ : 4½ Concrete, mixed with Different Percentages of Water, By Taylor and Thompson. (*See p.* 383.)

of concrete laid with a large excess of water. In certain cases, we have found as much as ⅛ inch rising from a layer of 1 : 2½ : 5 concrete less than five inches thick.

Chemical and microscopical analyses, which Mr. Clifford Richardson has very kindly made for us, show that this laitance has nearly the same chemical composition,* except for a large loss on ignition, as normal Portland cements, but consists largely of amorphous material of an isotropic nature,—that is to say, it does not affect polarized light, and has almost no setting properties.

It is evident, therefore, that when concrete or mortar is laid under water, or with a large excess of water, a portion of the cement is rendered incapable of setting, and the strength of the mass is consequently reduced in proportion to this loss. The conclusion is naturally reached that for concrete laid under water, or in locations where a large excess of water is required in mixing, a higher percentage of cement than usual, about one-sixth more, should be employed.

A lean mixture has been found to be more seriously injured by an excess of water than a rich one, probably because the water has a greater opportunity to penetrate the mass, and therefore to dissolve the cement.

GRAVEL VS. BROKEN STONE CONCRETE

Comparative tests of broken stone and gravel concretes, in the same proportions by volume, show almost invariably that concrete made from hard broken stone, such as trap, or hard limestone, gives higher compressive strength than concrete made from gravel. This appears to be the rule not only when the materials are mixed by measured volumes, regardless of the percentages of voids, but also when the broken stone and gravel are each screened to substantially the same sizes.

The relative values of gravel and broken stone concrete in the table which follows are based on the comprehensive series of a comparative test made by Mr. Candlot in France and tabulated on page 367.

Comparative Strength of Broken Stone and Gravel Concrete.

From Candlot's Experiments

Age.	Ratio of strength of broken stone concrete to gravel concrete. With equal voids	Broken stone 47.4% voids. Gravel, 40% voids.
7 days	1.30	1.33
1 month	1.26	1.19
6 "	1.18	1.20
1 year	1.12	1.09

Each ratio gives the extra strength of broken stone over gravel concrete of similar age. For example, if a concrete containing gravel having

*See page 302.

40 % voids tests 2 000 lb. per sq. inch at the age of six months, a concrete in similar proportions by volume containing broken stone with 47.4% voids should, according to Candlot's experiments, test 1.20 times greater or 2 400 lb. per sq. inch.

The last column is averaged directly from Candlot's table, and may be taken as applicable to average conditions. It is noticeable that the gravel concrete approaches the broken stone concrete as its age increases. Since in many cases the ultimate strength of concrete is determined by the strength of its coarse aggregate, it follows that at, say, the age of a few months a gravel concrete may reach or surpass the strength of a broken stone concrete having a coarse aggregate of soft stone of low strength.

Although the claim is frequently made that gravel concrete is stronger than broken stone concrete, the authors have failed to find substantial proof of this. On the other hand, various records, among them a number of tests at the Watertown Arsenal,* as well as the tests tabulated on page 388, tend to show the probable accuracy of Candlot's tests.

Another argument in favor of broken stone concrete lies in the fact that gravel is often covered with a film of dirt, difficult to remove, which lowers the strength. In experiments for the East Boston Tunnel† by Mr. Howard A. Carson, Chief Engineer, concrete beams made with washed gravel were about one-third stronger than beams made with gravel coated with a thin film of dirt.

Advocates of gravel concrete, among them Mr. R. Feret,‡ assert that as the rounded stones slip more readily into place, it is easier to make with them a compact mass. Loose rounded stones also contain a smaller percentage of voids than angular, but this is at least partly offset by the fact shown by the experiments of the authors, tabulated on page 171, that broken stone compresses more on ramming.

Although the weight of evidence apparently favors broken stone concrete, it by no means follows that broken stone always should be used to the exclusion of gravel. In many instances, the ultimate strength of the concrete is of minor importance because the proportions of the concrete are determined by other considerations. Often, where strength is the criterion, but gravel is cheaper than broken stone, an additional percentage of cement may be economical. Moreover, the ultimate strength of gravel concrete is undoubtedly greater than that of concrete made with a poor quality of broken stone. With fixed proportions, as discussed on page 15,

* Tests of Metals, U. S. A., 1898, pp. 649 to 654.
† Boston Transit Commission, 7th Annual Report, 1901, p. 39.
‡ Chimie Appliquée, p. 533.

gravel is cheaper for the contractor than broken stone, because a given loose volume makes a larger quantity of concrete.

As indicated on page 388, in mixtures of like proportions by volume, the gravel concrete will have less cement in a cubic yard of concrete than a broken stone concrete unless the stone is well graded. Under ordinary conditions to attain concretes of nearly equal strength, with gravel and with broken stone, the sand should be proportioned in each according to the volume and dimensions of the voids in the stone,* and the quantity of cement per unit volume of compacted concrete should be the same in each. The gravel concrete thus will be apt to be the denser, and this will tend to overcome the slight difference in strength due to the varying character of the surfaces of the particles of the gravel and broken stone.

Sometimes it is advantageous to mix a small percentage of gravel with broken stone.

In comprehensive tests at the U. S. Government Laboratories, St. Louis,† upon concrete beams, cylinders and cubes of different aggregates, a granite concrete was about 10 per cent stronger than a gravel concrete made of exceptionally clean hard gravel pebbles, while the gravel concrete showed a strength about 10 per cent greater than that attained by a limestone concrete.

Tests made by Messrs. William B. Fuller and Sanford E. Thompson‡ at Jerome Park Reservoir, New York City, in 1905, upon the density and strength of concrete with different aggregates are illustrated in the curves in Fig. 126.§ Because of the greater density, the proportions by volume being the same, the specimens made with gravel and sand contained, in the set concrete, a slightly larger percentage of cement, so that the strength of the gravel concrete is slightly higher than if allowance had been made for this. The relatively low strength of the concrete with broken stone and screenings may be due in part to the character of the screenings, since tests by other experimenters have sometimes given exceptionally high strength when screenings were used.

The following conclusion was drawn with reference to the relative strength of broken stone and gravel concrete.

A concrete with an angular coarse aggregate, such as broken stone, is stronger than one with a rounded coarse aggregate, like gravel, and the

* This can be better accomplished by trial mixtures, thoroughly compacted, of the dry aggregate, or, still better, of small batches of concrete, than by water measurements of the voids. The proportions of the aggregates giving the smallest bulk of concrete to a given weight of the mixture of aggregates will be the best. Also, see Chapter XI on Proportioning.

† U. S. Geological Survey, Bulletin No. 344, 1908.

‡ Transactions American Society of Civil Engineers, Vol. LIX, p. 67, 1907

§ *Engineering News*, May 30, 1907, p. 599.

same sand and cement—although the rounded aggregate produces greater density—thus indicating a stronger adhesion of cement to broken stone than to gravel. However, if the sand is also angular, like screenings, but

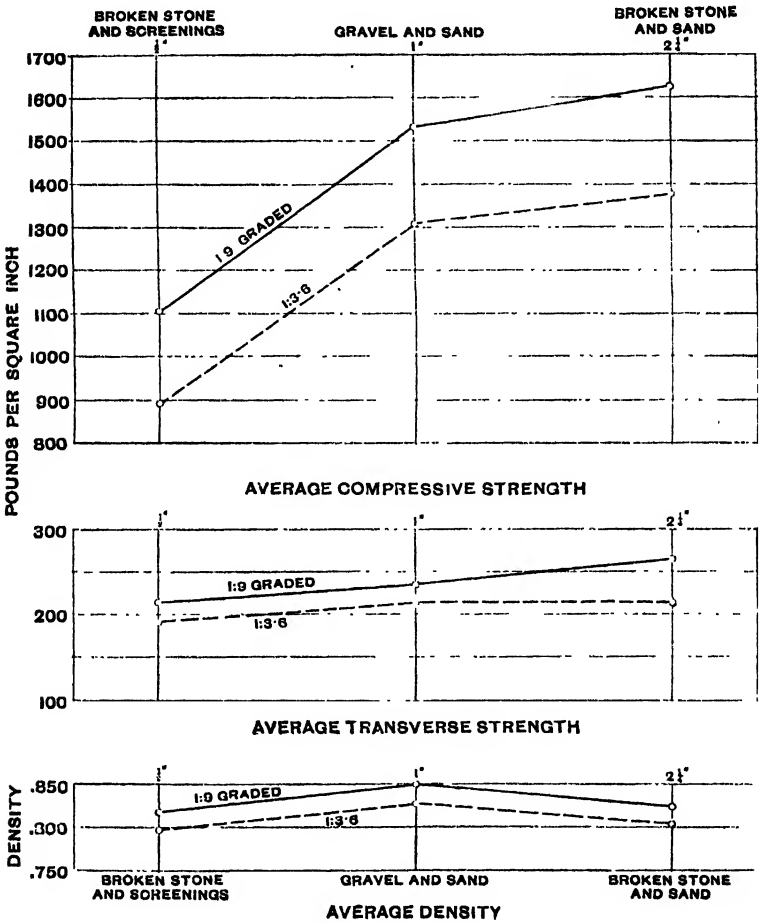


FIG. 126.—Comparative Density and Strength of Concrete made with Different Aggregates. By Fuller and Thompson. (*See p.* 387)

with its grains of the same sizes as the sand, the concrete with rounded coarse and fine aggregate is the stronger, probably because of its greater density.

EFFECT OF THE SIZE OF STONE OR GRAVEL UPON THE STRENGTH OF CONCRETE

The dimensions of the largest particles of stone and gravel which may be used in a concrete are often limited by practical considerations of mixing and placing. For ordinary work it is often specified that the stone shall pass through a 2-inch, or, more often, through a 2½-inch ring. For ordinary mass concrete of wet consistency the limit may be placed as high as 3

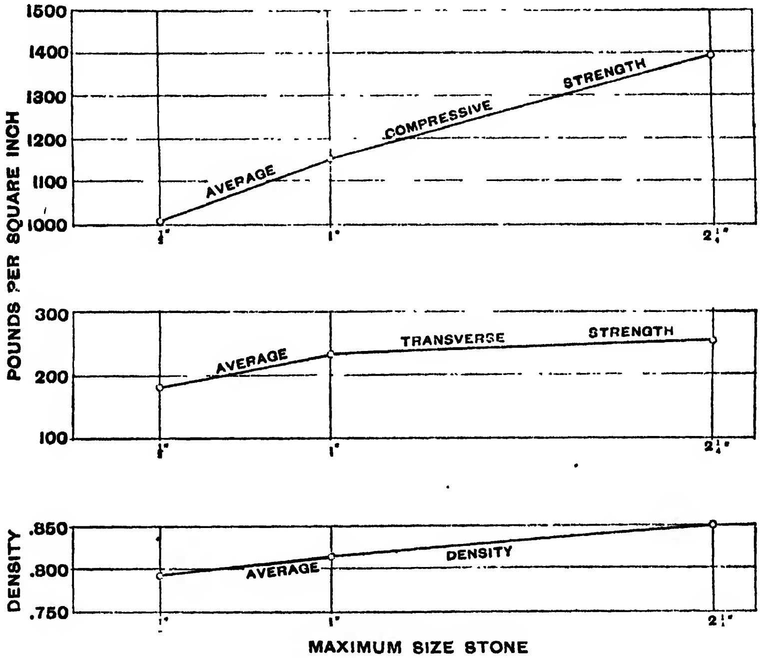


FIG. 127.—Comparative Density and Strength of Concrete made from Broken Stone of Different Maximum Sizes. Proportions 1:3:6. Age, 140 Days. (*See p.* 390.)

inches. In some cases, however, the stone must be small enough to pack readily around reinforcing metal, while in walls whose surface is to be picked or washed as described on page 289, a better appearance will result with stones under, say, one inch diameter, although the strength of concrete appears generally to increase with the size of the largest particles of stone in the mixture. This is illustrated with the gravel and the finer trap in experiments by Mr. Howard* at the Watertown Arsenal upon 12-inch

* Test on Metals, U. S. A., 1898, p. 654.

cubes of 1 : 1 : 3 concrete made with uniform stone of different sizes. The weights of the specimens indicate that the increase of strength is due primarily to the density The higher the limit of size the greater the variation in the sizes of material and therefore the greater the density of the mixture

John Kyle* nearly doubled the strength of 1 2 6 concrete made with 1½-inch stone by substituting 4 parts of 3½ inch stone for a like portion of the 1½ inch

Tests by Messrs Fuller and Thompson† showing the effect of aggregates of different maximum size are illustrated in the curves in Fig 127 From these tests the following conclusions were drawn

1 —Stone of the largest size makes the strongest concrete under both compression and transverse loading, i e, a graded aggregate in which the maximum size of the stone is 2¼ in in diameter gives stronger concrete than a graded aggregate with 1 in maximum size, and the 1 in stone gives a stronger concrete than ½ in stone A concrete in which the graded aggregate runs to 1 in in maximum size will require for equal strength about one sixth more cement, and with an aggregate running to ½ in maximum size, about one third more cement than concrete with an aggregate in which the maximum size is 2¼ in

2 —The largest stone makes the densest concrete Concrete made with graded stone having a maximum diameter of 2¼ in is noticeably denser than that with 1-in stone, and this is denser than that with ½ in. stone

EFFECT OF THE QUALITY OF THE STONE UPON THE STRENGTH OF CONCRETE

The ultimate strength of concrete is often limited by the texture or strength of the coarse aggregate This is evidently the case with cinder concrete Experiments by Mr Geo W Rafter‡ gave the strength of concrete made with hard broken sandstone and various proportions of mortar from 1 5 to 2 4 times the strength of similar mixtures of broken shale and mortar, and this discovery led to the rejection of the latter as a material for concrete.

Tests of the authors upon 12-inch cubes broken at the Watertown Arsenal lead them to believe that at least in certain cases the ultimate strength of a concrete is actually fixed by the shearing strength of the particles of stone which make up the aggregate Cubes in proportions 1 : 2¼ : 4¾,—based on a cement barrel of 3 8 cubic feet,—attained an ultimate strength of 5000 to 5500 pounds per square inch. On account of

* Proceedings Institution of Civil Engineers, Vol LXXXVII, p 88

† Transactions American Society of Civil Engineers, Vol LIX, p 67, 1907.

‡ Second Report on the Genesee River Storage Project, New York, 1894.

differences in the methods of mixing and ramming, some of the specimens reached this limit at the age of two months while others did not attain it for six months; but it was noticeable that at whatever period the ultimate strength was reached the planes of fracture were smooth, breaking through each piece of stone, whereas before the ultimate strength was reached many of the stones pulled out from the concrete, leaving jagged instead of smooth surfaces on the pyramids remaining after the cubes were broken to destruction. The stone employed for these specimens was a hard, dense trap. If a weaker stone had been used, it is probable that the pieces would have sheared at a much earlier period and the ultimate strength would have been lower.

Tests at the United States Government Laboratories at St. Louis§ upon 6-inch cubes of exceptionally good 1 : 2 : 4 concrete 26 weeks old, made with different coarse aggregates, show the following average ultimate strengths:

Granite concrete, 4750 pounds per square inch.

Gravel concrete (quartz pebbles), 3810 pounds per square inch.

Limestone concrete, 3460 pounds per square inch.

Cinder concrete, 2320 pounds per square inch.

If concrete is mixed in such proportions or by such methods that the ultimate strength is reached before the stones shear, the strength of the particles of stone is a much smaller factor in the result.

Tests of crushing strength of building stone made by Mr. Richard L. Humphrey * give the relative strength of specimens of several kinds of stone:

The average of a large number of tests of 2-inch cubes, part on edge and part on bed, by Gen. Q. A. Gillmore, and quoted in Burr's "Materials of Engineering,"† shows average results for granite and sandstone almost identical with the average of Humphrey's tests on these materials, while the average strength of specimens of limestone and marble was about 13 000 lb. per square inch. Tests at the Watertown Arsenal‡ give the crushing strength of 4-inch cubes of sound trap rock as 33 300 lb. per square inch, and of seamy trap as 19 400 lb.

The table giving results of Mr. Humphrey's test is especially interesting as showing in a general way that the heaviest rock is apt to have the highest strength. Of the 8-inch cubes tested on their bed, so as partially to eliminate the effect of cleavage planes, the specimen of quartzite is the only one which does not follow this rule. In Gillmore's tests mentioned above, the

§ U. S. Geological Survey Bulletin, No. 344, 1908.

* As tabulated by Edwin C. Eckel in *Engineering and Mining Journal*, June 20, 1902, p. 921.

† Edition of 1903, p. 433.

‡ Tests of Metals, U. S. A., 1898, p. 577.

variation in the same kind of stone from different localities is large, but in each kind the heavier rocks usually give the higher resistances. We may state, therefore, as a general rule in comparing rocks of the same kind, that those of the highest specific gravity are apt to be the strongest, and this rule may be extended in many cases to the comparison of different kinds of rock

Crushing Tests of Cubes of Stone.

By Richard L. Humphrey. (See p. 391.)

Location.	Kind of Stone.	lb Weight per cubic foot	Specific Gravity	Absorption	Average Crushing Strength			
					2 inch cube		8 inch cube	
					Bed lb per sq in	Edge lb per sq in	Bed lb per sq in	Edge lb per sq in
Chester, Pa..	Gneiss	165.71	2 69	0 355	6 097	5 446	9 505	6 426
Germantown, Pa.....	Gneiss	176 23	2 825	0 135	19 891	15 555	11 636	13 984
French Creek, Pa. ...	Granite	190.46	3 085	0 155	19 997	14 348	17 274	7 910
Conshohocken, Pa ...	Mica schist	177.76	2 91	0 155	20 038	15 680	10 417	7 532
Curwensville, Pa.	Sandstone	146 00	2.40	2.335	10 218	8 013	7 513	4 463
Lumberville, Pa......	Quartzite	158.19	2 63	0 098	no test	no test	14 841	8 637

EFFECT OF PERCENTAGE OF CEMENT UPON THE STRENGTH OF CONCRETE.

The strength of concretes of the same density made with similar materials varies approximately with the percentage of cement, so that the comparative strength of concrete in different proportions sometimes may be estimated sufficiently close for practical purposes The following table gives the results of certain of the Jerome Park tests* by Messrs Fuller and Thompson, where the density of the concrete was maintained nearly constant.

DESTRUCTIVE AGENCIES

The effect of sea water, frost, fire, and rust, are treated in Chapters XVI, XVII and XVIII.

Effects of Acids. Experience shows that after concrete has thoroughly hardened, it resists the attack of diluted acids, such as are found in sewage, and that it is only seriously affected by strong acids which injure nearly all other materials. Concrete has proved to be the most successful lining for digesters in pulp mills, where sulphurous acid is present under high heat pressure.

* Transactions American Society of Civil Engineers, Vol. LIX, p. 67, 1907.

Effect of Manure. Concrete of good quality after hardening is not affected by manure, although it may be injurious to green concrete.*

Effect of Oils. Tests† indicate that mineral oils do not injure concrete even if applied to it when only a week or two old. Animal fat and vegetable oils tend to disintegrate it if applied when the concrete is green, but these appear to be successfully resisted if the concrete has thoroughly hardened. Hardened concrete may be affected by the vapor from the melting of animal fat, probably because of the acid which it contains. Mr. Toch‡ states that

Comparative Density and Strength of Similar Concrete with Different Percent of Cement and 2½-inch Stone Graded as an Ellipse and Straight Line

BY FULLER AND THOMPSON. (*See p.* 392.)

Materials.		Density with Different Percentages of Cement*				Modulus of Rupture at 90 Days, Different Percentages of Cement.*				Compressive Strength at 140 Days, Different Percentage of Cement.			
Stone.	Sand.	8%	10%	12½%	15%	8%	10%	12½	15%	8%	10%	12½%	15%
Crushed	Screenings	0.829	.	...	.....	[illegible]	...	...	...	980	....	.....	......
"	"	....	0.840	. .	....	....	250	...	...	....	1 129	....	......
"	"	....	....	0 [illegible]	...	.	....	245	...	...	.....	[illegible]	.....
"	"	....	....	....	0.839	..	....	....	326	..	.....	..	1 034
"	"	0.871	....	...	....	[illegible]	....	....	....	990	.....	.....	.....
Gravel	Sand	...	0 855	..	...		215	..	....	...	1 715	.....	......
"	"	....	....	0 865	..	.	....	307	....	....	......	1 890	......
"	"	.. .	.....	..	0.867	.	....	....	339	...	.....	......	2 040
Averages		0 850	0.850	0 848	0 853	170	248	276	332	985	1 428	1 654	1 837
Strength computed as proportional to the percentage of cement, based on strength with 8% cement						176	220	275	330	985	12 30	1 540	1 85

* In gravel and sand mixtures the percentage by weight of cement was increased in each case to balance the difference in specific gravity between this and the crushed material.

the action of fat or vegetable oil is due to expansion caused by the formation of crystals of stearate and oleate of lime. Light oils, like kerosene or naphtha, penetrate any substance very readily, so that if concrete tanks are used for their storage, special precautions must be taken in their construction.

Effect of Electrolytic Action. Tests§ and experience indicate that concrete is injured by electrolysis. However, there is less danger for plain concrete or for reinforced concrete than for structural steel even if the latter is incased in concrete or other masonry.

* See "Investigation of Collapse of Filter Roof during Construction at Lawrence, Mass.," by Sanford E. Thompson, Journal New England Water Works Association, Vol. XXII, No. 2.

† James C. Hain in Engineering News, Apr. 20, 1905, p. 279.

‡ Engineering News, Apr. 20, 1905, p. 419.

§ By A. A. Knudsen, American Institute Electrical Engineers, Vol. 26, p. 133, by Maximilian Toch, Engineering Record, June 30, 1906, p. 794, and by N. J. Nicholas, Engineering News, Dec. 14, 1908, p. 710.

STRENGTH AND ELASTICITY OF CINDER CONCRETE

Tests at the Watertown Arsenal* on 12-inch cubes of cinder concrete mixed in different proportions gives results arranged in the following tables:

Compressive Strength of 12-inch cubes of Cinder Concrete.
Watertown Arsenal. (*See p.* 394.)

Cement.	Proportions Cement	Sand.	Cinder	Age, 1 month. Mean weight lb. per cu. ft.	Compressive strength lb. per sq. in.	Age, 3 months. Mean weight lb. per cu. ft.	Compressive strength lb. per sq. in.
German Portland.............	1	1	3	112.1	1 466	110.4	2 001
	1	2	3	115.2	1 098	112.8	1 634
	1	2	4	111.2	901	107.9	1 325
	1	2	5	108.8	760	105.3	1 084
	1	3	6	107.6	529	103.5	788
American Portland............	1	1	3	117.2	1 965	115.2	2 624
	1	2	5	111.3	818	110.0	1 412

Note: Each value for German cement is an average of three 12 inch cubes. Each value for American cement is an average of six 12-inch cubes made from two brands of first class Portland cement. The exact age of the German cement specimens was 38 and 99 days, and of the American cement specimens 31 and 90 days.

Elastic Properties of Cinder Concrete, 12 inch cubes at three months.
Watertown Arsenal. (*See p.* 394.)

American Portland Cement.	Proportions Cement.	Sand.	Cinder.	Age when Tested	Modulus of Elasticity between loads per sq. in. 100 and 600 lb.	100 and 1000 lb.	1000 and 2000 lb.	Permanent sets after loads per sq. in. of 600 lb.	1000 lb.	2000 lb.	Compressive strength lb. per sq. in.
A	1	1	3	90	2 500 000	2 500 000	1 420 000	0.	.0001	.0006	2 780
	1	2	5	90	1 087 000	957 000		.0008	.0028		1 402
	1	2	5	90	1 471 000	1 286 000		.0002	.0010		1 715
B	1	1	3	90	4 167 000	3 214 000	1 190 000	0.	.0001	.0014	2 368
	1	1	3	90	2 083 000	1 875 000	1 351 000	.0001	.0002	.0017	2 580
	1	2	5	90	1 190 000	849 000		.0009	.0066		1 200
	1	2	5	90	1 087 000	865 000		.0024	.0089		1 263

*Tests of Metals, U. S. A., 1898, pp. 561 and 573.

MAKING CONCRETE SPECIMENS FOR TESTING

Complete and careful records must be made of the methods employed and the materials used in making concrete specimens for testing, in order to reach results of value for comparison with those of other experimenters. The lack of this care and accuracy has rendered the larger number of tests on concrete of only local significance.

The practical relation of the density of a concrete to its strength, as discussed in the preceding pages, indicates that it is not merely necessary to measure roughly the materials entering into the composition, but that the exact amount of solid matter, the coarseness of the particles, the character of the surfaces of the grains, the moisture in the materials, and the additional quantities of water used, must be very carefully recorded.

The cost of making and testing concrete specimens is so great, that the additional time required for entering notes full enough to produce results of scientific value is insignificant. The blank form with the values in an actual test filled out is presented on page 396 for recording data relating to the making of concrete specimens. On the same form may be added places for recording the results of the tests. In most cases it is advisable for greater exactness to make separate batches for each specimen.

In addition to the information outlined, mechanical analyses should be made of the aggregates as a part of the permanent records, and for the computations in the form, it is also necessary to determine the specific gravities of the materials.

The specific gravity of Portland cement in most cases may be assumed as 3.1, and, in fact, the specific gravity of the sand may also be assumed without appreciable error as 2.65. For the specific gravity of other aggregates special tests are necessary.

Concrete for experimental specimens should be mixed by experienced men. There is a certain knack in properly turning the materials so as to mix them thoroughly which can be acquired only by practice, and the amount and manner of ramming or puddling is so important that specimens may be rendered worthless by improper manipulation.

The molds for specimens should be made of metal or of good quality lumber, preferably white pine, so that it will not twist or get out of shape, and the surface next to the concrete should be planed, and all joints made water-tight. The mold should be wet or greased before placing the concrete. If metal, the grease or oil must cover every part of the surface. A wooden mold for two cubes is shown in Fig. 128.

Dimensions of Specimens. Compression specimens are limited in size

Expt. No.

File *Waltham Reservoir.*

Date *2/9/06.*

Form for Recording Data on Concrete Specimens

(Figures in ( ) refer to Item Numbers.)

ITEM.

1. Nominal Proportions . . . . *1 : 1.8 : 4.1*
2. Car No. *00*
3. Kind of Cement . . . . *Atlas*
4. Kind of Sand . . . . *¾ n.c. ¼ G*
5. Analysis No. . . . . *420 and 421*
6. Kind of Coarse Aggregate . . . . *W. Gravel*
7. Analysis No. *422*
8. Weight of Cement Used *3.12*
9. Weight of Sand Used *5.72*
10. Weight of Coarse Aggregate Used *12.85*
11. Weight of Water Used *1.77*
12. Per Cent Water to Weight of Cement plus Sand *20%*
13. Temperature of Water . . . . *60° F.*
14. Temperature of Laboratory . . . . *70° F.*
15. Total Weight of Material (8) + (9) + (10) + (11) *23.46*
16. Weight of Mold Empty *3.00*
17. Weight of Mold Filled *26.30*
18. Weight of Concrete Net *23.30*
19. Weight of Concrete Left Over *0.00*
20. Weight Unaccounted for—Assumed as Solid Material* *0.16*
21. Weight Unaccounted for—Assumed as Water *0.00*
22. Volume of Fresh Specimen (cu. ft.) *0.1527*
23. Weight of Specimen—Mold Removed *22.7*
24. Method of Storage . . . . *Air*
25. Weight of Specimen Before Testing *22.5*
26. Measurements of Specimen Before Testing . . . . *7.99″ × 8.02″ × 4.12″*
27. Date and Hour Specimen Made . . . . *2/9–3 p.m.*
28. Date Tested . . . . *3/9–10 a.m.*
29. Specific Gravity Cement . . . . *3.15*   30. Sand . . . . *2.65*   31. Stone *2.75*
32. Weight of Cement in Fresh Concrete $(8) \times \frac{(18)}{(18) + (19) + (20)}$ *3.09*
33. Weight of Sand in Fresh Concrete $(9) \times \frac{(18)}{(18) + (19) + (20)}$ *5.68*
34. Weight of Coarse Aggregate in Fresh Concrete
$$(10) \times \frac{(18)}{(18) + (19) + (20)} \quad \ldots\ldots\ldots\ldots \ 12.76$$
35. Weight of Water in Fresh Concrete $(11) \times \frac{(18)}{(18) + (19) + (20)}$ *1.76*
36. Absolute Volume Cement in Fresh Concrete (assume 1 cu.ft.water, 62.4 lb.)
$$\frac{(32)}{(22) \times 62.4 \times (29)} \quad \ldots\ldots\ldots\ldots \ 0.103$$
37. Absolute Volume Sand in Fresh Concrete
$$\frac{(33)}{(22) \times 62.4 \times (30)} \quad \ldots\ldots\ldots\ldots \ 0.225$$
38. Absolute Volume Coarse Aggregate in Fresh Concrete
$$\frac{(34)}{(22) \times 62.4 \times (31)} \quad \ldots\ldots\ldots\ldots \ 0.487$$
39. Absolute Volume Water in Fresh Concrete $\frac{(35)}{(22) \times (62.4)}$ *0.184*
40. Total Absolute Volume Materials (36) + (37) + (38) + (39) . . *0.999*
41. Density (36) + (37) + (38) *0.815*
42. Remarks . . . .

Computed by *G. B.*

Checked by *S. E. T.*

***Adhering to Tools and Trays. Divide the Total Loss, (15) — [(18) + (19)], by Estimation into Items (20) and (21).**

by the capacity of the testing machine. The Emery Machine at the Watertown Arsenal, one of the largest in the world, has a capacity of 800 000 pounds, and the authors have had 12-inch concrete cubes tested there which reached this limit, so that 12 inches on a side may be fixed in general as the maximum size for specimens. For a lower limit it is doubtful if specimens less than 6 inches square can be made to give accurate results. A series of comparative tests by the authors upon 8-inch and 12-inch cubes gave much higher breaking strength per square inch for the larger size

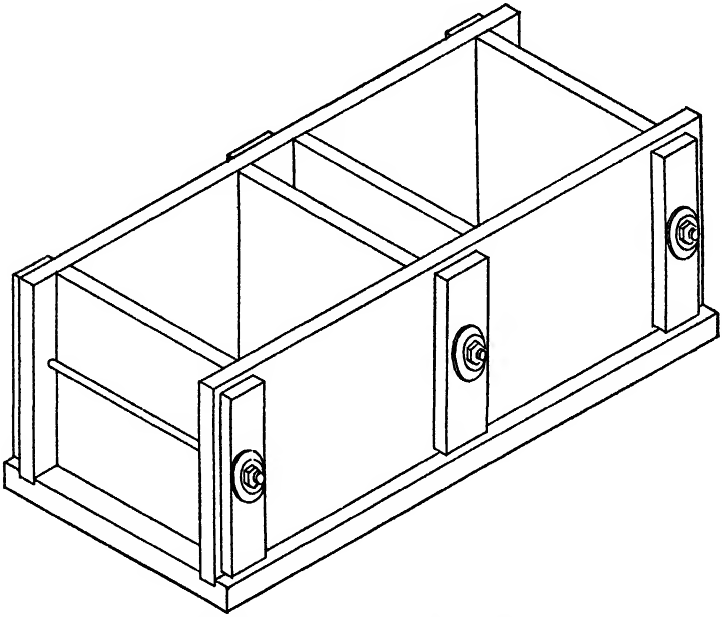


FIG. 128.–Mold for Concrete Cubes. (*See p.* 395.)

specimens. It was evident from the lower unit weight of the smaller specimens, that the difference was due, at least in part, to variation in homogeneity.

Cubes have been the common form of compression specimens and are suitable for comparative tests of ultimate breaking strength, but for studying the real value of concrete in compression, or for determination of elastic properties, long prisms are preferable.

For column tests, the length of a specimen should be at least five times the largest lateral dimension. Both theory and practice show that beyond this point there is but little variation in the strength per square inch, providing the loading is central. See p. 369.

The specimen recommended for crushing tests by the Joint Committee on Concrete and Reinforced Concrete, and used at the U. S. Government Laboratories at St. Louis, is a cylinder 8 inches diameter by 16 inches long.

For reinforced concrete beams the Committee recommended 8 by 11 inches by 13 feet long, testing this on a 12 foot span.

Beams for testing the transverse strength of concrete are usually made from 6 to 12 inches square. The smaller size is satisfactory provided the mixture is a fairly wet one so that the corners and surfaces of the molds can be filled. For specimens 6 inches square a convenient length is 6 feet, to be broken on a 60 inch span. The halves of the specimens may be afterwards broken to average with the full beam test or to compare the strength at different periods. Experiments prove that the ultimate fiber stress in the half beams will be practically, as well as theoretically, the same as that in the whole beams.

Specimens for crushing must be faced with some material which will transmit the strain to all points in the surfaces. At the Watertown Arsenal plaster of Paris or neat cement is employed. After spreading the surface with a coat of plaster or cement, a block of polished steel is placed upon it, and it is allowed to set. Before crushing, the surface is tested with a straight edge, and any irregularities are smoothed off with its sharp edge.

Specimens for Rough Tests. If the quality of sand is questioned and a laboratory is not available, a rough test may be made by mixing up a block of mortar or concrete, using the same aggregates mixed in the same proportion and to the same consistency that is to be employed in the work and examining the specimens from day to day. If kept in a warm room under a moist cloth, the mortar or concrete should harden after 24 hours so as to resist the pressure of the thumb and at the end of a week in the air it should be hard and sound.

Method of Quartering. To obtain an average sample from a pile of sand, gravel, or stone, the method of quartering is useful. Shovelfuls of the material are taken from the various parts of the pile, mixed together and spread in a circle. The circle is quartered, as one would quarter a pie, two of the opposite quarters are shoveled away from the rest, thoroughly mixed, spread, and quartered as before. The operation is repeated until the quantity is reduced to that required for the sample.

CHAPTER XXI

REINFORCED CONCRETE DESIGN

Reinforced concrete is concrete in which steel or other metal is imbedded to increase its strength. Although it has been employed generally as a building material for only a few years, the laws governing the effective combination of concrete and steel are now sufficiently well established to enable the engineer to design a structure with assurance of permanent strength and durability.

Occasional failures have occurred in reinforced concrete construction through neglect of essential principles. The causes have been (1) poor design, particularly in the details which do not occur in steel design; (2) poor materials, especially poor sand; (3) misplacement of reinforcement; and (4) too early removal of forms. These are all readily preventable causes under careful engineering and superintendence. Some of the more important points to guard against are outlined in Chapter II, page 28a.

Until recently there has been considerable divergence in the theory of beam design and of column design. Authoritative reports were brought out in Europe in 1907 and 1908. In America, the Joint Committee on Concrete and Reinforced Concrete presented its first Progress Report early in 1909. This Joint Committee is composed of members selected from the American Society of Civil Engineers, the American Society for Testing Materials, the American Railway Engineering and Maintenance of Way Association, and the Association of American Portland Cement Manufacturers, and therefore represents the highest authority in the United States. Its recommendations have tended to standardize general practice.

In this chapter the recommendations on design of this American Joint Committee have been followed, not only because of their general acceptance as a standard, but because they agree with the views of the authors and represent the most satisfactory rules thus far formulated. This has necessitated no changes in the methods of analysis given in the first edition, since the theory of stress there presented has since been generally adopted.

Results of recent tests have made possible a more complete treatment of the details of design, and extensive study and investigation have led to the addition of simple working formulas and practical recommendations.

In general, only brief discussions together with the rules and principal formulas for design are given in the text, the analytical treatment of each

subject being transferred to the Appendix or printed in footnotes for the use of readers interested in the theory.

In the following pages, then, are discussed:

Fundamental principles of the combination of steel and concrete.	400 to 416
General principles of design and formulas for rectangular beams and slabs	416 to 422
Simple formulas for T-beams	423 to 426
Design of the ends of continuous beams next to the supports	427 to 430
Reinforcement for diagonal tension and shear	441 to 456
Bond of steel to concrete	456 to 461
Details of beam design	441 to 461
An example of floor design	468 to 475
Theory of the design of flat slabs	483
Bending moments and shears from an elementary standpoint	433
Distribution of loads	431
Tables and curves for beam and slab design	507 to 526
Tests of reinforced beams	477
Columns of plain concrete, vertically reinforced, and hooped	488
Reinforcement for temperature contraction	500
Types of reinforcement	504
Analyses for the derivation of beam formulas. including:	
Simple rectangular beams	751
T-beams	754
Beams with steel in both tension and compression	757
Beams with concrete bearing tension	760
Simple beams treated by the parabolic theory	762

In other parts of the treatise are discussed various special types of reinforced concrete construction and details of design, including:

Arch design	533
Retaining wall design	659
Footings	644
Building construction	607
Chimney design	630
Analysis for circular beams and chimneys	765
Conduits	679
Tunnels	689
Dams	674
Reservoirs and tanks	695
Specifications for first-class or high carbon steel	38
Protection of metal from corrosion and fire	327

The notation adopted in the formulas is the Standard Notation as adopted by the Joint Committee 529

GENERAL PRINCIPLES OF REINFORCED BEAMS

A concrete beam, when reinforced with iron or steel rods properly placed, develops a capacity for carrying loads several times greater than its carrying capacity when without reinforcement. It is evident that the location of the reinforcement in the beam must conform to the principles of mechanics so that the concrete shall be strengthened in its weakest part. Hence, since concrete is comparatively weak in its resistance to pull, reinforcing metal

should be placed where it will aid the concrete in carrying tension. In a beam or slab the metal should be as near to the surface on the tension side of the beam as is consistent with properly imbedding it and providing a sufficient thickness of concrete to protect it from rust and fire.

Since concrete is a brittle material and steel a comparatively ductile one, it might be expected that the stretching of the tension surface of a beam would result in the formation of cracks on the under surface of the concrete, and that all the pull would be imposed upon the steel. Tests by Prof. Frederick E. Turneaure* and others have shown that cracks in the concrete are actually produced by the tension and that the tension load is thus transferred to the metal. However, while these cracks reduce the strength of the concrete, they are so minute, being at first invisible to the naked eye, and so distributed over the section, that the reinforcing metal, as shown by tests, is protected by the concrete from corrosion even up to the point of the elastic limit of the steel.†

Not only must the steel be correctly located, but it is essential to have the proper quantity of metal in the beam. It is obvious that if the cross section of the metal is too large as compared with the area of the concrete in compression, the beam, in case of failure, will give way by compression in the concrete, whereas, if the area of the metal is too small, weakness will show itself as soon as the metal has reached its yield point, which is usually not far from one-half the actual breaking strength of the steel. The area of the reinforcing metal in rectangular beams and slabs is apt to vary according to the conditions from about ½% to 1½% of the area of the cross section of the reinforced beam above the steel. For example, a beam 10 inches wide and 11 inches deep with steel one inch above its bottom surface (100 square inches net area) requires, according to circumstances, from ½ square inch to 1½ square inches section of steel. In any given design this area of reinforcement should be determined from the character of the member and the strength and elasticity of the concrete and the steel. More than 1% of steel is not usually economical in a rectangular beam unless the concrete is allowed to be stressed beyond the high pressure of 750 pounds per square inch.

In designing a beam composed of concrete with steel imbedded in it, the bending moment produced by the superimposed load,—which is termed the live load,—plus the weight of the beam itself, the dead load, must be no greater than the moment of resistance of the beam (*i.e.*, the moment of the internal resisting forces of the strength of the concrete and steel) divided by a proper factor of safety.

* Proceedings American Society for Testing Materials, 1904

† See page 410.

That which differentiates the study of a reinforced concrete beam from that of a beam composed of a single homogeneous material is the determination of the pull, which is borne by the steel alone, and of the compression, sustained entirely by the concrete. The problem is rendered the more complex because the strength and elasticity of concrete vary through a wide range according to the nature of its ingredients and their proportions. Current practice, borne out by experiments made at various American universities, indicates that beams may be designed on the assumption that the concrete in the upper part of the beam resists all the compression and the steel in the bottom of the beam takes all of the pull. This is always on the safe side, since the concrete assists the steel in tension to a slight degree. The theories of the distribution of the stresses in reinforced concrete, which are based on the elasticity of the concrete and the steel, are sufficiently accurate for the practical purposes of design. Before giving formulas and tables to be used in the design of reinforced beams, the principles governing the assumption of the distribution of stresses and the properties of the materials will be considered.

A Plane Section Before and After Bending. While experiments at the Massachusetts Institute of Technology indicate that the law of plane sections before and after loading does not apply exactly to reinforced concrete beams, nevertheless, it is sufficien ly accurate for practical purposes to assume it correct, viz: that if a plane section is taken through a beam before loading, after loading, this section, even though inclined to its original position by the bending due to the load, remains a plane section. From this it follows, as in the common theory of beams, that the stretching or shortening per unit of length of any fiber which cuts the section considered may be assumed as proportional to the distance of this fiber from the neutral axis of the section.

MODULUS OF ELASTICITY OF STEEL

The modulus of elasticity of steel varies from 28 000 000 pounds per square inch to 31 000 000 pounds per square inch; 30 000 000 is customarily taken as an average value, and is the value adopted in this treatise.

All Steel, irrespective of its Ultimate Strength, Elastic Limit or Chemical Composition, has Substantially the Same Modulus of Elasticity. It follows therefore from the principles of elasticity that the stretch under a given pull is independent of the character of the steel.

MODULUS OF ELASTICITY OF CONCRETE

The modulus of elasticity is an important item in reinforced concrete design and is discussed at length in the pages which follow. **For practical design it is recommended that the ratio of the modulus of elasticity of steel to that of concrete be taken at 15,** corresponding to a concrete modulus of 2 000 000.

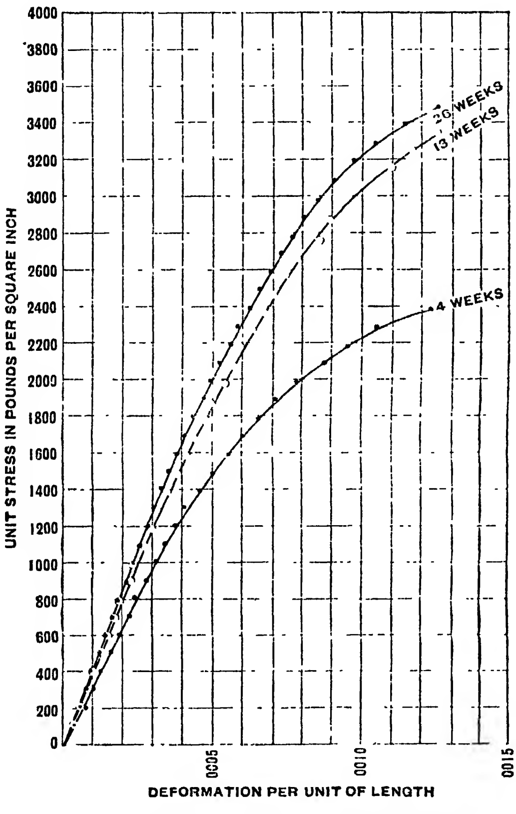


FIG. 129. Stress Deformation Diagram, Limestone Concrete Cylinders of Medium Consistency and Extra Good Quality.* (See p. 404).

Determination of Modulus of Elasticity. The modulus of elasticity, E, may be taken as the quotient of the stress per unit of area divided by the deformation (that is, the elongation or the shortening) in a unit length. In

* Bulletin No. 344, U. S. Geological Survey, p. 33.

customary English units where the modulus is in pounds per square inch,

$$E = \frac{\text{stress per square inch}}{\text{deformation per linear inch}}$$

It is determined in the laboratory by measuring the deformation for the loads successively applied and plotting them as shown in Fig. 129. The curves in the diagram represent the deformations at different stages of the loading, for a typical cylinder 8 inches in diameter by 16 inches high of extra strong 1 : 2 : 4 concrete, tested at the St. Louis Government Laboratory in 1907. The set, which is the permanent deformation when the load is released, is not indicated in the diagram because the total deformation is that which must be used in reinforced concrete analysis.

The form of the deformation curve is approximately a parabola,* but the tests at St. Louis† indicate that for first-class concrete the modulus is nearly constant for about one-third of the ultimate strength. The modulus at this point is $\frac{800}{0.00025}$, or 3 200 000 pounds per square inch, in the four weeks old concrete tested.

Results of Tests. Numerous tests have been made to determine the modulus of elasticity of concrete which indicate as large a range in results obtained by different experimenters, even with concrete of the same proportions of cement to aggregate, as from 1 500 000 to 5 000 000 per square inch. The reasons for this are not yet fully determined; it has been conclusively proved, however, that the age of concrete, its richness and its density have undoubtedly a large influence on this variation.

The following table, compiled from various tests, may be of value as suggesting approximate values of the modulus for different proportions of concrete based upon the total deformation at one-third the crushing strength of cylinders at an age of thirty days. Two columns are given, one for ordinary wet concrete of medium quality, and one for concrete very carefully made with a dense mixture of mushy consistency and kept wet during hardening. The "ordinary" values are slightly below those which should be expected in practice on construction work.

The modulus of elasticity of concrete probably bears a definite relation to its ultimate strength, but the factors which enter into this relation probably will never be determined exactly. Plotting the results of a large number of tests made at the Watertown Arsenal, at the Government Labora-

* See discussion by Prof. Talbot in University of Illinois Bulletin, No. 10, Feb. 1, 1907, p. 21.
† Bulletin No. 344, U. S. Geological Survey, pp. 36-53.

tory at St. Louis, and at many of the colleges, indicates an approximate ratio of 1300 between the modulus of elasticity and the ultimate strength.

Kimball's Tests. The moduli at different loads from tests of Mr. George A. Kimball made at the Watertown Arsenal upon 12-inch cubes are given

Moduli of Elasticity of Concrete of Different Proportions. Approximate Average Values. (*See p.* 404.)

	PROPORTIONS.	ORDINARY WET CONCRETE. Crushing Strength at 30 days. lb. per sq. in	ORDINARY WET CONCRETE. Modulus of Elasticity lb. per sq. in.	EXCEPTIONALLY STRONG CONCRETE. Crushing Strength at 30 days. lb per sq in	EXCEPTIONALLY STRONG CONCRETE. Modulus of Elasticity lb. per sq. in.
Broken stone or gravel concrete	1 : 1½ : 3	2300	2 500 000	2800	3 600 000
	1 : 2 : 4	1700	2 000 000	2500	3 200 000
	1 : 2½ : 5	1500	1 800 000	2200	2 800 000
	1 : 3 : 6	1300	1 600 000	1900	2 500 000
	1 : 4 : 8	900	1 300 000	1500	2 000 000
	1 : 2 : 5	700	900 000	1000	1 300 000

NOTE—A modulus of 2 000 000, corresponding to a ratio of 15, is recommended for general use.

in table below. The moduli are computed with the set deducted from the deformation, so that the values are slightly higher than would be obtained from total deformation.

Elastic Properties of Broken Stone Concrete 12-inch Cubes.

Portland cement,* bank sand and broken conglomerate stone.

BY GEORGE A. KIMBALL at Watertown Arsenal. (*See p.* 405.)

COMPOSITION Cement	COMPOSITION Sand	COMPOSITION Broken Stone	Age	MODULUS OF ELASTICITY BETWEEN LOADS PER SQUARE INCH OF 100 and 600 lb.	MODULUS OF ELASTICITY BETWEEN LOADS PER SQUARE INCH OF 100 and 1 000 lb.	MODULUS OF ELASTICITY BETWEEN LOADS PER SQUARE INCH OF 1 000 and 2 000 lb.	Compressive Strength per sq. in. lb.
1	2	4	7 days	2 593 000	2 034 000	1 351 000	1 730
1	2	4	1 mo.	2 662 000	2 445 000	1 [illegible]2 000	2 567
1	2	4	3 mos.	3 671 000	3 170 000	[illegible]58 000	2 975
1	2	4	6 mos.	3 646 000	3 567 000	2 582 000	3 989
1							
1	3	6	7 days	1 869 000	1 530 000		1 511
1	3	6	1 mo.	2 438 000	2 135 000	1 219 000	2 260
1	3	6	3 mos.	2 976 000	2 656 000	1 805 000	2 741
1	3	6	6 mos.	3 608 000	3 503 000	1 863 000	3 068
1							
1	6	12	1 mo.	1 376 000			1 146
1	6	12	3 mos.	1 642 000	1 364 000		1 359
1	6	12	6 mos.	1 820 000	1 522 000		1 592

Various other tests of modulus of elasticity may be found in Tests of Metals, U. S. A., during the years 1898 to 1907.

Tests of Mortar Prisms. Elastic properties of prisms of neat Portland cement and cement mortar, from tests made by Mr. Howard* at the Watertown Arsenal, are presented in the following table:

Elastic Properties of Cement and Mortar Prisms 6 by 6 by 18 inches.

Watertown Arsenal. (*See p.* 406.)

Brand of Cement	Composition: Cement	Composition: Sand	Age Days	Modulus of elasticity between loads per square inch of 100 and 600 lb.	100 and 1 000 lb.	1 000 and 2 000 lb.	Permanent sets after loads per square inch of 600 Inch	1 000 Inch	2 000 Inch	Compressive strength per square inch lb.
Alpha	Neat	0	7	7 143 000	5 000 000	8 333 000	0.	0.	0.	4 783
			7	1 167 000	3 000 000	3 118 000	0.	0.	.0002	5 000
Alpha	1	1	15	3 125 000	2 812 000	2 326 000	-.0002	-.0002	.0007	3 846
			36	2 381 000	2 500 000	2 011 000	0.	.0002	.0012	4 763
			56	2 632 000	2 727 000	3 030 000	.0001	.0002	.0010	4 018
Alpha	1	2	15	1 724 000	1 175 000		.0005	.0023		1 376
			30	2 273 000	2 105 000	1 538 000	.0001	.0006	.0010	2 184
			38	2 778 000	2 812 000	2 325 000	0.	.0001	.0020	2 755

Gaged length, 10 inches.

Modulus of Elasticity in Beams vs. Columns. The modulus of elasticity in beams as determined by measurements and computations by Professor Talbot is approximately the same or possibly slightly lower than in columns.

Effect of Consistency of Concrete upon the Modulus of Elasticity. An excess of water in the concrete not only decreases the strength (see page 382), but also affects the deformation curve so as to show a more variable modulus near the beginning of the test. The moduli of concrete of different consistencies and at different ages are shown in the tables from tests of the authors on following page.

Relation of Stress Deformation Curve to the Theory of Beams. The theory of beams is worked out under the assumption that a section plane before bending remains plane after bending so that the deformation or stretch at any point in the compressive portion of the beam is proportional to the distance of this point from the neutral axis. According to this assumption the distribution of stresses is also proportional to the distance from the neutral axis so long as the modulus of elasticity is constant. This distribu-

* Tests of Metals, U. S. A., 1898.

tion may be then represented by a straight line as shown in Fig. 131, p. 417. When, however, the modulus of elasticity changes Hook's law—that stress is proportional to deformation—is no longer applicable, since the intensity of stress is no longer proportional to the distance from the neutral axis but changes according to the relation of the moduli of elasticity at different loadings, and the line representing the distribution becomes a curve.*

Modulus of Elasticity of Concrete of Different Consistencies.† *Proportions by Volume* 1, : 2½ : 4½

By Taylor and Thompson. (*See p.* 406.)

Approximate age in months.	Dry. Compressive strength. Pounds per sq. in.	Dry. Modulus at ⅓ ultimate strength. Pounds per sq. in.	Medium. Compressive strength. Pounds per sq. in.	Medium. Modulus at ⅓ ultimate strength. Pounds per sq. in	Very Wet. Compressive strength. Pounds per sq. in.	Very Wet. Modulus at ⅓ ultimate strength Pounds per sq. in.
1	4,370	4 050 000	3,360	4 500 000	2110	2 100 000
2	5430	4 070 000	3910	4 550 000	2770	3 400 000
6	5170	5 255 000	5170	3 760 000	3,350	2 880 000
17	5510	3 920 000	4720	3 750 000	2430	2 080 000

Since the modulus is nearly constant within the working limits the authors have adopted the straight line theory of distribution of stress as simplest and most practical.‡

Formerly the parabolic distribution of pressure in concrete above the neutral axis was used in preference to the straight line theory because it corresponds somewhat more nearly to actual test. The two theories, however, require practically identical percentages of steel and the only difference is in the determination of the unit stress in the concrete. When using the parabola theory, about 15% lower compressive stress in the concrete must be used than when figuring by the straight line theory to obtain similar results. For example, 650 pounds per square inch safe compression by the straight line theory corresponds to about 565 pounds per square inch by the parabola theory.

* A comprehensive analytical discussion of the effect of a varying modulus of elasticity upon the pressure in a beam under different loadings is presented by Prof. Talbot in Journal Western Society of Engineers, Aug. 1904.

† "The Consistency of Concrete," by Sanford E. Thompson, American Society for Testing Materials. Vol. VI, 1906.

‡ It is also recommended by the Joint Committee, 1909.

Value to Use for the Ratio of Elasticity in Compression. For beam and slab design and also for column design, tests indicate that a practical value of 15 for the ratio of the moduli of steel to concrete corresponding to a concrete modulus, $E_c = 2\ 000\ 000$, best satisfies the conditions for ordinary 1 : 2 : 4 concrete, and without serious error may be used for all classes of concrete, and is therefore recommended for general use.* For calculations relative to deflections where the tensile strength of the concrete is taken into account, a ratio of elasticity of 8 to 12 may be used as giving results corresponding more nearly to actual conditions. The value of 15 has been adopted in the American, British, German and Austrian rules up to 1909. The French rules for 1907 authorize a range from 8 to 15, according to conditions.

A lower modulus of elasticity for concrete (that is, a higher ratio) should be used in determining the location of the neutral axis in beam design than the values obtained at working loads in compression tests, to compensate for the neglect, in the ordinary formulas, of the effect of tension in the concrete. The use of a high ratio is generally on the safe side also, since it lowers the apparent location of the neutral axis and increases the amount of steel required. These reasons explain the selection of a ratio of 15, which is a higher value than is obtained in compression tests. On the other hand, when the modulus is to be used to determine the deflection of a beam, a lower ratio (i. e., a higher modulus) should be used to make up for the omission of the tensile stress unless this is allowed for in the formulas.

In column design, while the use of a low ratio is most conservative, a high ratio (i. e., a low modulus) corresponds more nearly to actual co di tions, because if there is a weak spot in the column or unusual loading, the steel will be brought into action to an amount indicated by the lower modulus.

The ratio of modulus of elasticity within working limits in beams figured by the parabola and by the straight line methods is indicated by Prof. Talbot's studies† to be in the ratio of about 13 to 12.

Modulus of Elasticity in Tension. But few tensile tests of concrete have been made, but these indicate‡ that the elastic modulus in tension is probably the same as the modulus in compression.

ELONGATION OR STRETCH IN CONCRETE

According to tests of Professor Turneaure, already mentioned, reinforced concrete under a pull, as in the lower portion of a beam, will usually stretch

* It is thus recommended by the Joint Committee, 1909.
† University of Illinois, Bulletin No. 4, April 18, 1906.
‡ Prof. W. Kendrick Hatt, Journal Association Engineering Societies, June 1904, p. 32.

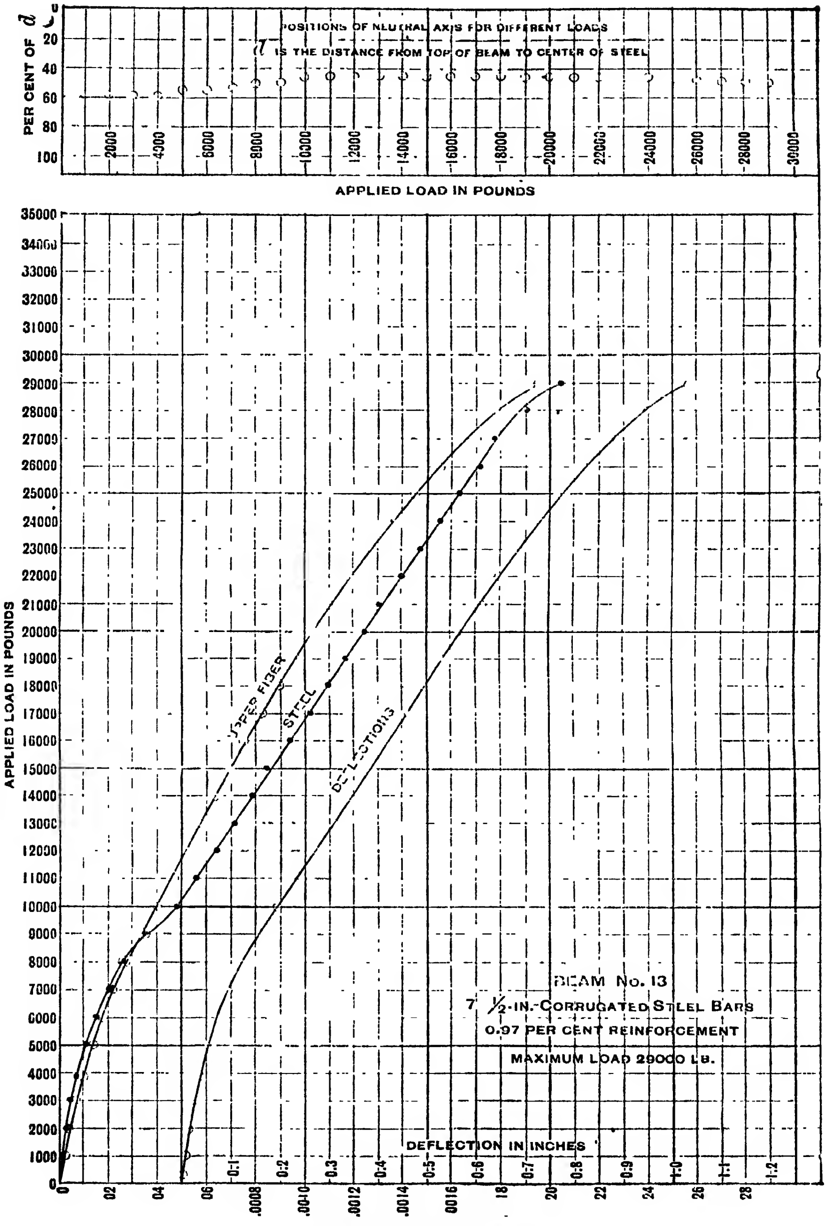


FIG. 130. Typical Deformation and Deflection Curves of a Reinforced Beam By Prof. A. N. Talbot. (*See p.* 410.)

0.0001 to 0.0002 of its length, that is, 0.01 per cent to 0.02 per cent, before showing minute cracks or "water-marks." Cracks become noticeable at a stretching varying in different specimens from 0.0003 to 0.0010 of their length. At this stretch, the steel imbedded in the concrete will have a stress of 9 000 to 30 000 pounds per square inch. Even then, however, the cracks are still so small and are so well distributed by steel properly placed that they are not apt to be noticed in a reinforced structure until the steel has nearly reached its elastic limit.

The concrete in a reinforced beam stretches similarly to the concrete in a plain beam except that the plain concrete beam breaks when the limit of stretch is reached, while if reinforced, the pull is borne partly by the steel and partly by the concrete, and they both stretch together up to the point where cracks, so minute at first as to be almost invisible, occur in the concrete.

The action of the reinforced concrete is shown in the deflection curve in Fig. 130. The inclination of this curve changes at about the same load that is required to break a similar beam of plain concrete.

The diagram shows a typical result of Prof. Talbot's tests of the deformation of the concrete and the deformation of the steel, the deflection of the beam, and the various measured positions of the neutral axis during flexure. Among other conclusions, Prof. Talbot draws the following:

1. The composite structure acts as a true combination of steel and concrete in flexure during the first or preliminary stage, and this stage lasts until the steel is stressed to, say, 3 000 pounds per square inch, and the lower surface of the concrete is elongated about $\frac{1}{10\,000}$ of its length.

2. During the second or readjustment stage there is a marked change in distribution of stresses, the neutral axis rises, the concrete loses part of its tensional value, and tensile stresses formerly taken by the concrete are transferred to the steel. During this stage minute cracks probably exist, quite well distributed, and not easily detected.

3. In the third or straight-line stage the neutral axis remains nearly stationary in position and the concrete gradually loses more of its tensional value. Visible cracks appear and gradually grow larger, though no change in the character of the load-deformation diagram results. It would seem probable that at these cracks the stress in the steel is more than is indicated by the average deformation for the full gage length.

Professor Talbot states that at the load when the curve changes character,—which in the beam shown in the diagram is about 8 000 pounds total load,—there are probably invisible cracks in the lower portion of the beam. This change in direction of the curve, indicating a suddenly increased load upon the steel, is strong proof of the loss in tensional resistance of the con-

crete. Professor Turneaure, moreover, in his experiments, at loads somewhat beyond the point of change in direction, actually discovered these minute cracks. He tested his beams upside down, that is, the load was applied upward, and the minute cracks or water-marks were shown by hair lines on the wet surface of the concrete. Professor Turneaure* says:

It has been found that by testing the beams when somewhat moist, a crack is made visible when exceedingly small, it appearing first as a narrow, wet streak perhaps $\frac{1}{8}$ inch wide and a little later as a dark hair-like crack. It was not necessary to search for the lines with a microscope as under these conditions they were readily found.

That the wet streak, called a "watermark" hereafter, shows the presence of an actual crack was demonstrated last year by sawing out a strip of the concrete containing such a watermark; the strip fell apart at the watermark.

In the plain concrete no watermarks or cracks were observed before rupture. Comparing the observed and calculated elongations of the reinforced concrete with those for the plain concrete at rupture, it will be seen that the initial cracking in the former occurs at an elongation practically the same as in the latter.

The significance of these minute cracks is an open question. It has been supposed that concrete reinforced by steel will elongate about ten times as much before rupture as will plain concrete. These experiments show very clearly that rupture begins at about the same elongation in both cases. In the plain concrete total failure ensues at once; in the reinforced concrete rupture occurs gradually, and many small cracks may develop so that the total elongation at final rupture will be greater than in the plain concrete. In other words, the steel develops the full extensibility of a non-homogeneous material that otherwise would have an extension corresponding to the weakest section.

These results are somewhat at variance with the conclusions reached by Mr. Considère† in France. He was not able to locate these fine cracks and therefore concluded that while the stretch of plain concrete was about 0.0001 of its length or about 0.01%, in combination with steel it could actually attain a stretch twenty times this, or 0.2%. Because of this apparent action of the concrete, Mr. Considère in his formula for beams assumes the concrete to resist a certain amount of tension.

The stretch, or deformation, in the concrete of a reinforced beam may be estimated approximately from the pull, or stress, upon the steel and the modulus of elasticity of the latter, since

$$\text{elastic deformation} = \frac{\text{stress}}{\text{modulus of elasticity}}$$

* Proceedings American Society for Testing Materials, 1904.

† Considère's Reinforced Concrete, p. 35.

For example, if the steel is pulled to 16 000 pounds per square inch, the stretch per unit of length (disregarding initial tension) is

$$\frac{16\ 000}{30\ 000\ 000} = 0.00053$$

Knowing the stretch in the concrete (and therefore the stretch in the steel imbedded in it) the stress in the steel is readily computed from the same formula.

Tensile Resistance in the Concrete. Professors Talbot and Turneaure both concluded from their tests in 1904 that the tensile strength of concrete may be disregarded in the consideration of the ultimate load carried by a beam. This has since been adopted as current practice in design and is in accordance with the recommendations of 1908 and 1909 in America and Europe. The tensile resistance of the concrete affects the deformation and deflection of the beam under the smaller loads, but if, as is customary, the working strength is taken as a definite fraction of the resistance at the elastic limit of the steel, **the tensile resistance of the concrete need not be considered in the design of reinforced beams.**

Prof. Turneaure says:

The presence of the cracks of course seriously affects the tensile strength of the concrete, and, as they appear at an elongation corresponding to a stress in the steel of 5 000 pounds per square inch or less, it would seem that no allowance should be made for the tensile resistance of the concrete. Furthermore, if such cracks are present the calculation of the tensile resistance of reinforced concrete by the method used by Considère leads to no useful result. In his tests Considère determines the stress in the steel from measurements of its elongation and then assumes the concrete to carry the remainder of the load. Assuming the value of E to be uninfluenced by the concrete, this would be correct so long as the stress in the steel and in the concrete is uniform between points of measurement. As stated by Considère himself, such results are only average values. But the concrete may be cracked entirely through and yet possess a very considerable average tensile strength over a length of several inches. Obviously in that case an average is of no value; the strength of the concrete is really zero.

In practical design the most important question which arises is how far a concrete beam may be cracked without exposing the steel to corrosive influences. In this respect it seems to the writer that the minute cracks which appear in the early stages of the tests can have very little influence. However, the entire question of the effect of cracks and pores in the concrete on the corrosion of the steel needs careful investigation.*

*For later information on this point, see p. 328.

QUALITY OF REINFORCING STEEL

It is generally recognized in reinforced beam design that the yield point of the steel should be considered as the point of failure of this material. Tests show that when the metal reaches its yield point, the beam sags, and this deflection, due to the stretch of the steel and in some cases to the slipping of the steel because of its reduced cross-section, is likely to produce crushing in the concrete.

The yield point of ordinary mild steel purchased in the open market, as determined by the drop of the beam in testing (the true elastic limit is several thousand pounds lower) cannot safely be fixed at a higher value than 30 000 pounds per square inch, although frequently, and in fact in the majority of cases, a value of at least 36 000 pounds, and in many cases 40 000 pounds will be found.

High steel, that is, steel containing a high percentage of carbon, has a much higher yield point than mild steel. If of first-class quality,* a minimum yield point may be placed at 50 000 or 55 000 pounds per square inch and much of it will reach 60 000 pounds. The ultimate strength should be not less than 85 000 pounds per square inch. Thus, if it can be safely employed in reinforced concrete, it is adapted to carry much higher stress than mild steel, and, conversely, a smaller percentage of it is required for the same moment of resistance. Many engineers do not approve of the use of high steel because of its brittleness when of poor quality, and the danger of sudden accident, and because of the fact that it is prohibited in ordinary structural steel work.

Brittleness in steel, however, is less dangerous in reinforced concrete than in many classes of structural steel work because the concrete protects it from shock, and also because smaller sections of steel are used in concrete beams than in steel beams, and the large and irregular shapes of the latter render them much more sensitive to irregular cooling during the process of their manufacture.

Mild steel, that is, ordinary market steel, is manufactured and sold under such standard conditions that for unimportant structures it often may be used without other test than the bending test given on page 415. High steel, on the other hand, must be very thoroughly tested. When tested, however, as per our specifications, page 38, it is entirely safe and to be preferred to mild steel. The objection to it for reinforced concrete is based largely upon the use of a poor quality of material and the extra cost. Another objection which has been raised is that before the elastic

* See Specifications for First-class Steel, p. 38.

limit is reached, the stretch in the high steel may produce excessive cracking in the concrete in the lower portion of the beam, and thus expose the steel to corrosion. The mere fact that cracks are visible does not prove that they are dangerous, because the steel is always designed to take the whole of the tension. Mr. Considère's and Professors Talbot's and Turneaure's tests indicate that there is no dangerous cracking even with high steel until the yield point of the steel is reached.

Tests made in Europe in 1907 (see p. 336) prove quite conclusively that the cement protects the steel from ordinary and even extraordinary corrosive action until the elastic limit of the steel is nearly reached. In cases where very minute cracking of the concrete may cause anxiety (even although not dangerous), the steel, whatever its quality, should not be stressed beyond the ordinary limits of, say, 16 000 pounds per square inch.

A yield point in steel of 30 000 pounds per square inch corresponds to a stretch of 0.0010 of its length and a yield point of 50 000 to a stretch of 0.00167. (See p. 411.)

If steel could be made with a high modulus of elasticity it would be particularly serviceable for reinforced concrete, because the higher the modulus of elasticity of a material, the less is the deformation under any given loading. Unfortunately, however, all steel, whether high or low in carbon, has substantially the same modulus of elasticity (30 000 000 lb. per sq. in.).

It may be stated, then, that high carbon steel, say, 0.56% to 0.60% carbon, of the quality used in the United States for making locomotive tires, is better than mild steel for reinforced concrete provided the steel is well melted and rolled, and is comparatively free from impurities, such as phosphorus. However, a high carbon steel, unless limited by chemical analysis, and made under careful inspection, is in danger of being more brittle than low carbon steel. Its use, therefore, should be limited strictly to work important enough to warrant the ordering of a special steel and the taking of sufficient trouble on the part of the purchaser to insure strict adherence to the specifications. Since manufacturers cannot always be depended upon to exactly follow specifications of this nature, it is necessary that an inspector be sent to the works either by the dealer or the purchaser.

The specifications for first-class steel on page 38 are sufficiently explicit so that steel which comes up to them can be safely used and a working stress of 20 000 lb. per sq. in. will not be excessive. A steel which can be employed with safety for all the locomotive and car wheels of the country certainly cannot be discarded as unsafe for concrete, provided similar precautions are taken in its purchase; on the other hand the extra cost may prohibit its use except under special conditions.

Bending Test for Steel. The most important test in the specifications is the bending test and **no steel which fails to pass this bending test should be used under any circumstances.** The bending test is as follows: Test specimens for bending shall be bent cold to the following angles without fracture on the outside of the bent portion:

Around twice their diameter.	Around their own diameter.
Specimens 1 inch thick, 80°.	Specimens $\frac{1}{4}$ inch thick, 130°.
Specimens $\frac{3}{4}$ inch thick, 90°.	Specimens $\frac{3}{16}$ inch thick, 140°.
Specimens $\frac{1}{2}$ inch thick, 110°.	Specimens $\frac{1}{8}$ inch thick, 180°.

Steel with high elastic limit, whether due to high carbon or to manipulation in manufacture, should be purchased with these reservations even if the working stress is to be no higher than is used with mild steel, say, 16 000 pounds per square inch, because it is liable to be brittle. In case a lot of steel has been delivered without previous test by the purchaser, one bar selected at random in every 100 should be subjected to this test and if it fails to pass, the portion from which it is taken should be rejected.

THE STRAIGHT LINE THEORY

For reasons discussed in the preceding paragraphs, the authors have selected the straight line theory of distribution of stress with the concrete taking no tension.

This theory assumes the following hypothesis as a basis for practical design:

(1) A plane section before bending remains plane after bending.

(2) Tension is borne entirely by the steel.

(3) Initial tension or compression is absent in the steel.

(4) Adhesion of concrete to steel is perfect within working limits.

(5) Modulus of elasticity of concrete within the usual limits of stress is a constant.

Our reasons for selecting this theory may be briefly recapitulated as follows:

(a) Beams designed by it and properly built will be unquestionably safe.

(b) Fine cracks are formed in the tension portion of the beam at an early stage in the loading which actually destroy nearly all of the tensile resistance of the concrete.

(c) The modulus of elasticity in many tests has been shown to be approximately a constant within working loads.

(d) This theory is the simplest, and the most easily understood.

(e) It has been adopted by the highest authorities in America and Europe.

(f) The results from it may be readily compared with other theories.

These assumptions lead to the formulas given in this chapter.

During the period termed by Prof. Talbot the first stage (see p. 410) it is necessary in scientific computations involving the deflection of the beam to take the tension of the concrete into account by the methods given on page 760.

LOCATION OF NEUTRAL AXIS

The location of the neutral axis after the load has been transferred to the steel, is given in formula (6) on page 420 and numerical values for different moduli of elasticity and different percentages of steel on page 521. As is evident from the formula, it varies with the strength and elasticity of both the concrete and steel. Because of the peculiar action of the deformations, as illustrated in Fig. 130, page 409, the location of the neutral axis changes as the load is applied. The question is much simplified in practice by assuming a constant ratio of moduli of 15.

An empirical formula suggested by Prof. Talbot for the location of the neutral axis under normal loading is given on page 479.

Tests show that the neutral axis for small loadings is just below the center line of the beam. For greater loadings it moves gradually nearer to the compression side. As the first cracks develop, the change in position of the neutral axis is more sudden, and the distance of the neutral axis from the compression side soon reaches its minimum, which for usual percentages of steel is 3/10 to 4/10 of the depth from the top, after which the change for additional loading is inappreciable. At failure the change is sudden again.

DESIGN OF A RECTANGULAR BEAM

In a simple rectangular beam we may represent the stresses by the diagram shown in Fig. 131, page 417. At any vertical section through the beam the concrete in the upper portion resists the forces which tend to compress it, and the steel in the lower part of the beam resists the forces which tend to stretch and break it in tension. The compressive resistance acts in one direction and the tensile resistance in another direction, as designated by the large arrows in the diagram. The center of tension in the steel is at the center of the rod, or, if there is more then one tier of rods, through the center of gravity of the set of rods. The center of pressure of

the concrete passes through the center of gravity of the triangle which represents the compressive stresses. The reason for the assumption of the uniformly increasing pressure from the neutral axis to the outside fiber is discussed above.

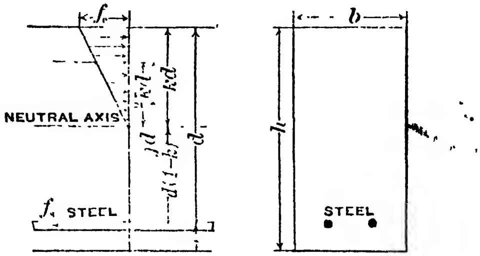


FIG. 131. Resisting Forces in a Reinforced Concrete Beam. (*See p.* 417 *and* 420)

On page 420 are given simple formulas to review a beam already designed and the various letters in Fig. 131 are there defined.

A complete analysis of rectangular beams is presented in Appendix II, and the reader is referred to the discussion of the theory there given and the derivation of the formulas. (See p. 751.)

The theory of beams requires that the total internal pressure be equal to the total tension or pull in the steel. The safe resisting moment of the beam, which of course must be equal to or greater than the bending moment, is the product of the moment arm (that is, the distance between centers of tension and compression) times either the total safe pressure or the total safe pull. In case the design is unbalanced so that the beam is stronger in compression than in pull, the strength of the beam is limited by the safe moment of resistance determined from the allowable tension or pull in the steel. If, on the other hand, the tensile resistance is greater than the compressive resistance, the concrete governs the strength of the beam.

A beam, then, must have breadth and depth sufficient to prevent excessive compression in the concrete in the top of the beam and enough steel to take all the pull without exceeding the working stress of the steel. Rules for this are given in the simple formulas which follow. The steel must also have sufficient bond (see p. 456) and in many cases inclined or vertical reinforcement is required as treated in connection with diagonal tension, pages 448 to 459. Continuous beams also require reinforcement over the supports, as described in pages 427 to 431.

Having computed the maximum bending moment due to the loads (see

p. 439) the breadth of the beam, b, is assumed and the depth of the steel is found from the following formula:

Let

d = depth of beam from compressed surface to center of steel in inches. (See Fig. 131, p. 417)

jd = moment arm or distance between centers of tension and compression.

b = breadth of beam in inches.

p = ratio of cross-section of steel to cross-section of beam above the center of gravity of the steel.

A_s = area of cross section of steel in square inches.

M = moment of resistance or bending moment in general in inch-pounds.

C = a constant for a given steel and a given concrete.

$$d = C\sqrt{\frac{M}{b}} \tag{1}$$

$$\text{and} \quad A_s = pbd \tag{2}$$

The constants C and p to be substituted in the above formulas may be taken from the table on page 519 corresponding to the allowable working stresses in steel and concrete and to the ratio of their moduli of elasticity.

Selected working stresses for tension in steel, f_s, and compression in concrete, f_c, require a definite percentage of steel, and the percentage cannot be altered without changing the ratio of these working stresses.*

For a working compression in the concrete of 650† pounds per square inch, a working pull in the steel of 16 000† pounds per square inch, and a ratio of modulus of steel to concrete of 15†, for concrete having a compressive stress in cylinder form of 2 000 pounds per square inch at 28 days.

$$d\ddagger = \frac{1}{10}\sqrt{\frac{M}{b}} \tag{3}$$

$$A = 0077\ bd \tag{4}$$

Example 1: What depth of beam having a span of 18 feet and what area of steel are required, using above unit stresses, for a freely supported beam with a load of 600 pounds per running foot?

* See page 752, formula (5).

† Recommended by the Joint Committee, 1909

‡ More exactly, $d = 0.096\sqrt{\frac{M}{b}}$

Solution: Bending moment, M, for $\frac{wl^2}{8}$ is $\frac{600 \times 18 \times 18 \times 12}{8} = 291600$ inch pounds, and using formula (3)

$$d = \frac{1}{10}\sqrt{\frac{291600}{8}} = 19 \text{ inches}$$

With 2 inches of concrete below the steel, the total depth of beam is thus 21 inches.

The area of steel from formula (4) is

$$A = .0077 \times 8 \times 19 = 1.17 \text{ square inches,}$$

thus (from Table 1, page 507) requiring four ⅝ inch round bars, or their equivalent.

Depths and Loads for Different Bending Moments. The depth may be obtained in terms of the unit load, if desired, by substituting for M in formula (1) its value in terms of the load and the span. This may be readily transposed also to give the load, w, which a given beam will carry.

The following table is used for determining on the one hand the depths of a beam to be designed, and on the other hand the safe loads of a beam already designed, for uniformly distributed loads. In the formulas in the table:

d = depth of beam from compressive surface to center of steel in inches.
b = breadth of beam in inches.
w = load in pounds per running foot of beam (including weight per linear foot of beam).
l = span in feet.
C = a constant from Table 10 on page 519.

Formulas for Depth and Loading of Rectangular Beams for Different Bending Moments. (See p. 519 for values of C.)

DEPTH, d, AND LOAD PER FOOT, w.	$\frac{wl^2}{12}$	$\frac{wl^2}{10}$	$\frac{wl^2}{8}$	$\frac{wl^2}{4}$	$\frac{wl^2}{2}$
d	$Cl\sqrt{\frac{w}{b}}$	$Cl\sqrt{\frac{1.2w}{b}}$	$Cl\sqrt{\frac{1.5w}{b}}$	$Cl\sqrt{\frac{3w}{b}}$	$Cl\sqrt{\frac{6w}{b}}$
w	$\frac{d^2b}{C^2l^2}$	$\frac{d^2b}{1.2C^2l^2}$	$\frac{d^2b}{1.5C^2l^2}$	$\frac{d^2b}{3C^2l^2}$	$\frac{d^2b}{6C^2l^2}$

Formulas to Review a Beam already Designed. To review a beam already designed, the following formulas may be used, the derivation of which is given in Appendix II, page 751.

f_c = unit compressive stress in concrete per square inch.
f_s = unit tensile stress in steel per square inch.
d = depth of beam from compressive surface to center of steel in inches.
j = ratio of distance between the centers of compression and tension to depth of beam.
$jd = d\left(1 - \frac{k}{3}\right)$ = distance between the centers of compression and tension.
b = breadth of beam in inches.
A_s = area of cross-section of steel in square inches.
p = ratio of cross-section of steel to cross-section of beam above center of gravity of steel.
k = ratio of depth of neutral axis to depth of beam d.
M = bending moment in inch pounds.
n = ratio of elasticity of steel to concrete

Then

$$p = \frac{A_s}{bd} \qquad (5) \qquad\qquad k = \sqrt{2pn + (pn)^2} - pn \qquad (6)$$

$$f_s = \frac{M}{A_s jd} \qquad (7) \qquad\qquad f_c = \frac{2M}{bd^2 jk} \qquad (8)$$

The values of j and k are dependent upon the percentage of steel and the ratio of moduli of elasticity of steel and concrete and may be taken from table, pages 520 and 521.

For a beam with about 0.8 per cent of horizontal steel (in which case, the tension in steel f_s is about 16 000 pounds per square inch and the compression in concrete f_c about 650 pounds per square inch) the distance between the centers of compression and tension jd, is about $\frac{7}{8}$ d and the above formulas may be expressed with scarcely appreciable error as

$$f_s = \frac{M}{0.87\, A_s d} \qquad (7a) \qquad\qquad f_c = \frac{6M}{bd^2} \qquad (8a)$$

Neither the allowable tension in steel nor the allowable compression in concrete should be exceeded. Tables for determining the dimensions and loading of rectangular beams are given on pages 509 to 511, and the methods of practical computation and details of design are illustrated in Example 6, page 469. T-beams are treated on page 423.

The selection of bending moments to use in design of continuous beams is treated on p. 439.

DESIGN OF SLABS

A slab, as far as the computation is concerned, is a rectangular beam and the depth and percentage of steel are therefore obtained by the formulas just given.

Since the bending moment is figured for a width of slab equal to one foot, b in formulas (1) and (2) becomes 12 inches and the formulas change (using notation on page 418) to

$$d = 0.29C\sqrt{M} \quad (9)$$
$$A_s = 12\,pd \quad (10)$$

For stress recommended by the Joint Committee, 1909

$f_c = 650$ pounds per square inch, $f_s = 16\,000$ pounds per square inch and $n = 15$, substituting corresponding value for C from Table 10, page 519, the above formulas become

$$d = 0.028\sqrt{M} \quad (11)$$
$$A_s = 0.092d \quad (12)$$

The use of these formulas is illustrated in Example 6, page 469.

Slabs which are continuous over the supports, such as those in a floor or in a buttressed retaining wall, must be designed with provision for the negative moment at the supports. For uniformly loaded spans continuous over 2 or more intermediate supports, a moment $M = \frac{1}{12}wl^2$ may be used both in the centers of the spans and also at the supports, while for end spans a moment $M = \frac{1}{10}wl^2$ is necessary.

In practice to provide for the moments over the supports some designers bend up all the bars near the $\frac{1}{4}$ point, but a better way, in order to be sure that no point in tension is unprovided with steel, is to bend up one-half, two thirds or three-quarters of the bars and run them over the supports allowing the remainder to continue at the bottom of the slab. To provide the rest of the steel at the support, the bars in the adjoining span can be carried back over the support. Where the bars are so long as to extend over several spans, they can be arranged to break joints at different places, and so keep as much steel over top of supports as at center of span.

The bend in the bars should be near the $\frac{1}{4}$ points in the span, and usually at an angle of about 30 degrees with the horizontal. Too sharp an angle may tend to crack the slab, while, on the other hand, they must be brought to the top of the slab far enough from the support to properly provide for the negative moment.

Tables for determining the dimensions and loading of slabs can be found on pages 512 to 515 and the methods of practical computation and details of design are illustrated in Example 6, page 469.

Cross Reinforcement of Slabs. Cross reinforcement, that is, bars at right angles to the principal bearing rods, is customarily used to prevent shrinkage and temperature cracks, and to give added strength. Although this reinforcement is not absolutely essential, it stiffens the construction floor and often renders expansion joints unnecessary.

The amount of steel to use for this usually is selected somewhat arbitrarily, a cross-sectional area of bars equivalent to 0.2 per cent to 0.4 per cent (p = 0.002 to 0.004) of the cross-section of the floors being the most usual practice.

The top of the slab over a girder or beam which is parallel to the principal reinforcement bars should be reinforced transversely not only for stiffening the T-beam (see p. 443) but also to provide for the negative bending moment produced with the bending of the slab next to the beam or girder. This reinforcement is also necessary even when the beam is simply a small stiffener.

Computing Ratio of Steel. The ratio of steel in a slab is most readily found by dividing the cross section of one bar by the area between two bars, this area being the spacing of the bars times the depth of steel below top of slab. For example, a slab with steel 4 inches below the top and ½ inch round bars spaced 6 inches apart has a ratio, $p = \frac{0.196}{24} = 0.0082$, or 0.82 per cent steel.

Square and Oblong Slabs. Flat plate design by the elastic theory is treated on page 483. A rule for ordinary cases is to require that when the length of the slab exceeds 1½ times its width, the entire load should be carried by transverse reinforcement. For slabs more nearly square the following table represents the proportion of steel which should be run across the slab. These values, while not exact, are on the safe side.

Steel in Oblong Slabs

Ratio of length to breadth of slab.	Ratio of steel across the slab in terms of the total steel.
1	0.50
1.1	0.59
1.2	0.67
1.3	0.75
1.4	0.80
1.5	0.83

Thus if a slab is square, the reinforcement may be placed half in one direction and half in the other. If the bending moment is $\frac{wl^2}{12}$, the reinforcement in each direction must satisfy $\frac{wl^2}{24}$. The total amount of reinforcement thus determined may be reduced 25 per cent by gradually increasing the rod spacing from the one-third point to the edge of the slab.

DESIGN OF T-BEAM

The quantity of concrete in a beam may be reduced when it is built at the same time as the slab so that there is no joint between them, by considering it to be a T-section, that is, computing a portion of the slab as acting with the upper part of the beam in compression. In Appen-

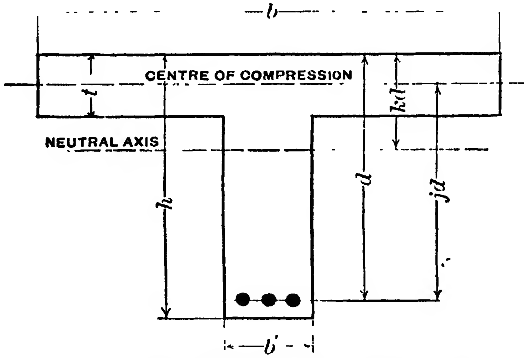


FIG. 132. Section of T-beam. (*See p.* 423.)

dix II, pages 754 to 756 inclusive, we present analyses of the two cases which may occur, depending upon the location of the neutral axis: Case I, neutral axis below the slab or flange; Case II, neutral axis at the underside of the flange; or within the flange. These analyses are not required for design and therefore only the working formulas are here reproduced.

The theory of the design is similar to the theory of a rectangular beam, namely, that the total compression in the concrete in the upper part of the beam is equal to the total tension or pull in the steel at the bottom of the beam.

In the design of a T-beam, the thickness of the flange is fixed by the thickness of slab required to support its load, and the width of flange to use is selected in accordance with rules given below. The values to be determined by computation are then the depth of the beam, the width of stem or web, and the amount of reinforcement.

The width of the slab, b, to use for the flange of the T-beam in compression is selected somewhat arbitrarily. In no case of course can it be taken greater than the distance between beams. The Joint Committee has recommended a width not exceeding one-fourth the span length of the beam and also has limited the width to use on either side of the web to four times the thickness of the slab. It is probably safe to use a somewhat greater ratio of width to thickness than this in many cases.

Cross-section of Web as Determined by the Shear. The width of the web of a T-beam is governed by the layout of the tension rods (see p. 459) and by a study of the shearing stresses (see p. 446).

The total vertical unit shear in a beam effectively reinforced with bent bars or stirrups, or both, is limited by the Joint Committee to 120 pounds per square inch for ordinary concrete having a compressive strength (in cylinders) of 2000 pounds per square inch at 28 days. This is conservative but was selected to prevent the opening of diagonal cracks.

To determine, then, the area of reinforced web required for shear involving diagonal tension, let

b' = breadth of the stem.

$d - \frac{t}{2}$ = moment arm, the depth from center of slab to steel, the thickness of slab being t.

V = total vertical shear.

then from formula (30) for determining the horizontal unit shear

$$b'\left(d - \frac{t}{2}\right) = \frac{V}{120} \qquad (13)$$

That is, the area of web at any point in the beam (considering this up to the middle of the slab) must not be less than the total shear divided by the maximum allowable unit shear for the beam with its reinforcement.

The design is illustrated in Example 6, page 477.

Minimum Depth of T-Beam. The minimum depth is the depth at which concrete and steel are stressed simultaneously to their working limits. It is governed by **the compression in the flange which must not exceed the working compressive strength of the concrete.** Greater depth than the minimum is generally used for economical reasons.

The minimum allowable depth may be found from the folding diagram, page 525. If preferred, the rectangular beam formula (1), page 418, may be used where the depth of the beam is not greater than four times the thickness of slab, using in this formula the breadth of the flange, b, for

the breadth of the beam. For ratios of depth of T-beams to thickness of slab larger than four the rectangular beam formula gives unsafe results and the formulas given in Appendix II, page 755, must be used.

The methods are illustrated in Example 6, page 470.

Economical Depth for a T-Beam. Usually a greater depth than the minimum is desirable for economy, because deepening the beam reduces the area of steel proportionally. Professors Turneaure and Maurer* analyze the depth for maximum economy and suggest from this the most economical values.

Using the notation

d = depth of T-beam from compressed surface to center of steel in inches.
t = thickness of flange in inches.
b' = breadth of the stem in inches.
M = bending moment in inch pounds.
f_s = allowable unit tension in steel in pounds per square inch.
r = ratio of unit cost of steel in place to unit cost of concrete in place (using same units for steel and concrete).

$$d - \frac{t}{2} = \sqrt{\frac{r\,M}{f_s b'}} \qquad (14)$$

From this formula the most suitable depth may be selected after two or three trial computations for different widths of stem. The ratio of costs, r, ranges between 38 and 75. For cost of concrete† in place 20 cents per cubic foot, and cost of steel in place 3 cents per pound, the ratio of costs equals 75, while for concrete at 40 cents per cubic foot and cost of steel 3 cents this value will be reduced to 38. In calculations where no unit costs are given, a value of 60 may be selected for r.

The depth of the T beam should not be made too great in proportion to the breadth of stem. Many designers make the ratio of the depth of a T-beam to its width of web between 2 and 3. For very deep and large beams a ratio of 4 may be accepted, while, on the other hand, if head room is limited, the depth of the beam may be fixed and the width of stem be determined by area required for shear, so that ratio, $\frac{d}{b}$ may be even less than 2

Another plan sometimes followed in studying designs is to make the depth

* Turneaure and Maurer's "Principles of Reinforced Construction," Second Edition, p. 238.

† The cost of concrete need not include form construction since a variation in depth affects this but slightly.

of T-beam an arbitrary ratio to its span. Comparison of a number of representative designs shows an average ratio of span to depth of beam as about 10 to 12, which suggests the approximate rule to make the depth in inches equal to the span in feet.

Sectional Area of Steel in a T-Beam. The area of cross-section of steel in tension may be obtained very closely by the following formula:

Let

A_s = cross-section of steel in square inches.
M = bending moment in inch-pounds.
f_s = allowable unit tension in steel in pounds per square inch.
d = depth of T-beam in inches.
t = thickness of flange in inches.

then

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} \tag{15}$$

This formula assumes that the center of compression of beam is at the center of the slab. This gives slightly high results for a T-beam with very thin flange in proportion to the dimensions of the web, and too low results for a shallow T-beam with thick flange; ordinarily the error is so slight as to be inappreciable but if d is less than $3\,t$ use formula (4), page 418, taking b as breadth of beam. Formulas for more exact computations or for reviewing T-beams are given in Appendix II, page 749, and quoted below* It is recommended that an inexperienced designer check his results obtained by approximate formulas by the more exact ones.

From the diagram, page 525, the area of steel may be obtained directly and comparisons made between different designs.

Details of Design. The design of a T-beam must also be studied for shear reinforcement (see p. 448), bond of steel to concrete (see p. 456), and especially for the design at the support, which must be adapted to the negative bending moment (see p. 428).

The example on page 470 illustrates the use of the formulas and the principles of design. The selection of bending moments is treated on page 439.

* Let kd = depth of neutral axis; n = ratio of elasticity; b = breadth of flange; f_c = outside fibre compression in concrete. Then,

$$kd = \frac{2\,nd\,A_s + b\,t^2}{2\,n\,A_s + 2\,bt};\quad z = \frac{3\,kd - 2\,t}{2\,kd - t}\cdot\frac{t}{3};\quad jd = d - z;\quad f_s = \frac{M}{A_s\,jd};\quad f_c = \frac{M\,kd}{bt\left(kd - \frac{1}{2}t\right)jd}.$$

BEAMS WITH STEEL IN TOP AND BOTTOM

Although concrete is always cheaper than steel to use for compression, it is frequently desirable to place steel in the compression as well as in the tension side of the beam. In a continuous beam, for example, the steel is carried horizontally into the support and may be figured with the concrete to assist it in taking the compression provided its length is sufficient to provide bond.

Analysis of the design with steel located to take compression and tension, with no tension considered in the concrete, is presented in Appendix II, page 757. For convenience, the diagram, Fig. 133, is here reproduced, and the working formulas are given.

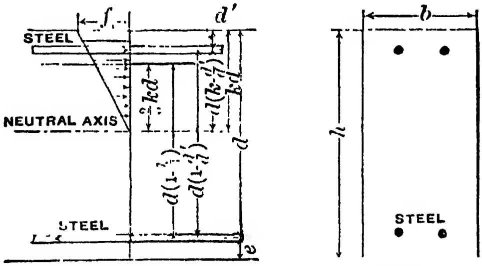


FIG. 133. Resisting Forces with Steel in Top and Bottom of Beam. *(See p. 427 and p. 757.)*

Let

b = breadth of beam in inches.

d = depth of beam from compressed surface to center of steel in inches.

a = ratio of depth of compressive steel to depth of beam.

p = ratio of cross-section of steel in tension to cross-section of beam bd above this steel.

p' = ratio of cross-section of steel in compression to cross-section of beam above the steel in tension.

f_c = unit compressive stress in outside fiber of concrete in lb. per sq. in.

f_s = unit tensile stress, or pull, in steel in lb. per sq. in.

f_s' = unit compressive stress in steel in lb. per sq. in.

M = moment of resistance or bending moment in general in in. lb.

C_c, C_s, C_s' = constants from Table 8, pages 516, 517.

The location of the neutral axis varies greatly with the location and the area of the steel, so that an approximate formula cannot easily be made.

The allowable stresses must not be exceeded either in the concrete or the steel. The bending moment, therefore, must not exceed the moment of resistance of the concrete and the steel and the beam must satisfy the equations (see p. 757 for derivation):

$$M = f_c bd^2 C_c \qquad (17) \qquad \text{or} \qquad f_c = \frac{M}{bd^2 C_c} \qquad (18)$$

and

$$M = f_s bd^2 C_s \qquad (19) \qquad \text{or} \qquad f_s = \frac{M}{bd^2 C_s} \qquad (20)$$

These formulas may be solved readily by introducing values of C_c or C_s from the table on page 516. The constants are dependent upon the values of p, p', a and n and therefore vary with the reinforcement of the beam.

The working strength of the steel in compression cannot be reached without exceeding the compressive strength of the concrete in which it is imbedded, but, if its value is desired, it may be determined from the formula (39), page 759, which, with the substitution of C_s' for the square brackets, becomes

$$f'_s = \frac{M}{bd^2 C'_s} \qquad (21)$$

The value of C_s' is obtained directly from the table on page 516. The use of the formulas is illustrated in Example 6, page 470.

DESIGN OF A CONTINUOUS BEAM AT THE SUPPORTS

The formulas and table just given for a beam with steel in top and bottom are of the greatest value in designing the ends of a continuous beam.

A number of concrete buildings have been built in the past with beams having insufficient steel through the top of the supports to take the pull and insufficient concrete at the bottom of the ends of the beam to take the compression, and when these have been loaded as designed, cracks, and in many cases serious ones, have occurred at the supports. Just as much care, therefore, is necessary in designing the end of a reinforced beam as the middle.

The tendency to overstress the supports is due to the T-beam design. In the middle of the T-beam the slab takes the compression, but at the support, the compression being in the bottom of the beam because of the negative bending moment, there is only the web of the beam to resist it.

In designing, a slightly higher compression may be allowed in the con-

crete at the end than at the middle of the beam, using 750 pounds per square inch for 2 000 pounds concrete instead of 650, (see page 528) because the negative moment decreases so rapidly that only a short section is under maximum stress. Besides this, the steel in the lower part of the beam (if sufficiently bonded to the concrete) may be reckoned in compression by the formulas just given. If this is not sufficient to fulfill the requirements, the lower surface of the beam near the support may be dropped so as to form a flat haunch.

By bending up half of the horizontal steel in the beams on each side of the support, and carrying it across over the support, lapping far enough to attain its full strength in bond, the tension in the top of the support will be provided for, since this gives the same tension steel as in the center of the beam. If desired, the stress, f_s, in the steel may be figured from formula (20) above.

Although bond tests with hooked bars (p. 467) indicate that a right angle 5 diameters in length or a semi circular bend of similar length, properly imbedded, will develop the elastic limit of the steel before giving way, it is the safest plan in ordinary construction to rely upon a straight lap of the required length (see p. 464). However, where this is impossible, as at the wall line in a building, or in a retaining wall, the effectiveness of the hook permits thorough bonding of the members together.

Since the bending moment is a maximum at or near the center of the support, the moment at the edge of the support is slightly less and it is, therefore, frequently worth while to recompute it or estimate it by curves on page 436.

If compression in concrete at the bottom of the member as obtained by formula (18), p. 428, exceeds the working strength, the steel in the bottom of the beam or else the concrete or both must be increased in area. The simplest plan in most cases is to make the beam deeper next to the support by forming a flat haunch. When this is not permissible, extra horizontal steel may be inserted instead. While the forms for this haunch are somewhat troublesome to construct, their cost for beams and girders of usual size should not exceed 25c. to 50c. each.

The amount of increased depth required may be obtained by trial from formula (18) above, assuming a new depth, and then with the aid of the table on page 516, determining whether the conditions are as specified. This is illustrated in the example on page 472.

Under ordinary conditions the computation need be made only at one point, that is, next to the support, since the point to end the slope can be readily figured from the following formula:

For a uniformly loaded beam, let

M_b = negative bending moment next to the support.
M_r = moment of resistance of the inverted T-beam without the haunch, governed by the concrete.
x = length of haunch.
l = span of beam.

Then

$$x = \frac{l}{5}\,\frac{M_b - M_r}{M_b} \quad \text{(approximately)*} \qquad (22)$$

An illustration of the use of this formula is given in Example 6, page 472.

EFFECT OF VARYING MOMENT OF INERTIA UPON THE BENDING MOMENT

However the bending moment may be computed, if the beam is built continuously with the next bay, pull or tension is bound to occur over the support with compression at the bottom of the beam. The assumption is sometimes made that if the middle of the beam is designed as freely supported, that is, on a basis of $\frac{wl^2}{8}$, the supports will be relieved and a readjustment will take place. This is only partially true, and usually should not be counted upon in design.

The assumption of reduced bending moment at the support is based on the smaller moment of inertia at the support, but a thorough study by the authors of different conditions shows that a very large difference in the moment of inertia, as great a difference as it is possible to have in any ordinary floor design, causes a reduction in bending moment of less than 10% and under most conditions the reduction is even much less than this. Consequently a beam at the support should be designed, as suggested in the preceding paragraph, for the full negative bending moment as required by the formula $\frac{wl^2}{12}$.

* This formula is based upon the fact that the point of zero moment is at approximately $\frac{1}{5}$ of the span, and from the curves of bending moment on p. 436, it is evident that the variation in the moment between the support and the $\frac{1}{5}$ point is very nearly a straight line. Hence the difference between the bending moment and the moment of resistance is in approximately the same ratio to the bending moment as is the ratio of the distance from the point where the haunch is needed to the point of zero bending moment. When the point of zero moment is not approximately at $\frac{1}{5}$ span the fraction may be altered accordingly.

SPAN OF A CONTINUOUS BEAM OR SLAB

It is customary to consider the span of a continuous beam or slab as the distance between the centers of its supports. In general this is the simplest plan to follow and one which is always on the side of safety. If the support is exceptionally wide, as when a slab runs into a wide beam, or a beam or girder into a large column,* an arbitrary length of span may be taken, if desired, as the net span between supports plus the total depth of the member which is being designed. The maximum negative bending moment may be considered then either at the center of the support or, if the width of the support is greater than the depth of the member, at a point within the support equal to half the depth of the member.

DISTRIBUTION OF SLAB LOAD TO THE SUPPORTING BEAMS

If slabs are reinforced in both directions, the loads carried to the beams supporting them will not be uniformly distributed over the length of the beam, but may be assumed to vary in accordance with the ordinates of a triangle.

Assuming that the slab transmits a load to its nearer support, we have the following formulas for determining the moment to use in computing the long and the short supporting beams.

Let

l_l = the longer span of a rectangular slab in feet.

l_s = the shorter span of the slab in feet.

w = load per linear foot of beam if the slab is considered as supported by longer beams only.

M_l = bending moment in foot pounds of longer beam.

M_s = bending moment in foot pounds of shorter beam.

Then the moments of the two beams, assuming them as freely supported, are found by the application of simple mechanics, to be

$$M_l = \frac{1}{8} w l_l^2 \left(1 - \frac{1}{3} \frac{l_s^2}{l_l^2} \right) \quad (23) \qquad \text{and } M_s = \left(\frac{1}{8} w l_s^2 \right) \frac{2}{3} \quad (24)$$

For continuous or fixed beams the fraction $\frac{1}{8}$ may be changed to its proper ratio.

Formula (24) does not apply to girders supporting one or more beams. This case is treated under the heading which follows.

* The deflection and the bending moment of a member are changed as soon as it enters the support because of the change in the moment of inertia.

Example 2: What will be the bending moments in the two continuous beams supporting an oblong panel the whole length of which is twice the breadth and which is reinforced so as to transmit its load both ways?

Solution: Using $\frac{1}{12}\,wl^2$ for the continuous beams instead of $\frac{1}{8}\,wl^2$ and substituting:

$$\text{Moment in longer beam, } M_l = \frac{1}{12}\,wl^2\left(1 - \frac{1}{3}\,\frac{(\frac{1}{2}l)^2}{l^2}\right) = \frac{11}{144}\,wl_l^2$$

in terms of the longer span, and

$$\text{Moment in shorter beam, } M_s = \frac{1}{12}\cdot\frac{2}{3}\,wl_s^2 = \frac{1}{18}\,wl_s^2$$

DISTRIBUTION OF BEAM AND SLAB LOADS TO GIRDERS

When one or more beams run into a girder, the load upon the girder consists of the concentrated live and dead loads from the beams acting at their points of intersection with the girder, the uniformly distributed weight of the girder itself, and the unsymmetrically distributed weight of a small portion of the floor slab, with its live load, which bears directly upon the girder.

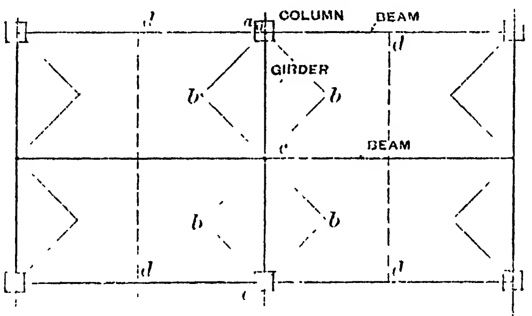


FIG. 134. Distribution of Beam and Slab Loads to Girder. (*See* p. 432.)

To avoid the computation of several moments, a series of studies have been made by the authors for different conditions, and it has been found that the maximum bending moment of a girder may be obtained without appreciable error by considering, as a uniformly distributed load, the weight of the girder plus the weight of slab and its live load, for an area whose length is the length of the girder and whose width is the average length of the beams running from each side into the girder. The sum of these loads divided by the length of the girder gives a uniformly distributed load for which the ordinary formula may be used.

Thus in Fig. 134, instead of computing the moment on the girder as the

sum of the moments produced by loads of the triangles, *a b c*, plus the concentrated loads from the beams at *c*, the entire load *d d d d* may be considered as uniformly distributed over the girder in the length *a a*.

With only one condition is there an appreciable variation from the exact maximum moment, and this is a case where two beams run into a girder at the one-third points. Here the maximum moment obtained by the uniformly distributed method gives slightly too conservative results, and may be reduced by 10%.

Moments in a girder other than the maximum must be computed for individual conditions.

BENDING MOMENTS AND SHEARS

Bending moments and shearing forces have to be computed so frequently in reinforced concrete design that the more common rules and formulas are given here, and besides this elemental matter diagrams are presented for estimating the moments and shears in various kinds of loading, and recommendations are made for the computation of bending moments in design. Shear and diagonal tension in beams are taken up at length.

Rule to Find Reactions at Supports. The reaction at a support must be found in order to determine the bending moment. The sum of the upward forces, which in ordinary beams are the reactions at the supports, is equal to the sum of all the downward vertical forces or loads. In a simple beam supported or fixed at the two ends, the reaction at either end is found by taking moments of all forces about the other support and solving for the reaction desired.

Expressed as a formula, if

R = desired reaction.
P = any vertical load.
l = span.
x = distance of load from the support at which the reaction is desired.
Σ = sum, using − for downward and + for upward forces, then

$$R = \frac{\Sigma P\,(l - x)}{l} \qquad (25)$$

Example 3: In Fig. 135, where there is a uniform load over the entire span and also concentrated loads $P_1 = 200$ and $P_2 = 350$ at the $\frac{1}{3}$ points, what is the left reaction?

Solution: $R = \dfrac{(200 \times 8) + (350 \times 4) + (100 \times 12)6}{12} = 850$ pounds.

The determination of reactions and moments of continuous beams is referred to on page 439.

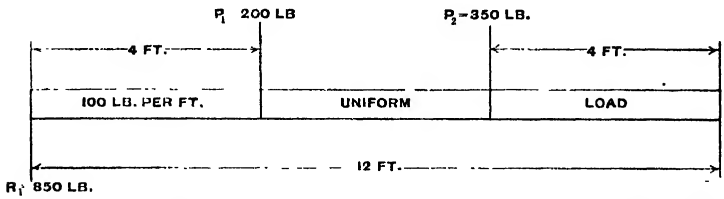


FIG. 135. Beam Loaded with Distributed and Concentrated Loads (*See pp.* 438 and 439.)

Rule to Find Bending Moment at Any Point in a Beam. Consider either side of the vertical section passing through the point and disregard the other side. Multiply each load and reaction by its average distance from the section and add the products, taking loads acting downward as negative and those acting upward as positive.

This sum is the bending moment at the section.

Moments in English measure are usually taken in inch-pounds. Hence, the distance must be in inches and the weights in pounds.

Example 4: In Fig. 135, what is the bending moment in inch-pounds at the middle of the span?

Solution: $M = R_1 (6 \times 12) - P_1 (2 \times 12) - (100 \times 6) \times 3 \times 12 - 34\,800$ inch pounds.

Rule to Find Shear at any Point in Beam. Consider either side of the section passing through the point and disregard the other side. Add the loads and reactions, taking the loads acting downward as negative and those acting upward, such as a reaction, as positive. The sum is the shear at the section.

Example 5. In Fig. 135 what is the shear at the left support and at the center?

Solution: $R_1 = 850$ pounds at left support and at the center the shear is $R_1 - P - (100 \times 6) = 50$ pounds.

Table of Common Bending Moments and Shearing Forces. The following table for convenient reference gives values of the shearing forces and bending moments for common cases. The values for external forces are independent of the structure of the beam.

Bending Moments and Shearing Forces.*

Description	Loading.	Section Considered.	Shearing Force. At distance x from support.	Shearing Force. Greatest.	Bending Moment. At distance x from support.	Bending Moment. Greatest.
Beam fixed at one end, unsupported at other.	At end		W	W	$W(l-x)$	Wl
	Uniform.		$\frac{W}{l}\left(l-x\right)$	W	$\frac{W}{2l}\left(l-x\right)^2$	$\frac{Wl}{2}$
Beam supported at both ends.	At middle.	Between support and middle.	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{W}{2}x$	$\frac{Wl}{4}$
		Beyond middle.	$-\frac{W}{2}$		$\frac{W}{2}\left(l-x\right)$	
	Uniform.		$\frac{W}{l}\left(\frac{l}{2}-x\right)$	$\frac{W}{2}$	$\frac{W}{2l}\left(lx-x^2\right)$	$\frac{Wl}{8}$
	At distance *a* from support.	Between support and load.	$\frac{W(l-a)}{l}$	$\frac{W(l-a)}{l}$	$\frac{W(l-a)}{l}x$	$\frac{Wa(l-a)}{l}$
		Beyond load.	$-\frac{Wa}{l}$	$\frac{Wa}{l}$	$\frac{Wa}{l}\left(l-x\right)$	

* W = total load; l = span; x = distance of section considered from support. If moment is in inch pounds, l and x must be in inches and W in pounds. If load is distributed so as to be in terms of weight per unit length, substitute wl for W in the formulas.

Table of Moments of Inertia. The table on page 438 gives the moment of inertia for beams of a few sections which might be used in concrete construction. The reinforcement, if any, may be considered as replaced by an area of concrete which is the area of the steel times the ratio of elasticity, n, and is located at the same distance from the neutral axis.

SHEAR AND BENDING MOMENT DIAGRAMS

The diagrams in Figs. 136 and 137, pages 436 and 437, give bending moments and shears for beams continuous over four spans. In diagram 136, various distributions of uniform loading are given; in the first place, at the top of the page, with all the spans loaded and ends fixed; next, all spans loaded and the ends supported; and below these curves, different spans loaded in such a way as to produce maximum and minimum bending

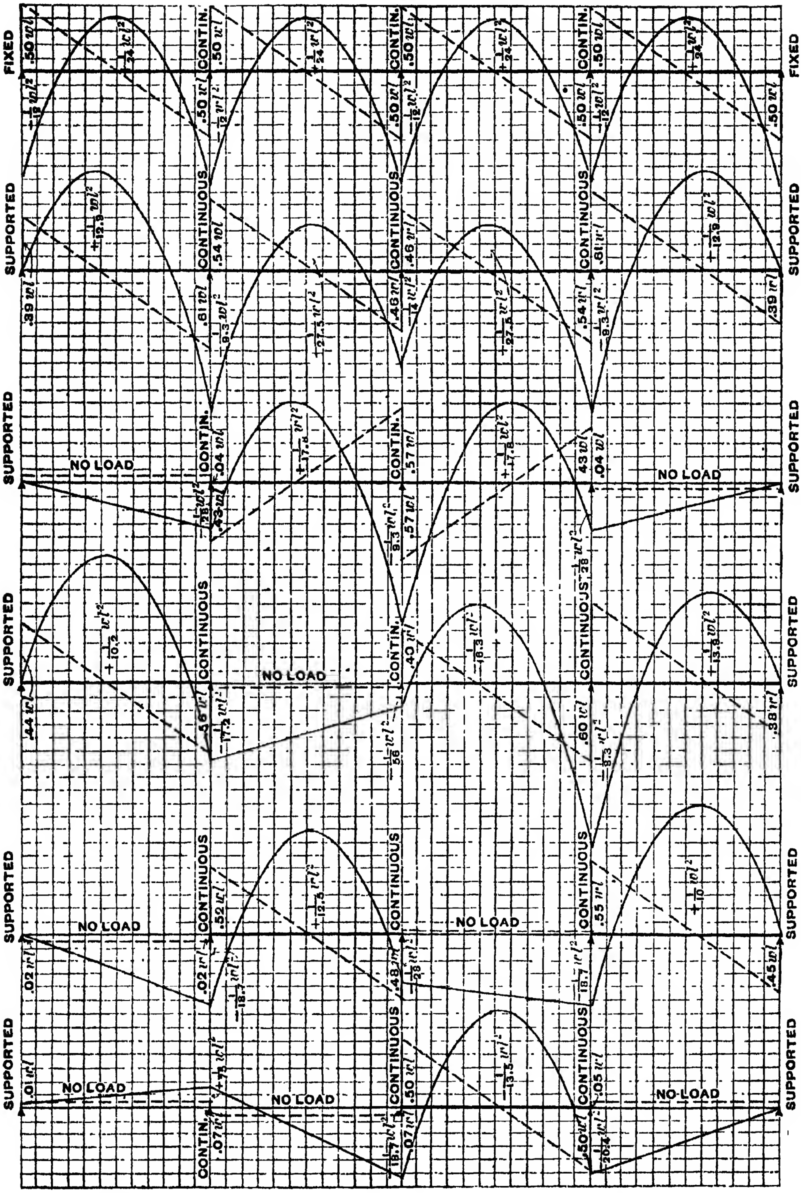


FIG. 136.—Bending Moments and Shears for Continuous Beams, Distributed Loads.
(See p. 435.)

moments and shears. The cases chosen are sufficiently representative to be used without appreciable error as maximum and minimum values for beams of any number of spans and any distribution of uniform loading.

As stated with the diagrams, the curves are all drawn to scale on cross-section ruling so that proportionate values may be read. The loads are given in terms of w, the load per unit of length. The horizontal scale has twelve divisions per span, so that the moments and shears can be readily

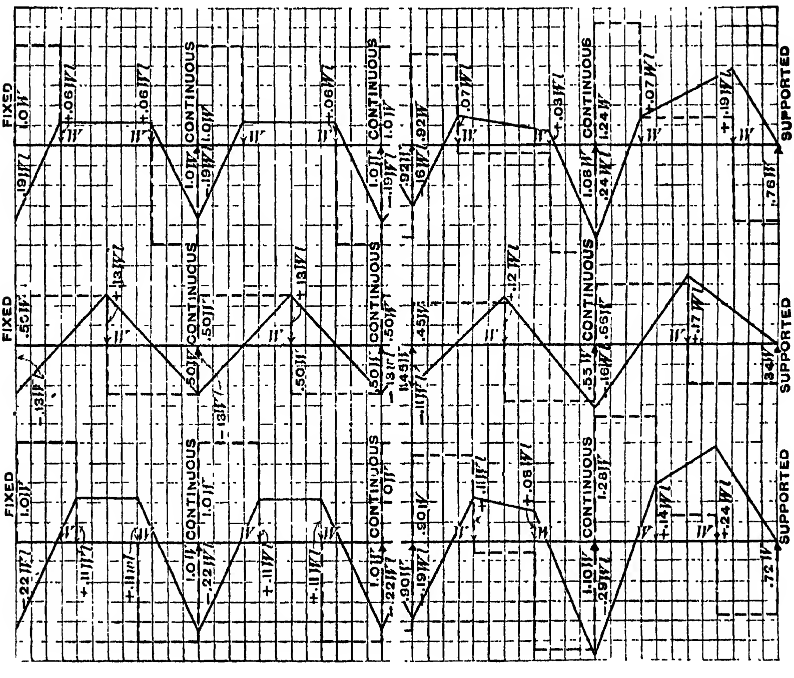
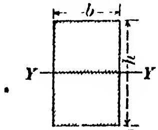
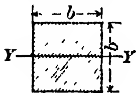
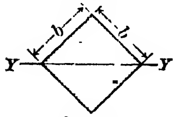
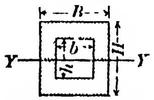
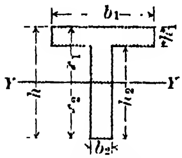
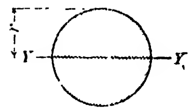
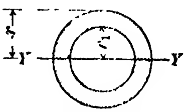
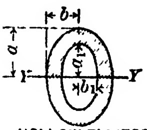


Fig. 137.—Bending Moments and Shears for Continuous Beams, Concentrated Loads
(*See p.* 437.)

estimated at $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ points. The values which are printed for the bending moments are in common fractions for convenience of comparison, although in order to scale them they must be changed from the common fraction to a decimal. Each vertical division for the bending moment scale represents 0.01 and for the shear scale represents 0.1. Bearing this in mind, the bending moment and shear can be scaled at any part of the span.

Concentrated loads are treated in Fig. 137, the loads being located at the

Moments of Inertia. (*See p.* 435.)

Figure.	Area A	Moment of Inertia. I	Distance of Neutral Axis from most Strained Fiber* Y
 RECTANGLE	bh	$\frac{bh^3}{12}$	$\frac{h}{2}$
 SQUARE	b^2	$\frac{b^4}{12}$	$\frac{b}{2}$
 SQUARE	b^2	$\frac{b^4}{12}$	$\frac{1}{2}b\sqrt{2}$
 RECTANGULAR CELL	$BH - bh$	$\frac{1}{12}(BH^3 - bh^3)$	$\frac{H}{2}$
 T SECTION	Area of flange + area of web $= A_1 + A_2$	$\frac{A_1h_1^2 + A_2h_2^2}{12} + \frac{A_1A_2(h_1+h_2)^2}{4(A_1+A_2)}$	$x_1 = \frac{h}{2} - \frac{A_1h_2 - A_2h_1}{2(A_1+A_2)}$ $x_2 = \frac{h}{2} + \frac{A_1h_2 - A_2h_1}{2(A_1+A_2)}$
 CIRCLE	πr^2	$\frac{\pi r^4}{4}$	r
 HOLLOW CIRCLE	$\pi(r^2 - r_1^2)$	$\frac{\pi(r^4 - r_1^4)}{4}$	r
 HOLLOW ELLIPSE	$\pi(ab - a_1b_1)$	$\frac{\pi a^3 b}{4} - \frac{\pi a_1^3 b_1}{4}$	a

*Applicable only to homogeneous (not to reinforced) beams.

quarter points, middle points, and third points respectively. This diagram is of special use in studying girders supporting cross beams. The stresses are computed for a beam of four spans and as the curves are symmetrical at each end, the diagram is broken in two, one-half being shown with fixed end and the other half with end supported. The results with a larger number of spans will not be appreciably different.

The vertical scale for concentrated loading is 0.05 per division for bending moments and 0.2 per division for shears.

The concentrated loads are given in terms of W, the load which is concentrated at each point.

The continuous beam is statically indeterminate, so that the moments and reactions have to be found by the theory of flexure, using the formula of three moments first evolved by Clayperon.*

In applying this to the various cases, the assumption is made that the moment of inertia of the beam is constant throughout its length. While this is not strictly true, extensive studies of various cases in reinforced concrete show that a large change in the moment of inertia makes a very small change in the bending moment, so that the relations are substantially correct until a member enters a much larger member.

BENDING MOMENTS TO USE IN DESIGN OF REINFORCED BEAMS

An examination of the curves in the diagram of bending moments for different loads, Fig. 136, page 436, indicates that in concrete beams built continuously it is safe to use for the positive bending moment in the center of the beam, except for the end spans, and also for the negative bending moment at the ends of the beams,

$$M = \frac{wl^2}{12}$$

and for end spans, for the center and also for the adjoining support

$$M = \frac{wl^2}{10}$$

the customary American and English units being adopted, viz:

M = bending moment in inch pounds.
w = load uniformly distributed in pounds per inch of length
l = length of beam in inches.

* See Lanza's "Applied Mechanics."

In case the load is in pounds per foot of length and l is in feet, the moment in inch pounds to satisfy the former equation is simply the product of the load per foot times the square of the length in feet.

The value of $\frac{wl^2}{12}$ for the bending moment has been widely adopted in Continental Europe, is being used in general practice in Germany, and is recommended in the 1909 recommendations of the American Joint Committee and in the 1907 French rules. However, it is absolutely necessary, when designing by this formula, that the beam be really continuous both in design and construction; that the stresses due to negative bending moment at the support be provided for; that the steel be accurately located; and that, to obtain the best workmanship, the concrete be laid by a responsible builder and superintended by a man experienced in concrete construction.

An examination of the diagrams referred to will show that under these conditions the value is conservative, since a uniformly distributed load, except in the end spans, does not exceed $\frac{wl^2}{24}$ and the worst panel loading shown for the middle of a span gives $\frac{wl^2}{12.5}$.

Many of the building laws in the United States, to provide for the possibility of poor construction or unforeseen conditions, give the more conservative figure,

$$M = \frac{wl^2}{10} \tag{26}$$

and for this reason and also because other assumptions may be made by multiplying by a decimal, this value is used in many of the tables in this book, and in fact it is advised for constructors who are not thoroughly familiar with reinforced concrete.

The same diagram, Fig. 136, shows that the negative bending moments are usually greater than the moments at the middle of spans. However, partial floor loading greatly reduces the negative moment, and as a live load is scarcely ever uniform over two full panels, it is considered safe to use the same value for negative moment as is used for the positive moment in the center of the beam, that is

$$M = \frac{wl^2}{12} \tag{27}$$

At the end support, the beam, if it runs into a column or heavy wall girder,

may be practically "fixed" and thus require top reinforcement for the negative moment, $-\frac{wl^2}{12}$, and the rods running into the support must be bent or otherwise anchored. Sometimes, if the slab has cross reinforcement running into the wall girder, it may be assumed to assist in connecting the beam and girder.

Some designers, in making more exact computations, separate the dead and live load, considering the dead load extended over all panels and finding the most unfavorable position for the live load. Unless the live load is a very exact quantity this is needless refinement.

Bending Moments for Independent Concentrated Loads. If the principal live loads on a beam are concentrated, as they often are upon a girder bridge, the moments and shears at all points must be specially computed. For occasional concentrated loads in connection with uniform live and dead loads, and for loads produced by beams running into girders, it is suggested that the maximum moment under the load be computed as if the beam or girder was supported, and this be reduced by the same ratio used in the distributed loading. Thus, since the maximum moment for a concentrated load at the middle of a supported beam is $\frac{1}{4}Wl$, if $\frac{1}{12}wl^2$ is used in distributed loading instead of the $\frac{1}{8}wl^2$ required for a supported beam, $\frac{8}{12}$ of $\frac{1}{4}Wl$, or $\frac{1}{6}Wl$, may be used for concentrated center loads. The negative bending moment with concentrated loading usually may be taken the same as the maximum positive moment due to concentrated loading, reduced as indicated, except that with loads at $\frac{1}{3}$ or $\frac{1}{4}$ points, this gives for the support next to the end a negative moment which is slightly low (see diagram, Fig. 136, page 436), and in some cases it should be separately figured or else estimated from this diagram.

SHEARING FORCES IN A BEAM OR SLAB

The bending of a beam produces a tendency of the particles to slide upon each other or shear. It is therefore necessary to study

(1) Vertical shear.

(2) Horizontal shear.

(3) Diagonal tension.

Most important of all is the resultant of the shearing forces with the tension which produces the pull in a diagonal direction termed diagonal tension.

Vertical Shear in a Beam. Concrete is strong in direct shear (see p. 382) and capable of standing a working shearing stress of at least 200 pounds per square inch, so that a concrete girder or beam or slab, unless perhaps

of hollow or tapered construction, always has sufficient area of section to withstand this direct shear. However, since the direct shear is a measure of the diagonal tension (see p. 446), which is excessive when the direct shear is comparatively low, it must always be computed in a beam or girder for use in the computation of diagonal stresses, as described on page 447.

The vertical shear is a maximum at the support, where it is equal to the reaction. Maximum shears for various loads are given in the diagram (Fig. 136, page 436), in terms of the loads. While with uniform or symmetrical loading the reaction, and therefore the maximum vertical shear, is one-half the total load upon the beam, it will be noticed from the diagram that where the end beams in a series of continuous beams are supported, which is very nearly the case when a beam runs into a light wall girder the shear at the first support away from the end may be 25 per cent greater than normal, and should be specially provided for in cases like a warehouse where the full live load is liable to be constantly maintained. A further study of the two diagrams (Figs. 136 and 137, pp. 436 and 437) will illustrate the cases where allowances should be made.

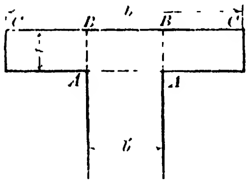


FIG. 138.– Section of a T-Beam (*See p.* 441)

In case the concrete in a beam or slab has cracked vertically next to the support because of accident or poor design, the bearing value of the horizontal rods may have to be estimated.

Longitudinal Vertical Shear in Flange of T-Beam. Vertical shear in a longitudinal direction is present in the wings of a T beam due to the load upon a beam being maximum next to the flange, as shown by lines *BA* in Fig. 138.

The area of concrete in a solid horizontal floor slab is generally sufficient to take care of this shear, but the following method may be used for computing it if desired:

Let

v_h = unit horizontal shear at AA.

v_v = unit vertical shear at BA

b' = breadth of stem.

b = breadth of flange.

t = thickness of flange.

The shear along the two planes BA may be considered as caused by the external forces acting not on the whole breadth, but only on the projecting flanges of the T-beam BC.

Then it is readily shown* that

$$v_v = \frac{v_h\, b' (b - b')}{2\, t\, b} \tag{28}$$

Although this vertical shear through the flanges is readily borne by the concrete, it is advisable, as stated on page 422, to place horizontal rods across the top of the beam, even if the bearing rods in the slab run parallel to the beam, in order to resist unequal bending moment which is liable to occur and to assure T-beam action.

Fillets at the angles between the flange and the beam, that is, between the slab and the beam, are not theoretically necessary, but they may be used for appearance sake and as an additional security in a deep beam with relatively shallow flanges or slabs. Small fillets are also advisable to aid in the removal of forms.

Horizontal Shear. The concrete in a solid rectangular beam or in a T beam is nearly always sufficient to sustain the direct horizontal shear which at any part is equal to the direct vertical shear. In a skeleton beam the horizontal shear may be excessive, but the reinforcement for diagonal tension will also take care of this, so that the direct horizontal shear as such need never be considered. Formerly, before tests of direct shear proved the high strength of concrete in shear, horizontal shearing stress was determined when designing a beam and the vertical stirrups or bent up rods were spaced to act in shear, using a value of 10 000 pounds per square inch in the steel to resist it. More recent tests have proved the stirrups and bent up rods to be in tension instead of shear.

Diagonal Tension. Not only does the high strength of concrete in direct shear indicate that cracks which form in the web of a beam are not caused by this, but tests of beams themselves show that such cracks are diagonal and in the direction which would be expected from the theory of diagonal tension. A typical crack due to diagonal tension is shown in Fig. 139, page 444.

Such cracks as these can be due only to a combination of the shearing stress with the horizontal tensile stress, whose resultant forms diagonal

* The above principle may be expressed by the equation $v_v\, 2t = v_h b' \frac{b-b'}{b}$, which solved for v_v will give formula (28).

tension. It is this diagonal tension which must be sustained in a reinforced concrete beam by the area of concrete or by bent up rods and stirrups, as indicated in paragraphs which follow.

Tests by Prof. Talbot* and Prof. Withey† indicate that for 1 : 2 : 4 concrete the first diagonal crack in a beam without stirrups or inclined reinforcement is apt to occur when the maximum shear is from 100 to 200 pounds per square inch. Since failure by diagonal tension is sudden, it is advisable to provide a high factor of safety. In a beam with diagonal

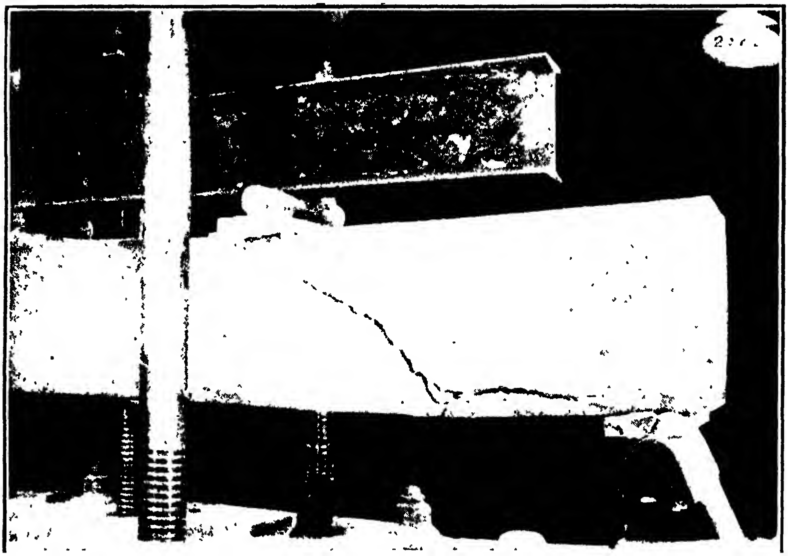


FIG. 139.—Beam under Load at University of Illinois Cracked by Diagonal Tension. (*See p.* 443.)

tension reinforcement, the first diagonal crack occurs at a period but slightly later than in a beam with horizontal rods alone, but in this case it is very small and not dangerous if the steel is designed to take the stress. However, it is desirable that there should be always a sufficient area of concrete, even when reinforced, to prevent the diagonal tension from exceeding the cracking point in the concrete.

* Bulletin No. 12, University of Illinois, 1907.
† Bulletin of University of Wisconsin, Vol. 4, No. 2, 1907.

REINFORCEMENT TO PREVENT DIAGONAL CRACKS IN BEAMS

The failure of a beam from diagonal tension is more sudden than from ordinary tension or compression, and therefore must be guarded against even more carefully. Formerly when beams were designed with full rect

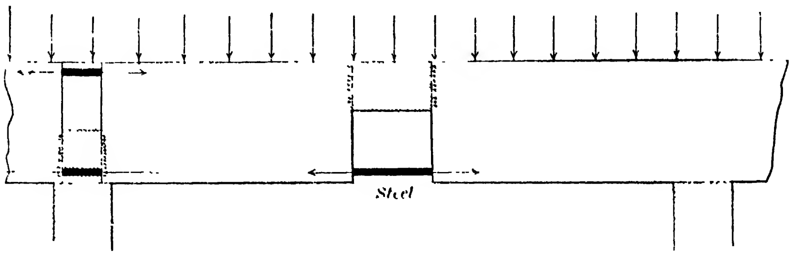


FIG. 140.—Beam with Break in Center illustrating no Shear. (*See p.* 446.)

FIG. 141.—Beam with Break near End, illustrating Action of Vertical Stirrup. (*See p.* 446.)

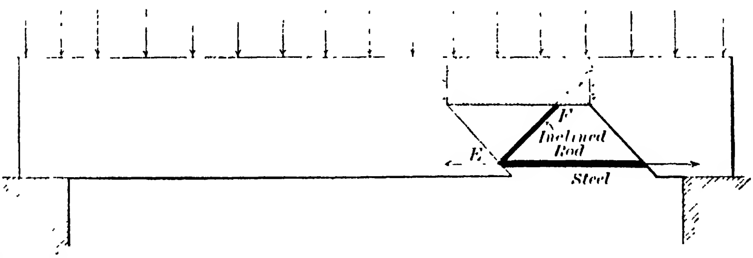


FIG. 142.—Beam with Break near End, illustrating Action of Inclined Rod. (*See p.* 446.)

angular section the concrete often had sufficient area to resist the diagonal tension without assistance from the reinforcement. With the advent of the T-section and the consequent reduction in the width of the stem, it nearly always becomes necessary to introduce stirrups or inclined rods to take the diagonal tension.

An elementary illustration of the action of the stirrups or inclined rods is shown in Figs. 140 to 142, page 445. In Fig. 140 the uniformly loaded beam is cut through in the middle, leaving simply a compression block at the top and tension rod at the bottom. There is pull in the rod at the bottom but no shear or tendency for one side of the section to slide upon the other. In Figs. 141 and 142, on the other hand, where a section of concrete *ABCD* is cut out nearer the end of the beam (this being cut in a diagonal direction to illustrate better the effect of diagonal tension) leaving a compression block *AB* at the top and a rod *CD* at the bottom, the load to the left of the break being heavier will tend to drop, and this downward force or shear, combined with the pull, may be resisted either by the vertical rod *BC*, Fig. 141, or in Fig. 142 by the inclined rod *EF*.

Computation of Shear and Diagonal Tension. Beams may be designed safe against diagonal tension failure by application of the formula for shear given below, because the shear may be taken as a measure of the diagonal tension.* A working stress for shear is therefore selected based, not upon tests of direct shear, but upon tests where the failure was by diagonal tension.

Prof. Talbot in the analysis of shearing stresses† in a reinforced concrete beam has presented a formula for the unit shear at any point in a beam which is a very close approximation.

Let

V = total shear.

v = unit shear.

b = breadth of beam.

b' = breadth of web of T-beam.

jd = depth between center of compression and center of tension (approximately, in a T beam, distance between center of slab and steel).

* The relation between the shear, as determined by the above formula, and the diagonal tension varies with the horizontal forces. From Merriman's "Mechanics of Materials," p. 265 1905 edition,

If

f_d = diagonal tensile unit stress.

f'_c = horizontal tensile unit stress in concrete.

v = horizontal or vertical shearing unit stress.

Then

$$f_d = \frac{1}{2}f'_c + \sqrt{\frac{1}{4}f'^2_c + v^2}$$

The direction of this maximum diagonal tension, as Prof. Talbot points out, makes an angle with the horizontal equal to one-half the angle whose co-tangent is $\frac{1}{2}\frac{f'_c}{v}$. If there is no tension in the concrete the last formula reduces to $f_d = v$. The maximum diagonal tension makes an angle of 45 degrees with the horizontal and is equal in intensity to the vertical shearing stress.

† Bulletin No. 14, University of Illinois, 1906, p. 20.

Then

$$v = \frac{V}{bjd} \quad (29) \qquad \text{or for a T-beam, } v = \frac{V}{b'jd} \quad (30)$$

That is, at any section of a beam, the unit shear, either vertical or horizontal, is the total shear at the section produced by the loads divided by the product of the breadth times the moment arm.

The Joint Committee recommends that beams be reinforced against diagonal tension when the shear exceeds a limit of 2 per cent of the compressive strength at 28 days or 40 pounds for 2000 pounds concrete. The laws governing the internal stresses in a beam with a reinforced web are not yet clearly defined, but it is established that a comparatively small amount of reinforcement by bent-up bars appreciably increases the strength of the beam and, therefore, where a part of the horizontal reinforcement is bent up in a scientific manner and arranged with due respect to the shearing stresses, a value of 3 per cent of the strength at 28 days, or, for 2 000 pounds concrete, 60 pounds per square inch may be allowed.*

Since tests, however, show that web reinforcement can be introduced to increase shearing resistance to a value at least three times as great as when the bars are all horizontal, for beams thoroughly reinforced for shear a limiting value based on the section of the beam of 6 per cent of the strength at 28 days, or 120 pounds for 2 000 pounds concrete, may be used.

In calculating web reinforcement, when the total shear is limited as above, the concrete may be counted upon as carrying $\frac{1}{3}$ of the shear† that is, for concrete having a crushing strength of 2 000 pounds at 28 days, 40 pounds per square inch may be allowed on the concrete and the balance of the shear taken by the reinforcement.

Following these recommendations of the Joint Committee and assuming that the distance between centers of compression and tension, jd, is approximately $\frac{7}{8}d$:

a. Stirrups are required with horizontal bars only when $\frac{V}{bd}$ is greater than $\frac{7}{8}$ (40) = 35.

b. Stirrups are required in rectangular beams where a part of the horizontal reinforcement is bent up and arranged with due respect to the shearing stresses (but not computed as taking diagonal tension) when $\frac{V}{bd}$ is greater than 52.

* This is an arbitrary assumption based on observations of experiments by members of the Joint Committee.

† Although it might be expected that the concrete, since it is assumed to have no tensile value, should not be assumed to assist in carrying diagonal tension, tests by Prof. Withey at the University of Wisconsin (Bulletin, Vol. 4, No. 2) indicate that the rule given is amply safe.

c. Since total shear should not exceed 120 pounds per square inch, bd should not be less than $\frac{V}{105}$.

For T-beams the same rules apply except that only the web of the beam is effective; hence, b' must be substituted for b.

Vertical and Inclined Reinforcement. When the allowable working strength of the concrete in shear, as indicated in the preceding paragraphs, is exceeded, web reinforcement must be introduced. This may consist of the bent up portion of the horizontal bars or of inclined or vertical members attached to or looped about the main reinforcement. Where inclined members are used, their connection with the horizontal reinforcement must be such as to insure against slipping.

Let

A_s = cross sectional area of bars of a vertical stirrup.

V, V_1, V_2 = total vertical shear at different sections.

v' = allowable unit shearing stress (or diagonal tension) in concrete alone.

s = distance between any two stirrups.

d = depth from top of beam to center of steel.

jd = distance from center of compression to center of tension in the beam (approximately $\frac{7}{8}d$).

b = breadth of beam. (In a T-beam, breadth of web or stem).

f_s = unit tensile stress in steel.

α = angle of inclination with the horizontal.

Since the shear at any place in the beam is used as a measure of the diagonal tension (see p. 446), the determination of the shear will indicate the diagonal tension or pull to be resisted by the stirrups and inclined bars.*

Now the vertical shearing unit stress, v, at any section, is of course the total vertical shear produced by the loads divided by the area of the vertical section. Thus, at section A, Fig. 143, the vertical shearing unit stress (and also the horizontal shearing unit stress, since the two are equal) is $\frac{V_1}{bjd}$ and the shear in the full breadth, AA, of the beam for a unit of length is $\frac{V_1}{jd}$ while similarly, at section B, it is $\frac{V}{jd}$. The total shear, therefore, on the horizontal plane A_1B_1 between the two vertical sections B and A, which are a distance, s, apart, is $\frac{1}{2}\frac{(V_1+V_2)}{jd}s$, or, when V at C is the average shear, is $\frac{V}{jd} \times s$. Since, as has been stated, the

* For more complete discussion of the relation of stirrup stress to shear, see page 764.

shear is used as a measure of the diagonal tension, the allowable working strength in tension of the concrete and the steel to resist this diagonal tension must equal this shear stress.

If the unit shear on a section does not exceed 120 pounds per square inch (for concrete testing 2000 pounds per square inch at 28 days), tests indicate (see p. 447) that the concrete may be assumed to take one-third of this, *i.e.*, 40 pounds per square inch, while steel must be provided to take the balance. In this case if a stirrup is placed at C and the distance between stirrups is s, the area of cross section of the steel in the vertical stirrup, A_s, must be sufficient to resist two-thirds of the diagonal tension over the plane AA_1B_1B, or, $A_sf_s = \frac{2}{3}\frac{sV}{jd}$.

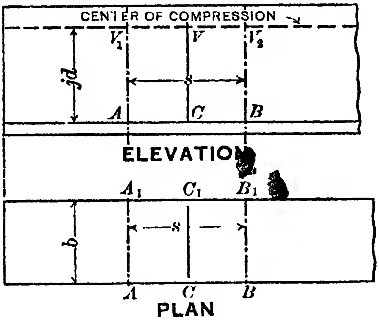


FIG. 143.—Illustration of Shearing Stresses. (*See p.* 448.)

As a more general case, the shear (involving diagonal tension) to be taken by the concrete, $v'bjd$, may be deducted from the total shear, using $(V-v'bjd)$ instead of V. Formulas are presented for both cases, those at the left to be used only when the concrete is assumed to take one-third of shear involving diagonal tension, and the steel two-thirds; those at the right when shear exceeds three times allowable strength of concrete.

$$A_s = \frac{2}{3}\frac{sV}{f_sjd} \quad (31) \qquad\qquad A_s = \frac{(V-v'bjd)s}{f_sjd} \quad (31a)$$

The spacing of the stirrups is treated under a separate heading which follows.

For bars inclined at 45 degrees, it may be assumed that the stress in any single reinforcing member is $\frac{A_sf_s}{0.7}$ and the required area of bar, (assuming the steel in formula (32) to take two-thirds of the shear, and in formula (32a) to take all in excess of that assumed for the concrete) is

$$A_s = \frac{2}{3}\frac{0.7\,sV}{f_sjd} \quad (32) \qquad\qquad A_s = \frac{0.7\,(V-v'bjd)s}{f_sjd} \quad (32a)$$

For other angles of inclination, it may be assumed as approximately correct for the present to use the formula

$$A_s = \frac{2}{3}\frac{\sin\alpha\, sV}{f_sjd} \quad (33) \qquad\qquad A_s = \frac{\sin\alpha\,(V-v'bjd)s}{f_sjd} \quad (33a)$$

The use of these formulas is illustrated in the example 6, page 473.

Formerly, as already stated, stirrups were figured to take direct shear across the rod, but this has been proved by tests to be incorrect.

Stirrups in a Continuous Beam. In a continuous beam in the part near the support subjected to negative bending moment, the diagonal tension acts in the opposite direction to that in the part subjected to positive bending moment. The maximum stress then is in the upper end of the stirrups, so that they should be inverted near the supports. In any case the stirrups should be anchored in the tensile portion of the beam with their free ends (straight or preferably hooked) extending into the compressive part of the beam.

Spacing of Stirrups. The spacing of stirrups must be less than the effective depth of the beam, and a practical limit for spacing is suggested as three-fourths the depth of the beam. Closer spacing than this, however, may be required in order to make the rods small enough to have sufficient bond, as given in the following paragraphs.

Let

x = distance in feet from left support to point at which required spacing is desired.

x_1 = distance in feet from left support to point beyond which stirrups are unnecessary.

l = span of beam in feet.

w = uniform load in pounds per foot.

V = total vertical shear at section x feet from left support in pounds.

v = total unit shear at section in pounds per square inch.

v' = allowable unit shear (or diagonal tension) on concrete alone.

A_s = cross-sectional area of vertical stirrup in square inches. (In a U-stirrup this is the sum of the area of the two legs).

f_s = allowable unit stress in stirrups in pounds per square inch.

jd = depth of beam in inches from center of compression to center of horizontal reinforcement. (In a T-beam this may be taken as distance between center of slab and steel; in a rectangular beam as 0.87 of the total depth to steel.)

b = breadth of beam in inches.

s = spacing of stirrups in inches at point x feet from left support.

The required spacing of stirrups of given size in any part of the beam from formulas (31) and (31a), is

$$s = \frac{3 A_s f_s j d}{2 V} \quad (34) \qquad\qquad s = \frac{A_s f_s j d}{V - v' b j d} \quad (34a)$$

Use left equation only when the concrete is assumed to take one-third of the shear involving diagonal tension (see page 449).

These equations become for a uniformly loaded beam*

$$s = \frac{3 A_s f_s j d}{w(l - 2x)} \quad (35) \qquad s = \frac{2 A_s f_s j d}{w(l - 2x) - 2v' b j d} \quad (35a)$$

For f_s = 16 000 lb. per sq. in., formulas (34) and (34*a*) become

$$s = \frac{24000 A_s j d}{V} \quad (36) \qquad s = \frac{16000 A_s j d}{V - v' b j d} \quad (36a)$$

id formulas (35) and (35*a*) change to

$$s = \frac{48000 A_s j d}{w(l - 2x)} \quad (37) \qquad s = \frac{32000 A_s j d}{w(l - 2x) - 2v' b j d} \quad (37a)$$

See also page 452 and table on page 518*b*.

The above formulas, while applying strictly to supported beams, may be used for continuous beams with safety.

Stirrups should thus be spaced by equation (34) or (35) up to a section where the unit shear equals the working shearing strength of concrete, bearing in mind, however, that the maximum spacing should not exceed three fourths the depth of the beam. The distance from the support to the point where no stirrups are required, for uniform loading is†

$$x_1 = \frac{l}{2} - \frac{v' b j d}{w} \qquad (38)$$

From formulas (34) and (36) it is evident that the necessary spacing of stirrups is inversely proportional to the total shear V at any point and therefore is the smallest at the end of the beam and increases toward its middle.

Many constructors advise the insertion of occasional stirrups throughout the entire length of the beam even if they are not theoretically necessary.

For a small beam where the stirrups are spaced uniformly, for convenience, only the minimum value of s needs to be figured by substituting for V in equation (34) and (36) the total shear at an arbitrary distance $\frac{1}{2}\,d$ from the support, or in equation (35) and (37) substituting $\frac{1}{2}\,d$ for x.

* By substituting in equation (34), $V = \frac{wl}{2} - wx$ as the total shear at any point in a uniformly loaded beam.

† The unit shear $v = \frac{V}{bjd}$. Stirrups are unnecessary at section where $v = v'$ or less, or $v' = \frac{V'}{bjd}$. For the case of uniform load $V' = \frac{wl}{2} - wx_1$. Substituting this for V' and solving for x_1, we have $x_1 = \frac{l}{2} - \frac{v' bjd}{w}$.

Graphical Method for Spacing Stirrups.* In a large and important beam, the spacing should vary with the shear. The following graphical method will be of use in such cases:

Lay out half of the span $\frac{l}{2}$ to any convenient scale as shown in Fig. 144. Compute the values of s at three or four points (point 1, 2 and 3 in the figure) and lay them out on the perpendiculars erected at the respective points to the same scale as the span. Draw a smooth curve located by the points on the perpendiculars. From point a on the perpendicular at the

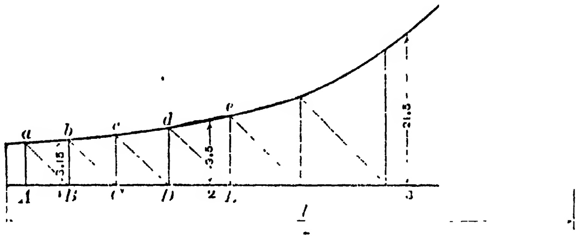


FIG. 144.—Graphical Method for Spacing Stirrups. (See p. 452.)

point where the first stirrup will be placed, draw a line at 45 degrees to intersect with the line representing the span and erect at the point of intersection, B, a perpendicular to cut the curve in point b. A line drawn from b at 45 degrees will intersect the span in point C, where the above process is repeated. The points, A, B, C, D, E, thus obtained are the points in which stirrups are required.

For uniformly loaded beams it is only necessary to compute the minimum spacing of stirrups, that is, at the support. The spacing at two other points may be obtained from the fact that the spacing for $x = \frac{l}{8}$ is four thirds the minimum and for $x = \frac{l}{4}$ is twice the minimum. For $x = \frac{l}{2}$ the spacing is infinity.

Types of Shear Reinforcement. Fig. 145 illustrates different types of diagonal tension reinforcement, showing beams reinforced with stirrups alone, with bent bars, and with a combination of bent bars and stirrups. The method of providing for the negative bending moment over the support is also indicated.

* For spacing with uniform loading, see page 518*b*.

Fig. 146, page 455 shows different types of stirrups.

Diameter of Stirrups. The diameter to select for stirrups is governed by the limiting spacing of the stirrups as given in the preceding paragraphs, by the bond of the stirrup prongs, and by convenience in selecting and placing the reinforcement. The effective length of the stirrup should be taken less than the total length because of the slight change in the intensity of shear below the neutral axis and because also a lower bond strength may be expected there.

Tests by Prof. Talbot indicate that it is safe to use up to at least $\frac{6}{10}$ of the total length of the stirrup in figuring the bond.

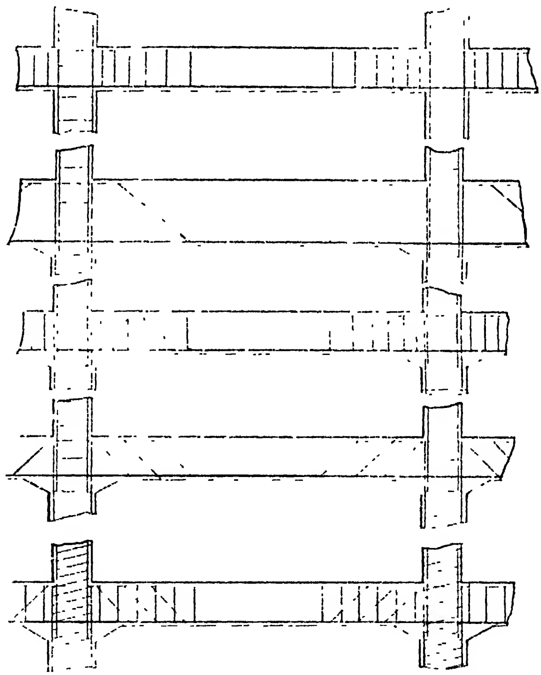


FIG. 145. Reinforcement of a Continuous Beam. (*See p.* 452.)

The maximum diameter of stirrups which can be used by these assumptions without danger of slipping is determined by the bond and can be figured by the formulas given below.

Let

i = diameter of stirrup bar.

A = area of stirrup bar.

o = circumference of stirrup bar.

d = depth from surface of beam to center of tension steel.
u = allowable bond stress per unit of surface of bar.
C_s and C_b = constants to use in formulas (39) to (42).

Then for vertical stirrup with straight upper end.*

$$\frac{A}{o} < 0.6\frac{u}{f_s}d \text{ or } \frac{A}{o} < \tfrac{1}{4}C_s d \tag{39}$$

For round or square stirrups $\frac{A}{o}$

Hence

$$i < C_s d \tag{40}$$

For rods inclined at 45° the above formulas change to†

$$i < C_b d \text{ for round or square sections} \tag{41}$$

$$\text{and } \frac{A}{o} < \tfrac{1}{4}C_b d \text{ for other shapes.} \tag{42}$$

The table below gives the values of C_s and C_b for different values of tension and bond when units are inches and pounds

Values of Constants to Use in Formulas (39) to (42) (See p. 454.)

Allowable unit bond stress	C_s					C_b				
	VERTICAL BARS Allowable unit tension in stirrups in lb. per sq. in.					BARS INCLINED 45° Allowable unit tension in bars in lb. per sq. in.				
Lb. per sq in.	12 000	14 000	16 000	18 000	20 000	12 000	14 000	16 000	18 000	20 000
80	0.016	0.014	0.012	0.011	0.010	0.022	0.019	0.017	0.015	0.014
100	0.020	0.017	0.015	0.013	0.012	0.028	0.024	0.021	0.019	0.017
120	0.024	0.020	0.018	0.016	0.014	0.033	0.028	0.025	0.022	0.020
150	0.030	0.026	0.023	0.020	0.018	0.042	0.036	0.031	0.028	0.025

* $f_s A < 0.6\, dou$, hence $\frac{A}{o} < 0.6\frac{u}{f_s}d$ Call $0.6\frac{u}{f_s} = \frac{1}{4}C_s$ and obtain $\frac{A}{o} = \frac{1}{4}C_s d$

† For rods inclined at 45° substitute for d in the above equation $\sqrt{2}\ d$.

Hence $\frac{1}{4}C_b = 0.6\sqrt{2}\frac{u}{f_s}$ and $\frac{A}{o} < \frac{1}{4}C_b d$

The above formulas and table apply directly only to straight rods.

The bond stress between concrete and plain reinforcing bars may be assumed at $\frac{1}{25}$ of the compressive strength at 28 days, or 80 pounds for 2 000 pound concrete (see p. 528), which for the allowable tension in steel, f_s = 16 000 pounds per square inch gives a diameter of i = 0.012 d. For deformed bars the bond may be increased to 100 or 150 pounds per square inch, varying with the character of the bar. Using the highest figure and 16 000 pounds per square inch as the allowable tension in steel, a beam 20 inches deep to center of steel, making no allowance for the value of a bent end, would require stirrups not to exceed 0.5 inch or $\frac{1}{2}$-inch diameter if deformed bars are used, or $\frac{1}{4}$ inch diameter plain stirrups. Deformed bars are therefore useful for stirrups to permit larger diameters, although the total quantity of stirrup steel required with a given allowable tensile stress is not changed.

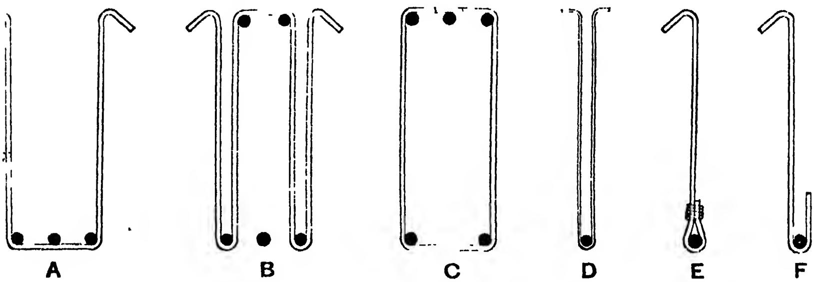


FIG. 146. Types of Stirrups. (*See p.* 453.)

Recent tests (p. 467) show that a right-angle bend of 5 diameters or a semi circular bend of similar length is sufficient to stress the steel to its elastic limit provided the hook is well imbedded in the concrete so that it cannot kick out. With an imbedment in concrete in all directions equal to 8 diameters of the bar, a hook of 5 diameters may be assumed to develop the elastic limit of the steel and larger stirrups can be used than the table indicates.

Ratio of Span to Depth in Rectangular Beam which Renders Stirrups Unnecessary. In a T-beam stirrups are almost always needed and every case must be computed by rules already given. For a beam which is rectangular throughout its length stirrups are unnecessary if at the section of maximum shear the intensity of diagonal tension does not exceed the allowable stress in the concrete. For a rectangular beam uniformly loaded we may deduce the following expression for the ratio of span to depth which will render stirrups unnecessary.

Let

l = span of beam in inches.
d = depth from surface of beam to steel in inches.
v = maximum unit shear at end of beam in lb. per sq. in.
C = a constant from Table 10 on page 519.
α = denominator in formula $M = \frac{wl^2}{\alpha}$

Then it may be shown* that, when $M = \frac{wl^2}{\alpha}$

$$\frac{l}{d} > \frac{\alpha}{1.74\,C^2 v} \qquad (43)$$

Adopting values of working compression in concrete of 650 pounds per square inch, working unit shear of 40 pounds per square inch, working tension in steel of 16,000 pounds per square inch, and a ratio of elasticity of 15; for $\alpha = 12$, $\frac{l}{d} = 15.6$.

If the ratio of span to depth (both in same units) is therefore equal to or less than the value of $\frac{\alpha}{0.77}$ as given by this formula, no stirrups are needed.

BOND OF STEEL TO CONCRETE IN A BEAM.

The bonding of the steel to the concrete is discussed on page 461, the values being based on the resistance to withdrawal of a steel rod imbedded in concrete. **In a reinforced concrete beam the bond of the tension steel per unit of length must not exceed its safe working value.** The concrete surrounding the steel acts as a web between its tensile and compressive parts,

* In addition to above notation let, w = load per linear inch of span, b = breadth of beam in inches, M = bending moment.

With uniform load the shear is a maximum at the support and is equal to $\frac{wl}{2}$. Taking 0.87 d as the approximate depth from the center of compression to the center of tension, the maximum intensity of shear (and consequently of diagonal tension) in the concrete is therefore (See formula (29) p. 447) $v = \frac{wl}{2 \times 0.87\,bd}$. From page 754, formula (11), it is evident that for $M = \frac{wl^2}{\alpha}$, $wl = \frac{\alpha\,bd^2}{C^2 l}$ Substituting this value of wl in the above expression for v and solving for $\frac{l}{d}$, bearing in mind that the maximum unit shear must not be exceeded, we obtain $\frac{l}{d} > \frac{\alpha}{1.74\,C^2 v}$,

Similarly when $M = \frac{wl^2}{12}$, $\frac{l}{d} > \frac{6.8}{C^2 v}$. **For a cantilever where** $M = \frac{wl^2}{2}$, $\frac{l}{d} = \frac{1.15}{C^2 v}$

and the pull in the rods as it becomes less and less, because of the reducing bending moment, passes into the beam, thus producing a bond stress between the steel and the concrete. If the bond is insufficient the rod will slip.

Care must be taken, therefore, to see that the size of horizontal bars in a beam is not too large to give sufficient bond surface between the steel and the concrete. Using the formula suggested by Prof. Talbot.*

Let

V = total shear.
v = unit shear in pounds per square inch.
u = unit bond in pounds per square inch of surface area.
o = perimeter of bar in inches.
Σo = sum of perimeters of all bars.
m = number of bars in tension.
jd = distance between centers of tension and compression.
d = depth from surface to center of tension steel.

Then

$$u = \frac{V}{jd\,\Sigma o} \tag{44}$$

The unit bond stress recommended by the Joint Committee for concrete whose strength is 2 000 pounds at 28 days is 80 pounds per square inch, and assuming also as a close approximation that $jd = \frac{7}{8}d$, the total perimeter of bars which are required at any point of a beam is

$$\Sigma o = \frac{V}{70\,d} \tag{45}$$

In a continuous beam this formula applies to the steel which is in tension whether it is located in the top or the bottom of the beam. Since the negative bending moment decreases quite rapidly, the bond stress at the support of a continuous beam is more apt to exceed the safe working limit than in the middle of the beam, thus requiring more attention and frequently limiting the diameter of the bars.

The above formula does not apply to the compression steel and therefore has no relation to the steel in the bottom of a continuous beam at the support.

* Bulletin No. 4, University of Illinois, 1906, p. 19.

The formula may be derived from the relation of the bond to the shear.

The tendency to slip, or the bond stress, is equal to the shear because the measure of both of them is the increment of the moment. Hence $u\Sigma o = vb$, from which, since

$$v = \frac{V}{bjd} \text{ then } u = \frac{V}{jd\,\Sigma o}$$

Tests by Prof. M. O. Withey* indicate that the bond of tension bars in a beam is much less than shown by tests in which bars are pulled out from blocks of concrete, probably because of the compression on the head of the block in the latter case. For 1 : 2 : 4 concrete, the ultimate bond strength at the age of 60 days averaged 276 pounds per square inch.

POINTS TO BEND HORIZONTAL REINFORCEMENT

The bending moment in a reinforced concrete beam decreases toward the ends, reducing in the same ratio the pull in the tension bars. Since these must be designed to take the maximum moment at the center of the beam, the steel at the ends, when the bars are carried horizontally through the whole length of the beam, is stressed away below its working strength. By bending up a part of the bars not required for tension, the inclined portion assists in providing for the diagonal tension, and by carrying the ends horizontally over the top of the supports the tension due to negative bending moment may be resisted there.

If part of the rods are bent up at a certain point, those remaining must have sufficient sectional area to carry the tension beyond this point, and must also have sufficient length imbedded to prevent slipping. The limiting locations for bending the rods may therefore be found as follows:

Let

m = number of bars at the center.

m_1 = number of bars to be bent.

M = maximum moment $= \frac{wl^2}{a}$ in which

a = denominator in the expression for bending moment.

x_1 = distance from support to point where m_1 bars may be bent up leaving sufficient steel to carry the pull.

Then it may be proved that the distance in feet from support to point where m_1 bars may be bent up and still leave sufficient steel to take the pull is†

*Proceedings American Society for Testing Materials, 1909.

† The ratio of pull at the middle to that at the point under consideration equals the ratio of moments in these points. Thus if the steel is stressed equally at both points,

$$M_{x_1} : M = (m - m_1)\frac{t^2\pi}{4} : \frac{m\,t^2\pi}{4}$$

Substituting

$$M = \frac{wl^2}{a} \text{ and } M_{x_1} = \frac{wl^2}{a} - w\left(\frac{l}{2} - x_1\right)^2$$

and solving for x_1

$$x_1 < \frac{l}{2}\left(1 - \sqrt{\frac{8\,m_1}{a\,m}}\right)$$

$$x_1 \overline{<} \frac{l}{2}\left(1 - \sqrt{\frac{8m_1}{m\alpha}}\right) \tag{47}$$

The last equation may be used for beams designed for any bending moment $M = \frac{wl^2}{\alpha}$. For $a = 8$, the formula changes to

$$x_1 < \frac{l}{2}\left(1 - \sqrt{\frac{m_1}{m}}\right) \tag{48}$$

Having thus determined x_1 see that the remaining horizontal bars are secure against slipping by the use of formula (44), page 457.

The use of the formulas are illustrated in the Example 6, page 472.

SPACING OF TENSION BARS IN A BEAM

The tension bars in a beam must be a sufficient distance apart to properly transmit the pull to the concrete in the beam and prevent cleaving the concrete between them.* At most points in a beam, with bars of ordinary size, the bond stress as determined from formula (44) page 457, is low, and there is therefore but little tendency to slip and the bars may be placed as close together as proper placing of the concrete between them will permit. At points where a part of the rods are bent up, and especially in the top of the beam over the supports, the bond stress may be high, and it is advisable to make a rule that the rods shall not be spaced nearer together in the clear than 1½ times their diameter. To permit the concrete to be readily placed between them and to give sufficient concrete on the sides of the beam for fire protection, it is advisable further to make the minimum spacing between the rods 1 inch and the minimum distance of the rods from the sides of the beam 1½ inches in the clear.

There is less danger of vertical splitting, and where two layers of rods are used the rods in a vertical plane may be placed directly over each other, and with sufficient space simply to permit the mortar to run between them. The Joint Committee specifies a limiting clear space of ½ inch.

Prof. McKibben has suggested a mathematical demonstration for determining the width of concrete required between the rods in order to make the resistance in shear equivalent to the adhesion of the concrete to the steel.

*The relation of bond to shear is discussed on the following page.

Let

l = length of rod considered in inches.

s_b = distance in the clear between two rods in inches.

i = diameter of rod in inches.

u = adhesion or bond between concrete and steel per square inch of surface of steel.

v = direct shearing strength of concrete in pounds per square inch.

If the beam splits at the rods, it is apt to shear through the concrete between the rods, and break the adhesion between the upper half of the rod and the concrete. When such splitting occurs the shearing strength of the concrete between the rods, on a plane with their centers, is equal to or less than the adhesion of the concrete to the half circumference of one of the rods and the minimum spacing is then*

$$s_b = 1.57 \frac{u}{v} i \tag{49}$$

If, for example, the working bond stress, u, is assumed as 80 pounds per square inch and the working strength of the concrete in direct shear is taken at 120 pounds per square inch, the formula becomes

$$s_b = 1.05 D \tag{50}$$

that is, the minimum net distance in the clear between the rods is approximately equal to the diameter of the rod. Since the concrete is not easily placed between the rods it may have a lower strength there and hence a clear spacing of $1\frac{1}{2}$ diameters (with a minimum of one inch), as suggested above is advisable unless it is determined by computation that the bond stress is much lower than is assumed here. Deformed bars, if stressed to their full bond value should be spaced farther apart than plain bar.

In the middle of a beam the bond stress is low, so that the formulas are most useful in considering the rods in the top of the beam over the support.

DEPTH OF CONCRETE BELOW RODS

The selection of the thickness of the concrete below the rods is governed more by the proper fire and rust protection of the metal than by the stresses in the beam.

* For a short length of rod l, equate the strength in shear of the concrete between the rods to the adhesion between the concrete and the upper half circumference of the rod.

Hence

$$s_b l v = \frac{\pi i l u}{2} \qquad s_b = 1.57 \frac{u}{v} i$$

Prof. Charles L. Norton, who has made a careful study of the subject, considers a thickness of 2 inches essential for efficient fire protection. (See p. 333.) Since an excessive thickness adds to the danger of cracking, because the tension in the concrete increases with the depth below the steel, with but slight corresponding gain in strength to the beam, this thickness, measured from the lower surface of the steel, and not from its center of gravity, may be taken as a maximum. Thus, in important members which are liable to severe fire, 2 inches may be considered the standard requirement, while for secondary members and floor slabs, a less thickness, ranging from ½ inch to 2 inches, is probably warranted.

The following thicknesses of concrete below the steel may be employed under ordinary conditions:

Thickness of Concrete below Steel.

Depth of slab or beam, inches	Thickness below lower surface of rods,* inches
1½ to 2	½
2½ to 4	¾
4½ to 8½	1
9 to 12	1¼
13 to 18	1½
19 to 20	1¾
Greater than 20	2

*Values up to depth of 20 inches are from tables of Mr. Edwin Thacher, except that his depths are taken below center of gravity of steel.

The Joint Committee, 1909, recommends slightly greater thickness than given in the above table, and its recommendations should be followed wherever the conditions are especially hazardous. The Committee suggests that "the metal in girders and columns be protected by a minimum of 2 inches of concrete; that the metal in beams be protected by a minimum of 1½ inches of concrete, and that the metal in floor slabs be protected by a minimum of 1 inch of concrete."

BOND OF CONCRETE TO STEEL TO RESIST DIRECT PULL

Tests by different experimenters show that with similar materials the bond is proportional to the area of the surface in contact, and varies with the character of the surface and the nature of the concrete or mortar.

Feret† found that the bond of concrete to iron is nearly proportional to the percentage of cement in a unit volume of concrete, and that there is an

† Thonindustrie-Zeitung 25 (153)2, 213–2, 215, translated in Cement, July 1902, p. 213.

increase of strength of about 50% in concrete two years old over that three months old. The best consistency for concrete he considers to be so plastic as to be almost sloppy. He found that the bond of a very dry concrete was only about one-fourth that of an almost sloppy concrete which gave the maximum density. The bond increased rapidly as the proportion of water increased until the concrete reached its maximum density, when for larger proportions of water there was a slow decrease in bond strength, very wet concrete showing a bond about three-fourths that of the slightly dryer concrete having maximum density.

In tests made by Prof. M. O. Withey at the University of Wisconsin* in 1906, the results of which are contained in the following table, the bond developed by different specimens averaged about 0.3 of the compressive strength and increased nearly proportionally with it.

*Variation of Bond with the Compressive Strength.**

1 : 2 : 4 Concrete—Age 28 days. (See p. 462.)

By PROF. MORTON O. WITHEY.

Size of rod.	Area of Rod.	Depth imbedded.	Maximum load.	Unit stress in rod.	Elastic limit of steel.	Bond	Compressive strength of concrete.
In.	Sq. in.	In.	Lb.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.	Lb. per sq. in.
9/16	0.248	6½	7200	29 000	36 400	627	2155
9/16	0.248	6⅛	5500	22 200	36 400	500	2000
9/16	0.248	6¼	6700	27 000	36 400	607	1850
9/16	0.248	6¼	4625	18 600	36 400	418	1485
9/16	0.248	6¼	4250	17 100	36 400	384	1435
9/16	0.248	6	4100	16 500	36 400	387	1150
½	0.196	6	3600	18 400	38 600	382	1150
½	0.196	5¾	1500	7 600	38 600	166†	795†
½	0.196	6¼	1840	9 400	38 600	187†	584†

As shown in the table, which gives in a condensed form the results of tests made by Prof. Talbot at the University of Illinois‡ in 1905, a 1 : 2 : 4 concrete realized a bond resistance averaging about 13 per cent. higher than a 1 : 3 : 5½ concrete. The effect of the surface of the bar upon bond resistance is also indicated. Plain round mild steel bars developed a bond resistance ranging from 355 to 465 pounds per square inch of contact sur-

* Bulletin, University of Wisconsin, Vol. 4, No. 2, Nov. 1907.

† Made upon concrete in which sand and limestone screenings containing 40% of dirt were used as aggregate.

‡ Bulletin No. 8, University of Illinois, Sept. 1906.

face, a bond about 2.7 times that of cold rolled shafting and tool steel and also much greater than that of flat mild steel bars. Deformed bars give a much higher bond resistance. In cases* where the embedment was not so great as to stress the bars beyond their elastic limit, the results indicate a

Tests of Bond of Union Between Concrete and Steel.
Age of Concrete, 60 days. (*See p.* 462.)
By Prof. Arthur N. Talbot.

Type of rod.	Size.	Proportions.	Encased length.	Surface in contact.	Maximum load.	Bond.	Frictional Resistance. Total Maximum.	Frictional Resistance. Unit Lb. per sq. in.	Ratio of Frictional Resistance to bond.
	Inches.		In.	Sq. in.	Lb.	Lb. per sq. in.	Lb.		
Plain Round	½	1:2:4	6	9.4	3893	412	2135	227	55.2
Plain Round	⅝	1:2:4	6	11.8	5370	465	3485	297	64.0
Plain Round	½	1:2:4	12	18.8	7605	404	4982	266	65.5
Plain Round	⅝	1:2:4	12	23.5	9736	414	5284	223	54.0
	Average	1:2:4			6652	424	3971	253	59.7
Plain Round	½	1:3:5½	6	9.4	3498	372	1983	210	57.0
Plain Round	⅝	1:3:5½	6	11.8	4170	355	2700	227	64.0
Plain Round	½	1:3:5½	12	18.8	7035	373	5066	268	72.0
Plain Round	⅝	1:3:5½	12	23.5	9458	402	5366	228	56.8
	Average	1:3:5½			6040	375	3779	233	62.4
Cold Rolled Shafting	1	1:3:5½	6	18.8	2570	136	1256	67	49.2
Cold Rolled Shafting	½	1:3:5½	6	9.4	1476	157	466	50	31.8
Mild Steel	$\frac{3}{16}$–1½	1:3:5½	6	20.2	2536	125	1713	84	67.1
Round Tool Steel	¾	1:3.6	6	14.1	2077	147	.....	.....	.....

bond strength for deformed bars in ordinary 1 : 2 : 4 concrete of from, say, 400 to 700 pounds per square inch of contact surface.†

Tests by Mr. Frank A. Bone‡ indicate that the bond of bars in

* Bulletin No. 1, University of Illinois, Sept. 1904, Table 7; Proceedings American Society for Testing Materials, Vol. VII, 1907, p. 467; Engineering Record, Dec. 22, 1906, p. 694.
† Bulletin, University of Wisconsin, Vol. 4, No. 2, Nov. 1907.
‡ Engineering News, May 21, 1908, p. 571.

concrete which is being stressed to a high compression is much greater than in unstressed concrete.

The results of many bond tests are without value because the yield point of the steel was exceeded.

The bond stress of the tension bars in a beam is determined by methods discussed on page 457.

LENGTH OF BAR TO PREVENT SLIPPING

In a reinforced concrete member it is necessary not only to have at a given section the required amount of steel to take the pull or the compression but it is also essential for each bar to have sufficient length of imbedment in the concrete so that the bond (that is, the resistance to slipping) of the bar in the concrete is great enough to develop the necessary direct pull or compression in the body of the bar. Unless a bar is bent up or anchored by some mechanical means (see p. 466) its resistance to slipping is determined by the length of imbedment and the value of the unit bond between the concrete and the steel.

The length of imbedment necessary to develop a required holding power through mere bond between the concrete and steel may be determined thus:

Let

f_s = working tensile or compressive stress per square inch in the body of the bar.

i = diameter of bar in inches.

u = bond in pounds per square inch of surface.

l_1 = necessary length of imbedment of bar in inches.

Then*

$$l_1 = \frac{i f_s}{4 u} \tag{46}$$

This formula holds for square as well as for round bars. Using the limiting bond stress suggested by the Joint Committee for round bars of mild steel, 80 pounds per square inch, the length of imbedment when the steel is stressed to 16 000 pound per square inch is fifty diameters. Deformed

* If the bar is round the total force to be developed in the body of the bar is $\frac{\pi i^2}{4} f_s$ while the holding power of the bar, or its resistance to slipping is $\pi i u l$.

Equating these and solving for l, we obtain

$$l_1 = \frac{i f_s}{4 u}$$

bars may be given a greater bond stress, while, as indicated in the preceding paragraphs, a smooth steel of the nature of tool steel must be given less. It has been suggested that the bond of deformed bars be taken the same as the bond of plain bars except using for their diameter the diameter of a cylinder based on the longest projections, that is, of a cylinder which would be sheared out by the deformed bar. Ordinarily then, as indicated on page 528, a bond stress for deformed bars varying with the character of the bar from 100 to 150 pounds per square inch may be used, corresponding to an imbedment of 40 to 27 diameters. For smooth metal of the nature of tool steel not over 30 or 40 pounds per square inch, that is 133 to 100 diameters should be permitted. Flat steel or structural steel with flat surfaces should be given a similarly low value per square inch.

The bond in all cases is based on the surface area of the imbedded bar. Later tests by Prof. Withey, which show much lower bond strength when the concrete in the specimens is not affected by the compression, are referred to on page 458. These indicate that a strength in bond for 1 : 2 : 4 concrete cannot be assumed greater than 273 pounds per square inch of surface of round bars, so that the value of 80 pounds per square inch suggested above, is a fair working unit bond stress.

All of these bond values are based on a concrete whose strength at the age of thirty days; when tested in cylinders, is 2 000 pounds per square inch. The length of imbedment also varies with the working stress in the steel in tension or compression. In compression, steel is not apt to be stressed over 8 000 to 10 000 pounds per square inch, so that a shorter length of imbedment is required. Furthermore, the concrete in compression provides a greater grip.

The following table gives the length of imbedment of round or square bars for different unit stresses in the steel:

Length of Imbedment Required for Round and Square Bars.

Unit stress in steel. f_s.	Length of bars to imbed in terms of the diameter.					
	Allowable bond stress, pounds per square inch.					
Lb. per sq. in.	40	60	80	100	120	150
8 000	50	33	25	20	17	13
12 000	75	50	37	30	25	20
16 000	100	67	50	40	33	27
20 000	125	83	62	50	41	33

NOTE: The length of imbedment may be obtained by multiplying the value selected from this table by the diameter of the bar.

VALUE OF HOOKED BARS IN BOND

The results of a valuable series of tests on hooked bars, made for the Eastern Concrete Construction Company at the Massachusetts Institute of Technology under the direction of Prof. H. W. Hayward are presented in a table which follows.

In all the tests ¾-inch round bars were imbedded in blocks 12 inches square and 15 inches long, to a depth of 12 inches with an additional bend of different lengths. In one case the straight portion of the bar was greased. Right-angle bends and semi-circular bends on a 3-inch diameter were tested. Seven specimens of each type were tested, the individual results being extremely uniform except in type 1, where the straight bar test gave the usual variations expected in ordinary bond tests.

Tests of Hooked Bars in Bond, Massachusetts Institute of Technology, 1909, *for Eastern Concrete Construction Company*

Proportions of concrete 1 : 2 : 4. Age 28 days. Crushing strength concrete 1130 lb. per sq. in. at 34 days. Round bars ¾-inch diameter.

Yield point of steel 35 900 lb. per sq. in. or 15 870 lb for ¾-inch bar.

Elastic limit of steel 29 000 lb. per sq. in. or 12 820 lb. for ¾-inch bar.

Type	Length of straight bar	Length and Kind of hook	First slip* lb.	Maximum load lb.	nd. lb p n. of sur	Condition at maximum load
		0″	7 770		275	Bars pulled out
		4″ square bend	16 000	16 000		Concrete crushed at bend finally split block.
		4″ " "	18 700	21 150		Concrete crushed; bars partly straightened.
		2″ " "	not recorded	17 770		Bars straightened out; Block not split.
		6″ " "	17 000	19 730		Bars pulled out, or split concrete by kicking back
		4″ " " (no backing of concrete)	9 600	15 590		End of bars raised up and crushed concrete; bars pulled through
		3″ return bend	17 100	22 660		Concrete split
		3″ " " (with 3″ extra length	17 300	23 630		" "

* As shown by drop of beam except in type (6), where the bent ends begin to straighten, the drop of the beam coming either at the same period or at about 1 000 lb. later.

In all the tests except those with straight bars (type 1) and those where there was no concrete back of the bend, the elastic limit of the steel was passed before the bars began to pull out as indicated by the drop of the beam.

The crushing strength and also the bond strength of the concrete was low because the specimens were stored at a low temperature, but this does not affect appreciably the value of the results.

The following conclusions may be drawn from these tests:

(1) A 4-inch right-angle bend in a $\frac{3}{4}$-inch round bar (5 diameters) is sufficient to stress the steel to its elastic limit. A longer bend than this is not necessary.

(2) A semi-circular bend on a diameter 4 times the diameter of the bar appears to be even more effective than the square bend.

(3) No crushing of the concrete occurs until the elastic limit of the steel is passed.

(4) A backing of concrete whose thickness is 4 times the diameter of the bar appears to be effective to prevent kicking back before the elastic limit of the steel is reached, provided the area of section is large enough to prevent cracking on a plane with the bend.

Tests at the Case School of Applied Science indicate that a section of concrete six inches square is not enough to prevent a $\frac{3}{4}$-inch bar from cracking the concrete before the steel reaches its elastic limit.

The most important conclusion from the Institute experiments is that a bend 5 diameters in length in a $\frac{3}{4}$ inch rod and probably,- from comparison of the results from the 2-inch hooks with the others,- -that even a bend of $2\frac{1}{2}$ diameters is sufficient, when the hooks are properly imbedded in concrete, to permit the steel to reach its elastic limit before starting to pull out. In a number of the tests the deformations were measured and show no initial slip previous to the periods given in the table. The result agrees with tests also made at the Institute upon hooks not imbedded in concrete where the elastic limit of the steel was reached before the hooks lost their grip.

EXAMPLE OF BEAM AND SLAB DESIGN

The use of the formulas given in the preceding pages can be best illustrated by the design of a floor bay consisting of slabs, beams and girders. The design of reinforced concrete structures permits of so many variations by locating steel in different ways that more than one type of design for the same member is almost always possible. The dimensions and reinforcement shown illustrate common methods, and the arrangement of details in the different members is also given as typical. The principles of design follow the recommendations of the Joint Committee on Concrete and Reinforced Concrete, 1909.

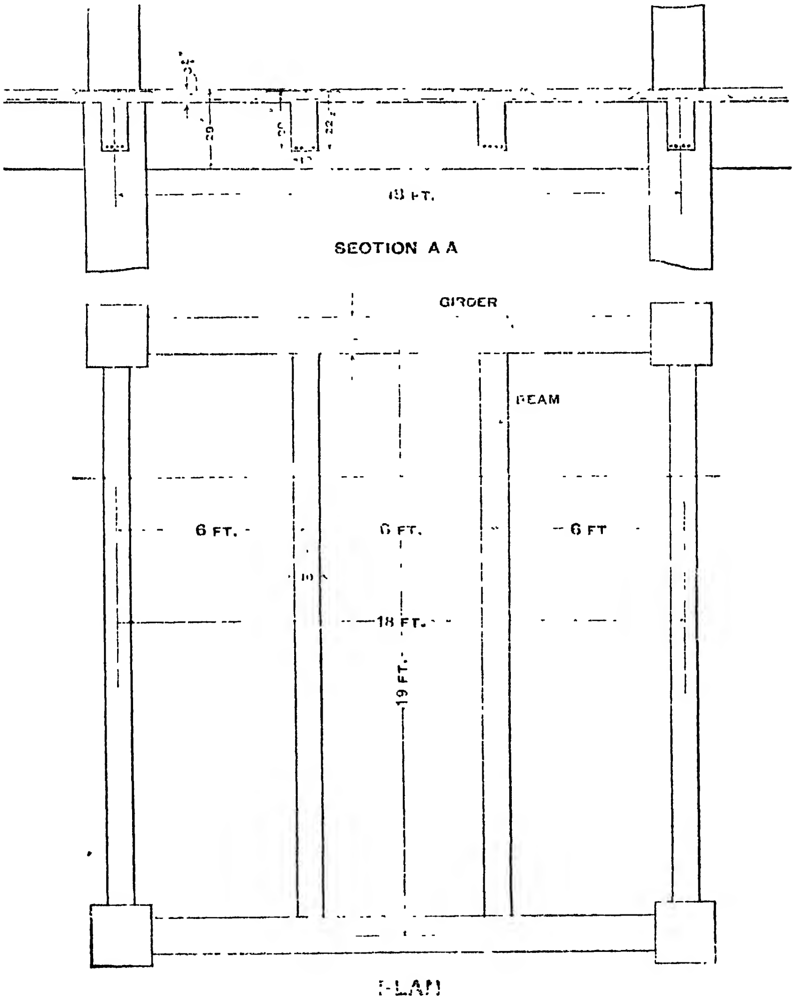


FIG. 147.—Design of Floor System (*See page* 469)

The computations are given with but few comments, but references are entered to the pages upon which each part of the calculation is based.

Example 6: Design a typical slab, beam and girder for a reinforced floor to support a live load of 250 pounds per square foot with columns spaced 18 by 19 feet on centers.

Solution: The girder will be made 18 feet long and the distance between centers of beams 6 feet. The beams are 19 feet long on centers.

	Refer to page
Take allowable fiber stress in concrete, 650 lb. per sq. in.	528
Take allowable tension in steel, 16 000 lb. per sq. in.	529
Take ratio of elasticity of steel to concrete, 15	520
Take direct shear in concrete, 120 lb. per sq. in.	528
Take shear in concrete involving diagonal tension, 40 lb. per sq. in.	528
Take bond between concrete and plain bars, 80 lb. per sq. in.	528
Notation used in Example is Joint Committee standard	529

Slab. Span of slab is 6 ft.
Live load, 250 lb. per sq. ft.
Assumed dead load, 50 lb. per sq. ft.
Total loading, 300 lb. per sq. ft.

Use for moment, $M = \frac{wl^2}{12}$, then $M^* = \frac{300 \times 6^2 \times 12}{12}$ 10 800 in. lb. 440

Same value may be found directly from curves 524
Since $f_c = 650$, $f_s = 16\ 000$ and $n = 15$, then
$C = 0.096$ and $p = 0.0077$, from table 10 519

Hence, depth to steel is, $d = 0.29 \times 0.096 \sqrt{10800} = 2.9$ in. 421

Taking ¾ in. concrete below steel, thickness of slab is 3¾ in. 461
Area steel, $A_s = 2.9 \times 12 \times 0.0077 = 0.268$ sq. in. 421
Round rods ⅜ inch in diameter spaced 5 inches on centers will give required area. Table 1. 507
The same results may be obtained by using the slab table 5: 513

Since this table is based on $M = \frac{wl^2}{10}$ and we use here $M = \frac{wl^2}{12}$ the total unit weight of 300 pounds per square foot may be reduced ⅙ or to 250 pounds and this value treated in the table, which gives a 3¾-inch slab.

Rods must be bent up to give same steel at top of slab over supports.

Beams. Span 19 feet.
Distance between beams, 6 feet.
Dead and live loads of the slab per foot of length of beam, $6 \times 300 = 1800$ pounds.
Assumed dead load of stem of the beam, 200 pounds per foot of length
Total unit loading, 2000 pounds.

Use for moment $M = \frac{wl^2}{12}$, then $M = \frac{2000 \times 19^2 \times 12}{12} = 722\ 000$ inch-pounds.

Reaction at support, which is the maximum shear, is

$$V = \frac{2000 \times 19}{2} = 19\ 000 \text{ pounds.}$$

* Only one 12 is inserted in the numerator to change the 6 ft. to inches because the 300 is pounds per foot.

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Breadth of Flange. Taking 8 times the thickness of slab plus the breadth of stem of beam (assumed as 10 inches) $b = (8 \times 3\frac{3}{4}) + 10 = 40$ inches.

Minimum Depth. Referring to Table on page 525, since the area of flange times the working strength of concrete, $f_c\, b\, t$, is 97 500 lb. and the assumed ratio of depth of beam to thickness of flange is 3.5 $\left(i.\ e., t = \frac{1}{3.5}d\right)$ the minimum distance from center of slab to steel in beam, jd, for a moment of 722 000 is 12 in., or adding $\frac{1}{2}t$, the depth d is 14 in. 426

A larger value of d however will be used for economical reasons as given below, since it reduces both the stress in concrete and the amount of steel. The decrease of depth of beam on the other hand would increase the stresses in concrete above the permissible working strength. 425

Cross-section of Web as Determined by the Shear. 424

$$V = 19\ 000 \text{ pounds (see above) hence}$$

$$b'\left(d - \frac{t}{2}\right) \overline{>} \frac{19\ 000}{120} \text{ or } 158 \qquad \text{formula (13), } 424$$

Economical Depth. From formula (14), $d - \frac{t}{2} = \sqrt{\frac{r\,M}{f_s \times b'}}$ if the ratio of unit cost of steel to cost of concrete, $r = 70$ 425

$$\text{for } b' = 8,\ d - \frac{t}{2} = 19.85 \text{ inches or } d = 21.7 \text{ inch.}$$

$$b' = 9,\ d - \frac{t}{2} = 18.7 \text{ " or } d = 20.6 \text{ "}$$

$$b' = 10,\ d - \frac{t}{2} = 17.8 \text{ " or } d = 19.7 \text{ "}$$

For convenience in placing steel take
$b' = 10$ inches, $d = 20\frac{1}{2}$ inches, $h = 22\frac{1}{2}$ inches 459

Sectional Area of Steel. From formula (15) 426

$$A_s = \frac{722\ 000}{18.625 \times 16000} = 2.4 \text{ square inches}$$

4 round bars $\frac{7}{8}$ inches diameter will be sufficient. Two of these may be bent up and lap over the top of the support 429

Steel at Top and Bottom. Negative bending moment at support equals positive M at middle or $-M = 722$ 000 inch pounds. 440

At support the flange of T-beam being in tension is negligible and since four $\frac{7}{8}$-in. round bars are in tensile and two in compressive part of beam, the T-beam changes into a rectangular beam with steel in top and bottom.

The ratios of steel in tension and compression are respectively

$$p = \frac{2.4}{10 \times 20.5} = 0.0117 \text{ and } p' = \frac{p}{2} = 0.0058$$

With these values of p and p' and for $n = 15$, and $a = 0.1$ we obtain from table (p. 516) $C_c = 0.227$ and $C_s = 0.0103$. Maximum pressure in concrete is

$$f_c = \frac{722\ 000}{10 \times 20.5^2 \times 0.227} = 760 \text{ lb. per sq. in., formula (18)} \qquad 428$$

$$f_s = \frac{722\ 000}{10 \times 20.5^2 \times 0.0103} = 16\ 700 \text{ lb. per sq. inch, formula (10)} \qquad 428$$

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Allowable compression in concrete at the support may be 15% larger than that at middle, hence, no haunch necessary. 429

Girder. Span 18 feet, breadth to use for T-beam, 44 in. (assuming breadth of stem as 14 in.) 424

Concentrated loads at ⅓ points.

Assumed dead load of the stem of the girder, 360 pounds per linear foot.

Load transmitted by the beams is considered as concentrated. 441

Reaction of concentrated loads, $V = 38\ 000$ pounds. 434

Maximum moment of concentrated loads with ends of beam simply supported would be, $M = 38\ 000 \times 6 \times 12 = 2\ 740\ 000$ inch pounds. 439

This corresponds to formula $M = \frac{wl^2}{8}$; to correspond to $M = \frac{wl^2}{12}$ it may be reduced by the ratio $\frac{8}{12}$ or 441

$M = 2\ 740\ 000 \times \frac{8}{12} = 1\ 827\ 000$ inch pounds.

Moment of dead load, $M = 116\ 600$ inch pounds 440

Total moment, $M = 1\ 943\ 600$ inch pounds.*

Minimum Depth. From Diagram 4, p. 525, since the area of flange times the working strength of concrete, $f_c bt = 107\ 250$ pounds and the assumed ratio of the depth of beam to the thickness of flange equals 6, the minimum depth, $d = 24$ inches. 525

A somewhat greater depth is economical as shown below. 425

Cross-section Determined by Shear. $V = 38\ 000 + 3600 = 41\ 600$ pounds 424

Using a limit of 120 pounds for total shear

$$b'\left(d - \frac{t}{2}\right) = \frac{41\ 600}{120} = 347 \text{ square inches} \qquad 424$$

Select by judgment

$b' = 14$ inches, $d = 26.5$ inches, $h = 29$ in. (to allow for 2 layers of steel).

Steel Area. For $M = 1\ 956\ 000$ from Diagram 4, p. 525, $A_s = 4.90$ square inches, 8 round bars ⅞ inch diameter will satisfy the moment.

Check of Results by Exact Formulas (14) to (17) 755

(This check is unnecessary in practice for an experienced designer.)

$b' = 14$ inches, $b = 30 + 14 = 44$ inches, $t = 3\frac{3}{4}$ inches.

$A_s = 4.90$ square inches.

$$kd = \frac{4.90 \times 2 \times 15 \times 26.5 + 44 \times 3.75^2}{2 \times 4.90 \times 15 + 44 \times 3.75 \times 2} = \frac{3900 + 620}{147 + 330} = \frac{4520}{477} = 9.5 \text{ inches}$$

$$z = \frac{3 \times 9.5 - 7.5}{2 \times 9.5 - 3.75} \cdot \frac{3.75}{3} = 1.72 \text{ inches.}$$

$$jd = 26.5 - 1.72 = 24.78 \text{ inches.}$$

The value for jd would be a bit lower, when the compression in the stem is also considered. It is evident that the approximate value used in previous figuring, $d - \frac{t}{2} = 24.63$ inches, is practically identical with the more exact moment arm.

Girder at Support. —$M = 1\ 943\ 600$ inch pounds. 440

Reinforcement at supports consists of ⅞ inch round bars.

Eight bars are in tensile and four in compressive part of beam, hence

$$\text{ratio tension steel, } p = \frac{4.90}{14 \times 26.5} = 0.0132$$

$$\text{ratio compression steel, } p' = \frac{0.0132}{2} = 0.0066$$

* By method suggested on page 433 the result would be 2 159 000 less 10 per cent or 1 943 000 pounds, a result almost identical with the more exact one.

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From Table 8 (p. 516), $C_c = 0.241$ and from formula (18) maximum compression in concrete, 428

$$f_c = \frac{1\,043\,600}{14 \times 26.5^2 \times 0.241} = 820 \text{ pounds per sq. in.}$$

which is excessive. 429

Depth and Length of a Haunch. For depth try $a = 0.1$, $d = 28$ inches 429
For this depth of beam the ratios of steel in tension and compression change to $p = 0.0132 \times \frac{26.5}{28} = 0.0125$, $p' = \frac{0.0125}{2} = 0.0063$ the corresponding values $C_c = 0.236$ and $C_s = 0.0109$
Maximum compression in concrete, 428

$$f_c = \frac{1\,043\,600}{14 \times 28^2 \times 0.236} = 750 \text{ pounds per sq. inch}$$

and maximum tension in steel 428

$$f_s = \frac{1\,043\,600}{14 \times 28^2 \times 0.0109} = 16\,250 \text{ pounds per sq. inch; formulas (18) and (19)}$$

This stress is allowable and the depth of haunch from top of beam of 28 inches will be accepted.

Length of haunch may be approximated. Moment of resistance of beam without haunch, allowing 15% excess compression or 750 lb. per sq. in., 428
$MR = 750 \times 14 \times 26.5^2 \times .241 = 1\,780\,000$ inch pounds Formula (17)
$MB = 1\,043\,600$ inch pounds. 428
Hence from formula (22) length of haunch 430

$$x = \frac{176\,000}{1\,943\,600} \cdot \frac{18}{5} \times 12 = 3.9 \text{ inches}$$

Since maximum negative moment occurs in middle of column and necessary length of haunch is only 3.9 inches, no haunch will be introduced outside of the column.

Diagonal Tension Reinforcement of Beam. *Vertical stirrups*

Take into consideration the beam designed on page 460 for which
$V = 19\,000$ pounds $w = 2000$ pounds
$b' = 10$ inches $jd = 18.625$ inches and the unit shear

$$v = \frac{19\,000}{10 \times 18.625} = 102 \text{ pounds per sq. in (formula 30),}$$ 447

The allowable unit shear in concrete equals 40 pounds, hence stirrups are necessary. 447

Diameter of Stirrups. From formula (40) and Table on page 454 for a bond stress of 80 pounds diameter of a straight pronged stirrup should not exceed $t = 0.012 \times 20.5$ inches. However, since in the present case the upper ends of stirrups are to be bent, ⅜ inch round bars may be considered as secure against slipping. 467

Location of Stirrups. Stirrups are unnecessary with the unit load $w = 2000$ pounds, at a distance from support

$$x_1 = \frac{18}{2} - \frac{40 \times 10 \times 18.625}{2000} = 5.3 \text{ feet (formula 38)}$$ 451

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Spacing of the ⅜-inch stirrups, the area of both prongs being $A_s = 0.22$ square inches, is obtained by plotting in Fig. 148 values of s from formula (36). At support, $s_1 = \frac{24\,000 \times 18.6 \times 0.22}{10\,000} = 5.2$ inches. 451
For $x = 2$ feet, $s_1 = 6.5$ inches, for $x = 4$ feet, $s_2 = 8.9$ inches. A smooth curve drawn through the points determines the spacing at any part of the beam. The first stirrup is placed half of the minimum spacing from the edge of the support and the last stirrup must not be farther distant from the limiting point, where stirrups are unnecessary, than half of the distance between the last two stirrups. The graphical determination of points for the stirrups is shown in Fig. 148. 452

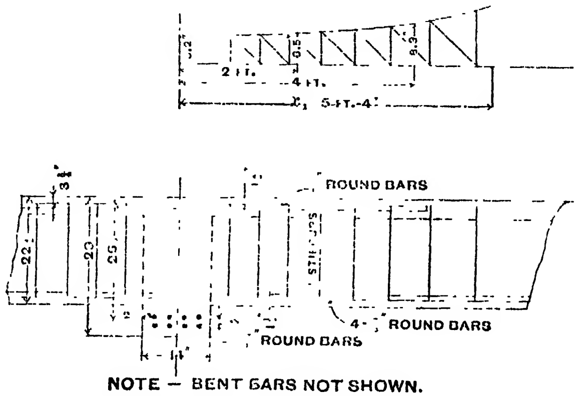


FIG. 148. Spacing of Vertical Stirrups (*See p.* 452).

Bent-up Bars as Diagonal Tension Reinforcement for Girder. In the girder designed on page 471 four of the eight bars are intended to be bent. If properly placed the bars may be used for taking the diagonal tension. The maximum total shear, $V = 41\,600$. Since the load is concentrated at points at which the beam runs into the girder the shear at the left of that point will be (See Fig. 149, p. 474).
$V = 41\,600 - (6 \times 360) = 39\,440$ pounds and the unit shear, $v = 116$ pounds; at right of the point, V will be $41\,600 - 6 \times 360 - 38\,000 = 1440$ pounds and $v = 4$ pounds. Theoretically, shear reinforcement is needed to the point only where the beam intersects the girder.
The diagonal tension equivalent to the horizontal shear at the support is $\frac{41\,600}{24.6} = 1700$ pounds per one inch of length of beam, at point A is $\frac{39\,440}{24.6} = 1600$ pounds per one inch of length of beam. The total diagonal tension is represented by a trapezoid, the parallel sides of which are 1700 and 1600 and the length 6 feet. Hence total diagonal tension $= \frac{1700 + 1600}{2} \times 6 \times 12 = 118\,800$ pounds. One-third of the shear, or 39 600 pounds, is assumed to be taken by the concrete, hence the tension to be taken by the shear reinforcements is 79 200 pounds. Since six ⅞-inch round rods are to be bent, their area is 3.60 square inches and their tensile value from page 449 is $\frac{A_s \times 16\,000}{0.7}$ = 82 000 pounds. Now comparing the above values it is seen that

tensile value of bars is in excess of stress to be provided for. It is also necessary that the bent bars be properly distributed and since shear is nearly uniform between the supports and the intersection of the beam, the inclined bars should be spaced at points *a*, *b*, *c*.

These points were found by dividing the distance on the center line *A B* into equal parts. They should be laid off on the neutral axis, but since the neutral axis changes for the positive and negative moment, the center line, as lying between the two neutral axes, was selected.

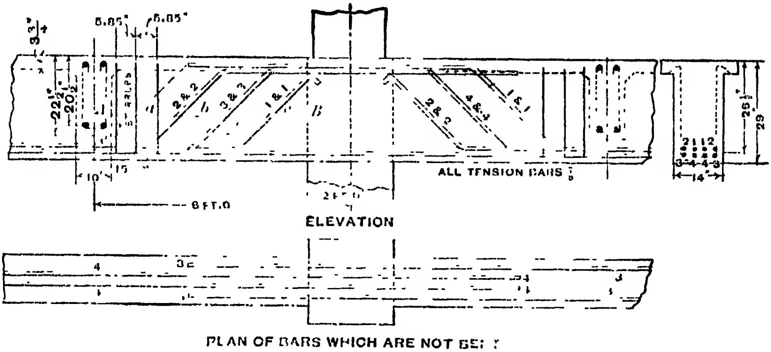


FIG. 149. Reinforcement for Girder (*See p. 474*)

A study must be made to see whether the tensile stresses in the bottom of the beam will permit this. In this case the girder is loaded by concentrated loads and the moment at the point where the beam intersects the girder is nearly the maximum. Approximate figuring of tensile stresses shows that the first two bars may be bent about 15 inches from the center of the intersection of the beam, while to resist diagonal tension the bar to intersect the center line at *a* should be bent at *c* as shown by the dotted line. To provide for the diagonal tension, between point *a* and the beam stirrups will be introduced. Using ½-inch rods for stirrups, the tensile value of which is $2 \times .196 \times 16\,000 = 6\,270$ pounds, it is necessary to space them $\frac{6270}{1065} = 5.85$ inches apart, as shown in Fig 149, the shear to be provided for in one inch of length of beam being 1 065 pounds.

EXAMPLE OF BENT BARS AS REINFORCEMENT FOR DIAGONAL TENSION

As indicated in the design for the girder in the example just given it is possible to provide for the diagonal tension by bent bars without stirrups. When the loading is uniformly distributed instead of concentrated, the location of the bends in the different bars as well as the size of the bars to

use should be governed by the distribution of the shear. This is illustrated in the example which follows.

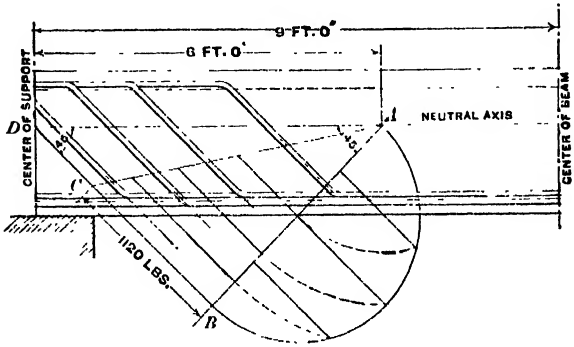


FIG. 150.—Spacing of Bent Bars. (*See p.* 475.)

Example 7—Suppose the 18-foot girder in previous example is loaded uniformly with 4 600 pounds per foot of length, find the locations of the points to bend up the bars to resist diagonal tension.

Solution —The load selected will require a beam of same section and tension reinforcement as the girder in previous example, where breadth of stem, $b' = 14$, depth to steel, $d = 26.5$, and depth from center of compression to tension, $jd = 24.6$. Then $V = 41\,400$ pounds and from page 447, the unit shear $vb' = \frac{V}{jd} = \frac{41\,400}{24.6} = 1\,680$ pounds per inch of length of beam. Two-thirds of this amount or 1 120 pounds per one inch of length of beam has to be provided for by diagonal tension reinforcement. The distance from the support of the limiting point where shear can be taken by concrete itself is $x_1 = 9 - \frac{40 \times 14 \times 24.6}{4\,600} = 6$ feet, formula (38), page 451. From this point to the right the shear increases from zero to its maximum value of 1 120 pounds at the support, and may be represented by the triangle $A\,B\,C$, Fig. 150. This triangle may be drawn in the following manner. From point A at the neutral axis draw a line $A\,B$ at 45 degrees, and from point D a perpendicular to line $A\,B$ through point of intersection B. Lay out the maximum shear $B\,C$. Now, suppose we intend to bend four bars, all of the same diameter, to take the diagonal tension, then each of them will take an equal part. Divide the area of the triangle into three equal parts, find centers of gravity of each part, and from these centers of gravity draw lines to represent the location of points to bend up the bars in the girder. The method of division of the triangle into an equal number of parts is clearly shown in the drawing where the line $A\ B$ is divided into equal parts and dotted arcs of cricles are drawn with centers at A.

MISCELLANEOUS EXAMPLES OF BEAM AND SLAB DESIGN.

Example 8: What is the value of C and the ratio of steel if pressure in concrete is limited to 400 pounds per square inch and pull in steel to 12 000 pounds per square inch, the ratio of moduli of elasticity being 15?

Solution: Approximate values, which are sufficiently exact, may be obtained from the Table 10 page 519, by exterpolation above item (1), from which C equals 0.123, and ratio of steel, $p = .0053$.

Example 9. What is the value of C for a beam in which the pressure in the concrete is 650 pounds per square inch, the pull in the steel 16 000 pounds, and the area of steel 1.2%, the ratio of moduli of elasticity being 15?

Solution: The requirements in the example are impossible. With the pressure in the concrete limited to 650 pounds per square inch, the pull in the steel, if 1.2% is used, cannot be as high as 16 000 pounds. From Table 11, page 520, when $p = 0.012$ and $f_c = 650$, $C = 0.090$ and the pull in the steel is 12 100 pounds. Furthermore, comparing this item with the line for 0.008 steel in the same table, it is evident that an increase of 50% in the area of the steel, *i.e.*, from ratio 0.008 to ratio 0.012, decreases the value C, and therefore the depth of beam, scarcely 7%.

Example 10: What safe load per square foot can be supported by a slab 5 inches thick and 10-foot span reinforced with ½-inch round bars placed 8 inches apart?

Solution. From slab table, page 514, since the given reinforcement from page 507 is equivalent to $0.196 \times 1\frac{1}{2} = 0.294$ square inches for one foot of width, we find by inspection that for a 5-inch slab the nearest area of steel in column (18) is 0.288. Hence, the total safe load for a 10-foot span is slightly more than 136 pounds, say, 140 pounds per square foot; and deducting the weight per square foot of the slab, column (15), gives $140 - 64 = 76$ pounds per square foot safe live load. If slab is square, continuous and reinforced in two directions, the safe load of 140 pounds may be multiplied by 2. Deducting the dead load of 64 pounds, the live load will be $280 - 64 = 216$ pounds per square foot.

Example 11: What safe load per square foot can be placed upon an 8-inch slab, 16 foot span, having steel reinforcement of 0.007?

Solution: Since by Rule 3, on page 513, total loads are inversely proportional to the squares of the span, the load for a 16-foot slab is ¼ the load for an 8-foot slab. For the total safe load of an 8-foot slab, we must interpolate between steel ratios of 0.006 and 0.008, thus obtaining

$$\frac{649 + 831}{2}$$

$= 740$ pounds per square foot. For the 16-foot slab the total safe load is therefore $\frac{740}{4} = 185$ pounds, and deducting the weight of the slab from column (15) gives a net live load of $185 - 103 = 82$ pounds per square foot.

Example 12: Using Table 4 of rectangular beams, page 510, what should be the dimensions and reinforcements for a beam 12 feet span, continuous and loaded uniformly with 1000 pounds per foot of length?

Solution: The assumed stresses are the same as those adopted in the Beam Table. Assuming a width of beam 12 inches, a total load per inch of width of $\frac{1000}{12} = 84$ pounds per running foot. Referring directly to the Beam Table, we find that the total depth corresponding to a 12-foot beam with this load is about 12 inches. The reinforcement from column (25) is $0.083 \times 12 = 1.00$ square inch.

Example 13: What total load per foot of length can be carried by a 12-foot simply supported beam 12 inches wide and 25 inches deep?

Solution: There is no value in the Table 4, page 511, for a beam whose total depth is 25 inches, but since, from rule 4, loads are proportional to the square of the depth of the steel, we may calculate the load in this case from the load for a 26-inch beam 12 inches wide. Assuming in both cases that the depth to steel, d, is 2 inches less than the total depth, we have $364 \times \frac{23^2}{24^2} \times 12 = 4\,000$ pounds per running foot of beam. Since the table is based on $M = \frac{wl^2}{10}$ for simply supported beams, deduct 20% from the above amount. Hence the safe load is $4000 - 800 = 3200$ pounds.

EXPERIMENTS UPON REINFORCED BEAMS

Tests upon reinforced concrete beams have been conducted at various universities in the United States, and by leading scientists in Europe. Valuable data with reference to the location of the neutral axis, the deformation and the ultimate loads with various percentages and classes of steel have been recorded* in the United States by Professors Hatt, Howe, Lanza, Marburg, Talbot, and Turneaure, and in Europe by Messrs. Considère, von Emperger, Feret, Rabut, Ramisch, Ribera and Sanders. An extensive series of tests has been carried on at the United States Government Structural Materials Testing Laboratories at St. Louis, using different materials, different methods of manufacture, and different types of reinforcement.

Special results of many of these tests have been mentioned in the preceding pages.

Tests of Prof. Arthur N. Talbot. At the University of Illinois, Prof. Talbot has made several valuable series of tests to investigate the laws of reinforced concrete, which cover an exceedingly wide range of percentages of steel and types of reinforcement. These are described in detail in various bulletins of the University.†

The fundamental principles of rectangular beams are illustrated in some of the earlier experiments which are summarized in the following table. Although a leaner mixture of concrete was used in these than in his later tests which, therefore, correspond more nearly to practical construction, the principles are not affected. The proportions in these beams were 1 : 3 : 6 based on loose measure of cement, or about 1 : 3½ : 7 based on a unit of 100 pounds cement per cubic foot. The beams were 15 feet 4 inches long, 12 inches wide, 13½ inches deep, with the reinforcement 12 inches below

* See also References, Chapter XXXI.

† Bulletin No. 1, Sept. 1, 1904; Bulletin No. 4, April 15, 1906; Bulletin No. 12, Feb. 1, 1907; Bulletin No. 29, Jan. 4, 1909.

the upper surface. These were tested on a span of 14 feet by two loads which divided the span into three equal parts. The exact proportions of the concrete were 96 pounds Portland cement to 3⅜ cubic feet sand to 6¾ cubic feet broken stone. The sand was well graded in size of grains and weighed 115 pounds per cubic foot loose and dry. The stone was Illinois limestone, with particles smaller than ¼ inch and coarser than 1½ inches screened out. The consistency was such that the water flushed to the surface under light ramming. The crushing strength of 6-inch cubes at the age of 60 days averaged 2030 pounds per square inch.

Typical deformation and deflection curves are given in Fig. 130, page 489.

Prof. Talbot gives the following description of the manner of failure of each beam except those numbered 27, 22, and 28, which crushed at the top at maximum load:

Tests of Reinforced Concrete Beams.

BY ARTHUR N. TALBOT. (*See p.* 479.)

Beam No.	Kind of Steel.	No. of Rods.	Size of Rods in.	Area of Steel. sq. in.	Ratio of area of steel to beam above steel.	Maximum Load. lb.	Load Considered lb	Total Elongation of Steel. in.	Ratio of depth of steel to depth of neutral axis k: As Measured.	Calculated by formula. (*p.* 420)	Talbot's formula. (*p.* 479)	Estimated Total* Bending Moment. in. lb	Moment of Resistance calculated from formula (7) or (8), *p.* 420. in. lb.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	
21	Round	3	½	0.59	0.0041	9 000	8 000	0.0665	0.33	0.33	0.33	261 000	226 800*b*	2 bars turned up
19	"	3	½	0.59	0.0041	9 200	9 200	0.0755	0.36	0.33	0.33	294 600	226 800*b*	2 bars turned up
16	Square	3	½	0.75	0.0052	9 000	9 000	0.065	0.37	0.36	0.35	313 200	284 700*b*	2 bars turned up
17	"	3	½	0.75	0.0052	10 000	9 500	0.059	0.37	0.36	0.35	302 000	284 700*b*	2 bars turned up
27	"	4	¾	2.25	0.0156	26 900	25 000	0.066	0.53	0.54	0.54	725 500	774 000*a*	2 bars turned up
9	Ransome	3	½	0.75	0.0052	22 800	18 000	0.142	0.34	0.36	0.35	540 000	474 500*c*	8 stirrups
15	Thacher	3	¾	1.20	0.0083	18 400	15 500	0.0715	0.41	0.43	0.41	466 000	443 300*b*	2 bars turned up
10	"	3	¾	1.20	0.0083	16 600	14 500	0.065	0.43	0.43	0.41	438 000	443 300*b*	2 bars turned up
22	Kahn†	3	¾	2.40	0.0167	24 400	22 000	0.064	0.57	0.55	0.56	641 000	786 200*a*	Bars sheared up
4	"	5	½	2.00	0.0139	23 000	21 000	0.069	0.47	0.52	0.51	615 000	714 800*b*	Bars sheared up
14	"	4	½	1.60	0.0111	17 200	17 000	0.062	0.46	0.48	0.46	505 500	580 400*b*	Bars sheared up
5	"	3	½	1.20	0.0083	15 000	13 000	0.0625	0.42	0.43	0.41	396 000	443 200*b*	Bars sheared up
28	Johnson	6	¾	2.19	0.0152	34 300	31 000	0.101	0.53	0.53	0.53	893 500	768 700*a*	4 bars turned up
13	"	7	½	1.40	0.0097	20 000	27 500	0.111	0.45	0.46	0.43	800 500	681 400*a*	4 bars turned up
20	"	5	½	1.00	0.0069	20 900	20 000	0.132	0.44	0.41	0.39	503 500	615 600*a*	3 bars turned up
2	"	5	½	1.00	0.0069	20 600	19 000	0.119	0.39	0.41	0.39	565 500	615 600*a*	Horizontal bars
7	"	3	½	0.60	0.0042	14 000	13 000	0.1175	0.33	0.33	0.33	401 000	384 400*c*	Horizontal bars
3	"	3	½	0.60	0.0042	14 000	12 000	0.1065	0.31	0.33	0.33	373 000	384 400*c*	2 bars turned up
								Average	0.418	0.422	0.411	506 006	507 388	

NOTE: — Columns (6) (11) (12) and (14) have been added by the authors.

*As calculated by Prof. Talbot. Based on "Load Considered" column (8).

a. Based on crushing strength of concrete of 2 030 lb. per square inch because the moment thus obtained is lower than the moment based on yield point of steel.

b. Based on yield point of steel as 36 000 lb. per square inch.

c. Based on yield point of steel as 60,000 lb. per square inch.

†Net areas of steel in Kahn bars at load points are lower than gross areas given, so that moments of beams, 4, 14, and 5, by corrected computation are much higher than shown in col. 13.

A portion of the data resulting from the experiments is tabulated above. Column (10) is taken from a separate table of Prof. Talbot's,* and columns (11), (12) and (14) are added by the authors to compare the actual tests and the theory adopted in this treatise.

Prof. Talbot suggests an empirical straight line formula† for the location of the neutral axis with different percentages of steel, which avoids the more intricate calculations necessary with the usual theoretical formulas involving the modulus of elasticity. Adopting the same notation employed throughout this treatise (see p. 420), let

k = ratio of depth of neutral axis to depth of center of gravity of steel.
p = ratio of area of section of steel to area of section of beam above center of gravity of steel.

Then with a slight change to conform to the use of a ratio of 15‡

$$k = 0.24 + 18\,p \tag{58}$$

Column (12) gives values of k calculated from this formula, using 0.26 for this concrete instead of 0.24 The formula is adapted to concrete beams with percentages of steel ranging from 0.006 to 0.012.

One of the most important conclusions in the authors' opinion, which, may be drawn from Prof. Talbot's tests, is the fact that computations made by the ordinary theory adopted in this treatise produce values for the neutral axis, and also for the ultimate moment of resistance, which are so near to the experimental results that these theoretical formulas (see p. 420) may be employed with confidence.

Calculating the location of the neutral axis by formula (6), page 420, and employing a ratio of the moduli of elasticity of steel to concrete of 20,— which Prof. Talbot's tests§ of elasticity show to be an average value between loads of 1 000 and 1 700 pounds per square inch (stresses which correspond to the compression in the beam when the neutral axis is as given), the theoretical distances given in column (11) agree almost exactly with the actual measurements in column (10). The moments of resistance calculated in column (14) also agree closely with the total bending moments in column (13).

T-Beam Tests by Prof. Frank P. McKibben. The T-beams tested at the Massachusetts Institute of Technology were made of concrete

* University of Illinois, Bulletin No. 1, September, 1904.

† Prof. Talbot gives the derivation of this formula and a theoretical discussion of his tests in Journal Western Society of Engineers, August, 1904

‡ The constant in Prof. Talbot's original formula was 0.26.

§ Journal Western Society of Engineers, August, 1904.

mixed in proportion 1 : 2 : 4 by volume based on a unit of 100 pounds cement per cubic foot. The stone used was crushed conglomerate well graded, the range of sizes of particles being from 1¼ to 3/16 inch, while the sand was a mixture of coarse and fine sands in equal parts. The steel reinforcement consisted of plain round bars ranging in size from 11/16 to 1 inch in diameter. The age of beams when tested was about 30 days. Their dimensions were as follows: span 12 feet, total depth 11 inches, depth to steel 9.5 inches, thickness of flange 3 inches, breadth of stem 8 inches, breadth of flange 2 feet. The percentage of reinforcement varied from 2.22 to 3.12 per cent based on the width of the stem, or from 0.74 to 0.104 per cent based on the width of the flange, using in both cases the depth to steel in computing the area of concrete. The following table gives the results of the tests.

Tests of Reinforced Concrete T-Beams

By Frank P. McKibben. (See p. 479.)

Massachusetts Institute of Technology

No. of Beams.	No. of Round Rods.	Size of Rods. In.	Area of Steel. Sq. in.	Percentage of Steel. In web width.	Maximum Load. Lb.	Load at Last Measurement. Lb.	Stress in Steel: At First Crack. Lb. per sq. in.	Stress in Steel: At Last Measurement: From Deformations. Lb. per sq. in.	Stress in Steel: At Last Measurement: Computed. Lb. per sq. in.	Ratio of Depth of Neutral Axis to Depth of Steel: Measured.	Ratio of Depth of Neutral Axis to Depth of Steel: Computed.	Bending Moment at Last Measurement. Inch-pounds.	Computed Moment of Resistance. Inch-pounds.	Computed Stress in Concrete. Lb. per sq. in.	Ultimate Strength of Prisms. Lb. per sq. in.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	2 1	5/8 13/16	1.61	2.22	24240	22000	3110	34500	37460	0.37	0.38	528000	515000*a*	1510	2220
2	2 1	7/8 13/16	1.71	2.25	26415	22000	8230	33300	36900	0.44	0.38	528000	503000*a*	1495	1740
3	2 1	7/8 15/16	1.90	2.50	24365	22000	6440	38100	33400	0.37	0.39	528000	636000*a*	1410	1700
4	3	15/16	2.07	2.73	27215	24000	5400	21600	33500	0.43	0.41	576000	671000*b*	1570	1680
5	2 1	1 13/16	2.37	3.12	30940	28000	2040	29600	34400	0.47	0.44	672000	671000*b*	1780	1610

a Based on stress in steel obtained from last measurement.
b Based on crushing strength of concrete, since beam failed by compression.
Note: In figuring the moment of resistance the computed depth of neutral axis for $n = 15$ was used. Percentage of steel in terms of width of flange is ⅓ of the values in col. (5).

The tests compare well with the results obtained from the formulas given on page 420. The stresses in steel, determined by measurements of stretch, do not vary appreciably from those obtained from the formulas. Beams

No. 4 and 5 failed by compression in the concrete, and the compressive stress in beam near to failure agrees quite closely with the strength of the prisms made of the same mix of concrete. A difference in deflection of the stem and the flange was detected by the tests, which indicates that the compressive stresses are not uniform throughout the whole width of the flange. This, however, in practice is undoubtedly more than balanced by assuming a width of flange smaller than the width of slab that actually assists in taking the compression. First cracks occur, as evident, at very low stresses, but they are very minute and almost invisible and their presence is not dangerous.

Tests of Repetitive Loading of Reinforced Concrete Beams by Prof. H. C. Berry. Fatigue tests of reinforced concrete beams made by Prof.

Fatigue Tests of Reinforced Concrete Beams. Size of Beams: 8″ × 11″. Span: 13 ft. Age: 6 Weeks

By H. C. Berry

University of Pennsylvania. (*See p.* 481)

Reinforcement	Number of Repetitions	Working Stress in Steel	Working Stress in Concrete	Breaking Load	Maximum Deflection
		lb. per sq. in.	lb. per sq. in.	lb.	in.
4, ½″ round rods . .	1			12 000	0 56
4, ½″ round rods . .	297 000	18 300	785	12 300	0.48
2, ⅝″ square bars . .	395 000	15 200	628	10 500	0.46
2, ¾″ diamond bars	2			13 000	0.62
2, ¾″ diamond bars	718 000	14 300	785		
	then				
	422 000	17 100	940	13 600	0.78
3, ¾″ corr. bars. . . .	0			20 000	0.66
3, ¾″ corr. bars . . .	295 000	10 800	940	17 700	0.55

H. C. Berry* at the University of Pennsylvania in 1908 indicate that as many as one million repetitions of high working stresses do not materially affect the ultimate strength of a reinforced concrete beam, its maximum deflection, or the position of its neutral axis. Duplicate beams were made of concrete mixed in the proportions of 1 part cement, 1½ parts bar sand and 4½ parts ¾-inch crushed granite and were reinforced with plain and deformed bars. These beams were tested when 6 weeks old, one being subjected to a repetitive loading sufficient to cause higher stresses than ordinarily allowed in

* *Engineering Record*, July 25, 1908, p. 90.

good practice, and then tested to failure, while the other was broken in the ordinary manner.

It was evident that the greater part of the set in the deformation in the plane of the steel occurred in the first few thousand applications of the load and that the set in the deformation on the compressive side of the beam was also relatively large for the first few thousand repetitions and increased with the stress applied and the number of repetitions.

The stresses realized and the deflections resulting from the repetitive loadings are shown in the accompanying table on page 481. The breaking strength of the beams sustaining the repetitive loading is substantially the same in every case as the corresponding beam with no appreciable repetitions.

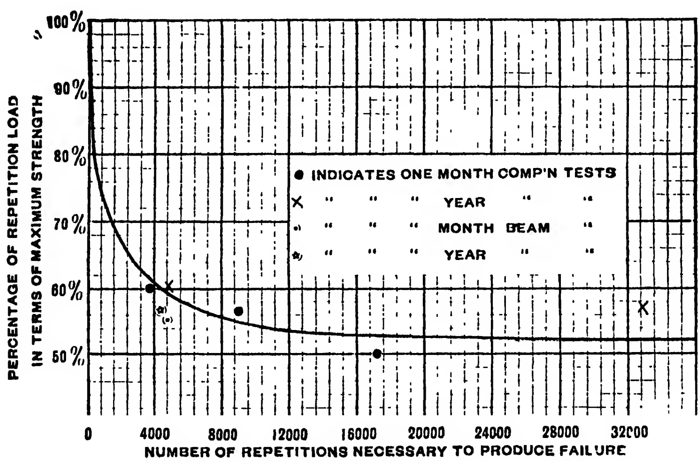


FIG. 151. Fatigue of Reinforced Concrete Beams. (*See p.* 482.)
By Prof. T. L. Van Ornum.

Compression tests by Prof. J. L. Van Ornum* at Washington University made in 1907 agree with the above tests for repetitive loadings under 50 per cent of the maximum strength of the concrete, but for repeated loads greater than this he found that beams will be subject to failure. He concluded that the number of repetitions required to cause this failure depended essentially upon the ratio of the test load to the ultimate strength of the concrete. In these tests, as will be seen from the curve in Fig. 151, which summarizes graphically the results of these experiments, the influence of

* Transactions American Society Civil Engineers, 1907, LVIII, p. 294.

the fatigue of concrete is limited to an intensity of about 50 per cent of the ordinary ultimate strength of the concrete.

Tests at Illinois University, at St. Louis,* and elsewhere confirm the principle illustrated and show that there is a fatigue limit to concrete corresponding in a general way to the elastic limit of metals. This varies with the character of the concrete from ½ to ⅔ the ultimate strength. Prof. Talbot finds in columns the deformation to be a measure of this fatigue limit, the latter usually occurring at about ½ the ultimate deformation.

This fatigue limit of concrete, while it does not influence the practice of conservative design, is a warning against the use of too high working stresses.

FLAT SLABS

Besides the usual systems for floors, using a combination of slabs, beams and girders, a floor system of a type of an entirely different design is sometimes employed, which consists of a flat unribbed slab continuous over the whole floor and supported by columns only. The type originally introduced by Mr. C. A. P. Turner of Minneapolis is sometimes termed the Mushroom System.

The reinforcement of the slab consists of bars running in four directions radially from the column, and the head of the column is usually enlarged in order to diminish the bending moment and increase the shearing resistance. The vertical steel in the column reinforcement or a portion of it may be bent and carried into the slab to add to the rigidity of the connection.

The moments and stresses in this system are statically indeterminate, but in order to make an application of the theory of flexure possible, the whole floor is considered as a series of flat circular slabs concentric with the columns and firmly clamped to them, supporting the rest of the floor. Thus the analysis of the whole floor is reduced to that of circular plates clamped to the columns, and flat slabs supported on all edges by these circular plates.

Let Fig. 152 represent a floor of this system, and consider the strip *ab* as separated from the rest of the floor. This strip when loaded will act as a fixed beam. The points of inflexion will be distant approximately one-fifth of the span from the circular lines of assumed support, which are the lines of maximum bending movement. The points of inflexion of the floor will thus be located on the dotted curve shown on the drawing. Instead of this curve we may assume the points to be on a circle, represented on the drawing by dash lines, and consider the area within this circle as a round plate, loaded with a uniform load over its area and in addition loaded around its circumference with a load which, per unit of

*See Bulletin 344, U. S. Geological Survey.

length, is equal to the remaining load of the panel divided by the circumference of the circle.

The part of the slab between the column and the points of inflexion will deflect downwards, while the rest of the slab will deflect as an ordinary supported beam.

The authors have adopted Prof. Eddy's analysis of stresses* in a homogeneous circular plate, and, from his general formulas, deduced formulas applying to circular slabs free on their outer edge and clamped round the column. In this analysis the effect of lateral stresses has been taken into account, this being expressed by Poisson's ratio, which is the ratio of the lateral deformation to that in the direction of stress. Very few tests have been made to determine the value of Poisson's ratio, and the results obtained vary considerably. Many of the earlier tests give as high as 0.2, but, since some of the best experiments in our American colleges indicate a value ranging, with concrete of different proportions and strength, from 0.05 to 0.15, the ratio of 0.10 is recommended for use with concrete where the correct value is unknown, as being undoubtedly safe for concrete of 1:2:4 proportions. It must be noted that the increase of Poisson's ratio tends to diminish the deflection and thus decrease the stress. A high Poisson's ratio therefore means a thinner slab and less steel.

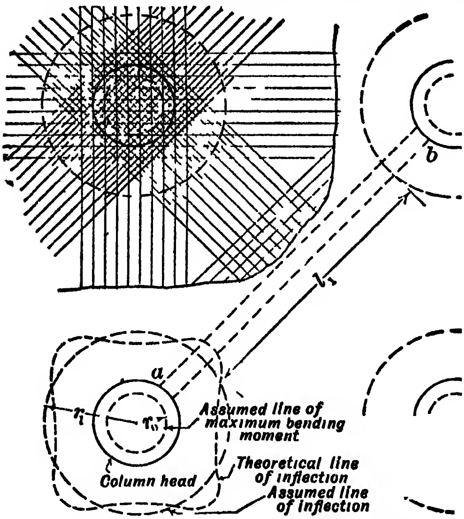


FIG 152. Plan of Flat Slab. (*See p.483*)

The meaning of Poisson's ratio as applied to a loaded column is the lateral deformation per unit of width divided by the longitudinal deformation per unit of length. For example, if a certain load causes a 10-inch column to expand laterally 0.0003 inches, while at the same time it shortens 0.03 inches in a gaged length of 100 inches, Poisson's ratio for that loading is $\frac{0.0003 \times 100}{0.03 \times 10} = 0.1$. In a slab supported on columns there is a similar condition of deformations caused by horizontal stresses at right angles to each other which are taken into account in the mathematical work involved in the derivation of the formulas.

The general formulas derived from the Eddy theory are complicated even after introducing a number of constants, but, disregarding the circumferential moments, which at the support are a minimum and

*Engineers' Society, University of Michigan, 1899.

negligible, it is possible to reduce the working formula for the moment at any circle whose radius is r to the following simple form.*

Let

q = uniformly distributed load around the outer edge of the plate in pounds per foot of length.

w = uniformly distributed load on surface of plate in pounds per sq. ft.

r_0 = radius in feet to line of maximum bending moment (which is within the column head).

r_1 = outer radius of assumed plate in feet.

r = any radius in feet where moment is to be computed, for critical section, r is radius of column head.

C_5, C_e = constants given in Table 9, p. 518.

M_r = total radial bending moment to be used ordinarily.

l_1 = distance in feet between lines of inflection.

Then total radial moment at any point of plate is $M_r = wr_0^2 C_5 + qr_0 C_e$

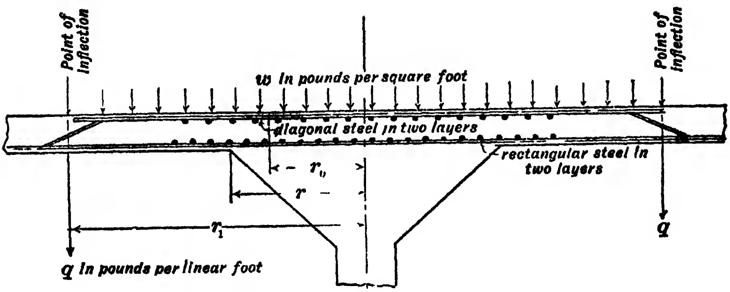


FIG. 152a Section of Flat Slab

The values of C_5 and C_e, as determined from r, r_1, and r_0, are obtained from Table 9, p. 518. If q is in pounds per foot of length, w in

* The more complete formulas which may be used with the aid of table on page 518a if outside of the limits of the values on page 518 are as follows

In addition to above notation, let

$C_1, C_2, C_3, C_4, C_a, C_b, C_c, C_d$, = constants given in Table 9 p. 518a

q = Poisson's ratio

M_1 = moment causing circumferential fiber stress } For loading uniformly distributed over plate
M_2 = moment causing radial fiber stress

M_a = moment causing circumferential fiber stress } For loading distributed along edge of plate
M_b = moment causing radial fiber stress

Then

$$M_1 = wr_0^2 \left\{ 0.1 \left(\frac{r}{r_0} \right)^2 - C_1 \left(\frac{r_0}{r} \right)^2 - C_3 \log \left(\frac{r}{r_0} \right) + C_4 \right\} \quad (54)$$

$$M_2 = wr_0^2 \left\{ 0.2 \left(\frac{r}{r_0} \right)^2 + C_1 \left(\frac{r_0}{r} \right)^2 - C_3 \log \left(\frac{r}{r_0} \right) + C_2 \right\} \quad (55)$$

$$M_a = qr_0 \left\{ -C_a \left(\frac{r_0}{r} \right)^2 - C_c \log \left(\frac{r}{r_0} \right) + C_d \right\} \quad (56)$$

$$M_b = qr_0 \left\{ C_a \left(\frac{r_0}{r} \right)^2 - C_c \log \left(\frac{r}{r_0} \right) + C_b \right\} \quad (57)$$

The M_r in text above is the sum of M_2 and M_b which are the only moments which need to be considered in practical design.

If $\frac{r}{r_0} = 1$, formulas (55) and (57) become

$M_2 = wr_0^2 (0.2 + C_1 + C_2)$ .... (52) $M_b = qr_0 (C_a + C_b)$ ...... (53)

pounds per square foot, and r_0 in feet, the moments are in foot pounds per foot or inch pounds per inch.

If the column head is enlarged so as to be comparatively thin at its circumference, a part of it is flexible and must be considered as a part of the slab so that the assumed line of support at which the slab may be considered as rigidly fixed is within the column head.*

The assumed line of support should coincide with the line of maximum bending moment. Its location depends upon the dimension and design of the column head and must be chosen by judgment rather than by computation. For ordinary conditions, it seems reasonable to consider this line of maximum moment as located within the column head at a distance from the circumference equal to the thickness of the slab; the radius r_0 in such a case would be a length, t, smaller than the radius of the column head.

The maximum *stress* will *not* be on this line of maximum bending moment because the strength is there increased by the greater thickness of concrete. The *maximum stress*, therefore, will be ordinarily at the circumference of the column head.† Hence, for computing the bending moment at line of maximum stress in slab, $r = r_0 + t$. (See Example 14, page 487.)

As in a fixed or continuous beam, the top of slab at support is in tension, and the bottom in compression, i. e. the moment is negative.

The thickness and reinforcement of the slab are found as in ordinary beam and slab design and illustrated in Example 14, page 487. The limiting thickness of slab is usually determined by the thickness required near the column to resist the negative bending moment there. It is advisable, then, to make the slab near the support as thin as possible by using a rich concrete and a larger amount of steel and by placing some steel in the bottom of the slab for compression.

It is common practice in flat slab floor construction to place the steel in the top of the slab in four layers, two diagonal and two rectangular. The authors advise, instead, placing only the two diagonal layers of steel in the top of the slab for tension, as shown in Fig. 152a, page 485 and illustrated in the example, and the two rectangular layers in the bottom of the slab. By this plan the bottom steel assists in taking compression, and the centers of tension and compression are brought farther apart, thus increasing the values of d and jd for a given thickness of slab and therefore permitting a thinner slab for a given loading. Tests show that the steel may be distributed over the full diameter of the column head plus at least a distance equal to the thickness of the slab on each side of it.

A value for compression in concrete, f_c, higher than in beam construction is permissible, and a lower value of n, because of the rich concrete mixture and because of the fact that the maximum stresses occur near

*Even in ordinary beam and slab design, the line of support is customarily assumed within the structural support.

†Since evolving this analysis, the actual stresses in the Minneapolis building tested by Mr. A. R. Lord and described before the National Association of Cement Users in December, 1910, have been compared with the stresses computed by the above formulas and the results tend to confirm, from a practical standpoint, the correctness of our formulas and assumptions.

the support, where the concrete bears on a larger area, and for this reason is able to stand, say, 15 per cent higher stresses than in the middle of the beam. It is advisable, however, to fix a maximum stress of 800 pounds per square inch even with a rich concrete of proportions say 1 : 1½ : 3.

The slab between the circular plates may be considered as supported on all edges. From Fig. 152 it is evident that the largest deflection and the largest *positive* bending moment occur in the middle of the panel, and may be safely taken as those of a square plate supported on all edges, the side of which is the diagonal distance between the lines of inflection. This distance, l_1, *between lines of inflection* may thus be taken as the span, and thickness and reinforcement at the middle computed very conservatively by the formula* $M = \frac{wl_1^2}{24}$.

EXAMPLE OF FLAT SLAB DESIGN

Example 14: Design a flat slab to support a live load of 200 pounds per square foot; spacing of columns 17 by 17 feet; diameter of head 54 inches. Assume working stress in steel, f_s = 16 000 pounds per square inch; in concrete at support, f_c = 700 + 15% = 800 pounds per square inch; and Poisson's ratio, g = 0.1, allowing for a rather rich concrete.

Solution: The slab will be considered as a flat circular plate fixed to column and supporting at its circumference the rest of floor, as outlined on page 483. The radius, r_0, to line of maximum bending moment will be taken as the radius of the column head minus the thickness of slab, $r_0 = 2.25 - .67 = 1.58$ feet, and the outer radius of plate, r_1, will be taken as the average distance of the points of inflection of the slab from centers of columns. The radius, r_1, is thus one-fifth of the distance between lines of maximum bending moment plus r_0, hence, r_1 minimum $= \frac{17 - 3.16}{5} + 1.58 = 4.35$ feet, and r_1 maximum $= \frac{24 - 3.16}{5} + 1.58 = 5.75$ feet, the average value of r_1 is $\frac{4.35 + 5.75}{2} = 5.05$ feet and the ratio of radii is $\frac{r_1}{r_0} = \frac{5.05}{1.58} = 3.20$.

Live load = 200 lb. per sq. ft., assumed dead load = 100 lb. per sq. ft., giving a total unit load, w, of 300 lb. per sq. ft.

Area of slab is 17 by 17 = 289 sq. ft. and area of circular plate $5.05^2 \times 3.14 = 80.1$ sq. ft.; hence, the difference of the two areas, 208.9 sq. ft., is the area of slab tributary to each column outside of assumed circular plate. The loading of this area is supported around the circumference of the flat plate, and equals $208.9 \times 300 = 62\,700$ lb. Dividing this value by the circumference of the outer edge of the plate gives the circumferential unit loading, q = 1980 lb. per lin. ft.

Line of maximum stress is at circumference of column head hence $r = 2.25$ and ratio of r to r_0 = 1.42.

From the corresponding constants in table on page 518, the bending moment at the circumference of column head is $M = (300 \times 1.58^2 \times 1.89) + (1980 \times 1.58 \times 2.54) =$ 9400 in. lb. per inch of circumference. This is a negative moment, the top of slab being in tension and the bottom in compression, as in any fixed or continuous member at the support.

If steel is used only in top of slab, the depth, reinforcement, and thickness of slab may be determined from the ordinary slab formula, page 421, using the total M given above, after changing it to inch pounds per foot. If steel is used in both top and bottom, the required depth and reinforcement may be determined by formulas (18) and (20), page 428. In the present case, 1⅜% of steel will be placed diagonally in two

*Tests of the Minneapolis building (see foot-note p. 486) show that stresses at middle of span are small so that this formula is conservative.

layers at top and 0.7% of steel rectangularly in two layers at bottom; hence, using formula (18), page 428, and table on page 517, with ratio $a = 0.15$, $d = \sqrt{\frac{9400}{800 \times 0.24}}$ = 7 inches, requiring a slab thickness of about 8 inches.

The total amount of steel required in top of slab at column is $A_s = 27 \times 2 \times 3.14 \times 7 \times .0137 = 16.3$ sq. in. Each of the two layers is effective on both sides of column, hence each layer must have $16.3 \div 4 = 4.08$ sq. in. Total width of layer or band is taken as the diameter of column head plus twice the thickness of slab, or 70 in., and, using ½-inch round bars, they would be placed 3½ in. apart At the bottom of slab at column, the ½-inch round bars would be placed 7 in. apart.

Several trials must usually be made to determine the most economical relation of the amount of steel and concrete. It should be borne in mind that the increase of reinforcement for a short length over the support decreases the thickness of entire slab, reducing the amount of material and at the same time the dead load and the moment. Hence, a larger percentage of steel than used in beam and slab design and the introducing of steel at the bottom usually will prove economical.

The diagonal distance, l_1, between lines of inflection is $24 - 10.1 = 13.9$ feet, and bending moment in middle of slab (see p. 487) is

$M = \frac{300 \times 13.9 \times 13.9 \times 12}{24} = 29000$ in. lb. per foot of width. The effective depth of slab, d, as determined by the necessary depth at support is $8 - 1 = 7$ in.

Then, from page 418, $C = \sqrt{\frac{12 \times 49}{29000}} = 0.142$.

In Table 11 (p. 520), $p = 0.0035$ corresponds to $C = 0.142$, hence, 0.35 per cent of steel in each diagonal direction will be necessary, or ½-inch round rods 8 in. apart. In this case the rods would naturally be placed 7 in. apart as a matter of convenience.

CONCRETE COLUMNS

Columns of short length, essentially piers, the length of which is not more than six times the least lateral dimension, may be built of plain concrete with no reinforcement, provided the loading is central. Columns longer than this should be reinforced for safety in construction and also to guard against the possibility of eccentric loading and the danger of sudden failure. It is desirable to further limit the use of reinforced columns to a length of 15 diameters.

Although concrete is especially adapted for sustaining compression, its compressive strength is so much lower than that of steel that in a building it is frequently difficult to keep the columns in the lower stories within the limits required by the uses for which the building is constructed.

To reduce the size of the column, four distinct methods are used either separately or in combination:

(1) Rich proportions of concrete.
(2) Vertical steel bars designed to assist in taking the compression.
(3) Hooping or banding.
(4) Structural steel shapes in combination with the concrete.

These will be considered in the order given.

While as a general proposition concrete in compression is always cheaper than steel, the limits of size of column frequently make steel reinforcement necessary not only to resist bending caused by eccentric loading or lateral pressure, but to take a part of the vertical compression load.

Whatever the type of construction, the effective area to use in figuring the compression should usually be less than the total area to allow a certain thickness on the surface for fire protection. The Joint Committee recommends that the protective covering shall be taken to a depth of $1\frac{1}{2}$ inch on all surfaces, since in a severe fire the concrete to this depth may be affected by the heat and its strength destroyed. A less thickness than this should be sufficient where the contents of a building are not especially inflammable, a decrease in the total diameter or width of a column of 1 to 2 inches being frequently a fair allowance when computing the effective area.

The steel, however, should in all cases be imbedded at least $1\frac{1}{2}$ to 2 inches, and when circular hooping is used to add strength and ductility the effective area must be taken as that within the hooping.

Rich Proportions of Concrete. The compressive strength of concrete is approximately proportional to the amount of cement which it contains (see page 392), so that increasing the proportion of cement is an effective means of strengthening the column to permit smaller section. A rich concrete also has a higher modulus of elasticity and there is consequently less deformation under load. Besides this, a rich concrete works smoother in placing and makes it easier to produce a homogeneous column, provided the aggregates are properly graded. The strength of concrete for different mixtures is indicated on page 360, and working stresses are suggested on page 527. Before permitting the use of high column stresses in a structure, actual compressive tests should be made upon cylinders 8 inches diameter by 16 inches high composed of the same materials to be used and mixed in the required proportions with the same wet consistency.

Vertical Steel Bar Reinforcement. Tests of long columns made at the Watertown Arsenal,* the Massachusetts Institute of Technology,† and the University of Illinois,‡ indicate conclusively that vertical steel bars imbedded in concrete may be counted upon to take their portion of the loading. As a column takes its load, it is shortened in height, the concrete and steel, shortening equally because they are bonded together. The concrete, however, has so much lower strength that it receives its allowable load before the steel can reach its full working strength. Consequently, the working load upon the steel must be figured at a low value, which is determined by the amount of shortening it has undergone up to the point where the concrete is shortened so as to reach its working strength. Since, with a given load, the shortening or deformation is proportional to its

* Tests of Metals, U. S. A., 1904, 1905, 1906, 1907.
† Transactions American Society of Civil Engineers, Vol. L, p. 487.
‡ University of Illinois Bulletin 20, December 25, 1907.

modulus of elasticity (see p. 529), the working stress in the steel must be the working stress in the concrete times the ratio of the moduli of elasticity of steel to concrete, as indicated below.

Although tests indicate that if vertical steel is placed at least 2 inches from the surface of the column, the elastic limit of the steel may be reached without danger or buckling, it is nevertheless advisable in almost all cases to place occasiona horizontal loops around the steel spaced at distances apart not greater than the width of the column as an additional precaution against the buckling of the rods, and also for the purpose of keeping the bars in place during the pouring of the concrete. The size and location of such loops are discussed in connection with column design on page 624.

Joints in the vertical steel when small diameter rods are used, say up to 1¼ inch, may be provided for by lapping as indicated on page 464. Large diameter rods should have their ends planed true and butted with a sleeve around the joint, or should have some other positive connection. In footings where the length of imbedment is not sufficient to take all the stress, a horizontal bearing plate must be provided.

Since the relative loading upon the steel and the concrete at any period is theoretically in direct proportion to the ratio of their moduli of elasticity at that period, and since full size column tests have borne out this assumption, the allowable loading, that is, the allowable unit pressure, is readily obtained as follows:*

---

* From mechanics

$$\frac{\text{stress per square inch}}{\text{modulus of elasticity}} = \text{deformation}$$

hence $\dfrac{f'_s}{E_s}$ = deformation of steel and $\dfrac{f_c}{E_c}$ = deformation of concrete.

Since with perfect adhesion between concrete and steel all parts of the column must undergo the same deformation,

$$\frac{f'_s}{E_s} = \frac{f_c}{E_c} \text{ or } f'_s = f_c n$$

The allowable stress in steel is therefore the allowable stress in the concrete times the ratio of elasticity. For practical purposes the total loading must be introduced. Since the total pressure in the column must be the sum of the pressure in the concrete plus the pressure in the steel,

$$fA = f_c A_c + f'_s A_s \quad \text{or} \quad fA = f_c A_c + f_c n A_s$$

and since $A_c = A - A_s$ we have

$$f = f_c \left[\left(\frac{A - A_s}{A} \right) + n \frac{A_s}{A} \right]$$

or since $p = \dfrac{A_s}{A}$ we reach the result

$$f = f_c [(1 - p) + np]$$

Let

f = allowable unit pressure upon the reinforced column, equal to the total load divided by the effective area.

f_c = allowable unit pressure upon the concrete of the column.

f_s' = allowable unit pressure upon the vertical steel in the column.

n = $\frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to modulus of elasticity of concrete.

P = load to be sustained by the column.

A = area of total effective* cross-section of column.

A_c = area of concrete in cross-section.

A_s = area of steel in cross-section.

p = $\frac{A_s}{A}$ = ratio of cross-section of steel to total cross-section of column.

For determining the total allowable unit compression, f (which is the total load, P, divided by the effective area A) with fixed area of concrete and steel, we have

$$f = \frac{f_c A_c + f_c n A_s}{A} \tag{59}$$

In terms of the percentage of steel,

$$f = f_c [1 + (n - 1) p] \tag{60}$$

The percentage of steel to use to obtain total unit stresses when the compression on the concrete is limited to f_c is

$$p = \frac{f - f_c}{f_c (n - 1)} \tag{61}$$

and the effective cross-section of column is

$$A = \frac{P}{f_c [1 + (n - 1) p]} \quad (62) \qquad \text{or } A = \frac{P}{f} \quad (63)$$

To this area must be added the protective covering as indicated above.

* See page 497.

The table below gives values of f for different stresses and different moduli of elasticity

Working Loads on Concrete Columns Reinforced With Longitudinal Rods
(*See p* 492)

RATIO OF STEEL	ALLOWABLE UNIT LOAD ON COLUMNS IN LB PER SQ IN											
p	Ratio of Moduli, $n = 10$				Ratio of Moduli, $n = 15$				Ratio of Moduli, $n = 20$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	$f_c =$ 450	$f_c =$ 550	$f_c =$ 650	$f_c =$ 750	$f_c =$ 450	$f_c =$ 550	$f_c =$ 650	$f_c =$ 750	$f_c =$ 450	$f_c =$ 550	$f_c =$ 650	$f_c =$ 750
0 01	490	599	708	817	513	627	741	855	535	654	773	892
0 02	531	649	767	885	576	704	832	960	621	759	897	1035
0 03	571	698	825	952	639	781	923	1065	706	863	1020	1177
0 04	612	748	884	1020	702	858	1014	1170	792	968	1144	1320

NOTE—Use column (6) ordinarily for first class 1 : 2 : 4 concrete

Examples on page 498 illustrate the use of these formulas.

The table on p. 493 from tests by Mr James E Howard gives the relation of actual tests to theoretical computations based on a ratio of elasticity of 15. It is noticeable that the actual strength is almost always more than the theoretical, and this is especially the case with the leaner mixtures because the modulus of elasticity of the leaner concrete is lower, and therefore the ratio of 15 is very conservative.

An excellent analytical treatment of columns reinforced with vertical steel is given by Professor Talbot in one of his University Bulletins.* The problem is discussed briefly by one of the authors in a paper before the Boston Society of Civil Engineers †

The analysis of the action of combined compression and bending, such as is produced in columns loaded eccentrically, and the method of computing the reinforcement in such cases is treated in pages 560 to 574.

Hooped or Banded Columns. Mr A Considère in France was the first to apply to reinforced concrete the principle that if a material is confined laterally, it will deform or shorten less under vertical loading, and therefore can sustain a heavier load before it crushes. This is the principle involved in the hooped or banded column. It is carried out in practice by placing steel bands or spiral hooping within the column designed to resist the lateral deformation.

* University of Illinois, Bulletin No. 12, Feb. 1, 1907.
† Sanford E. Thompson in Journal Association Engineering Societies, June 1907, p. 316.

Tests at the Watertown Arsenal,* the University of Illinois† and the University of Wisconsin,‡ 1906-1907, prove that while hooping or banding increases the crushing strength of the column, the deformation, that is, the shortening of the column, is so great at a comparatively early period in the loading that the safe strength cannot be based directly upon the breaking strength.

A perfect fluid like water will transmit pressure equally in all directions. Concrete, on the other hand, under ordinary loading expands laterally a very small percentage of its vertical deformation or shortening (see p. 484); so that, even from a theoretical standpoint, the hoops should not come into play until the concrete has shortened so much that its elastic limit, or the period corresponding to this, has been passed.§

Strength of Plain vs Vertically Reinforced Concrete and Mortar Columns. Columns 12″ × 12″. Height 8 feet Age of Mortar and Concrete 6 months Watertown Arsenal (see p 492)

PROPORTIONS			Plain Concrete or Mortar Columns Actual Strength lb per sq in	REINFORCED COLUMNS			Computed Strength based on col (4) and a ratio of $n = 15$ lb. p. sq in.	REFERENCE TO "TESTS OF METALS" U. S. A.
				Reinforcement				
Cement.	Sand	Stone		Description	Ratio Area Steel to Area Column	Actual Strength lb per sq in		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	2	0	3070	8-¾″ round bars	0.029	4200	4290	1905 p 377
1	3	0	2380	8-¾″ round bars	0.029	3840	3320	1905 p. 377
1	4	0	1520	8-¾″ round bars	0.029	3380	2120	1905 p 377
1	5	0	1080	8 ¾″ round bars	0.029	2810	1510	1905 p. 377
1	5	0	1080	13 ¾″ round bars	0.046	3900	1780	1905 p 377
1	1	2*	1720	4-¾″ twisted bars	0.014	2890	2060	1904 p 386
1	2	3*	1769	4-¾″ twisted bars	0.014	2010	2100	1904 p. 386
1	2	4	1413	4 0″ 0.74″ × 0 74″ trussed bars	0.014	1900	1689	1906 p 538
1	2	4*	1710	4-¾″ twisted bars	0.014	1990	2050	1904 p 386
1	2	4†	2400	8-¾″ twisted bars	0.029	3700	3360	1907 p 242
1	3	6	1450	8-⅝″ corr bars	0.019	2290	1840	1904 p 379
								1906 p. 535

* ½″ to 1½″ pebbles.
†Age 17 months 22 days.

The action of the hooped column as established by tests on long columns is discussed by one of the authors as follows:‖

* Tests of Metals, U. S. A., 1906.
†University of Illinois. Bulletin No. 20, Dec. 15, 1907.
‡Transactions American Society for Testing Material, Vol. IX 1909.
§ See discussion by Sanford E. Thompson in Journal Association Engineering Societies, July, 1907, p. 320. The effect of lateral expansion based on the action of plain columns is here treated before the publication of the tests of hooped column which established the principle.
‖ Sanford E. Thompson in Transactions American Society of Civil Engineers, Vol. LXI, 1908, p. 47.

When a load is placed upon the top of any column, it causes vertical compression or deformation, with, at the same time, a lateral expansion. The lateral expansion in concrete columns, as shown by tests upon plain and upon reinforced columns by Mr. J. E. Howard at the Watertown Arsenal* and by A. N. Talbot, M. Am. Soc. C. E., at the University of Illinois,† is at first very small. Any stress produced in the steel hooping must be proportional to its deformation or stretching; hence, with small lateral expansion of the concrete, there can be but slight stress in the hoops. For this reason, and also because of the initial shrinkage of the concrete, which the lateral expansion must first overcome scarcely any stress or pull comes upon the hoops until the concrete within them has reached a loading equal to the breaking load in plain concrete. As this load is approached, the modulus of elasticity of the concrete becomes very much lower, and consequently both the vertical and lateral deformations become much greater. Then, and not until then, does the lateral pressure begin to act appreciably upon the hoops. In other words, up to the very crushing strength of plain concrete, the value of the hooping is actually negligible. From then on, the reinforcement takes practically all the load, and a high ultimate strength may be attained, although coincident with great shortening of the column.

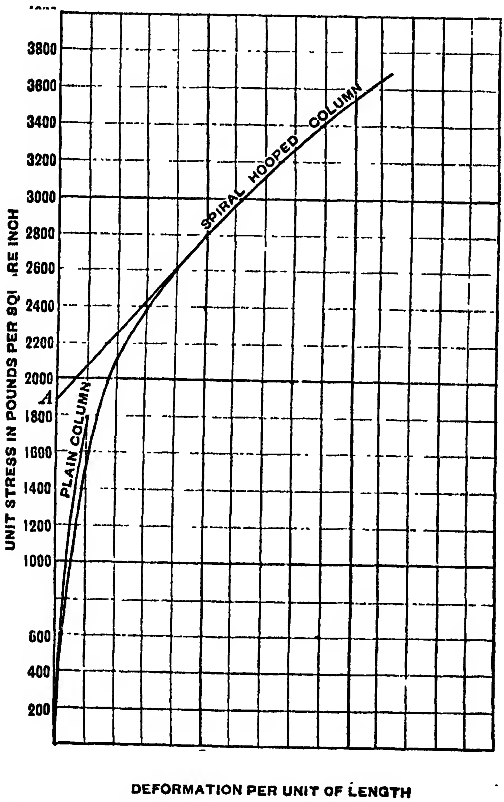


FIG. 153. Deformation of a Hooped and of a Plain Column.‡ (*See p.* 494.)

Even with the concrete restrained within the hoops, the shell of concrete outside of them, which is necessary for fireproofing and for the protection

* Tests of Metals, U. S. A., 1905, pp. 293–336.

† Proceedings American Society for Testing Materials, Vol. VII, 1907, p. 382.

‡ Columns 109 and 182 from Bulletin No. 20, University of Illinois, December 15, 1907.

of the steel, begins to crack and peel at about the same load as that which will cause complete failure in unreinforced concrete. Professor Talbot, in fact, states as a general proposition that: "Cracking and peeling of the concrete appear at loads corresponding to the ultimate strength of the concrete."

Tests also indicate that the shortening of the column is so great that the elastic limit of any vertical steel rods is passed at a load but slightly greater than that corresponding to the crushing strength of plain concrete.

The typical deformation of a column reinforced with spiral hooping as compared with a column having no reinforcement is shown by the curves Fig. 153. Although the ultimate strength of the hooped column shown is 3700 pounds per square inch, it will be seen that at a load of 1800 pounds per square inch, the crushing strength of the plain column, the curve drops off very rapidly and the line produced back to the axis of ordinates at A agrees very closely with the crushing strength of the plain column. At 2000 pounds per square inch the deformation per unit of length is 0.0017. At this deformation vertical steel in such a column would be stressed to 51 000 pounds per square inch. In other words, at a load only 10% higher than that to be expected of a plain column, even steel of a high elastic limit would have reached its yield point.

The entire subject is treated very fully by Professor Talbot in the description of his tests in the Bulletin from which the diagram is taken.

Quoting again from Mr. Thompson's Discussion before the American Society of Civil Engineers:

Tentative conclusions with regard to hooped column design at the present stage of tests may be summarized as follows:

(1) Hooping, if properly applied, increases the ultimate breaking strength under a single loading to double or treble the breaking strength of a plain column.

(2) The surface of concrete outside of the hooping will begin to crack at a loading corresponding to the breaking load of an unhooped column.

(3) Hooping, if not continuous or rigid, will peel off with surface concrete so that the effective strength of the column will be no greater than a similar one of plain concrete.

(4) The total vertical deformation of a hooped column is so great at the period of first external crack that any vertical steel, unless designed to carry the entire load, is stressed beyond its safe strength.

(5) The ultimate breaking strength of a hooped column is no measure of its safe strength, and formulas based on the ultimate strength are useless.

Notwithstanding these conclusions it must not be inferred that hooping is of no value. It does increase the crushing strength, and thus adds

ductility to the column and permits of a somewhat higher unit stress upon the concrete. The hoops also appear practically to affect the shearing stress so that the column acts more like a cube than like a long prism, with consequently higher strength. The Joint Committee conclude:

> The general effect of bands or hoops is to increase greatly the "toughness" of the column and its ultimate strength, but hooping has little effect upon its behavior within the limit of elasticity. It thus renders the concrete a safer and more reliable material, and should permit the use of a somewhat higher working stress. The beneficial effects of "toughening" are adequately provided by a moderate amount of hooping, a larger amount serving mainly to increase the ultimate strength and the possible deformation before ultimate failure.

The loadings suggested for use by the Joint Committee are referred to on page 527.

A type of formula suggested by Considère for determining the ultimate strength of hooped columns is as follows·

Let

f = ultimate unit pressure upon the reinforced column, equal to the total load divided by the effective area in pounds per square inch.

f_c = ultimate unit pressure upon the concrete of the column in pounds per square inch.

p = ratio of sectional area of longitudinal reinforcement to the area of concrete core.

p'' = ratio of volume of steel hooping in a given height of column to the volume of the concrete core in this height.

Then

$$f = 1.5\, f_c + 2\,400\, p + 5100\, p' \tag{64}$$

Professor Talbot suggests the following formulas for ultimate crushing strength:

$$f = f_c + 65\,000\, p'' \tag{65}$$

for columns reinforced with bands, and for those reinforced with spirals

$$f = f_c + 100\,000\, p. \tag{66}$$

The above formulas cannot be safely used, however, for computing the working strength of hooped columns.

The Joint Committee suggest with reference to hooping:

The effective area of the column shall be taken as the area within the protective covering (see page 489); or, in the case of hooped columns or columns reinforced with structural shapes, it shall be taken as the area within the hooping or structural shapes.

The Joint Committee also specify that the hoops or bands should not be counted upon directly as adding to the strength of the column. They suggest:

Where bands or hoops are used, the total amount of such reinforcement shall be not less than 1% of the volume of the column disclosed. The clear spacing of such bands or hoops shall not be greater than one-fourth the diameter of the enclosed column. Adequate means must be provided to hold bands or hoops in place so as to form a column, the core of which shall be straight and well centered.

Hooping then may be considered not as adding to the working strength in proportion to the amount of steel in the hoops, but rather as increasing the toughness of the column and reducing the danger of sudden failure, so that a lower factor of safety is permissible. In practice, to gain the benefit of this, a higher working stress may be permitted in hooped columns when reinforced with steel bands or hoops the total volume of which in a given length of column is at least 1 per cent of the volume of concrete within the hooping.

Adopting the Joint Committee recommendations:

Columns with reinforcement of not less than 1 per cent in bands or hoops may be given a working stress 20 per cent higher than for plain concrete columns. If working stress in plain concrete is taken as 450 pounds per square inch, the hooped concrete may be thus given 540 pounds per square inch.

Columns reinforced with not less than 1 per cent and not more than 4 per cent of longitudinal bars and with not less than 1 per cent in bands or hoops may be given a working stress 45 per cent higher than plain concrete columns. If the working stress in plain columns is taken as 450 pounds per square inch, the hooped and vertically reinforced column may be thus given 650 pounds per square inch plus the working value of the longitudinal rods as indicated on page 492.

STRUCTURAL STEEL REINFORCEMENT

If the structural steel is designed to take all the load and then is simply fireproofed with a concrete covering, it is not reinforced concrete. When the structural steel is designed so that it takes a load in combination with

the concrete it may be termed reinforcement. In this case the steel is figured in the same way as vertical bars and the stresses determined from formula (60), page 491. If, for example, the allowable stress on the concrete is 450 pounds per square inch and a ratio of 15 is used, the steel can be figured only for a compressive stress of 6750 pounds per square inch.

To utilize the full working strength of the steel, the plan has sometimes been followed of separating the structural steel core from the concrete so that they will work independently, and designing the columns in the lower stories so that the steel will take the entire weight of the upper stories while the concrete surrounding the steel supports the weight of the lower stories.

Structural steel reinforcement is sometimes in the form of a cross in the center of the column, or, as in the case of the McGraw building,* channels connected by riveted latticing are placed and concrete poured within the reinforcement as well as providing a protective layer around it. Stresses for this type are suggested on page 528.

Tests† of columns reinforced with structural steel shapes frequently show lower ultimate strength than similar columns reinforced with the same quantity of steel in the form of vertical round bars. This probably is due in part to the difficulty in properly placing the concrete around the structural steel.

COLUMN EXAMPLES.

Example 15: What size of square column reinforced with 2 per cent of longitudinal bars without bands will be required to support a load of 94 000 pounds?

Solution: By paragraph (a), page 527, the allowable compression on 2000 pounds concrete is limited to 450 pounds per square inch For this allowable stress, using 2% of longitudinal reinforcement and a ratio of moduli of elasticity of 15, the area of column from formula (62), page 491, is

$$A = \frac{94\ 000}{450\,(1 + 14 \times 0.02)}$$

= 163 square inches, corresponding to 12.8 inches square. The denominator of this expression may be obtained directly from table on page 492. Allowing 2 inches for protective covering gives 14.8 inches, or, say, 15 inches square.

Example 16: Find the diameter of a round column reinforced by 1 per cent of hooping only, designed to support a load of 120 000 pounds. Assume the allowable pressure on plain concrete as 450 pounds and a ratio of moduli of elasticity, $n = 15$.

Solution: The allowable unit compression on hooped columns may be increased 20 per cent over that on plain concrete (see paragraph (b), page 527),

*William H. Burr, Transactions American Society of Civil Engineers, Vol. LX, 1908, p. 433.
†M. O. Withey in *Engineering Record*, July 10, 1909, p. 41.

hence $f = 450 + 20\% = 540$. The area of section from formula (63) is

$$A = \frac{120\ 000}{540}$$

= 222 square inches, giving an effective diameter of 16.8 inches. Adding 3 inches for protective covering gives a total diameter of 20 inches.

Example 17: What sectional area of vertical steel will be required for a square column limited to 36 inches diameter, which has to bear 1 000 000 pounds with pressure in plain concrete limited to 450 pounds per square inch?

Solution: By paragraph (c), page 528, in a column reinforced with vertical bars and 1% of bands or hoops, the allowable pressure on the concrete may be increased 45% over that on plain concrete, hence $f_c = 450 + 45\% = 652$ pounds per square inch. Considering the area within hooping equal to $33^2 =$ 1090 square inches as effective, the unit pressure from page 491, will be

$$f = \frac{1\ 000\ 000}{1090}$$

= 918 pounds per square inch. Assume $n = 15$, then from formula (61), page 491,

$$p = \frac{918 - 652}{14 \times 652}$$

= 0.029, and area of steel, $A = 1090 \times 0.029 = 31.6$ square inches. From table on page 507, it is found that 18 round rods 1½ inches diameter will give the required area.

Example 18: What should be the area of a column 10 feet high supporting 1 000 000 pounds, reinforced with 3.5 % of longitudinal reinforcement and 1% of hooping for $n = 15$ and an allowable compression in plain concrete limited to 450 pounds?

Solution: Since the column is reinforced with longitudinal and hooping reinforcement, the unit compression on concrete may be taken as $f_c = 450 + 45\% = 652$ pounds per square inch (paragraph c, page 528). Then from formula (62), page 491, the column area is

$$A = \frac{1\ 000\ 000}{652\ (1 + 14 \times 0.035)}$$

= 1030 square inches. The denominator of this expression may be obtained directly from table on page 492.

REINFORCEMENT FOR TEMPERATURE AND SHRINKAGE STRESSES

All masonry is subject to temperature cracks, but when they are distributed in the many joints between bricks or stones they do not show so plainly as on the smooth surface of concrete.

Expansion from a rise in temperature rarely causes trouble except at angles where the lengthening of the surface may produce a buckling or a sliding of one portion of the wall past the end of the other. In a building, the walls and floors are generally so well bonded together and free to move

as a unit, that no provision need be made for expansion. In a structure like a square reservoir, the effect of expansion must be taken into account in the design to prevent failure at the corners.

Contraction is often more serious, although cracks are by no means necessarily dangerous. To prevent cracking due to the shrinkage of the concrete in hardening (see p. 287) or to the lowering of the temperature, reinforcement should be inserted or joints formed to localize the cracks. (See p. 285.)

Reinforcement properly placed distributes the contraction stresses so as to make the cracks very small, practically invisible, but it does not prevent them entirely.

The steel must be sufficient in quantity, and should be of small diameter and placed as close as practicable to the surfaces to distribute the cracks and thus make them very fine. Deformed bars, that is, bars with irregular surfaces which provide a mechanical bond with the concrete, are more effective than smooth bars, and steel of high elastic limit also is advantageous.

In practice, from $\frac{2}{10}$ of 1% to $\frac{4}{10}$ of 1% (a ratio of 0.002 to 0.004) of steel, based on the cross section of the concrete, is commonly used as temperature or shrinkage reinforcement.

The tensile strength of concrete is so low that a small change in temperature will crack it. For example, the coefficient of expansion of concrete is 0.0000055 (see p. 287) and the modulus of elasticity is generally assumed as 2 000 000, therefore, the stress (see p. 404) per degree Fahrenheit is $0.0000055 \times 2\,000\,000 = 11$ pounds per square inch, and a fall in temperature of $\frac{300}{11} = 27°$ is sufficient to crack a concrete the tensile strength of which is 300 pounds per square inch.

It is evident, and it has been proved by experience, that there is less cracking in concrete laid in cold than in warm weather.

Longitudinal reinforcement is especially necessary in conduits which must be water-tight.

Shrinkage cracks due to the hardening of the concrete may be prevented by keeping the concrete wet. (See p. 287.)

It has been suggested by Mr. Charles M. Mills that the relation between the tensile strength of the concrete and the bond with the bars is an important factor in governing the size of the cracks, and the following analysis, based on his suggestions, gives a means of estimating the size and distance apart of the cracks so as to form a basis for judgment as to the sizes and percentages of steel to use.

The tensile stress in the steel at a crack tends to pull out the bars from

the concrete, and referring to Fig. 154, the bond stress of the bar in the length *ab* must equal the tensile stress in the whole cross-section of the concrete at *b* caused by the contraction of the concrete.

Let

x = distance apart of cracks.
D = diameter of round bar or side of square bar.
p = ratio of cross section of steel to cross-section of concrete.

Then,* if, as is sufficiently accurate for practical purposes, the strength of concrete in tension is assumed to be equal to the bond between plain steel bars and concrete, the distance apart of cracks is

$$x = \frac{D}{2p} \text{ for square or round bars}$$

The distance apart is inversely proportional to the unit bond*, so that a deformed bar having twice the bond strength would space the cracks one-half as far apart and allow them to be only one-half as wide

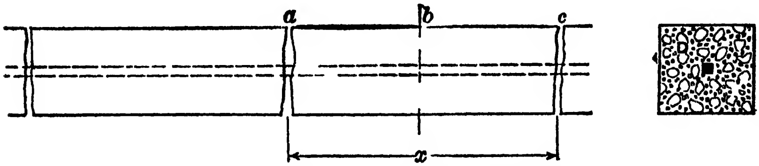


FIG. 154 Reinforcement for Temperature Stresses (*See p* 501)

It is evident that the distance apart of the cracks is proportional to the diameter of the reinforcing bars, and inversely proportional to the percentage of steel.

From this formula is tabulated the estimated percentage of reinforcement for different spacing of cracks and different sizes of bars, assuming the bonding strength of the steel to the concrete to equal the tensile strength of the concrete.

* In addition to above notation, let

A_c = area of section of concrete
A_s = area of section of steel
o = perimeter of steel bar
f'_c = tensile stress in concrete
u = unit bond between plain steel and concrete.
f_s = unit tensile stress in steel
D = diameter of bar

Then $A_c f_c' = \frac{1}{2} uox$, or $x = \frac{2 A f_c}{u o}$. If $f'_c = u$, $x = \frac{2 A_c}{o}$, and since $p = \frac{A_s}{A_c}$

$x = \frac{2 A_s}{o p}$. Also, $\frac{A_s}{o} = \frac{D}{4}$ for both round and square bars, hence $x = \frac{1}{2} \frac{D}{p}$.

Estimated Percentage of Reinforcement for Different Spacing of Cracks

		Distance apart of cracks with					
Plain bars		12″	18″	24″	36″	48″	60″
Deformed bars *		8″	12″	16″	24″	32″	40″
		%	%	%	%	%	%
Diameter of round or side of square bar	1/4″	1.04	0.70	0.52	0.35	0.26	0.21
	3/8″	1.56	1.04	0.78	0.52	0.39	0.31
	1/2″	2.08	1.39	1.04	0.69	0.52	0.41
	5/8″	2.60	1.74	1.30	0.87	0.65	0.52
	3/4″	3.12	2.08	1.56	1.04	0.78	0.62
	7/8″	3.65	2.44	1.82	1.22	0.91	0.73
	1″	4.17	2.78	2.08	1.39	1.04	0.83

NOTE: To express the steel as the ratio of area of cross-section of steel to cross-section of concrete, divide the percentages by 100; thus 1.04 becomes $p = 0.0104$.

* Assuming the bond of deformed bars to be 50% greater than plain.

The size of the crack is governed by the amount of shrinkage and for cracks due to temperature changes may be estimated as the product of the coefficient of contraction (0.0000055) by the number of degrees fall in temperature by the distance between cracks.

Estimated Width of Cracks for Different Distances Apart

	Distance apart					
Width for different temperature changes	12″	18″	24″	36″	48″	60″
30° Fahr.*	0.0020	0.0030	0.0040	0.0059	0.0079	0.0099
50° "	0.0033	0.0050	0.0066	0.0099	0.0132	0.0165
70° "	0.0046	0.0069	0.0092	0.0139	0.0185	0.0232

From this, if it can be determined how large a crack will be allowable, the corresponding spacing can be obtained.

To avoid large cracks it may be necessary to use enough steel to prevent its passing its elastic limit. If the bars are continuous for such a length that the ends are practically immovable, as in a long retaining wall, a drop

* 30° corresponds to a shrinkage of 0.017%; 50° to 0.028%; 70° to 0.038%.

in temperature, tending to shorten them, produces a tensile stress which is independent of the distance between the restrained ends. Assuming the coefficient of expansion of steel the same as concrete and the modulus of elasticity of steel as 30 000 000, this stress is 30 000 000 × 0.0000055 = 165 pounds per square inch per degree of temperature, or for 50° Fahr. is 8250 pounds per square inch. This is well within the elastic limit of the steel and would not, of itself, cause the steel to take a permanent set. However, since the concrete surrounding the steel will be continuous except at certain cracks, the stretch in the steel may be unevenly distributed and largely confined to the immediate vicinity of the cracks. If cracks occur while steel is unstressed, through the concrete shrinking, the steel tends to resist the shrinkage by tension at the crack and compression at the center of the block of concrete, and the tensile stress will be equal to the compressive and each equal to one-half the tensile strength of the concrete. This may be expressed by the following formula, using the foregoing notation:*

$$f'_s = \frac{1}{2p} f'_c$$

Since the tensile stress in the concrete is liable to be low at the time shrinkage cracks are formed, it may be assumed, for illustration, as 200 pounds per square inch making

$$f'_s = \frac{100}{p}$$

This represents the stress due to local cracks which is additional to the temperature stresses above described. The total stress is, therefore, for 50° change of temperature $8250 + f'_s$ or $8250 + \frac{100}{p}$. If the elastic limit of the steel is 40 000 pounds per square inch, and we must keep below this,

$$40\,000 = 8250 + \frac{100}{p} \text{ and } p = 0.0031$$

For steel, the elastic limit of which is 50 000 pounds per square inch,

$$50\,000 = 8250 + \frac{100}{p} \text{ and } p = 0.0024$$

These values of p represent the lowest theoretical ratio of area of cross section of steel to area of cross-section of concrete which can be used without the steel passing its elastic limit at certain of the cracks when the ends are restrained or the length is so great that intermediate parts are practically restrained.

* $\frac{A_c f'_c}{2} = A_s f'_s$ or $f'_s = \frac{A_c}{2A_s} f'_c$ hence $f'_s = \frac{1}{2p} f'_c$

In view of the very slight stretch required to relieve the stress in the bars when the elastic limit is exceeded, and the probability of its distribution by the restraint to movement by the mass, it is not always essential to consider the elastic limit.

SYSTEMS OF REINFORCEMENT

One of the earliest recorded examples of the application of reinforced concrete is a boat of concrete and iron, built by Mr. L. J. Lambot in France, and shown at the Paris International Exhibition in 1855.* In 1861 Mr. Coignet began his investigations, and in 1866 Mr. Monier, to whom the invention of reinforced concrete is often attributed, applied the combination of concrete and iron to various structures, and laid the foundation for its future widespread applications.

As long ago as 1872, Mr. W. E. Ward,† at Port Chester, N. Y., built a house entirely of concrete, reinforced with iron I-beams and round rods.

The rapid development of reinforced concrete has resulted in the introduction of numerous systems, many of them covered by patents, for arranging the metal in the concrete, or for special forms of metal. These systems are fully described in the various French works on reinforced concrete.‡

A few of the systems, representing both the arrangement and the form of the metal, are described below, and forms of metal extensively used in the United States are illustrated in Fig. 155.

Systems of Reinforcement

Bonna. Metal of cruciform cross-section.

Bertini. Girder Frame. Horizontal tension members with vertical stirrups shrunk on to them.

Chaudy and Degon. Cross rods passing under bearing rods, but looped up between them.

Coignet. Round bars in top and bottom of beam connected by diagonal wire lacing.

Columbian. Vertical steel plates with horizontal ribs.

Cottacin. Round rods interlaced in the same manner as in wire netting.

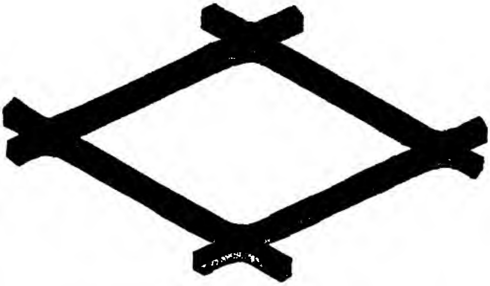
Cummings. Bars of different lengths having their ends bent to an incline and formed into a loop to resist internal stresses.

Cup Bar. Special rolled section with longitudinal ribs connected at frequent intervals by cross ribs forming cup depressions.

* Christophe's Beton Armé, 1902, p. 1.

† Transactions American Society Mechanical Engineers, Vol. IV, p. 388.

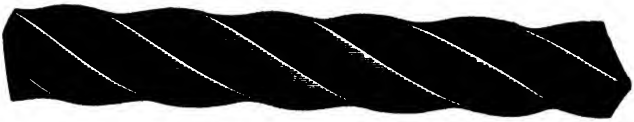
‡ See among others Christophe's Beton Armé, 1902, pp. 10–71, and Morel's Ciment Armé, 1902, pp. 88 to 152.



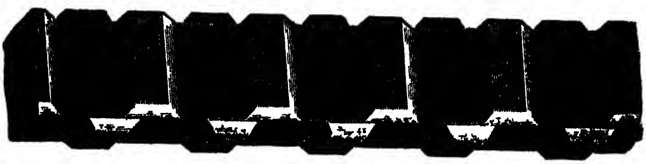
Expanded Metal.

Kahn Trussed Bar.

Thacher Bulb Bar.



Ransome Twisted Bar.



Johnson Corrugated Bar.

De Man Undulated Bar.

FIG. 155.—Types of Reinforcing Steel. (*See pp.* 504 *and* 506.)

De Man. Undulated Bars. (See Fig. 155.)

Diamond Bar. Bars rolled round with parallel ribs passing along and around the bar forming diamond-shaped shoulders on its surface.

Donath. Inverted T-beams or I-beams connected by horizontal diagonals of light, flat metal on edge.

Expanded Metal. Sheet steel, slit and expanded, so as to form a diamond mesh. (See Fig. 155, p. 505.)

Ferroinclave. Sheet steel with inversely tapered corrugations to be covered on both sides with concrete.

Gabriel. Deformed tension members with trussing of hard drawn wire.

Habrich and Düsing. Flat metal twisted hot.

Hennebique. A combination of alternate straight bars and bars with ends bent up at an angle, with vertical U-bars, or stirrups, of flat iron passing around the straight bars and reaching nearly to the top of the beam.

Herringbone Frame. Horizontal tension member with special attachments for stirrups.

Holzer. Metal in form of I-beams.

Hyatt. Flat plates or bars set on edge and pierced with holes through which pass small round rods to form the cross reinforcements.

Johnson. Corrugated bars. (See Fig. 155, p. 505.)

Kahn. Horizontal flanged bars with flanges sheared up at intervals. (See Fig. 155, p. 505.)

Lock-Woven Steel Fabric. Steel wire mesh, locked at intersections.

Lug Bars. Twisted bars with projecting lugs at intervals in the surface.

Melan. Steel ribs, either I-beam or 4 angles latticed, imbedded in the concrete of the arch.

Monier. Two series of round parallel bars at right angles to each other.

Mushroom. Flat floor slabs supported by columns with enlarged heads.

Parmley. Bars with bent ends, to place in the sides of a conduit or the haunches of an arch to resist tension.

Rabitz. Various combinations employing galvanized wire.

Ransome. Square steel rods twisted cold. (See Fig. 155, p. 505.)

Roebling. Flat steel bars set on edge, clamped to supporting beams, and held in alignment by flat bar separators.

Schülter. Like Monier System except rods are placed diagonally.

Scofield. An oval bar with projecting shoulders.

Thacher. Bulb bars. (See Fig. 155, p. 505.)

Triangle Mesh. Wire mesh reinforcement with transverse metal placed diagonally.

Trussit. Expanded metal or herringbone lath bent to V-shaped section.

Visintini. Beams of concrete, cored out so as to form lattice girders.

Welded Wire Fabric. Wire mesh reinforcement with wires at right angles to each other and welded at intersections.

TABLE 1. AREAS, WEIGHTS AND CIRCUMFERENCES OF BARS.

Areas and Weights of Square and Round Rods and Circumferences of Round Rods.

One cubic foot weighs 490 lb.

Thickness or Diameter in inches.	Area of Square Rod in square inches.	Area of Round Rod in square inches.	Circumference of Round Rod in inches.	Weight of Square Rod One Foot Long.	Weight of Round Rod One Foot Long.	Thickness or Diameter in inches.	Area of Square Rod in square inches.	Area of Round Rod in square inches.	Circumference of Round Rod in inches.	Weight of Square Rod One Foot Long.	Weight of Round Rod One Foot Long.
0						**2**	4.0000	3.1416	6.2832	13.60	10.68
1/16	0.0039	0.0031	0.1963	0.013	0.010	1/16	4.2539	3.3410	6.4795	14.46	11.36
1/8	0.0156	0.0123	0.3927	0.053	0.042	1/8	4.5156	3.5466	6.6759	15.35	12.06
3/16	0.0352	0.0276	0.5890	0.119	0.094	3/16	4.7852	3.7583	6.8722	16.27	12.78
1/4	0.0625	0.0491	0.7854	0.212	0.167	1/4	5.0625	3.9761	7.0686	17.22	13.52
5/16	0.0977	0.0767	0.9817	0.333	0.261	5/16	5.3477	4.2000	7.2649	18.19	14.28
3/8	0.1406	0.1104	1.1781	0.478	0.375	3/8	5.6406	4.4301	7.4613	19.18	15.07
7/16	0.1914	0.1503	1.3744	0.651	0.511	7/16	5.9414	4.6664	7.6576	20.20	15.86
1/2	0.2500	0.1963	1.5708	0.850	0.667	1/2	6.2500	4.9087	7.8540	21.25	16.69
9/16	0.3164	0.2485	1.7671	1.076	0.845	9/16	6.5664	5.1572	8.0503	22.33	17.53
5/8	0.3906	0.3068	1.9635	1.328	1.043	5/8	6.8906	5.4119	8.2467	23.43	18.40
11/16	0.4727	0.3712	2.1598	1.608	1.262	11/16	7.2227	5.6727	8.4430	24.56	19.29
3/4	0.5625	0.4418	2.3562	1.913	1.502	3/4	7.5625	5.9396	8.6394	25.00	20.20
13/16	0.6602	0.5185	2.5525	2.245	1.763	13/16	7.9102	6.2126	8.8357	26.90	21.12
7/8	0.7656	0.6013	2.7489	2.603	2.044	7/8	8.2656	6.4918	9.0321	28.10	22.07
15/16	0.8789	0.6903	2.9452	2.989	2.347	15/16	8.6289	6.7771	9.2284	29.34	23.04
1	1.0000	0.7854	3.1416	3.400	2.670	**3**	9.0000	7.0686	9.4248	30.60	24.03
1/16	1.1289	0.8866	3.3379	3.838	3.014	1/16	9.3789	7.3662	9.6211	31.89	25.04
1/8	1.2656	0.9940	3.5343	4.303	3.379	1/8	9.7656	7.6699	9.8175	33.20	26.08
3/16	1.4102	1.1075	3.7306	4.795	3.766	3/16	10.160	7.9798	10.014	34.55	27.13
1/4	1.5625	1.2272	3.9270	5.312	4.173	1/4	10.563	8.2958	10.210	35.92	28.20
5/16	1.7227	1.3530	4.1233	5.857	4.600	5/16	10.973	8.6179	10.407	37.31	29.30
3/8	1.8906	1.4849	4.3197	6.428	5.049	3/8	11.391	8.9462	10.603	38.73	30.42
7/16	2.0664	1.6230	4.5160	7.026	5.518	7/16	11.816	9.2806	10.799	40.18	31.56
1/2	2.2500	1.7671	4.7124	7.650	6.008	1/2	12.250	9.6211	10.996	41.65	32.71
9/16	2.4414	1.9175	4.9087	8.301	6.520	9/16	12.691	9.9678	11.192	43.14	33.90
5/8	2.6406	2.0739	5.1051	8.978	7.051	5/8	13.141	10.321	11.388	44.68	35.09
11/16	2.8477	2.2365	5.3014	9.682	7.604	11/16	13.598	10.680	11.585	46.24	36.31
3/4	3.0625	2.4053	5.4978	10.41	8.178	3/4	14.063	11.045	11.781	47.82	37.56
13/16	3.2852	2.5802	5.6941	11.17	8.773	13/16	14.535	11.416	11.977	49.42	38.81
7/8	3.5156	2.7612	5.8905	11.95	9.388	7/8	15.016	11.793	12.174	51.05	40.10
15/16	3.7539	2.9483	6.0868	12.76	10.02	15/16	15.504	12.177	12.370	52.71	41.40

BEAM AND SLAB TABLES

Beam Tables. Tables 2, 3, and 4, pages 509, 510 and 511, give the loading and reinforcement for beams, based on 1 inch of width under different conditions. For a beam 10 inches wide, for example, both the safe load per linear foot and the steel area will be ten times the values given in the tables.

The tables are for rectangular beams but may be used for T-beams which have a depth 3 or 4 times the thickness of slab by taking the width of flange as the breadth, b

Table 2 is for a simply supported beam and is based on a working compressive stress in concrete of 500 pounds per square inch and in steel of 14 000 pounds per square inch—lower values than are customarily used in construction, but required in many building laws If the compression in concrete is limited to 500 pounds, while 16 000 pounds is permitted in the steel, use the same loading but reduce the steel in the ratio of 16 to 14

Tables 3 and 4 are for ordinary design, approved by the authors and corresponding to recommendations of the Joint Committee All tables are based on a ratio of elasticity of $n = 15$ (See p 408)

For other working stresses than those given, the loads may be multiplied by ratios of the values of the constant C in Table 10, page 519, since C is proportional to the load

The uses of the tables are illustrated in Examples 12 and 13 As high steel is not recommended for ordinary work on a small scale, no table is presented for safe loads for concrete reinforced with it

Slab Table. Table 5 is for slab design with different working stresses in the steel and concrete Ordinarily, the series at the top of the second page of the table is used Note that the values are based on $\frac{wl^2}{10}$ For $\frac{wl^2}{12}$, generally used where the slabs are fully continuous over the supports, add 20% to the loads, leaving the area of steel as given For square slabs fully reinforced in both directions, the loads may be doubled, or if also fully continuous, they may be doubled and 20% added also

Table 6 is more convenient for review of beams already designed It is computed by using formulas (7) and (8) on page 753, and selecting the lower value of M. The most economical ratio of steel for the limiting stresses is $p = 0.0077$. For ratios lower than this the safe loads on the slabs are governed by the tensile strength of the steel, while for larger ratios they are limited by the working strength of the concrete in compression.

TABLE 2. USE FOR SIMPLY SUPPORTED BEAMS FOR EXTRA CONSERVATIVE DESIGN

Safe Loading and Reinforcement for Rectangular Beams One Inch in Width. 1 : 2 : 4 Concrete. Mild Steel.

Based on $M = \frac{wl^2}{8}$ $n = 15$. $f_c = 500$ $f_s = 14000$ (*See p. 508 and item 8, p. 519*)

For $M = \frac{wl^2}{10}$ add 25% to the safe loads using same steel area. For $M = \frac{wl^2}{12}$ add 50% to the safe loads using same steel area.

Depth of Beam (h) in.	Total Safe Load (w) per Linear Foot for Beam One Inch Wide including Weight of Beam. *For safe live load deduct weight of beam in column (22)* (See important footnotes) Span in Feet (l): 5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	Weight of Beam One Inch Wide per Linear Foot lb (22)	Depth to Steel (d) in (23)	Depth below Steel (e) in (24)	Steel Area in a Beam One Inch Wide * sq in (25)	Safe Moment of Resistance (See p 753) (M) in lb. (26)
5	33	23	17	13	10	8														5.4	4.0	1.0	0.024	1230
6	51	36	26	20	16	13	10	9												6.4	5.0	1.0	0.030	1925
7	74	51	38	29	23	18	15	13	11	9										7.5	6.0	1.0	0.037	2770
8	101	69	51	39	31	25	21	17	15	13	11									8.6	7.0	1.0	0.043	3770
9	123	86	63	48	38	31	25	21	18	16	14	10								9.6	7.75	1.25	0.047	4620
10	157	109	80	61	48	39	32	27	23	20	1	15								10.7	8.75	1.25	0.053	5890
11	195	135	99	76	60	49	40	34	29	25	22	19	17							11.8	9.75	1.25	0.059	7310
12	237	165	121	93	73	59	49	41	35	30	26	23	20							12.8	10.75	1.5	0.066	8900
13	274	188	138	106	84	68	56	47	40	35	30	26	23	21						13.9	11.50	1.5	0.070	10100
14	321	223	164	125	99	80	66	56	47	41	36	31	28	25						15.0	12.50	1.5	0.076	12030
15	374	260	191	146	115	93	77	65	55	48	41	36	32	29	26					16.0	13.50	1.5	0.082	14030
16	431	300	220	168	132	105	89	75	64	55	48	42	37	33	30					17.1	14.50	1.5	0.088	16170
17	493	342	251	193	152	123	102	86	73	63	55	48	43	38	34					18.1	15.50	1.5	0.094	18500
18	557	388	265	218	172	139	115	97	8	71	62	54	45	43	39					19.2	16.50	1.5	0.101	20960
19	593	411	303	232	183	148	122	103	86	77	66	58	51	46	41	37				20.3	17.00	2.0	0.104	22250
20		462	339	260	205	166	137	115	98	85	74	65	57	51	46	41				21.4	18.00	2.0	0.110	24940
22			419	321	253	205	169	142	121	104	91	80	71	63	57	51	33			23.6	20.00	2.0	0.122	30800
24			506	388	306	248	205	172	147	127	111	97	86	77	69	62	40			25.7	22.00	2.0	0.134	37260
26				462	365	295	244	205	175	151	131	115	102	91	82	74	47	33		27.9	24.00	2.0	0.146	44350
28				532	428	346	287	241	205	177	154	135	120	107	96	87	55	38		30.0	26.00	2.0	0.159	52020
30					496	402	332	279	238	205	179	157	139	124	111	100	64	45		32.1	28.00	2.0	0.171	60360
36						576	476	400	340	293	256	225	199	178	159	144	92	64	46	38.5	33.50	2.5	0.210	86400
42						800	661	556	473	408	355	312	277	247	222	200	128	89	64	44.9	39.50	2.5	0.240	120100
48							876	736	627	540	471	414	367	327	294	265	169	118	65	51.4	45.50	2.5	0.278	159400

RULES. 1. For safe load of any width of beam multiply by width in inches.
2. For area of cross-section of steel for any width of beam multiply column (25) by width in inches.
3. Total loads for other spans (l) and the same depth of steel are inversely proportional to the squares of the spans.
4. Total loads for other depths of steel (d) and the same span are proportional to the squares of the depths of steel.
5. The values in this table may apply to a very carefully graded 1 : 2½ : 5 mixture.

* This is for a ratio of steel $p = 0.0062$ (0.62 per cent) which is required for the given working stresses.

TABLE 3. USE ONLY FOR CONTINUOUS BEAMS

Safe Loading and Reinforcement for Rectangular Beams One Inch in Width. 1 : 2 : 4 Concrete. Mild Steel.

Based on $M = \frac{wl^2}{12}$ $n = 15$. $f_c = 650$. $f_s = 16000$. (*See p. 508 and item 18, p. 519.*)

Depth of Beam. (h) in.	Total Safe Load (w) per Linear Foot for Beam One Inch Wide including Weight of Beam. *For safe live load deduct weight of beam in column (22)* (See important foot-notes.) Span in Feet (l). 5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	Weight of Beam One Inch Wide per Linear Foot. lb. (22)	Depth to Steel. (d) in. (23)	Depth Below Steel. (c) in. (24)	Steel Area in a Beam One Inch Wide.* sq. in. (25)	Safe Moment of Resistance. (See p. 753.) (M) in.-lb. (26)
5	70	48	36	28	22	17	14	12												5.4	4.0	1.0	0.031	1740
6	109	76	55	43	34	28	23	19	16											6.4	5.0	1.0	0.038	2720
7	155	109	80	61	48	40	32	28	23	20										7.5	6.0	1.0	0.046	3910
8	214	149	109	84	66	53	44	37	31	28	24									8.6	7.0	1.0	0.054	5320
9	262	182	133	102	80	66	54	46	38	34	29	25								9.6	7.75	1.25	0.060	6520
10	334	232	170	131	103	84	70	58	49	43	37	32	29							10.7	8.75	1.25	0.067	8310
11	415	288	211	162	128	103	85	72	61	53	46	41	36	32						11.8	9.75	1.25	0.075	10320
12	504	350	257	197	156	126	104	88	74	65	56	49	43	38	35					12.8	10.75	1.25	0.083	12540
13	577	401	294	226	178	144	119	101	85	73	64	56	50	44	40					13.9	11.50	1.5	0.089	14350
14	682	473	348	266	210	170	140	119	101	86	76	67	59	53	47	43				15.0	12.5	1.5	0.096	16950
15	796	552	406	311	245	199	164	138	118	101	89	78	68	61	55	49				16.0	13.5	1.5	0.104	19780
16	917	637	468	359	283	229	190	160	136	116	102	90	79	71	64	58				17.1	14.5	1.5	0.112	22820
17	1049	728	535	409	324	262	216	182	155	133	116	102	91	80	72	66	42			18.1	15.5	1.5	0.119	26070
18	1188	826	606	464	367	296	245	206	175	151	132	116	103	91	83	74	48			19.2	16.5	1.5	0.127	29540
19	1261	876	643	492	389	316	260	218	186	161	140	124	110	100	91	81	50			20.3	17.0	2.0	0.131	31360
20	1414	982	721	552	437	353	292	245	209	180	157	138	122	109	98	89	56			21.4	18.0	2.0	0.139	35160
22		1212	890	682	539	436	360	302	258	222	194	170	151	134	121	109	70	48		23.6	20.0	2.0	0.154	43400
24		1466	1078	824	652	528	436	367	312	269	235	206	182	163	146	132	84	59		25.7	22.0	2.0	0.169	52520
26			1282	982	775	628	518	437	372	320	280	245	217	194	174	157	101	70		27.9	24.0	2.0	0.185	62500
28				1152	910	737	610	512	442	376	328	288	256	228	204	185	118	82	60	30.0	26.0	2.0	0.200	73360
30				1336	1055	854	707	594	505	436	380	334	296	264	238	214	137	95	70	32.1	28.0	2.0	0.216	85080
36				1913	1511	1224	1012	850	725	624	544	478	424	378	340	306	196	136	100	38.5	33.5	2.5	0.258	121800
42				2660	2101	1702	1406	1182	1007	868	756	665	589	526	472	426	272	190	139	44.9	39.5	2.5	0.304	169300
48						2261	1869	1570	1338	1153	1005	883	782	697	626	565	361	251	185	51.4	45.5	2.5	0.350	224600

RULES.

1. **For safe load of any width of beam** multiply by width in inches.
2. **For area of cross-section of steel** for any width of beam multiply column (25) by width in inches.
3. **Total loads for other spans** (*l*) and the same depth of steel are inversely proportional to the squares of the spans.
4. **Total loads for other depths of steel** (*d*) and the same span are proportional to the squares of the depths of steel.
5. The values in this table may apply to a very carefully graded 1 : 2½ : 5 mixture.

* This is for a ratio of steel $p = 0.0077$ (0.77 per cent) which is required for the given working stresses.

TABLE 4. USE FOR END SPANS OF CONTINUOUS BEAMS. FOR SUPPORTED BEAMS DEDUCT 20 PER CENT.

Safe Loading and Reinforcement for Rectangular Beams One Inch in Width. 1 : 2 : 4 Concrete. Mild Steel.

Based on $M = \frac{wl^2}{10}$, $n = 15$, $f_c = 650$, $f_s = 16\ 000$ (*See p. 508 and item 18, p. 519*).

For $M = \frac{wl^2}{8}$ deduct 20 per cent from safe loads, using same steel area.

Depth of Beam. (h) in.	Total Safe Load (w) per Linear Foot for Beam One Inch Wide including Weight of Beam. *For safe live load deduct weight of beam in column (22)* (See important foot-notes.) Span in Feet (l). 5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	Weight of Beam One Inch Wide per Linear Foot lb.	Depth to Steel. (d) in.	Depth Below Steel. (e) in.	Steel Area in a Beam One Inch Wide.* sq. in.	Safe Moment of Resistance. (See p. 753.) (M) in.-lb.
																				(22)	(23)	(24)	(25)	(26)
5	58	40	30	23	18	14	12	10												5.4	4.0	1.0	0.031	1740
6	91	63	46	36	28	23	19	16	13											6.4	5.0	1.0	0.038	2720
7	129	91	67	51	40	33	27	23	19	17										7.5	6.0	1.0	0.046	3910
8	178	124	91	70	55	44	37	31	26	23	20									8.6	7.0	1.0	0.054	5320
9	218	152	111	85	67	55	45	38	32	28	24	21								9.6	7.75	1.25	0.060	6520
10	278	193	142	109	86	70	58	45	41	36	31	27	24							10.7	8.75	1.25	0.067	8310
11	346	240	176	135	107	86	71	60	51	44	38	34	30	27						11.8	9.75	1.25	0.075	10320
12	420	292	214	164	130	105	87	73	62	54	47	41	36	32	29					12.8	10.75	1.25	0.083	12540
13	481	334	245	188	145	120	99	84	71	61	53	47	42	37	33					13.9	11.50	1.5	0.089	14350
14	548	394	290	222	175	142	117	99	84	72	63	56	49	44	39	36				15.0	12.5	1.5	0.096	16950
15	663	460	338	259	201	166	137	115	98	84	74	65	57	51	46	41				16.0	13.5	1.5	0.104	19780
16	764	531	390	299	236	191	155	133	113	97	85	75	65	59	53	48				17.1	14.5	1.5	0.112	22820
17	874	607	446	341	270	218	180	152	129	111	97	85	76	67	60	55	35			18.1	15.5	1.5	0.119	26070
18	990	688	505	357	306	247	204	172	140	125	110	97	86	76	69	62	40			19.2	16.5	1.5	0.127	29540
19	1051	730	536	410	324	263	217	182	155	134	117	103	91	83	75	66	42			20.3	17.0	2.0	0.131	31360
20	1178	818	601	460	364	294	243	204	174	150	131	115	102	91	82	74	47			21.4	18.0	2.0	0.139	35160
22		1010	742	568	449	363	300	252	215	185	162	142	126	112	101	91	58	40		23.6	20.0	2.0	0.154	43400
24		1222	898	687	543	440	303	300	260	224	190	172	152	136	122	110	70	49		25.7	22.0	2.0	0.169	52520
26			1068	818	646	523	432	364	310	267	233	204	181	162	145	131	84	58		27.9	24.0	2.0	0.185	62500
28				900	758	614	508	427	363	313	273	240	213	190	170	154	98	68	50	30.0	26.0	2.0	0.200	73360
30				1113	879	712	589	495	421	363	317	278	247	220	198	178	114	79	58	32.1	28.0	2.0	0.216	85080
36				1594	1259	1020	843	708	604	520	453	308	353	315	283	255	163	113	83	38.5	33.5	2.5	0.258	121800
42				2216	1751	1418	1172	985	839	723	630	554	491	438	393	355	227	158	116	44.9	39.5	2.5	0.304	169300
48						1884	1557	1308	1115	961	837	730	652	581	522	471	301	209	154	51.4	45.5	2.5	0.350	224000

RULES. 1. **For safe load of any width of beam** multiply by width in inches.
2. **For area of cross-section of steel** for any width of beam multiply column (25) by width in inches.
3. **Total loads of spans** (l) and same depth of steel are inversely proportional to the squares of the spans.
4. **Total loads for other depths of steel** (d) and same span are proportional to the squares of the depth of steel.
5. The values in this table may apply to a very carefully graded 1 : 2½ : 5 mixture.

* This is for a ratio of steel $p = 0.0077$ (0.77 per cent) which is required for the given working stresses.

TABLE 5. USE FOR DESIGNING SLABS, IF FULLY CONTINUOUS, ADD 20% TO LOADS

Safe Loadings per Square Foot and Reinforcement for Slabs for Various Working Stresses in Steel (f_s) and Concrete (f_c) (See pp 508 and 420)

Based on $M = \frac{wl^2}{10}$ For supported ends $\left(M = \frac{wl^2}{8}\right)$, deduct 20% from loads. For fully continuous $\left(M = \frac{wl^2}{12}\right)$ add 20%. For square slabs multiply by 2. Use same steel area always.

	Total depth of slab (h)	4	5	6	7	8	9	10	11	12	13	14	15	Weight of slab per square foot lb (14)	Depth to steel (d) in (15)	Depth below steel (e) in (16)	Steel area in section of slab one foot wide* sq in (17)	Safe Moment of resistance M in lb (18)
		Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab in column (14). (See important foot notes on opposite page.) Span in feet (l)															See p 755	
$f_s = 14000$, $f_c = 500$, $n = 15$, $p = 0.0062$	2½	147	94	66										3[illegible]	1¼	½	0.130	2830
	3	243	156	108	79									3[illegible]	2¼	½	0.167	4670
	3½	360	231	161	118	90								45	2½	½	0.205	6930
	4	507	32[illegible]	2[illegible]	16[illegible]	127	100	81						51	3¼	¾	0.242	9750
	4½	58[illegible]	378	26[illegible]	192	147	11[illegible]	94						55	3½	1	0.260	11320
	5	768	4[illegible]1	343	251	1[illegible]2	15[illegible]	1[illegible]3	10[illegible]					6[illegible]	4	1	0.298	14780
	6	1[illegible]01	[illegible]	536	393	30[illegible]	[illegible]	[illegible]	159	134	113			77	5	1	0.372	23100
	7	172[illegible]	1111	771	56[illegible]	432	342	27[illegible]	[illegible]	1[illegible]3	163	143	123	90	[illegible]	1	0.446	33260
	8	2353	1512	1050	77[illegible]	588	171[illegible]	3[illegible]	312	62[illegible]	2[illegible]	1[illegible]5	167	103	7	1	0.521	45270
	9	3081	1971	1369	100[illegible]	70[illegible]	[illegible]	493	407	31[illegible]	292	251	219	116	8	1	0.595	59140
	10	3898	2495	1733	1273	[illegible]5	7[illegible]0	624	515	4[illegible]	369	315	2[illegible]7	1[illegible]8	9	1	0.670	74840
$f_s = 14000$, $f_c = 600$, $n = 15$, $p = 0.0084$	2½	195	125	87	61									3[illegible]	1¼	½	0.176	3760
	3	322	207	143	105	8[illegible]								3[illegible]	[illegible]	½	0.227	61[illegible]0
	3½	480	308	214	157	120	[illegible]5							4[illegible]	[illegible]	½	0.277	9230
	4	675	434	301	221	1[illegible]	1[illegible]4	108						[illegible]	[illegible]	¾	0.[illegible]28	1[illegible]990
	4½	784	503	349	25[illegible]	1[illegible]	155	1[illegible]	104					58	3½	[illegible]	0.353	150[illegible]0
	5	10[illegible]3	65[illegible]	457	334	5[illegible]	[illegible]3	1[illegible]3	13[illegible]	114	[illegible]6			64	4	[illegible]	0.403	19680
	6	15[illegible]9	10[illegible]7	713	52[illegible]	400	317	[illegible]5		17[illegible]	150	13[illegible]	114	[illegible]	[illegible]	1	0.504	307[illegible]0
	7	[illegible]	117[illegible]	10[illegible]7	[illegible]5[illegible]	5[illegible]5	45[illegible]	3[illegible]7	305	[illegible]7	217	1[illegible]0		[illegible]0	[illegible]	1	0.60[illegible]	44[illegible]80
	8	3[illegible]34	2013	139[illegible]	10[illegible]	[illegible]3	621	500	417	34[illegible]	[illegible]5	59	2[illegible]	103	7	1	0.[illegible]06	60[illegible]70
	9	4101	[illegible]625	1823	1339	1025	[illegible]0	[illegible]	[illegible]4	45[illegible]	3[illegible]	33	2[illegible]	[illegible]	8	1	0.806	78720
	10	51[illegible]1	33[illegible]1	2307	1694	[illegible]5	10[illegible]5	830	6[illegible]	577	[illegible]91	4[illegible]	3[illegible]	1[illegible]	9	1	0.907	99630
$f_s = 14000$, $f_c = 700$, $n = 15$, $p = 0.0107$	2½	216	155	110	80									32	[illegible]	½	0.2[illegible]5	4730
	3	407	2[illegible]	181	133	10[illegible]	81							38	[illegible]	½	0.2[illegible]9	7830
	3½	6[illegible]4	[illegible]5	26[illegible]	1[illegible]7	1[illegible]	1[illegible]0	9[illegible]						4[illegible]	2[illegible]	½	0.353	11620
	4	850	[illegible]4[illegible]	[illegible]79	278	2[illegible]	168	13[illegible]	113	95				[illegible]	3¼	¾	0.417	16350
	4½	98[illegible]	633	410	[illegible]	2[illegible]	1[illegible]5	157	131	11[illegible]	93			55	3½	1	0.44[illegible]	1[illegible]60
	5	1[illegible]3	[illegible]	574	[illegible]	3[illegible]	255	205	171	143	121	106		64	4	1	0.514	24770
	6	[illegible]01	[illegible]	[illegible]8	[illegible]5[illegible]	503	3[illegible]	2[illegible]1	2[illegible]7	224	18[illegible]	1[illegible]	1[illegible]	77	5	1	0.642	38700
	7	[illegible]	1[illegible]	1[illegible]3	[illegible]47	[illegible]4	574	[illegible]2	38[illegible]	3[illegible]	273	23[illegible]	[illegible]0	90	[illegible]	1	0.770	55730
	8	394	5[illegible]	1[illegible]	12[illegible]	[illegible]	7[illegible]1	6[illegible]	5[illegible]	44[illegible]	37[illegible]	3[illegible]	[illegible]	[illegible]03	7	1	0.89[illegible]	75860
	9	5161	3[illegible]0[illegible]	2[illegible]94	1685	1[illegible]90	1019	926	[illegible]8	573	48[illegible]	4[illegible]	3[illegible]	1[illegible]	8	1	1.027	99080
	10	[illegible]53[illegible]	41[illegible]1	[illegible]904	133[illegible]	1[illegible]33	1290	1045	8[illegible]4	7[illegible]	[illegible]	[illegible]5	[illegible]	[illegible]	9	1	1.15[illegible]	125400
$f_s = 16000$, $f_c = 500$, $n = 15$, $p = 0.0050$	2½	137	8[illegible]											32	1[illegible]	½	0.105	2620
	3	220	145	101	[illegible]1									38	2¼	½	0.135	4340
	3½	33[illegible]	215	14[illegible]	10[illegible]	8[illegible]	66							45	[illegible]	½	0.1[illegible]5	6440
	4	[illegible]7	303	210	1[illegible]1	1[illegible]	[illegible]3	75						5[illegible]	[illegible]	¾	0.1[illegible]5	9060
	4½	5[illegible]	3[illegible]1	214	[illegible]9	[illegible]3	108	87	72	61				58	3½	1	0.210	10510
	5	[illegible]13	458	31[illegible]	233	175	141	114	95	[illegible]0				64	4	1	0.240	13720
	6	1115	[illegible]1[illegible]	498	361	7[illegible]	2[illegible]1	178	148	124	105	9[illegible]		77	5	1	0.300	21450
	7	1[illegible]0[illegible]	103[illegible]	716	5[illegible]	401	315	2[illegible]6	213	1[illegible]	151	133	114	90	6	1	0.360	30880
	8	2156	1404	975	[illegible]15	[illegible]4[illegible]	433	349	290	24[illegible]	206	1[illegible]1	155	103	7	1	0.420	42040
	9	2860	1831	12[illegible]1	931	715	565	457	378	318	271	233	203	116	8	1	0.480	54910
	10	3619	231[illegible]	160[illegible]	11[illegible]	905	715	579	479	402	343	295	257	129	9	1	0.540	69500
$f_s = 16000$, $f_c = 600$, $n = 15$, $p = 0.0067$	2½	183	117	82										32	1¼	½	0.141	3520
	3	303	195	135	99									38	2¼	½	0.181	5830
	3½	450	289	201	14[illegible]	112	89	72						45	2½	½	0.221	8650
	4	632	406	282	20[illegible]	158	125	101	84	70				51	3¼	¾	0.261	12160
	4½	734	471	327	240	183	145	117	97	82				58	3½	1	0.281	14110
	5	958	615	427	313	239	190	153	127	107	90	79		64	4	1	0.322	18430
	6	1497	962	668	489	374	297	239	199	167	141	124	106	77	5	1	0.402	[illegible]
	7	2156	1385	962	705	539	427	344	286	240	203	178	153	90	6	1	0.48[illegible]	[illegible]
	8	2935	1885	1309	959	734	581	468	389	327	277	245	[illegible]	103	7	1	0.563	[illegible]
	9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

	Total depth of slab (h) in	Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab, column (14) (See important footnotes) — Span in feet (l): 4	5	6	7	8	9	10	11	12	13	14	15	Weight of slab per square foot lb	Depth to steel (d) in	Depth below steel (e) in	Steel area in a section of slab one foot wide sq in	Safe moment of resistance. See p. 753 (M) in lb
$n=15$, $p=0.0077$, $f_s=16000$, $f_c=650$ (USE THESE ORDINARILY)	2½	206	132	92	67	52								32	1¾	¾	0.162	3052
	3	340	218	151	111	85	61	54						38	2¼	¾	0.208	6536
	3½	509	326	226	166	127	101	81	67					45	2¾	¾	0.254	9770
	4	711	455	316	232	178	140	114	94	79	67			51	3¼	¾	0.300	13650
	4½	824	528	366	269	206	163	132	109	92	78	67		58	3½	1	0.323	15830
	5	1077	690	[illegible]	352	269	213	172	142	120	102	88	77	64	4	1	0.370	20680
	6	1683	1077	748	550	421	332	269	223	187	159	137	120	77	5	1	0.462	32310
	7	2423	1551	1077	791	606	479	388	320	269	229	198	172	90	6	1	0.554	46520
	8	3297	2111	1466	1077	824	651	528	436	366	312	269	234	103	7	1	0.647	63320
	9	4308	2758	1915	1407	1077	851	689	570	479	408	352	306	116	8	1	0.739	82720
	10	5454	3491	2424	1781	1366	1077	873	721	606	516	445	388	128	9	1	0.832	104700
$n=15$, $p=0.0087$, $f_s=16000$, $f_c=700$	2½	231	148	103	[illegible]									32	1¾	¾	0.153	4140
	3	382	245	170	125	[illegible]	[illegible]	[illegible]						[illegible]	[illegible]	[illegible]	0.235	7340
	3½	567	364	253	185	[illegible]	112	[illegible]	75	63				45	2¾	¾	0.287	10900
	4	797	[illegible]	356	261	[illegible]	158	127	[illegible]	[illegible]	75			51	3¼	[illegible]	0.339	15330
	4½	[illegible]	[illegible]	413	302	231	[illegible]	148	123	[illegible]	[illegible]			[illegible]	3½	1	[illegible]	[illegible]
	5	1209	[illegible]	[illegible]	[illegible]	303	239	193	160	135	114	100		64	4	1	0.418	23430
	6	[illegible]	[illegible]	842	[illegible]	[illegible]	374	[illegible]	250	210	178	[illegible]	134	77	5	1	[illegible]	36300
	7	[illegible]	[illegible]	[illegible]	859	[illegible]	[illegible]	434	[illegible]	303	[illegible]	[illegible]	193	90	[illegible]		[illegible]	52280
	8	[illegible]	[illegible]	[illegible]	[illegible]	925	[illegible]	[illegible]	[illegible]	413	349	306	[illegible]	103	7	1	0.731	[illegible]
	9	4840	[illegible]	[illegible]	1581	[illegible]	[illegible]	[illegible]	[illegible]	539	458	395	314	[illegible]	8	1	0.835	92940
	10	[illegible]	[illegible]	[illegible]	[illegible]	153	1210	[illegible]	[illegible]	[illegible]	580	500	[illegible]	[illegible]	9	1	0.910	117600
$n=15$, $p=0.0034$, $f_s=20000$, $f_c=500$	2½	118	76	53										32	1¾	¾	0.071	2270
	3	195	[illegible]	87	64									38	2¼	¾	0.022	3760
	3½	290	186	129	95	[illegible]								45	2¾	¾	0.112	5580
	4	408	262	[illegible]	133	[illegible]	81							51	3¼	¾	0.133	7850
	4½	474	304	211	155	118	94	76						58	3½	1	0.143	9110
	5	619	398	276	[illegible]	[illegible]	[illegible]	99	82					64	4	1	0.163	[illegible]
	6	967	[illegible]	431	[illegible]	242	192	154	128	108	91	90		77	5	1	[illegible]	18600
	7	1392	894	621	455	348	[illegible]	[illegible]	185	155	131	115	[illegible]	90	6	1	0.245	26780
	8	1895	[illegible]	846	619	474	[illegible]	302	257	211	[illegible]	157	135	103	7	1	[illegible]	36450
	9	[illegible]	1587	1102	810	620	490	[illegible]	[illegible]	[illegible]	235	[illegible]	[illegible]	116	8	1	[illegible]	47610
	10	[illegible]	2009	1395	[illegible]	[illegible]	620	502	415	349	297	[illegible]	223	[illegible]	9	1	[illegible]	60260
$n=15$, $p=0.0047$, $f_s=20000$, $f_c=600$	2½	[illegible]	103	[illegible]	53									32	1¾	¾	[illegible]	3100
	3	267	171	[illegible]	87	67								38	2¼	¾	[illegible]	5130
	3½	396	254	177	[illegible]	99	78							45	2¾	¾	0.155	7610
	4	556	357	248	[illegible]	139	110	89						51	3¼	¾	0.183	10700
	4½	646	415	288	211	161	128	103	[illegible]					58	3½	1	0.197	12420
	5	844	542	376	276	[illegible]	167	135	112	94	79			64	4	1	0.226	16220
	6	1318	[illegible]	588	431	329	261	210	175	[illegible]	124	109	94	77	5	1	0.282	25350
	7	1898	1219	847	[illegible]	474	[illegible]	303	[illegible]	212	179	157	135	90	6	1	0.335	36500
	8	2584	1659	1152	844	646	572	412	343	[illegible]	243	214	184	103	7	1	0.395	49680
	9	3381	[illegible]	1502	1104	845	668	541	447	[illegible]	[illegible]	276	240	116	8	1	[illegible]	61900
	10	4279	[illegible]	1901	[illegible]	1070	845	[illegible]	[illegible]	475	405	[illegible]	304	128	9	1	0.508	82140
$n=15$, $p=0.0060$, $f_s=20000$, $f_c=700$	2½	202	130	90	66									32	1¾	¾	0.126	3890
	3	334	[illegible]	149	109	83								38	2¼	¾	0.162	6430
	3½	497	319	221	162	124	98							45	2¾	¾	0.198	9550
	4	698	449	311	229	175	138	111	93					51	3¼	¾	0.234	13430
	4½	810	520	361	265	202	160	129	107	90				58	3½	1	0.252	15580
	5	1058	679	472	346	264	210	169	140	118	100	87		64	4	1	0.288	20350
	6	1653	1062	737	540	413	327	[illegible]	219	184	156	137	117	77	5	1	0.360	31800
	7	2381	1530	1062	778	595	472	380	316	[illegible]	224	197	169	90	6	1	0.432	45790
	8	3240	2082	1445	1059	810	642	517	430	361	305	268	231	103	7	1	0.504	62330
	9	4241	2715	1885	1385	1060	838	678	561	471	402	346	302	116	8	1	0.576	81420
	10	5368	3435	2386	1753	1342	1060	859	710	596	508	438	382	128	9	1	0.648	103030

RULES

1. For load for any width of slab multiply by width in feet
2. For area of cross-section of steel for any width of slab multiply column (18) by width in feet
3. Total loads for other spans (l) and same depth of steel are inversely proportional to the squares of the spans
4. Total loads for other depths of steel (d) and same span are proportional to the squares of the depths of steel.

TABLE 6. USE FOR REVIEWING DESIGNS. IF FULLY CONTINUOUS ADD 20% TO LOADS.

Safe Loads per Square Foot and Reinforcement for Slabs. Proportions 1 : 2 : 4.

(See p. 508).

Based on $M = \dfrac{wl^2}{10}$

f_c = or < 650 $n = 15$

f_s = or < 16000

For supported ends, $\left(M = \dfrac{wl^2}{8}\right)$, deduct 20% from loads

For fully continuous, $\left(M = \dfrac{wl^2}{12}\right)$, add 20% to loads

For square slabs, $\left(M = \dfrac{wl^2}{20}\right)$, multiply loads by 2.

Ratio of cross-section steel to beam above steel.	Total depth of slab.	Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab in column (15). (See important footnotes.)												Weight of slab per square foot.	Depth to steel.	Depth below steel.	Steel area in a section of slab one foot wide.	Safe moment of resistance. See p. 753
		Span in feet (l.)													(d)	(e)		(M)
(p)*	(h) in.	4	5	6	7	8	9	10	11	12	13	14	15	lb.	in.	in.	sq. in.	in. lb.
														(15)	(16)	(17)	(18)	(19)
0.002	3	95	60	42										38	2¼	¾	0.054	1800
	4	198	125	88	64	49	39							51	3¼	¾	0.078	3760
	5	300	190	133	97	74	59	48						64	4	1	0.096	5702
	6	469	297	207	151	116	92	74						77	5	1	0.120	8910
	7	675	428	298	218	167	132	107						90	6	1	0.144	12830
	8	919	582	406	296	227	180	146	120					103	7	1	0.168	17460
	9	1201	760	531	387	296	235	190	157					116	8	1	0.192	22810
	10	1519	962	671	388	375	298	241	199	167				128	9	1	0.216	28870
0.004	3	185	117	82	60	46	36	29						38	2¼	¾	0.108	3510
	4	385	244	170	124	95	76	61	50					51	3¼	¾	0.156	7324
	5	584	370	258	188	144	114	92	77	64				64	4	1	0.192	11100
	6	913	578	403	294	225	179	144	120	100				77	5	1	0.240	17340
	7	1314	832	581	423	324	257	208	172	144				90	6	1	0.288	24970
	8	1788	1133	790	576	441	350	283	234	196	167	145	120	103	7	1	0.336	33980
	9	2336	1479	1032	752	576	458	370	306	257	219	189	164	116	8	1	0.384	44380
	10	2957	1873	1307	952	730	579	468	387	325	277	239	208	128	9	1	0.432	56180
0.006	3	272	172	120	87	67	52	43	36					38	2¼	¾	0.162	5160
	4	567	359	250	183	140	111	90	74	62				51	3¼	¾	0.234	10770
	5	858	544	379	270	212	168	136	112	94				64	4	1	0.288	16310
	6	1342	850	593	432	331	263	212	176	147				77	5	1	0.360	25490
	7	1932	1223	854	622	477	378	306	253	212				90	6	1	0.432	36700
	8	2630	1665	1162	847	649	515	416	345	289	246	213	185	103	7	1	0.504	49960
	9	3435	2175	1518	1106	848	673	544	450	377	321	278	242	116	8	1	0.576	65260
	10	4348	2753	1921	1400	1073	852	688	570	478	407	351	306	128	9	1	0.648	82600
0.008	3	348	220	153	112	86	68	55	46					38	2¼	¾	0.216	6610
	4	726	460	321	234	179	142	115	95	80				51	3¼	¾	0.312	13790
	5	1100	697	480	354	271	215	174	144	121				64	4	1	0.384	20900
	6	1719	1088	760	553	424	337	272	225	189				77	5	1	0.480	32650
	7	2475	1567	1094	797	611	485	392	324	272	231	200		90	6	1	0.576	47020
	8	3309	2134	1489	1085	831	660	533	441	370	315	272	237	103	7	1	0.672	64000
	9	4401	2787	1945	1417	1086	862	697	577	483	412	356	310	116	8	1	0.768	83600
	10	5570	3527	2461	1793	1374	1091	882	730	612	521	450	392	128	9	1	0.864	105800
0.010	3	374	237	165	120	92	73	59	49					38	2¼	¾	0.270	7100
	4	781	494	345	251	193	153	124	102	86				51	3¼	¾	0.390	14820
	5	1182	749	522	381	292	232	187	155	130				64	4	1	0.480	22460
	6	1847	1170	816	595	456	362	292	242	203	173	149		77	5	1	0.600	35090
	7	2660	1684	1175	850	656	521	421	348	292	249	215	187	90	6	1	0.720	50520
	8	3618	2292	1599	1165	893	709	573	474	397	339	292	255	103	7	1	0.840	68750
	9	4727	2993	2089	1522	1166	926	748	619	519	442	382	333	116	8	1	0.960	89800
	10	5986	3790	2645	1927	1477	1172	948	784	657	560	484	421	128	9	1	1.080	113700

* Percentages of steel are values in this column multiplied by 100.

Compression in concrete under tabular loads with the different percentages of steel:

Ratio of steel	0.002	0.004	0.006	0.008	0.010
Compression in concrete, lb. per sq. in.	370	500	610	650	650

RULES. 1. For load for any width of slab multiply by width in feet.

2. For area of cross-section of steel for any width of slab multiply column (18) by width in feet.

3. Total loads for other spans (1) and same depth of steel are inversely proportional to the squares of the spans.

4. Total loads for other depths of steel (4) and same span are proportional to the squares of the depths of steel.

A ratio of elasticity of $n = 35$ is used in the table below, although it is permissible to design with a ratio of 15 in very conservative practice.

The loads for slabs with a ratio of steel of 0.002 are limited by the working strength of the steel, and the values with the higher ratios by the working strength of the cinder concrete.

It is noticeable that less steel can be used economically for a given thickness of slab than with broken stone or gravel concrete, because the strength of the slab is more apt to be limited by the strength of the cinder concrete than by the strength of the steel.

Safe Loading and Reinforcement for CINDER CONCRETE SLABS One Foot in Width. Proportions 1 : 2½ : 5. *Mild Steel.* (See p. 515).

Based on $M = \frac{wl^2}{10}$, f_c = or < 225, f_s = or < 14 000, $n = 35$

Ratio cross-section steel to beam above steel. (p)*	Total depth of slab. in.	Total safe load (w^1) per square foot including weight of slab. For safe live load deduct weight of slab in column (12). (See important foot-notes.) Span in Feet (l) 4	5	6	7	8	9	10	Weight of slab per square foot. lb.	Depth to steel. (d) in.	Depth below steel. (e) in.	Steel area in a section of slab one foot wide. sq. in.	Safe moment of resistance. (See p. 753.) (M_R) in. lb.
									(10)	(11)	(12)	(13)	(14)
0.002	2½	48	31						24	1¾	¾	0.042	920
	3	70	51	35	26				29	2¼	¾	0.054	1520
	3½	119	76	53	39				34	2¾	¾	0.066	2280
	4	166	106	74	54	41			39	3¼	¾	0.078	3180
	4½	192	123	85	63	48			43	3½	1	0.084	3690
	5	251	161	112	82	63	50		48	4	1	0.096	4820
	6	392	251	174	128	98	78	63	58	5	1	0.120	7530
	7	565	361	251	184	141	112	90	68	6	1	0.144	10840
	8	768	492	341	351	192	152	123	77	7	1	0.168	14750
0.004	2½	76	48	34	25				24	1¾	¾	0.084	1460
	3	125	80	56	41	31			29	2¼	¾	0.108	2400
	3½	187	120	83	61	47	37		34	2¾	¾	0.132	3590
	4	261	167	116	85	65	52	42	39	3¼	¾	0.156	5020
	4½	303	194	135	99	76	60	48	43	3½	1	0.168	5820
	5	396	253	176	129	99	78	63	48	4	1	0.192	7600
	6	619	396	275	202	155	122	99	58	5	1	0.240	11880
	7	891	570	396	291	223	176	143	68	6	1	0.288	17110
	8	1213	776	539	396	303	240	194	77	7	1	0.336	23290
0.006	2½	86	55	38	28				24	1¾	¾	0.126	1640
	3	141	90	63	46	35			29	2¼	¾	0.162	2710
	3½	211	135	94	69	53	42	34	34	2¾	¾	0.198	4050
	4	295	189	131	96	74	58	47	39	3¼	¾	0.234	5660
	4½	342	219	152	112	85	68	55	43	3½	1	0.252	6570
	5	447	286	199	146	112	88	72	48	4	1	0.288	8580
	6	698	447	310	228	175	138	112	58	5	1	0.360	13400
	7	1005	643	447	328	251	199	161	68	6	1	0.432	19300
	8	1368	876	608	447	342	270	219	77	7	1	0.504	26270

* Percentages of steel are values in this column multiplied by 100.

RULES. 1. For load for any width of slab multiply by width in feet.

2. For area of cross-section of steel for any width of slab multiply column (13) by width in feet.

3. Total loads for other spans (e) and same depth of steel are inversely proportional to the squares of the spans.

4. Total loads for other depths of steel (d) and same span are proportional to the squares of the depths of steel.

TABLE 8. USE FOR BEAMS WITH STEEL IN TOP AND BOTTOM

Constants for Determining Depth of Beam. Moment of Resistance, and Fiber Stresses for Different Percentages of Steel. [See p. 428.] (See Example on page 470.)
Ratio of Elasticity of Steel to Concrete, $n = 15$.

Depth of beam $d = \sqrt{\dfrac{M}{b\, f_c\, C_c}}$ or $\sqrt{\dfrac{M}{b\, f_s\, C_s}}$ whichever is greater

Fiber stresses, $f_c = \dfrac{M}{C_c\, b\, d^2}$, $f_s = \dfrac{M}{C_s\, b\, d^2}$, $f_s' = \dfrac{M}{C'_s\, b\, d^2}$

Moment of resistance, $M = f_c C_c\, d^2$ or $f_s C_s b\, d^2$, whichever is less.

Rule 1. To determine Depth of Beam:
Assume p, ratio of tension steel, and p' ratio compression steel.
Assume a, ratio depth of steel in compression to depth in tension.
Locate these values in table and find C_c and C_s corresponding.
Substitute values C_c and C_s in formulas for depth, d (above.)
Accept the larger value as depth from compressed surface of beam to center tension steel.

Rule 2. To determine Fiber Stresses and Moment of Resistance in a given beam:
Compute p and p' and a.
Locate these values in table and find required constants.
Substitute values in formulas above and obtain required stresses or moment of resistance.

Rule 3. To determine Depth of Haunch at support of a beam or girder.
Decide tentatively amount of steel in tension and compression.
Assume a trial depth of haunch.
Determine by Rule 2 the fiber stresses.
If stresses are not as required, assume new depth of haunch and re-compute.
(See Example 6, page 470.)

Rule 4. To interpolate values of any C when required ratio of p to p' is given in table.
Example: Given $a = 0.15$, $p = 0.012$, $p' = 0.006$. Then $p' = 0.5\,p$, and interpolating in this group between $p = 0.1$, $p' = .005$ and $p = .015$, $p' = .0075$, gives $C_c = .23$ and $C_s = 0.0103$.

Rule 5. To interpolate values of any C when required ratio of p to p' is not given in table.
Example: Given $a = 0.1$, $p = 0.013$, $p' = 0.009$. Then $p' = 0.69\,p$, which lies between groups $p' = 0.5\,p$; and $p' = p$.
Find by interpolation in group $p' = 0.5\,p$; for $p = 0.013$, $p' = 0.0065$, $C_c = .24$ and $C_s = 0.0114$.
and in group $p' = p$; for $p = 0.013$ and $p' = 0.013$; $C_c = .29$ and $C_s = .0115$.
Interpolate between the two above values and find for $p = 0.013$ and $p' = 0.009$, $C_c = 0.26$ and $C_s = 0.0114$.

p = Ratio Cross Section of Steel in Tension to Concrete above it.
p' = Ratio Cross Section of Steel in Compression to Concrete.
k = Ratio Depth of Neutral Axis to Depth of Tension Steel.
C_c, C_s, C'_s = Constants in formulas above.

	p	p'	k	C_c	C_s	C'_s	p	p'	k	C_c	C_s	C'_s
	$a = 0.05$ — Ratio of Depth of Steel in Compression to Depth of Steel in Tension.						$a = 0.1$ — Ratio of Depth of Steel in Compression to Depth of Steel in Tension					
$p' = 0.25\,p$	0.005	0.00125	0.307	0.15	0.0046	0.0121	0.005	0.00125	0.310	0.15	0.0045	0.0148
	0.01	0.0025	0.394	0.20	0.0088	0.0154	0.01	0.0025	0.398	0.20	0.0088	0.0175
	0.015	0.00375	0.450	0.24	0.0130	0.0179	0.015	0.00375	0.454	0.23	0.0129	0.0199
	0.02	0.005	0.490	0.27	0.0172	0.0199	0.02	0.005	0.494	0.26	0.0170	0.0219
	0.025	0.00625	0.521	0.30	0.0214	0.0218	0.025	0.00625	0.526	0.29	0.0210	0.0235
	0.03	0.0075	0.546	0.32	0.0256	0.0234	0.03	0.0075	0.551	0.31	0.0250	0.0250
	0.035	0.00875	0.566	0.34	0.0298	0.0250	0.035	0.00875	0.571	0.33	0.0290	0.0265
	0.04	0.01	0.583	0.36	0.0342	0.0265	0.04	0.01	0.589	0.35	0.0330	0.0280
$p' = 0.5\,p$	0.005	0.0025	0.296	0.16	0.0046	0.0131	0.005	0.0025	0.299	0.16	0.0045	0.0158
	0.01	0.005	0.373	0.22	0.0090	0.0174	0.01	0.005	0.381	0.21	0.0088	0.0192
	0.015	0.0075	0.420	0.28	0.0134	0.0210	0.015	0.0075	0.428	0.26	0.0131	0.0227
	0.02	0.01	0.454	0.32	0.0178	0.0240	0.02	0.01	0.462	0.30	0.0174	0.0256
	0.025	0.0125	0.480	0.36	0.0222	0.0269	0.025	0.0125	0.488	0.34	0.0215	0.0284
	0.03	0.015	0.499	0.40	0.0266	0.0296	0.03	0.015	0.509	0.38	0.0258	0.0312
	0.035	0.0175	0.515	0.44	0.0310	0.0323	0.035	0.0175	0.524	0.41	0.0301	0.0339
	0.04	0.02	0.528	0.48	0.0355	0.0350	0.04	0.02	0.539	0.44	0.0343	0.0356
$p' = p$	0.005	0.005	0.271	0.18	0.0046	0.0149	0.005	0.005	0.281	0.17	0.0045	0.0176
	0.01	0.01	0.336	0.27	0.0092	0.0212	0.01	0.01	0.349	0.25	0.0089	0.0232
	0.015	0.015	0.372	0.35	0.0138	0.0266	0.015	0.015	0.386	0.32	0.0133	0.0285
	0.02	0.02	0.395	0.42	0.0184	0.0320	0.02	0.02	0.410	0.38	0.0177	0.0337
	0.025	0.025	0.412	0.49	0.0230	0.0373	0.025	0.025	0.428	0.44	0.0221	0.0381
	0.03	0.03	0.425	0.56	0.0275	0.0423	0.03	0.03	0.442	0.50	0.0265	0.0411
	0.035	0.035	0.435	0.63	0.0322	0.0473	0.035	0.035	0.452	0.56	0.0309	0.0481
	0.04	0.04	0.443	0.70	0.0368	0.0523	0.04	0.04	0.461	0.62	0.0353	0.0503
$p' = 1.5\,p$	0.005	0.0075	0.256	0.20	0.0046	0.0168	0.005	0.0075	0.268	0.18	0.0045	0.0197
	0.01	0.015	0.305	0.32	0.0093	0.0251	0.01	0.015	0.322	0.27	0.0090	0.0276
	0.015	0.0225	0.331	0.42	0.0140	0.0330	0.015	0.0225	0.350	0.37	0.0134	0.0348
	0.02	0.03	0.349	0.52	0.0186	0.0401	0.02	0.03	0.369	0.46	0.0178	0.0417
	0.025	0.0375	0.361	0.62	0.0232	0.0476	0.025	0.0375	0.382	0.54	0.0222	0.0489
	0.03	0.045	0.369	0.72	0.0280	0.0550	0.03	0.045	0.392	0.62	0.0267	0.0454
	0.035	0.0525	0.376	0.81	0.0326	0.0625	0.035	0.0525	0.399	0.70	0.0312	0.0626
	0.04	0.06	0.381	0.91	0.0372	0.0699	0.04	0.06	0.405	0.78	0.0357	0.0691

p = Ratio Cross Section of Steel in Tension to Concrete above it.
p' = Ratio Cross Section of Steel in Compression to Concrete.
k = Ratio Depth of Neutral Axis to Depth of Tension Steel.
C_c, C_s, C'_s = Constants in formulas above.

a = 0.15 = Ratio of Depth of Steel in Compression to Depth of Steel in Tension

	p	p'	k	C_c	C_s	C'_s
$p'=0.25p$	0.005	0.00125	0.312	0.15	0.0044	0.0189
	0.01	0.0025	0.402	0.19	0.0086	0.0206
	0.015	0.00375	0.458	0.23	0.0127	0.0223
	0.02	0.005	0.499	0.25	0.0167	0.0240
	0.025	0.00625	0.530	0.28	0.0207	0.0257
	0.03	0.0075	0.555	0.30	0.0247	0.0273
	0.035	0.00875	0.577	0.32	0.0286	0.0285
	0.04	0.010	0.595	0.33	0.0326	0.0297
$p'=0.5p$	0.005	0.0025	0.304	0.15	0.0044	0.0202
	0.01	0.005	0.386	0.21	0.0087	0.0226
	0.015	0.0075	0.435	0.25	0.0128	0.0251
	0.02	0.01	0.471	0.29	0.0169	0.0279
	0.025	0.0125	0.496	0.32	0.0210	0.0306
	0.03	0.015	0.518	0.35	0.0251	0.0328
	0.035	0.0175	0.535	0.38	0.0292	0.0353
	0.04	0.02	0.549	0.41	0.0333	0.0376
$p'=p$	0.005	0.005	0.292	0.16	0.0044	0.0222
	0.01	0.01	0.360	0.23	0.0087	0.0265
	0.015	0.015	0.398	0.29	0.0129	0.0313
	0.02	0.02	0.425	0.35	0.0171	0.0357
	0.025	0.025	0.444	0.40	0.0213	0.0402
	0.03	0.03	0.458	0.45	0.0255	0.0446
	0.035	0.035	0.469	0.50	0.0297	0.0490
	0.04	0.04	0.479	0.55	0.0338	0.0528
$p'=1.5p$	0.005	0.0075	0.280	0.17	0.0045	0.0247
	0.01	0.015	0.338	0.26	0.0087	0.0306
	0.015	0.0235	0.369	0.33	0.0129	0.0373
	0.02	0.03	0.389	0.40	0.0171	0.0439
	0.025	0.0375	0.403	0.47	0.0213	0.0505
	0.03	0.045	0.414	0.54	0.0256	0.0569
	0.035	0.0525	0.422	0.61	0.0298	0.0635
	0.04	0.06	0.429	0.68	0.0340	0.0697

a = 0.2 = Ratio of Depth of Steel in Compression to Depth of Steel in Tension.

	p	p'	k	C_c	C_s	C'_s
$p'=0.25p$	0.005	0.00125	0.313	0.14	0.0044	0.0272
	0.01	0.0025	0.404	0.18	0.0086	0.0243
	0.015	0.00375	0.460	0.22	0.0126	0.0204
	0.02	0.005	0.503	0.24	0.0165	0.0272
	0.025	0.00625	0.535	0.27	0.0204	0.0284
	0.03	0.0075	0.560	0.29	0.0243	0.0297
	0.035	0.00875	0.582	0.31	0.0282	0.0309
	0.04	0.010	0.600	0.32	0.0320	0.0320
$p'=0.5p$	0.005	0.0025	0.309	0.15	0.0044	0.0270
	0.01	0.005	0.392	0.20	0.0086	0.0272
	0.015	0.0075	0.442	0.24	0.0126	0.0291
	0.02	0.01	0.479	0.27	0.0166	0.0312
	0.025	0.0125	0.506	0.30	0.0206	0.0331
	0.03	0.015	0.527	0.33	0.0245	0.0354
	0.035	0.0175	0.544	0.36	0.0284	0.0378
	0.04	0.02	0.559	0.38	0.0323	0.0397
$p'=p$	0.005	0.005	0.290	0.16	0.0044	0.0313
	0.01	0.01	0.371	0.22	0.0086	0.0314
	0.015	0.015	0.411	0.27	0.0126	0.0352
	0.02	0.02	0.439	0.32	0.0166	0.0390
	0.025	0.025	0.460	0.36	0.0206	0.0428
	0.03	0.03	0.475	0.41	0.0246	0.0470
	0.035	0.035	0.487	0.45	0.0286	0.0511
	0.04	0.04	0.497	0.49	0.0326	0.0551
$p'=1.5p$	0.005	0.0075	0.292	0.16	0.0044	0.0339
	0.01	0.015	0.353	0.23	0.0086	0.0359
	0.015	0.0235	0.386	0.30	0.0126	0.0416
	0.02	0.03	0.409	0.36	0.0166	0.0464
	0.025	0.0375	0.424	0.42	0.0206	0.0531
	0.03	0.045	0.436	0.48	0.0246	0.0592
	0.035	0.0525	0.444	0.54	0.0286	0.0653
	0.04	0.06	0.452	0.59	0.0326	0.0709

a = 0.25 = Ratio of Depth of Steel in Compression to Depth of Steel in Tension.

	p	p'	k	C_c	C_s	C'_s
$p'=0.25p$	0.005	0.00125	0.316	0.15	0.0045	0.0458
	0.01	0.0025	0.408	0.19	0.0086	0.0320
	0.015	0.00375	0.465	0.22	0.0126	0.0311
	0.02	0.005	0.507	0.24	0.0164	0.0315
	0.025	0.00625	0.539	0.26	0.0202	0.0321
	0.03	0.0075	0.565	0.28	0.0240	0.0332
	0.035	0.00875	0.588	0.30	0.0278	0.0338
	0.04	0.010	0.606	0.31	0.0316	0.0349
$p'=0.5p$	0.005	0.0025	0.314	0.15	0.0045	0.0476
	0.01	0.005	0.398	0.19	0.0085	0.0347
	0.015	0.0075	0.450	0.23	0.0125	0.0342
	0.02	0.01	0.487	0.26	0.0164	0.0354
	0.025	0.0125	0.514	0.29	0.0202	0.0373
	0.03	0.015	0.537	0.32	0.0240	0.0386
	0.035	0.0175	0.556	0.34	0.0278	0.0404
	0.04	0.02	0.570	0.36	0.0315	0.0422
$p'=p$	0.005	0.005	0.308	0.15	0.0045	0.0523
	0.01	0.01	0.382	0.20	0.0085	0.0386
	0.015	0.015	0.425	0.25	0.0124	0.0406
	0.02	0.02	0.454	0.29	0.0162	0.0435
	0.025	0.025	0.475	0.33	0.0200	0.0466
	0.03	0.03	0.491	0.37	0.0238	0.0504
	0.035	0.035	0.504	0.41	0.0276	0.0542
	0.04	0.04	0.515	0.44	0.0314	0.0577
$p'=1.5p$	0.005	0.0075	0.304	0.15	0.0044	0.0567
	0.01	0.015	0.369	0.22	0.0084	0.0446
	0.015	0.0225	0.404	0.27	0.0123	0.0475
	0.02	0.03	0.428	0.32	0.0161	0.0519
	0.025	0.0375	0.444	0.37	0.0199	0.0576
	0.03	0.045	0.457	0.42	0.0237	0.0623
	0.035	0.0525	0.466	0.47	0.0275	0.0681
	0.04	0.06	0.475	0.52	0.0313	0.0732

a = 0.3 = Ratio of Depth of Steel in Compression to Depth of Steel in Tension.

	p	p'	k	C_c	C_s	C'_s
$p'=0.25p$	0.005	0.00125	0.318	0.14	0.0015	0.1687
	0.01	0.0025	0.411	0.18	0.0086	0.0155
	0.015	0.00375	0.468	0.21	0.0125	0.0395
	0.02	0.005	0.511	0.23	0.0163	0.0377
	0.025	0.00625	0.544	0.25	0.0201	0.0375
	0.03	0.0075	0.571	0.27	0.0239	0.0376
	0.035	0.00875	0.592	0.29	0.0275	0.0382
	0.04	0.010	0.611	0.30	0.0311	0.0389
$p'=0.5p$	0.005	0.0025	0.320	0.15	0.0015	0.1515
	0.01	0.005	0.404	0.19	0.0085	0.0159
	0.015	0.0075	0.450	0.22	0.0124	0.0433
	0.02	0.01	0.495	0.25	0.0161	0.0423
	0.025	0.0125	[illegible]	0.27	0.0199	0.0129
	0.03	0.015	[illegible]	0.29	0.0236	0.0134
	0.035	0.0175	0.564	0.31	0.0272	0.0148
	0.04	0.02	0.580	0.33	0.0308	0.0463
$p'=p$	0.005	0.005	0.317	0.14	0.0044	0.1790
	0.01	0.01	0.393	0.19	0.0084	0.0550
	0.015	0.015	0.438	0.24	0.0123	0.0499
	0.02	0.02	0.465	0.27	0.0160	0.0507
	0.025	0.025	0.490	0.31	0.0197	0.0529
	0.03	0.03	0.507	0.34	0.0233	0.0556
	0.035	0.035	0.521	0.37	0.0269	0.0585
	0.04	0.04	0.532	0.40	0.0305	0.0616
$p'=1.5p$	0.005	0.0075	0.313	0.14	0.0044	0.2360
	0.01	0.015	0.384	0.20	0.0084	0.0614
	0.015	0.0225	0.422	0.25	0.0122	0.0574
	0.02	0.03	0.447	0.30	0.0159	0.0595
	0.025	0.0375	0.464	0.34	0.0195	0.0635
	0.03	0.045	0.478	0.38	0.0231	0.0694
	0.035	0.0525	0.490	0.42	0.0266	0.0722
	0.04	0.06	0.497	0.46	0.0301	0.0770

USE THIS TABLE ORDINARILY

TABLE 9. FLAT SLABS SUPPORTED ON COLUMNS

Data for Computing Bending Moments. (See p. 485) See Example 14, p. 487.

Rule. To find bending moment in a flat plate loaded uniformly and supported on columns, or other fixed supports:

Assume radius of support, r_0, within column head, where bending moment is a maximum (see p. 485).

Determine radius of surface assumed to act as a fixed circular plate, r_1, (for a floor take this as $\frac{1}{2}$ diagonal distance between lines of maximum bending moment plus radius, r_0, of support) (See Fig. 152a, p. 485).

Radius r, used in table below, is radius to any point where bending moment is required. For critical section, r is radius of column head.

Compute load per linear foot, q, around circumference of plate having radius, r_1. (See p. 485.)

Take for w the live plus dead load per square foot of slab.

Then moment causing radial fiber stress at any distance r (see table below), from centre of column is:

$$M_r = w r_0^2 \; C_b + q r_0 \; C_e$$

Use M_r to find required depth of slab and amount of steel at edge of column head from ordinary beam or slab formulas. (See Example 14, p. 487.)

Note that if w is in lb. per sq. ft., q in lb. per foot of length, and r_0 in ft., the moment will be in ft.-lb. per foot of width or in in.-lb. per inch of width of circumference having a radius r.

Table below is computed from values on opposite page which should be used direct when Poisson's ratio is other than 0.1.

Values of constants C_b and C_e based on Poisson's ratio of 0.1.

$\frac{r_1}{r_0}$	Values of $\frac{r}{r_0}$: 1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
CONSTANTS C_b											
1.4	0.10	0.06	0.03	0.01	0.01	0.02	0.04	0.06	0.09	0.13	0.17
1.6	0.24	0.16	0.10	0.06	0.03	0.02	0.02	0.02	0.04	0.07	0.10
1.8	0.45	0.31	0.22	0.14	0.09	0.05	0.03	0.02	0.02	0.03	0.05
2.0	0.72	0.54	0.39	0.28	0.20	0.13	0.08	0.05	0.03	0.02	0.02
2.2	1.08	0.83	0.63	0.48	0.35	0.25	0.18	0.12	0.08	0.05	0.03
2.4	1.52	1.19	0.93	0.72	0.56	0.42	0.31	0.23	0.16	0.11	0.07
2.6	2.04	1.63	1.30	1.03	0.82	0.64	0.50	0.38	0.28	0.20	0.14
2.8	2.66	2.15	1.74	1.41	1.14	0.91	0.73	0.57	0.45	0.34	0.25
3.0	3.37	2.75	2.25	1.85	1.52	1.24	1.01	0.82	0.66	0.52	0.40
3.2	4.17	3.43	2.84	2.36	1.96	1.63	1.35	1.11	0.91	0.74	0.59
3.4	5.06	4.19	3.49	2.93	2.46	2.07	1.74	1.45	1.21	1.00	0.82
3.6	6.05	5.04	4.23	3.57	3.03	2.57	2.18	1.84	1.56	1.31	1.09
3.8	7.15	5.98	5.05	4.30	3.67	3.14	2.69	2.30	1.96	1.67	1.42
4.0	8.35	7.02	5.96	5.09	4.37	3.76	3.25	2.80	2.41	2.08	1.78
4.5	11.77	9.98	8.55	7.38	6.41	5.60	4.89	4.28	3.75	3.29	2.88
5.0	15.99	13.65	11.79	10.27	9.00	7.93	7.01	6.22	5.53	4.91	4.37
CONSTANTS C_e											
1.4	0.53	0.36	0.22	0.10	0.00	−0.09	−0.17	−0.24	−0.30	−0.36	−0.42
1.6	0.87	0.66	0.48	0.33	0.21	0.10	0.00	−0.09	−0.17	−0.24	−0.30
1.8	1.25	0.99	0.77	0.60	0.44	0.31	0.20	0.09	0.00	−0.09	−0.16
2.0	1.66	1.35	1.10	0.89	0.71	0.55	0.42	0.30	0.19	0.09	0.00
2.2	2.10	1.73	1.44	1.19	0.99	0.81	0.65	0.52	0.39	0.28	0.18
2.4	2.55	2.13	1.79	1.52	1.28	1.08	0.90	0.75	0.61	0.48	0.37
2.6	3.03	2.56	2.17	1.86	1.59	1.37	1.17	1.00	0.84	0.70	0.58
2.8	3.51	2.98	2.56	2.21	1.91	1.66	1.44	1.25	1.08	0.93	0.76
3.0	4.02	3.43	2.96	2.57	2.25	1.97	1.73	1.52	1.34	1.17	1.02
3.2	4.54	3.89	3.38	2.95	2.60	2.29	2.03	1.80	1.60	1.42	1.25
3.4	5.07	4.36	3.80	3.34	2.95	2.62	2.33	2.08	1.86	1.67	1.49
3.6	5.60	4.83	4.22	3.72	3.30	2.95	2.64	2.37	2.14	1.92	1.73
3.8	6.17	5.34	4.68	4.14	3.69	3.31	2.98	2.69	2.43	2.21	2.00
4.0	6.73	5.84	5.13	4.55	4.07	3.66	3.30	2.99	2.72	2.48	2.26
4.5	8.12	7.07	6.23	5.55	4.98	4.50	4.09	3.73	3.41	3.13	2.87
5.0	9.72	8.50	7.54	6.75	6.10	5.55	5.08	4.66	4.30	3.98	3.69

Data for Determining Bending Moments for Flat Slabs Supported on Columns for Various Values of Poisson's Ratio.

Rule. Proceed as indicated on opposite page, except using sum of moments M_2 and M_b:

$$M_2 = wr_0^{\,2}\left\{0.2\left(\frac{r}{r_0}\right)^2 + C_1\left(\frac{r_0}{r}\right)^2 - C_2\log\left(\frac{r}{r_0}\right) + C_3\right\} \quad \ldots\ldots\ldots\ldots\ldots (55)$$

$$M_b = q\,r_0\left\{C_a\left(\frac{r_0}{r}\right)^2 - C_c\log\left(\frac{r}{r_0}\right) + C_b\right\} \quad \ldots\ldots\ldots\ldots\ldots (57)$$

Formulas (54) and (56) (p. 485) for M_1 and M_a for circumferential fiber stresses are not usually required.

If $r = r_0$, $M_2 = wr_0^2\,(0.2 + C_1 + C_2)$ (52) and $M_b = qr_0\,(C_a + C_b)$ (53)

Poisson's ratio g	Ratio outer to inner radius $\frac{r_1}{r_0}$	Constants in formulas (52) to (57), pages 485 and 518a: For uniformly distributed loading				For circumferential loading			
		C_1	C_2	C_3	C_4	C_a	C_b	C_c	C_d
	1.4	0.21	−0.31	1.21	0.14	0.57	−0.04	1.73	0.61
	1.6	0.35	−0.30	1.58	0.29	0.78	0.10	1.98	0.84
	1.8	0.52	−0.27	2.00	0.48	1.00	0.26	2.23	1.09
	2.0	0.74	−0.21	2.47	0.72	1.24	0.44	2.47	1.36
	2.2	1.00	−0.11	2.99	1.01	1.48	0.63	2.72	1.64
	2.4	1.31	0.02	3.56	1.35	1.74	0.83	2.97	1.93
	2.6	1.67	0.19	4.18	1.75	2.01	1.04	3.21	2.24
0.075	2.8	2.08	0.40	4.85	2.21	2.29	1.26	3.46	2.55
	3.0	2.55	0.64	5.56	2.72	2.57	1.48	3.71	2.87
	3.2	3.06	0.93	6.33	3.30	2.86	1.72	3.96	3.20
	3.4	3.63	1.26	7.15	3.93	3.15	1.96	4.20	3.53
	3.6	4.25	1.64	8.01	4.63	3.45	2.21	4.45	3.87
	3.8	4.94	2.06	8.93	5.10	3.76	2.47	4.70	4.22
	4.0	5.67	2.53	9.89	6.23	4.06	2.72	4.95	4.57
	4.5	7.77	3.90	12.52	8.58	4.86	3.40	5.56	5.48
	5.0	10.25	5.59	15.45	11.37	5.67	4.09	6.18	6.40
	1.4	0.21	−0.31	1.24	0.14	0.55	−0.02	1.77	0.61
	1.6	0.34	−0.30	1.62	0.28	0.75	0.12	2.02	0.84
	1.8	0.51	−0.26	2.05	0.47	0.97	0.28	2.28	1.09
	2.0	0.72	−0.19	2.53	0.71	1.20	0.46	2.53	1.36
	2.2	0.97	−0.09	3.06	1.00	1.44	0.66	2.78	1.65
	2.4	1.27	0.05	3.64	1.34	1.69	0.86	3.04	1.94
	2.6	1.62	0.22	4.28	1.75	1.95	1.08	3.29	2.25
0.10	2.8	2.02	0.44	4.96	2.20	2.21	1.30	3.54	2.56
	3.0	2.47	0.70	5.69	2.72	2.48	1.54	3.80	2.89
	3.2	2.97	1.00	6.48	3.30	2.76	1.78	4.05	3.22
	3.4	3.52	1.34	7.31	3.94	3.05	2.02	4.30	3.55
	3.6	4.12	1.73	8.20	4.65	3.33	2.27	4.55	3.89
	3.8	4.78	2.17	9.13	5.42	3.63	2.54	4.81	4.25
	4.0	5.50	2.65	10.12	6.26	3.93	2.80	5.06	4.60
	4.5	7.52	4.05	12.80	8.61	4.71	3.41	5.70	5.53
	5.0	9.95	5.84	15.80	11.46	5.50	4.22	6.33	6.47
	1.4	0.20	−0.30	1.30	0.12	0.52	0.01	1.85	0.60
	1.6	0.32	−0.28	1.69	0.26	0.71	0.16	2.12	0.84
	1.8	0.48	−0.24	2.14	0.45	0.91	0.33	2.38	1.09
	2.0	0.67	−0.16	2.65	0.69	1.12	0.52	2.65	1.37
	2.2	0.91	−0.05	3.20	0.98	1.34	0.72	2.91	1.65
	2.4	1.20	0.11	3.81	1.33	1.58	0.93	3.17	1.95
	2.6	1.52	0.30	4.47	1.73	1.82	1.16	3.44	2.26
0.15	2.8	1.90	0.53	5.18	2.20	2.06	1.39	3.70	2.58
	3.0	2.32	0.81	5.95	2.72	2.32	1.64	3.97	2.91
	3.2	2.78	1.13	6.77	3.31	2.58	1.89	4.23	3.25
	3.4	3.30	1.50	7.65	3.95	2.84	2.14	4.50	3.59
	3.6	3.87	1.92	8.57	4.67	3.11	2.41	4.76	3.94
	3.8	4.48	2.38	9.55	5.45	3.38	2.68	5.03	4.29
	4.0	5.16	2.91	10.58	6.31	3.66	2.96	5.29	4.66
	4.5	7.06	4.42	13.40	8.72	4.38	3.66	5.96	5.57
	5.0	9.32	6.30	16.52	11.60	5.12	4.42	6.61	6.53

NOTE—All values are plus unless otherwise indicated.

TABLE 9a. NUMBER OF STIRRUPS IN UNIFORMLY LOADED BEAM

Number of stirrups, $N_s = \dfrac{l\,b}{A_s\,C_n}$ (See Example 20 below).

N_s = number of stirrups in each end of beam. b = breadth of web of beam in inches.
jd = distance from center of compression to center of horizontal reinforcement. (See p. 450.)
l = span of beam in feet. v = total shearing unit stress at end of beam in lb. per sq. in.
v' = allowable shearing unit stress (or diagonal tension) in concrete alone in lb. per sq. in.
A_s = cross-sectional area of vertical stirrup in sq. in. (In a U-stirrup, sum of areas of two legs.)
f_s = allowable unit tensile stress in the stirrup in lb. per sq. in. C_n = constant.

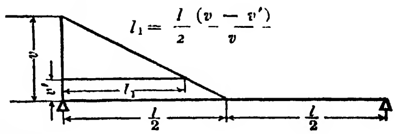
Values of Constant C_n for Finding Number of Stirrups in Each End of Beam.

v	f_s=12 000				f_s=14 000				f_s=16 000				f_s=18 000			
	v'=0	40	60	80	v'=0	40	60	80	v'=0	40	60	80	v'=0	40	60	80
70	57	311	2800		67	363	3267		76	415	3733		86	467	4200	
75	53	245	1333		62	286	1556		71	327	1778		80	367	2000	
80	50	200	800		58	233	933		67	267	1067		75	300	1200	
85	47	168	544		55	196	635		63	224	725		71	252	816	
90	44	144	400	3600	52	168	467	4200	59	192	533	4800	67	216	600	5400
95	42	126	310	1689	49	147	362	1970	56	168	414	2252	63	188	465	2533
100	40	111	250	1000	47	130	292	1167	53	148	333	1333	60	167	375	1500
105	38	99	207	672	44	116	242	784	51	133	277	896	57	149	311	1008
110	36	90	176	489	42	105	205	570	48	120	235	652	55	135	264	733
115	35	82	152	376	41	95	177	438	46	109	203	501	52	123	228	563
120	33	75	133	300	39	88	156	350	44	100	178	400	50	113	200	450
125	32	69	118	247	37	81	138	288	43	92	158	329	48	104	178	370
130	31	64	106	208	36	75	124	243	41	86	142	277	46	96	159	312
140	29	56	88	156	33	65	102	182	38	75	117	207	43	84	131	233
150	27	50	74	122	31	58	86	143	36	66	99	163	40	74	111	184
160	25	44	64	100	29	52	75	117	33	59	85	133	38	67	96	150
170	24	40	56	84	27	47	66	98	31	54	75	112	35	60	84	126
180	22	37	50	72	26	43	58	84	30	49	67	96	33	55	75	108

TABLE 9b. LOCATION OF VERTICAL STIRRUPS IN BEAM WITH UNIFORM LOADING

Rule. Find distance of each stirrup from end of beam by multiplying l_1, (obtained from formula) by values from Table 9b, selected by reading along horizontal line opposite proper value of N_s.

Values of Constant C_l for Fnding Distance of Each Stirrup From End of Beam.



N_s*	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th	17th	18th	19th	20th
1	.333																			
2	.146	.529																		
3	.092	.303	.615																	
4	.067	.213	.396	.667																
5	.053	.165	.296	.460	.702															
6	.044	.135	.238	.358	.507	.728														
7	.037	.115	.199	.295	.405	.544	.748													
8	.032	.099	.172	.251	.340	.444	.573	.764												
9	.029	.088	.151	.210	.294	.378	.476	.598	.778											
10	.026	.079	.134	.194	.259	.330	.410	.503	.618	.789										
11	.023	.071	.121	.175	.232	.294	.361	.437	.526	.636	.799									
12	.021	.065	.111	.150	.210	.265	.324	.389	.461	.546	.652	.808								
13	.020	.060	.102	.145	.192	.241	.293	.350	.413	.482	.564	.665	.815							
14	.018	.055	.094	.134	.177	.221	.268	.319	.374	.434	.501	.580	.677	.822						
15	.017	.051	.087	.125	.164	.204	.248	.293	.342	.395	.453	.518	.594	.688	.828					
16	.016	.048	.082	.116	.152	.190	.230	.271	.316	.363	.414	.471	.534	.607	.698	.833				
17	.015	.045	.077	.109	.143	.178	.214	.253	.293	.336	.382	.432	.486	.547	.618	.707	.838			
18	.014	.043	.072	.103	.134	.167	.201	.236	.274	.313	.355	.399	.448	.501	.560	.629	.716	.843		
19	.013	.040	.068	.097	.127	.157	.189	.222	.257	.293	.331	.372	.416	.463	.514	.572	.639	.723	.847	
20	.013	.038	.065	.092	.120	.149	.179	.210	.242	.276	.311	.348	.388	.430	.476	.526	.583	.648	.730	.851

* Number of stirrups in each end of beam as found from Table 9a.

EXAMPLE FOR NUMBER AND LOCATION OF STIRRUPS

Example 20: Given: l = 24 ft.; total load = 2400 lb. per lin. ft.; b = 12″; jd = 21″; v' = 40 lb. per sq. in.; f_s = 16 000. Use $\frac{7}{8}$-inch square twisted U-stirrups, i.e., A_s = 0.383.

Solution: Shearing unit stress at end of beam, $v = \dfrac{2100 \times 24}{2 \times 12 \times 21}$ = 115 lb. per sq. in. Taking values from Table 9a, opposite v = 115, under f_s = 16 000, with v' = 40, we find C_n = 109. Hence, $N_s = \dfrac{l\,b}{A_s\,C_n} = \dfrac{24 \times 12}{0.383 \times 109}$ = 6.9. Therefore use 7 stirrups in each end of beam.

To locate stirrups, we find $l_1 = \dfrac{24 \times 12}{2}\left(\dfrac{115 - 40}{115}\right)$ = 94 inches. Take values from Table 9b opposite N_s = 7, and, multiplying each value by l_1 = 94, distance of each stirrup from end of beam will give: 1st stirrup, 3.5″; 2d, 10.8″; 3rd, 18.7″; 4th, 27.8″; 5th, 38.1″; 6th, 51.1″; 7th, 70.3″.

TABLE 10. TABLE FOR CONSTANT C FOR BEAMS

Data for Determining Depth of Beam, Moment of Resistance and Reinforcement

To be used in formulas for Depth of rectangular beam or slab. $d = C\sqrt{\frac{M}{b}}$

and in formulas for Moment of Resistance $M = \frac{b\,d^2}{C^2}$

(*See pp.* 418 *and* 754.)

Based on dimensions in inches and moments in inch-pounds.

Item	Working Strength of Steel f_s lb. per sq. in.	Working Strength of Concrete f_c lb. per sq. in.	Ratio of Moduli of Steel to Concrete $n = 10$: Ratio Depth of Neutral Axis to Depth of Steel. k	Ratio of Moment Arm to Depth of Steel $(1 - \frac{k}{3})$ j	Ratio Area of Steel to Beam Above Steel. p	Safe Working Value of Constant C. C	Ratio of Moduli of Steel to Concrete $n = 15$: Ratio Depth of Neutral Axis to Depth of Steel. k	Ratio of Moment Arm to Depth of Steel, $(1 - \frac{k}{3})$ j	Ratio Area of Steel to Beam Above Steel. p	Safe Working Value of Constant C. C
1	12000	500	0.294	0.902	0.0061	0.123	0.384	0.872	0.0080	0.109
2		550	0.314	0.895	0.0072	0.114	0.407	0.864	0.0093	0.102
3		600	0.333	0.889	0.0083	0.106	0.428	0.857	0.0107	0.095
4		650	0.351	0.883	0.0095	0.100	0.448	0.851	0.0121	0.090
5		700	0.368	0.877	0.0108	0.094	0.467	0.844	0.0136	0.085
6		750	0.384	0.872	0.0120	0.089	0.484	0.839	0.0151	0.081
7		800	0.400	0.867	0.0133	0.085	0.501	0.833	0.0167	0.077
8	14000	500	0.263	0.912	0.0047	0.129	0.348	0.884	0.0062	0.114
9		550	0.281	0.906	0.0055	0.120	0.372	0.876	0.0073	0.106
10		600	0.299	0.900	0.0064	0.111	0.392	0.869	0.0084	0.099
11		650	0.318	0.894	0.0074	0.104	0.409	0.861	0.0095	0.093
12		700	0.333	0.889	0.0083	0.098	0.428	0.857	0.0107	0.088
13		750	0.348	0.884	0.0093	0.093	0.446	0.851	0.0120	0.083
14		800	0.364	0.879	0.0104	0.088	0.462	0.846	0.0132	0.080
15	16000	500	0.238	0.921	0.0037	0.135	0.319	0.894	0.0050	0.118
16		550	0.256	0.915	0.0044	0.125	0.339	0.887	0.0058	0.110
17		600	0.272	0.909	0.0051	0.116	0.358	0.881	0.0067	0.103
18		650	0.288	0.904	0.0058	0.109	0.378	0.874	0.0077	0.096
19		700	0.304	0.899	0.0067	0.102	0.397	0.868	0.0087	0.091
20		750	0.319	0.894	0.0075	0.096	0.414	0.862	0.0097	0.086
21		800	0.333	0.889	0.0083	0.092	0.429	0.857	0.0107	0.083
22	20000	500	0.200	0.933	0.0025	0.146	0.272	0.909	0.0034	0.127
23		550	0.217	0.928	0.0030	0.134	0.292	0.903	0.0040	0.118
24		600	0.232	0.923	0.0035	0.124	0.311	0.896	0.0047	0.109
25		650	0.246	0.918	0.0040	0.117	0.328	0.891	0.0053	0.103
26		700	0.259	0.914	0.0045	0.110	0.344	0.885	0.0060	0.097
27		750	0.272	0.909	0.0051	0.104	0.359	0.880	0.0067	0.092
28		800	0.285	0.905	0.0057	0.098	0.374	0.875	0.0075	0.087
29	24000	500	0.172	0.943	0.0018	0.157	0.240	0.920	0.0025	0.135
30		550	0.185	0.938	0.0021	0.145	0.256	0.915	0.0029	0.125
31		600	0.200	0.933	0.0025	0.134	0.272	0.909	0.0034	0.116
32		650	0.213	0.929	0.0029	0.124	0.288	0.904	0.0039	0.109
33		700	0.226	0.925	0.0033	0.117	0.303	0.899	0.0044	0.103
34		750	0.238	0.921	0.0037	0.111	0.319	0.894	0.0050	0.096
35		800	0.251	0.916	0.0042	0.105	0.334	0.889	0.0056	0.092

NOTE—For intermediate stresses, interpolate.

TABLE 11. DATA FOR DETERMINING DEPTH OF RECTANGULAR BEAM OR SLAB OR MOMENT OF RESISTANCE FOR DIFFERENT PERCENTAGES OF STEEL.

Ratio of elasticity, $n = 15$.

Rule 1. To find depth of beam or slab for a given percentage of steel:

On line with the given percentage, select the higher value of C. This, substituted in formula

$$d = C\sqrt{\frac{M}{b}}$$

(see p. 418), gives the smallest permissible depth. Thus for 0.004 steel ratio the value of C from column (9) must be used instead of from column (6) because the latter would stress the steel to 23 700 pounds, which would not be allowable. It is evident also that the ratio of steel is too low for economy, because concrete is stressed only to 440 pounds.

Rule 2. To find amount of steel for a given beam or slab and given loading with stress in concrete limited to 650 pounds per square inch and stress in steel to 16 000 pounds per square inch:

Compute value of C from formula $M = \frac{bd^2}{C^2}$ (see p. 754). Locate this value either in column (6) or (9), whichever satisfies the allowed stresses, and find the corresponding value of p in the first column. Thus, if $C = 0.097$, it must be located in column (9) instead of column (6), because the latter would give a higher stress in steel than is allowable. The desired ratio of steel is therefore 0.0077. If $C = 0.088$, it must be located in column (6) because column (9) would give too high a stress in concrete.

Ratio area of steel to beam above steel.	Ratio depth of neutral axis to depth of steel.	Ratio moment arm to depth of steel.	Working compressive strength of concrete Lb. per sq. in.	Maximum fibre stress in steel corresponding to $f_c = 650$	Constant in formula $d = C\sqrt{\frac{M}{b}}$ see page 418	Working tensile strength of steel Lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s = 16000$	Constant in formula $d = C\sqrt{\frac{M}{b}}$ see page 418
p (1)	k (2)	j (3)	f_c (4)	f_s (5)	C (6)	f_s (7)	f_c (8)	C (9)
0.002	0.217	0.928	650	32900	0.124	16000	290	0.183
0.003	0.258	0.914	650	28000	0.114	16000	370	0.151
0.004	0.292	0.903	650	23700	0.108	16000	440	0.132
0.005	0.320	0.893	650	20800	0.104	16000	500	0.118
0.006	0.344	0.885	650	18600	0.100	16000	560	0.108
0.007	0.365	0.878	650	16900	0.098	16000	610	0.101
0.008	0.384	0.872	650	15600	0.096	16000	670	0.095
0.009	0.402	0.866	650	14500	0.094	16000	720	0.089
0.010	0.418	0.861	650	13600	0.092	16000	760	0.085
0.012	0.446	0.851	650	12100	0.090	16000	860	0.078
0.014	0.471	0.843	650	11000	0.088	16000	950	0.072
0.016	0.493	0.836	650	10000	0.086	16000	1040	0.068
0.018	0.513	0.829	650	9300	0.085	16000	1120	0.065
0.020	0.531	0.823	650	8600	0.084	16000	1210	0.061

TABLE 12. PROPORTIONAL DEPTHS OF NEUTRAL AXIS

The table below gives the proportional depths of the neutral axis calculated from formula (6) on page 420 for various percentages of steel and moduli of elasticity. Its use is *not* advised for ordinary calculations of moments of resistance and dimensions of beams or slabs, because it presents no means of determining, without further calculation, the stress in the steel or the concrete, and therefore is liable to lead to uneconomical design. Its principal use is for determining the moment of resistance, and consequently the safe loading for beams already built.

Proportional Depth of Neutral Axis below top of Beam for different per cents of Steel and various assumptions of Elasticity. (*See p.* 310.)

p Ratio of area of steel to area of cross-section of beam above steel.	k Ratio of depth of neutral axis to depth of center of steel below most compressed surface of beam. Ratios of Modulus of Elasticity of Steel to Modulus of Concrete in Compression, $\frac{E_s}{E_c} = n$								
	6	7.5	10	12	15	20	30	35	40
0.001	0.104	0.115	0.132	0.143	0.158	0.181	0.217	0.232	0.246
0.002	0.184	0.159	0.181	0.196	0.217	0.246	0.292	0.311	0.328
0.003	0.173	0.191	0.217	0.235	0.258	0.292	0.344	0.365	0.384
0.004	0.196	0.217	0.246	0.266	0.292	0.328	0.384	0.420	0.428
0.005	0.217	0.239	0.270	0.292	0.320	0.358	0.418	0.442	0.464
0.006	0.235	0.258	0.292	0.314	0.344	0.384	0.446	0.471	0.493
0.007	0.251	0.276	0.311	0.334	0.365	0.407	0.471	0.497	0.519
0.008	0.266	0.292	0.328	0.353	0.384	0.428	0.493	0.519	0.412
0.009	0.279	0.306	0.344	0.369	0.402	0.446	0.513	0.539	0.562
0.010	0.292	0.320	0.358	0.384	0.418	0.463	0.531	0.557	0.584
0.012	0.315	0.344	0.384	0.402	0.446	0.493	0.562	0.588	0.611
0.014	0.334	0.364	0.407	0.436	0.471	0.519	0.588	0.614	0.638
0.016	0.353	0.384	0.428	0.457	0.493	0.542	0.611	0.637	0.660
0.018	0.369	0.402	0.446	0.476	0.513	0.562	0.631	0.657	0.680
0.020	0.384	0.418	0.463	0.493	0.531	0.580	0.649	0.675	0.697
0.030	0.446	0.483	0.531	0.562	0.599	0.649			
0.040	0.493	0.531	0.580	0.611	0.649	0.697			
0.050	0.531	0.569	0.618	0.649	0.686	0.732			

DIAGRAM 1. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{8}$$

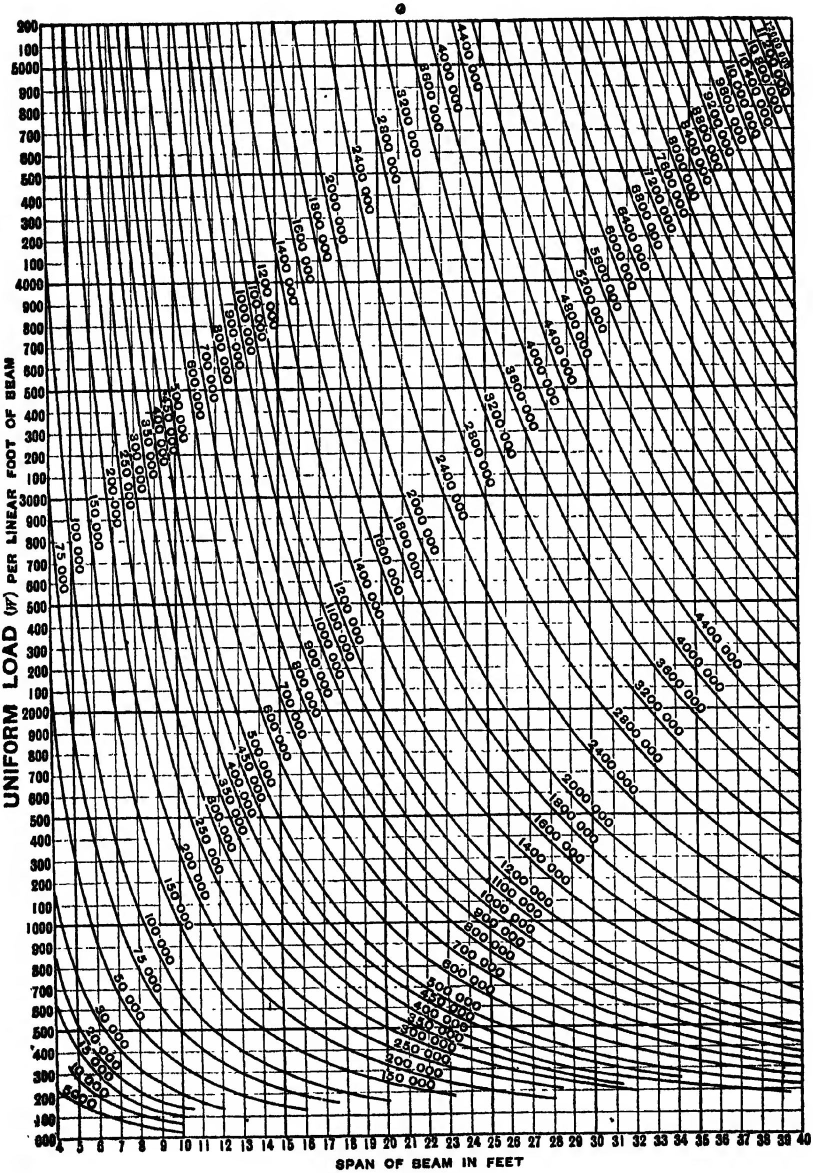


DIAGRAM 2. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{10}$$

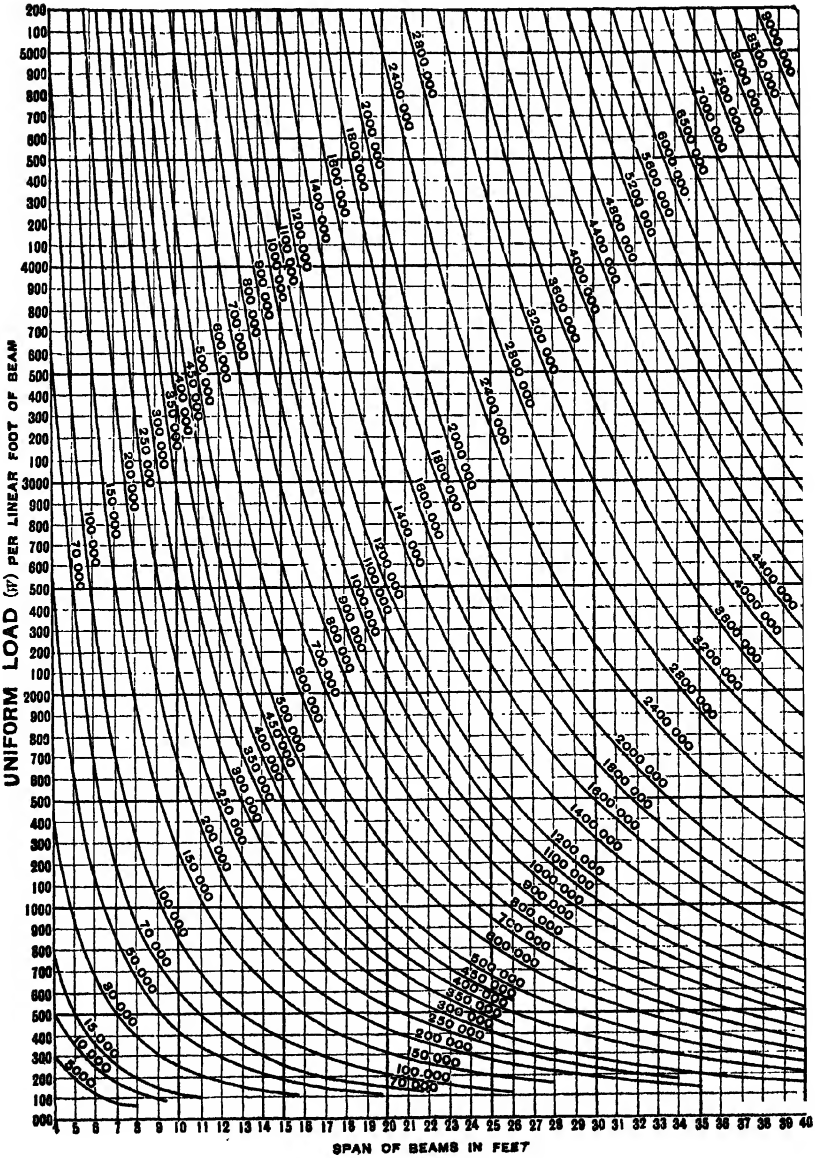


DIAGRAM 3. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{12}$$

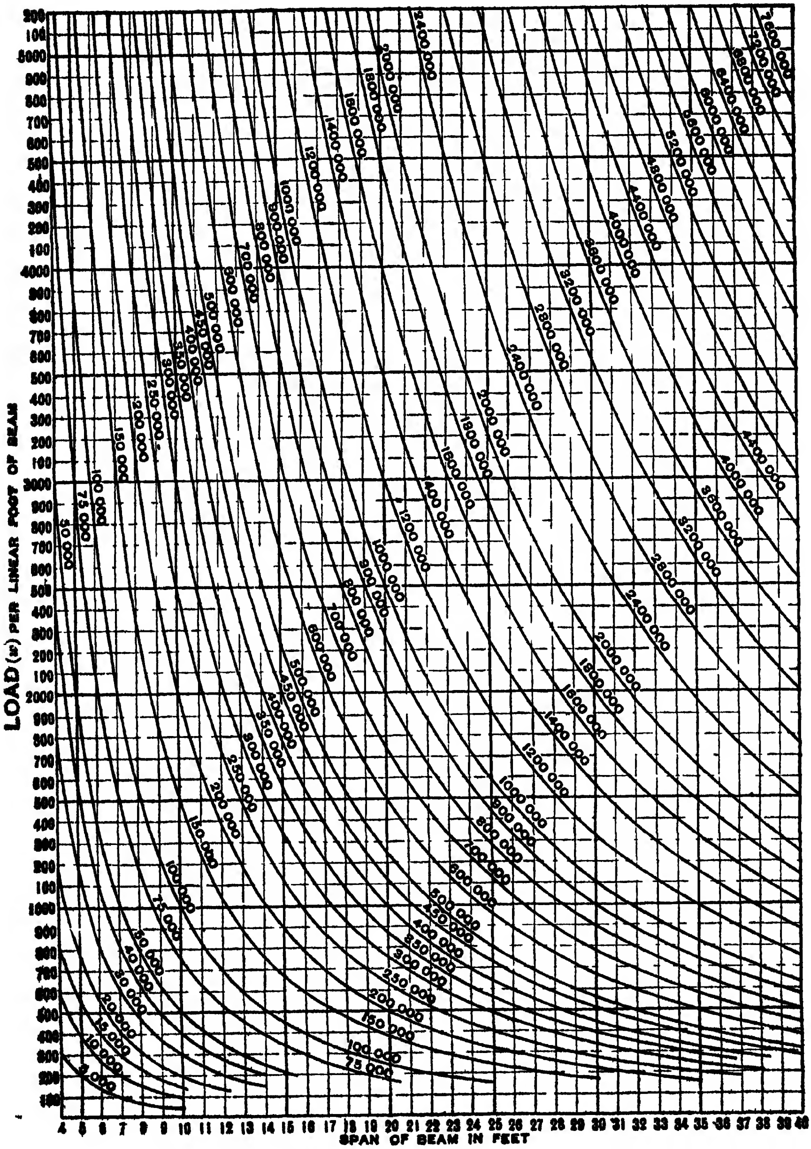


DIAGRAM 4. CURVES FOR DESIGN OF T-BEAMS.

Notation:

f_c = working compression in concrete in lb. per sq. in.
f_s = working tension in steel in lb. per sq. in.
b = breadth of flange in inches.
t = thickness of flange in inches.
d = depth of steel in T-beam in inches.

jd = moment arm, approximately equal to $d - \frac{t}{2}$.

k = ratio of depth of neutral axis to depth of steel.

Diagram is made from formulas (14) to (19), pages 755 and 756.

Curves at left, of Moment Arm, are applicable to all conditions.

Curves at right may be used for any combination of stresses having k as given in diagram. For other stresses find value of k in Table 10, page 519, and interpolate between the curves.

To Determine Minimum Depth of a T-beam Consistent with a Working Compression in Concrete. Enter Diagram 4 at top with value specified for maximum working compression f_c in concrete times bt, which is assumed breadth of flange of T-beam, times thickness of slab. Follow this line down vertically until it intersects the slant line representing the previously assumed relation of thickness of slab to depth of steel. From point of intersection of these two lines follow horizontal line across the diagram to the left till it intersects a vertical line corresponding to required moment in inch-pounds read from bottom of diagram. This will give **minimum value** of jd, the distance between center of gravity of steel and center of compression in concrete. The depth from surface of beam to steel is $d = jd + \frac{1}{2}t$.

If assumed relation between d and t does not correspond to actual, repeat the operation.

No smaller depth than the minimum can be used. Larger depths than the minimum are usually economical (see pp. 424, 425).

To Determine Area of Steel in a T-beam Consistent with a Working Tension in the Steel. Enter diagram at bottom of page with bending moment in inch-pounds and follow this line vertically upwards till it intersects slant line jd, which is the distance between the center of the steel and center of compression of the concrete and is approximately equal to $d - \frac{1}{2}t$. From intersection of these two lines follow horizontal line to left and read off directly the area of steel required in square inches corresponding to the specified working stress in the steel.

To Determine Total Compression in Flange of a T-beam $\left(f_c \frac{2kd - t}{2kd} bt\right)$

Enter table at bottom with moment in inch-pounds, follow this line up vertically till it intersects slant line jd. From point of intersection follow the horizontal line to the right hand column of figures, which will give the total compression in the flange of a T-beam.

To Determine Maximum Fiber Stress f_c. Determine total compression as above. Equate this value to $f_c \dfrac{2kd - t}{2kd} bt$. Assume a value for k and compute f_c. Determine value of f_s. Refer to Table 10, page 519, and see if value of k corresponds to f_c and f_s. If not, select a new k and recompute.

WORKING STRESSES IN REINFORCED CONCRETE

The Joint Committee on Concrete and Reinforced Concrete, 1909, recommend working stresses as follows.*

General Assumptions. The following working stresses are recommended for static loads. Proper allowances for vibration and impact are to be added to live loads where necessary to produce an equivalent static load before applying the unit stresses in proportioning parts

In selecting the permissible working stress to be allowed on concrete, we should be guided by the working stresses usually allowed for other materials of construction, so that all structures of the same class but composed of different materials may have approximately the same degree of safety.

The stresses for concrete are proposed for concrete composed of one part Portland cement and six parts aggregate, capable of developing an average compressive strength of 2 000 pounds per square inch at twenty-eight days when tested in cylinders 8 inches in diameter and 16 inches long, under laboratory conditions of manufacture and storage, using the same consistency as is used in the field In considering the factors recommended with relation to this strength, it is to be borne in mind that the strength at twenty eight days is by no means the ultimate which will be developed at a longer period and therefore they do not correspond with the real factor of safety On concretes in which the material of the aggregate is inferior, all stresses should be proportionally reduced, and similar reduction should be made when leaner mixes are to be employed On the other hand, if, with the best quality of aggregates, the richness is increased, an increase may be made in all working stresses proportional to the increase in compressive strength at twenty eight days, but this increase shall not exceed 25 per cent

Bearing † For compression on surface of concrete larger than loaded area

32 5 per cent of compressive strength at twenty-eight days, or 650 pounds per square inch on 2 000 pound concrete

Columns. (a) Plain columns or piers whose length does not exceed twelve diameters,

22½ per cent of compressive strength at twenty eight days, or 450 pounds per square inch on 2000 pound concrete

(b) Columns with reinforcement of bands or hoops,‡

27 per cent of compressive strength at 28 days, or 540 pounds per square inch on 2000 pound concrete.

Vertical steel reinforcement 8100 pounds per square inch.

* The form in which these are given corresponds with the 1909 Report of the Reinforced Concrete Committee of the National Association of Cement Users

† For beams and girders built into pockets in concrete walls, the lower compressive stress of 450 pounds per square inch should not be exceeded

‡ The amount of band or hoop reinforcement must be at least 1 per cent of the volume of the column enclosed, and clear spacing of the bands or hoops not greater than one-fourth the diameter of the enclosed column

(c) Columns reinforced with not less than 1% and not more than 4% of longitudinal bars and with bands or hoops spaced not greater than one-fourth the diameter of the enclosed column,

32½% of compressive strength at 28 days, or 650 pounds per square inch on 2000 pound concrete.

Vertical steel reinforcement, 9750 pounds per square inch.

(d) Columns reinforced with structural steel column units which thoroughly encase the core,

32½% of compressive strength at 28 days, or 650 pounds per square inch on 2000 pound concrete.*

Vertical steel, 9750 pounds per square inch.

Compression in Extreme Fiber. For extreme fiber stress of beams calculated for constant modulus of elasticity,

32.5 per cent of the compressive strength at twenty-eight days, or 650 pounds per square inch for 2000 pound concrete.

Adjacent to the support of continuous beams, stresses 15 per cent greater may be allowed.

Shear. Pure shearing stresses uncombined with compression or tension, 6 per cent of compressive strength at twenty-eight days, or 120 pounds per square inch for 2000 pound concrete.

Diagonal Tension. In beams where diagonal tension is taken by concrete, the vertical shearing stresses should not exceed

2 per cent of compressive strength at twenty-eight days, or 40 pounds per square inch for 2000 pound concrete.

Bond for Plain Bars. Bonding stress between concrete and plain reinforcing bars,

4 per cent of compressive strength at twenty-eight days, or 80 pounds per square inch for 2000 pound concrete.

For drawn wire.

2 per cent, or 40 pounds on 2000 pound concrete.

Bond for Deformed Bars.† Bonding stress between concrete and deformed bars may be assumed to vary with the character of the bar from

5 per cent to 7½ per cent of the compressive strength of the concrete at 28 days, or from

100 to 150 pounds per square inch for 2000 pound concrete.

* Lower stresses than these should be used unless the concrete is very carefully proportioned and placed. The authors recommend 500 lb. per sq. in. in general practice.

† No recommendation for deformed bars is given in the report of the Joint Committee; the values given are those suggested by the Reinforced Concrete Committee of the National Association of Cement Users.

Reinforcement. The tensile stress in steel should not exceed 16 000 pounds per square inch.* The compressive stress in reinforcing steel should not exceed 16 000 pounds per square inch, or fifteen times the working compressive stress in the concrete.

Modulus of Elasticity. It is recommended that in all computations the modulus of elasticity of concrete be assumed as $\frac{1}{15}$ that of steel; that is, that a ratio of fifteen be employed.

STANDARD NOTATION†

t = thickness of slab, *i.e.*, thickness of T-flange.
b = breadth of beam; in a T-beam, breadth of T-flange.
b' = breadth of stem of T-Beam.
d = depth from surface of beam to center of tension steel.
p = ratio of cross-section of steel in tension to cross-section of beam above this steel.
p' = ratio of cross-section of steel in compression to cross-section of beam above the steel in tension.
f_c = unit compressive stress in outside fiber of concrete.
f_s = unit tensile stress, or pull, in steel.
f'_s = unit compressive stress in steel.
f = average unit compression in a column.
n = $\frac{E_s}{E_c}$ = Ratio of modulus of elasticity of steel in tension to modulus of elasticity of concrete in compression.
k = ratio of depth of neutral axis to depth of steel in tension.
kd = distance from outside compressive surface to neutral axis in beam in which the depth to steel in tension is d.
j = ratio of lever arm of resisting couple to depth d.
jd = arm of resisting couple.
z = depth from surface of beam to center of compression.
M = moment of resistance or bending moment in general.
M_b = bending moment.
M_r = resisting moment.
M_2 = bending moment in a flat slab causing radial fibre stress for loading distributed along the edge of the fixed plate.
M_b = bending moment in a flat slab causing radial fibre stress for loading uniformly distributed over the plate.
V = total shear.
v = unit shear.
v' = unit working shear.
u = unit bond.
o = circumference of one bar.
Σo = total circumference of all bars in a beam.
A = total area.
A_s = area of steel.
w = unit loading for uniformly distributed load.
q = unit loading for circumferential loading.
i = diameter of bar.
r = ratio unit cost of steel to unit cost of concrete.
l = span of beam.

*If the steel has a high elastic limit and is of the exceptional quality called for by the specifications on page 38, the authors would frequently permit a stress as high as 20 000 pounds per square inch.

†Substantially as adopted by the Joint Committee on Concrete and Reinforced Concrete and as used in this Treatise.

l_l = longer span of a rectangular slab.
l_s = shorter span of a rectangular slab.
M_l = bending moment of longer beam.
M_s = bending moment of shorter beam.
α = denominator in bending moment formula $M = \frac{wl^2}{\alpha}$
m = number of bars at the center of beam.
m_1 = number of bars to be bent.
C = constant from Table 10, p. 519.
C_r, C_s, C'_s, = constant from Table 8, p. 516.
C_1, C_2, C_a, C_b. = constant from Table 9, p. 518.
C_s, C_b = constant from Table on p. 454.
r_0 = inner radius of flat plates in feet.

COMMON FORMULAS.

Rectangular beam:

Formula	Reference to Page
Depth of steel, $d = C\sqrt{\frac{M}{b}}$	418
Tension in steel, $f_s = \frac{M}{A_s jd}$	420
Unit shear, $v = \frac{V}{bjd}$	447
Ratio span to depth not requiring stirrups, $\frac{l}{d} > \frac{\alpha}{1.74\, C^2 v}$	456
Steel area, $A_s = pbd$	418
Compression in concrete, $f_c = \frac{2M}{bd^2 jk}$	420
Unit bond, $u = \frac{V}{jd\, \Sigma o}$	457

T-Beam:

Formula	Reference to Page
Horizontal steel, $A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)}$ (approx.)	426
Area required by shear, $b'\left(d - \frac{t}{2}\right) > \frac{V}{120}$	424
Economical depth, $d - \frac{t}{2} = \sqrt{\frac{rM}{f_s b'}}$	425
Length of haunch, $x = \frac{l}{5} \cdot \frac{M_b - M_r}{M_b}$ (approx.)	430
Depth of neutral axis, $kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt}$	755
Moment arm, $jd = d - z$	755
Tension in steel, $f_s = \frac{M}{A_s jd}$	756
$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3}$	755
Compression in concrete, $f_c = \frac{Mkd}{bt\left(kd - \frac{1}{2}t\right)jd}$	756

Beam with Steel in Top and Bottom:

Moment, $M = f_c bd^2 C_c$ or $= f_s bd^2 C_s$ } which ever is larger 428

Tension in steel, $f_s = \dfrac{M}{bd^2 C_s}$ 428

Compression in concrete, $f_c = \dfrac{M}{bd^2 C_c}$ 428

Compression in steel, $f'_s = \dfrac{M}{bd^2 C'_s}$ 428

Flat Plates

Radial loading,
Max. $M_2 = wr_0^2\,(0.2 + C_1 + C_2)$ 486

Circumferential loading,
Max $M_b = qr_0\,(C_a + C_b)$ 486

Column:

Ratio steel, $p = \dfrac{f - f_c}{f_c\,(n - 1)}$ 491

Area column, $A = \dfrac{P}{f_c\,[1 + (n - 1)\,p]}$ 491

Average unit load,
$\dfrac{P}{A} = f = f_c\,[1 + (n - 1)\,p]$ 491

Stirrups:

Area of stirrups, $A_s = \dfrac{2}{3}\,\dfrac{sV}{f_s jd}$ 449

Spacing of stirrups, $s = \dfrac{3}{2}\,\dfrac{A_s f_s jd}{V}$ 450

Distance from support where no stirrups needed $x = \dfrac{l}{2} - \dfrac{vbjd}{w}$ 451

Max distance from support where rods may be bent, $x_1 = \dfrac{l}{2}\left(\frac{1}{2} - \sqrt{\dfrac{8\,m_1}{\alpha\, m}}\right)$ 459

Diameter vertical stirrup for square or round bar, $i < C_s d$ 454

For other shapes, $\dfrac{A_s}{o} < \dfrac{1}{4}\,C_s d$ 454

Diameter inclined stirrup for square or round bar, $i < C_b d$ 454

For other shapes, $\dfrac{A_s}{o} < \dfrac{1}{4}\,C_b d$ 454

Distribution of slab load to supporting beams:

For longer beam
$M_l = \frac{1}{8}\,w\,l_l^2\left(1 - \dfrac{1}{3}\,\dfrac{(l_s)^2}{(l_l)^2}\right)$ 431

For shorter beam,
$M_s = \frac{2}{3}\,(\frac{1}{8}\,wl_s^2)$ 431

FIG. 156. Walnut Lane Bridge, Philadelphia.

CHAPTER XXII

ARCHES*

BY FRANK P. McKIBBEN

The treatment of arch design by what is termed the elastic theory, although generally considered a complicated problem, as a matter of fact is easily handled by one who is familiar with elementary mechanics and with the principles of reinforced concrete beam design. The process is necessarily somewhat lengthy, involving extended operations in simple arithmetic, but by following the analysis presented in the following pages it can be readily understood. It is doubtful whether in the whole category of the design of structures there is a prettier application of mechanics and mathematics than the design of a reinforced concrete arch bridge.

While in a volume of this size it is impossible to present all phases of the subject, the underlying principles are treated in sufficient detail and with a discussion thorough enough to permit an engineer to safely design an arch.

Following a brief historical introduction discussing the use of concrete versus steel construction, the different forms of arches are reviewed with suggestions for design; the loading for different conditions is scheduled (p. 541); the outer forces are analyzed, including the effect of temperature (p. 553); the method of procedure to be followed in arch design is taken up in a practical example item by item (p. 574); allowable unit stresses are suggested (p. 583); the design of abutments is outlined (p. 583); and a few illustrations of existing bridges are presented

Girder bridges are not treated specifically in this chapter, but they may be readily designed by applying the principles of reinforced concrete beam and slab construction as treated in Chapter XXI on Reinforced Concrete.

The treatment of conduit or sewer arches which are so deeply imbedded as to require computations for earth pressure is referred to on page 693.

Perhaps the most interesting feature of the present chapter is the complete analysis of a typical arch which is presented on page 574. The steps to be followed are outlined consecutively and the mathematical processes indicated in full.

The formulas for distribution of stress given on page 560 apply not only to arch design but also to column and beam design where there is eccentric

*The authors are indebted to Prof. McKibben for this chapter, which has been especially prepared by him for this treatise.

loading or thrust in place of or in addition to the ordinary loads. To facilitate the understanding of the formulas, a departure is made from the usual notation schedule, which must necessarily be several pages away from the work, by placing in addition, at the bottom of each page, a brief definition of all the symbols used on that page.

CONCRETE VERSUS STEEL BRIDGES

Reinforced concrete, either as arch or girder spans, is being used not only in preference to steel trusses or steel girders, where the stone arch is too expensive to be considered, but the concrete bridge is frequently replacing the old steel structure. The reasons generally conceded for this widespread growth may be briefly stated as: (1) greater durability; (2) less cost of maintenance; (3) less vibration and less noise; (4) more æsthetic effects.

The relative first cost for concrete and steel depends upon the local conditions. In many places a concrete bridge can be built for less than a first-class steel span, although it cannot so readily compete with the flimsy trussed spans frequently seen. The concrete may be laid with less skilled labor than the steel bridge, but since the concrete structure is built on the spot, while the steel is prepared in an established shop, even more careful supervision and inspection are necessary with the concrete. The foundations for a concrete arch are frequently more expensive than concrete abutments for a steel truss because of the greater area required to take the thrust, while on the other hand, in rock or other hard material, a less quantity of concrete may be required for the arch abutments. This part of the design may often be the determining feature from the economical standpoint.

The most serious objection to steel, especially for highway bridges, lies in the fact that unprotected it cannot resist for a great length of time the oxidation due to air, water and locomotive gases, and unless properly cared for and frequently painted, it rusts badly. The examination by the author of this chapter of approximately 600 highway bridges carrying electric railways proves that frequently these bridges are not properly maintained, many of them receiving little or no attention for years at a time, so that the structures are often badly corroded, and in fact, cases are on record where subordinate members of steel bridges have rusted away completely in less than fifteen years.

In a concrete bridge the steel is effectively prevented from rusting by the concrete in which it is imbedded (see p. 327), so that, when properly designed and built, no repairs whatever should be required, and no limit can be placed upon the life of the bridge.

Concrete is strongest in compression, and is therefore eminently suitable for use in arch spans where the stresses are largely compressive. The mass of the concrete and the quantity of earth filling or ballast over the arch so deaden the impact due to traffic that in many cases no impact allowance need be made, while at the same time the noise and vibration which occur in steel spans are avoided.

USE OF STEEL REINFORCEMENT

The use of steel reinforcement in a concrete arch is desirable but not absolutely necessary, as it is possible to construct a concrete arch like the Walnut Lane Bridge in Philadelphia (see pp. 532 and 592) with the concrete laid in blocks, each block forming a voussoir like the stones in a masonry arch. At the same time under ordinary conditions, while the introduction of steel does not, with the present knowledge of concrete arch design, permit great diminution in section, it does give considerable added strength at comparatively low cost and may prevent the formation of cracks in the concrete and take tension caused by any unforeseen action of the arch, such as settlement of foundations, improper allowance for temperature or shrinkage of the concrete while hardening.

The area of the cross section of the longitudinal steel bars in solid arch rings is to a certain extent arbitrary. Good practice sanctions ½% to 1½% of the ring at the crown and the exact quantity to use must first be selected by judgment, and then tested by the computation and revised if necessary.

As in column design (see p. 489), it is impossible to stress the steel in compression to an amount ordinarily proper in structural steel work, because in so doing the deformation would be so great as to overstress the concrete. The actual compressive stress in the steel, therefore, can never be greater than the working stress in the concrete multiplied by the ratio of the modulus of elasticity of steel to that of concrete. Under ordinary conditions this limit on the steel may be taken as 7500 pounds per square inch.

Since the beginning of this century there has been a remarkable development in methods of construction and in our knowledge of the principles of reinforced concrete arch bridges, but even yet engineers incline to employ a somewhat excessive quantity of concrete in the solid rings of ordinary highway concrete arches. This is frequently out of proportion to the quantity of material used in a reinforced concrete ribbed arch or a steel arch. Improvements in arch design evidently lie, as is indicated in subsequent pages, in the substitution of comparatively narrow ribs for solid arches and in the

use of hollow abutments with earth filling in place of solid concrete abutments. This will considerably reduce the cost of reinforced concrete arches.

HISTORY OF CONCRETE ARCH BRIDGES

In the development of concrete bridges it is natural that the arch rather than the beam should have been the first type of bridge to be constructed. It was a comparatively short step from the stone voussoir arch to the concrete voussoir or to the monolithic arch. One finds therefore many concrete arch bridges, and, until recently, few beam bridges, although for short spans beam bridges are now being constructed in considerable numbers, both in this country and abroad.

The first plain concrete arch of any importance was built in Europe in 1869 and is known as the Grand Maitre bridge at Fontainebleu Forest. It has a maximum span of 115.8 feet and carries the aqueduct of the Paris waterworks from Vanne. The first plain concrete arch in the United States was constructed in 1871 by John C. Goodridge in Prospect Park, Brooklyn, and has a span of 31 feet. The earliest reinforced concrete arch in Europe of which there is a well defined record was built in Copenhagen, Denmark, in 1879, with a span of 71.7 feet. It is probable, however, that Jean Monier of Paris was the inventor of the reinforced concrete arch and that he built some bridges before the dates mentioned. In the United States the first reinforced concrete arch on record was erected in 1889, with a span of 35 feet, by Ernest L. Ransome at Golden Gate Park in San Francisco.

When these structures are compared with the 233 feet span of the Walnut Lane Bridge in Philadelphia, which in 1908 was, with perhaps one exception, the longest plain concrete arch in existence, with the 230 feet, 3-hinge Grünwald Arch at Munich, Bavaria, or still more sharply with the Hudson Memorial design for an arch across the Spuyten Duyvil Creek with a span of 703 feet, a wonderful development is observed.

Although in a very few cases concrete bridges built during this development have failed, every such failure can be traced to a direct disregard of well known principles of design or construction. Moreover, as a matter of fact, accidents to concrete arches have been much fewer than the failures of wrought iron or steel bridges during the corresponding period of metal bridge development.

CLASSIFICATION OF ARCHES

Arches in general may be classified with reference to the material of which they are made, the arrangement of the spandrels and arch rings, or the

number of hinges. Reinforced concrete arches may be divided as to the arrangement of the reinforcement into three groups: the Monier, Melan and Wünsch types. The Monier arch in its developed form is the type most commonly used in the United States. This system of reinforcement was invented by Jean Monier about the year 1876. As first devised, a wire netting was imbedded in the concrete near the soffit, but later two nettings were used, one near the soffit, and the other imbedded in the concrete near the extradosal surface. Wire netting of small mesh with wires of equal size in both directions obviously is not well suited for use in an arch and considerable improvement was soon effected in this type by making the longitudinal bars of the reinforcement heavier than the transverse.

In the usual design a layer of longitudinal bars is imbedded near the intrados and an equal number near the extrados, the bars of the two layers being connected with small bars or stirrups. Transverse bars, at right angles to the longitudinal, form with them a netting both in the top and bottom of the arch. They serve to prevent cracks in the concrete and distribute the loads laterally. These cross bars also act with the stirrups in holding the longitudinal bars in place during construction.

The principal longitudinal bars are designed to carry tension due to the bending moment and to assist the concrete in compression caused by the thrust and the bending moment.

Melan Type. This system was invented by Joseph Melan of Brünn, Austria, in 1892. The reinforcement consists of curved steel ribs imbedded in the concrete and extending from abutment to abutment. For short spans the ribs are simply curved I-beams and for long spans each rib is made of two angles near the extrados latticed to two angles near the intrados. The built-up ribs thus formed are usually deeper at the springings than at the crown of the arch. The principal function of the lattice bars is to hold the angles in position when the latter are stressed, and to make a unit which is easy to handle during erection. By far the most important function of steel reinforcement is to carry bending moment, and the steel in the Melan type can be easily placed and kept in position during erection so as to fix positively its location in the finished structure. The material in the lattice bars of the ribs or in the webs of the I-beams is not economically placed. The first Melan arch in the United States, of 30 feet span, was erected at Rock Rapids, Iowa, in 1894, and many other bridges have since been built of this system.

Wünsch Type. Comparatively few bridges have been constructed on this system. The arch, which was invented by Robert Wünsch of Budapest, Hungary, in 1884, has a horizontal extrados and a curved intrados and the

reinforcement of the arch ring consists of steel ribs spaced from 1½ to 2 feet apart, with a horizontal upper member placed near the extrados and a curved lower member near the intrados. The two members are connected at each abutment to a vertical member imbedded in the concrete. The bridge at Sarajevo in Bosnia, of 83 feet span, is one of the largest built of the Wünsch system.

ARRANGEMENT OF SPANDRELS AND RINGS

The spandrel, which is the space between the roadway surface and the top or extrados of the arch ring, may be treated in one of two ways. First, it may be entirely filled with earth or with concrete which carries the roadway; or, second, it may be left more or less open, and the roadway supported upon a deck carried on a series of transverse walls, longitudinal walls, or columns resting upon the arch ring.

Filled Spandrels. In this form of construction the earth or concrete filling rests directly upon the arch ring, and is held in place laterally by retaining walls which also rest upon the arch ring. As the depth of these walls, unless they are of reinforced design, increases from the crown to the springing, their thickness, designed to resist the earth pressure, also increases until at the abutments the spandrels may be largely filled with the concrete composing the side walls.

If the side walls simply rest upon the arch ring, a crack is liable to form at the junction of ring and wall due to the deflection of the arch ring from the weight of the earth upon it. On the other hand, if the ring and wall are connected by sufficient steel to prevent the formation of this crack, indeterminate stresses are set up which are undesirable and which may result in transferring the crack to another place. This danger may be obviated by building the spandrel walls as gravity walls, leaving a vertical expansion joint at each junction of spandrel and wing walls and at some intermediate point between this joint and the crown.

Another plan is to build thinner reinforced side walls as vertical slabs tied together, with the lateral pressure resisted by reinforced cross walls. The principal objections to the use of solid fillings are as follows: (1) They increase the weight of the superstructure, and consequently thicker arch rings and larger foundations are required. (2) Unless the earth filling is carefully compacted by rolling, tamping or wetting, it will sink and allow the roadway to settle with it. (3) It is difficult to make the side walls and the arch ring act in unison, and unsightly cracks may be formed. Filled spandrels may be therefore limited properly to bridges with solid arch

rings of short span, say not over 80 feet, or to those having a rise of less than $\frac{1}{10}$ the span, where the cost of form construction prohibits an open design.

Open Spandrels. The objections just mentioned to the use of filled spandrels are of such importance that during the last few years the use of open spandrels in the larger structures has made rapid progress. In addition to being lighter, the open spandrel construction facilitates inspection and lends itself to more pleasing architectural treatment. It permits indeed a treatment peculiar to concrete, which does not follow the type of design used for so many centuries in stone arch bridges. With open spandrels the roadway may be laid upon small arches or upon I-beams carried by transverse or longitudinal walls which in turn rest upon the arch ring; or it may be laid with reinforced concrete beam and slab construction, making a floor similar to those used in reinforced concrete buildings. The beams in this case are placed longitudinally with the roadway, and rest upon transverse walls.

Upon the adoption of the open spandrel it was soon seen that considerable material was wasted in the transverse walls and in the solid arch rings. The next step, therefore, was to reduce the walls to columns and the ring to a series of longitudinal ribs spaced similarly to the ribs of a steel arch. In some cases these ribs are very wide, in fact, are really two independent arch rings as in the Walnut Lane bridge, Philadelphia,* and in other cases the ribs are narrow as in the Rock Creek bridge on Ross Drive in the District of Columbia.†

HINGES

The use of hinges in concrete arches is by no means of recent origin. As early as 1873, an arch was constructed near Erlach, Germany, with three asphalt "joints" and many others with hinges have been built since then. The chief object of the hinge in the arch rings or ribs is to render the structure more nearly determinate.

Although two or even one hinge can be used, three hinges offer the advantage of definitely fixing the pressure line throughout the ring so that it can be easily and accurately located. Except for the friction of the hinges, the stresses are practically independent of changes of temperature or of any reasonable settlement of the foundations. On the other hand, the hinges are often an expensive detail. It is sometimes claimed also that three-hinged arches are not so rigid as fixed arches, but because of their great weight this criticism does not appear to be well founded.

*See p. 592.
†See p. 590.

In the design of a hinged structure the moment is usually assumed to be zero at the hinge. This assumption is not strictly correct because as the structure deforms under its load it tends to rotate about its hinges and this produces friction at the hinge due to the thrust acting thereon.

The design of the hinge is a most important feature. One of the most instructive failures in arch construction was that of the Maximilian Bridge at Munich, a three-hinged voussoir masonry arch of two spans, each 144.3 feet, when during construction, both spans of the bridge slipped off the hinges at the springings and dropped about 12 inches. This failure was due to an error in the design of the hinges. The bearing surfaces of the hinges were not given sufficient curvature, and the friction which was relied upon to prevent slipping of the two parts composing each hinge was reduced to a minimum by the use of a lubricant, which gave a low coefficient of friction.

Three-hinged construction is best suited to arches of small rise where the center line of the rib can be made to fit closely the line of pressure resulting in small bending moments. Arches with one or two hinges are more indeterminate than three-hinged arches and have practically all of the disadvantages of both the fixed and the three-hinged types.

SHAPE OF THE ARCH RING

For hingeless arches the intrados should be either three-centered, five-centered or elliptical, while, if desired, the extrados may be the arc of a circle so placed as to give greater depth to the arch ring at the springings than at the crown. A segmental arch, that is an arch formed by the segment of a single circle cannot often be used to advantage, for it seldom can be made to fit the line of pressure. While many arches are elliptical in form, the three-centered intrados is perhaps the most common and it is pleasing to the eye, easily constructed and gives an economical design.

Ribs with three hinges should be deepest at sections nearly midway between the crown and spring hinges, decreasing in depth toward the hinges, since sections near the hinges take only thrust and shear with practically no moment, while the intermediate sections resist a moment in addition to the thrust and shear.

THICKNESS OF RING AT CROWN

The next step in the design of an arch after deciding on the shape of the intrados is to choose a trial thickness of the ring at the crown and at the springing. The choice may be made by judgment based on experience or

with the aid of one of the various empirical formulas in use. Since the crown thickness depends not only on the amount of thrust but also upon the bending moment, which varies greatly in a given arch due to the varying positions of the live load, it is difficult and in fact impossible to devise a rational formula for its determination.

The thickness of the arch ring should vary with the shape of the arch, with the span, rise, amount of filling over the ring, the amount of live load and the material of which the arch is made, and while there is no formula that will apply even approximately in all cases, the formula by Mr. F. F. Weld* gives fairly correct results in ordinary cases. It is as follows:

Let

h = crown thickness in inches.

L = clear span in feet.

w = live load in pounds per square foot, uniformly distributed.

w' = weight of fill at crown in pounds per square foot.

Then

$$h = \sqrt{L} + \frac{L}{10} + \frac{w}{200} + \frac{w'}{400}. \qquad (1)$$

Obviously the thickness for a hingeless arch should increase from the crown to the springing. The radial thickness of the ring at any section is frequently made equal to the thickness at the crown multiplied by the secant of the angle which the radial section makes with the vertical. For a 3-centered intrados and an extrados formed by the arc of a circle, these trial curves may be at the quarter points a distance apart of $1\frac{1}{4}$ to $1\frac{1}{3}$ times the crown thickness and at the springings 2 to 3 times the crown thickness.

These empirical rules should be used only in preliminary study and *never for the final design.* The true shape of the ring and the thickness at different sections must be fixed by computation based on the line of pressure as described in the pages which follow.

LIVE LOADS FOR HIGHWAY BRIDGES

For highway bridges the kind and magnitude of the live load depend upon the location of the structure. Each location should be studied and the live load chosen to fit the requirements. The following classification is sufficient for stone or concrete arches and may also be applied to beam and slab construction.

**Engineering Record,* Nov. 4, 1905, p. 529.

City Bridges. For *floors* of city or other bridges carrying heavy traffic, three types of loads are recommended as follows:

1. A uniform live load of 100 pounds per square foot on sidewalks and roadways.

2. On each street railway track, one 8-wheel electric car having a wheel spacing of 5, 15, 5 feet between centers of wheels along one rail; each wheel carrying 12,500 pounds. The car is assumed to cover an area 9 feet wide by 40 feet long.

3. One wagon weighing 20,000 pounds on each of two axles 12 feet apart.

In applying these loads to find the maximum stress in the floor, either of the loads mentioned, or that combination of any of the above loads which produces the maximum stress, should be used. If the uniform load is used simultaneously with either of the concentrated loads, the former should cover only that part of the roadway not covered by the latter.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1800 pounds per linear foot of each railway track together with a uniform load of 100 pounds per square foot of remaining area of roadway and sidewalks.

For spans of 200 feet or more, a uniform load of 1200 pounds per linear foot of each railway track together with a uniform load of 80 pounds per square foot of remaining area of roadway and sidewalks.

The load on each track should be assumed to cover a width of 9 feet, thus giving 200 pounds per square foot under the track for spans of 100 feet or less and 133 pounds per square foot for spans over 200 feet in length.

For spans between 100 and 200 feet, the loads are to be taken proportionally.

Suburban, Town or Heavy Country Bridges. For *floors* of suburban, town, or heavy country bridges, the same uniform load and electric car load as for floors of city bridges but with wagon weighing 10,000 pounds on each of two axles 10 feet apart.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1800 pounds per linear foot of each track, together with a uniform load of 80 pounds per square foot of remaining area of roadway and sidewalks.

For spans of 200 feet or more the values corresponding to the above are 1200 pounds per linear foot of each track and 60 pounds per square foot of remaining area.

The load on each track should be assumed to cover a width of 9 feet.

For spans between 100 and 200 feet, the loads are to be taken proportionally between the limits stated.

Light Country Bridges. For *floors* of light country bridges, sub-

jected to light highway or electric railway traffic, on each track one 8-wheel electric car carrying 9000 pounds on each wheel, or one wagon weighing 6000 pounds on each of two axles 10 feet apart. These two loads should be assumed to act together where necessary to produce the maximum stress in the floor.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1200 pounds per linear foot of each track, together with a uniform load of 80 pounds per square foot of remaining area of roadway.

For spans of 200 feet or more, the values corresponding are 1000 pounds per linear foot of each track, and 50 pounds per square foot of remaining area.

For spans between 100 and 200 feet the loads are proportional between the limits stated.

It is customary to see that the design is sufficient to carry a steam road roller. The heaviest roller usually specified weighs 30,000 pounds, 12,000 pounds on the front roller, which has a width of 4 feet, and 9000 pounds on each of the two rear rollers, each of the latter having a width of 20 inches. The axles are taken as 11 feet apart and the two rear wheels as 5 feet center to center.

LIVE LOADS FOR RAILROAD BRIDGES

For railroad bridges the loading depends upon the location of the line, and hence the future traffic which may be expected. Two consolidated locomotives, with 25 000 pounds on each driving wheel, followed by 5000 pounds per foot of each track, is a common loading. An alternate plan quite generally followed for the rings of stone or concrete arches where the filling is of sufficient thickness to distribute the concentrated loads over a considerable area of arch ring is to use 5000 pounds per foot of track with no concentrated load. This load of 5000 pounds per foot of track is equivalent to about 625 pounds per square foot of horizontal area. These values are satisfactory for spans, say, over 80 feet in length.

Generally speaking, the shorter the span the greater should be the assumed uniform load, and hence for spans of, say, 80 feet or less, a uniform load of 1000 pounds per square foot is frequently adopted, this being approximately equivalent to the heaviest locomotive loadings.

A concentrated load on top of a fill is generally assumed to be distributed downward at angles of 45°. The top of the distributing slope may be taken from the ends of the ties. Wheel loads may be taken as distributed over 3 feet of length of surface of fill and at 45° angles through the filling.

DEAD LOADS AND EARTH PRESSURE

With open spandrels having columns or transverse walls, the dead loads act vertically upon the arch ring and can be more accurately found than with filled spandrels.

With spandrels filled with earth the dead load carried by the arch ring is that due to the weight of the roadway, of the filling, and of the arch ring itself. The earth filling is usually assumed to act vertically, in which case the forces acting on the arch are easily computed. For arches in which the ratio of rise to span is small, such an assumption is sufficiently correct. A common assumption for weight of earth fill where the actual value is unknown is 100 pounds per cubic foot.

Since the pressure produced by the earth filling against the extradosal surface of the ring is really inclined, being nearly vertical near the crown and considerably inclined near the springings, it is sometimes advisable in an arch of large rise to take account of the horizontal component of the pressure near the springings. The earth pressure acting against an inclined plane may be found either algebraically or graphically. The algebraic solution is given under the subject of retaining walls, page 665, and in the example of arch design the inclined pressure is taken into account for illustration, although it is really unnecessary in the case selected. (See p. 576.)

OUTLINE OF DISCUSSION ON ARCH DESIGN

The method of designing an arch by the elastic theory is illustrated by the example on pages 574 to 582. The steps to be taken are there stated in full.

In the following pages the reactions at the supports, which in an arch are not simple vertical forces, and the relations between the outer loads and the internal stresses, are first treated briefly so as to understand the theory in a general way. Next (p. 553), the working formulas are given for finding the thrust, shear and bending moment at the crown, and at intermediate points in the arch ring. From these, the force polygon and the line of pressure, which is an equilibrium polygon drawn for a pole distance equal to the horizontal thrust, may be drawn (p. 555). The method of determining the stresses due to temperature and rib shortening is given (p. 556). Since the lines of pressure do not ordinarily pass through the center line of the arch ring, the pressures on the various sections are eccentric, and the distribution of stress in an arch under different conditions is discussed at length, the same analyses applying also to any other member like a column, subjected to eccentric pressure (p. 558 to 574). Diagrams are presented

to aid in the determinations. Following the example, the design of arch abutments is given (p. 583), and beyond this are general directions with reference to construction details. Several typical arches are illustrated (p. 589).

RELATION BETWEEN OUTER LOADS AND REACTIONS AT SUPPORTS

An arch differs from a beam in that under vertical loads the reactions at the supports of the arch are inclined, while for a beam the reactions are vertical. The loads acting on the arch, together with the reactions caused by the loads, constitute the entire system of forces acting, and for a complete analysis of the arch the relation between these forces should be determined. This relation is more simply deduced if for each reaction there are substituted its horizontal and vertical components.

For arches symmetrical about the center line of span the following analysis is applicable. For unsymmetrical arches, methods similar to those presented in the following pages are to be employed although the necessary formulas are too long to be given here.

NOTATION

H_1 and V_1 = horizontal and vertical components of the left reaction.
H_2 and V_2 = horizontal and vertical components of the right reaction.
M_1 and M_2 = moments at left and right supports respectively.
M = moment at any point on arch axis having coördinates x and y.
M_c, H_c, V_c = moment, thrust and shear at the crown.
M_L = moment at any point on left half of arch axis of all loads between the point and the crown.
M_R = moment at any point on right half of arch axis of all outer loads between the point and crown.
m = number of divisions into which the half length of arch axis is divided.
s = short length of arch axis.
I = moment of inertia of cross section about the gravity axis.
L = horizontal span of arch axis.
r = rise of arch.
E_c = modulus of elasticity of concrete.
n = ratio of moduli of elasticity of steel to concrete.
R = resultant force acting on any section of the arch ring.
N = thrust = normal component of resultant R.
V = shear = radial component of resultant R.

H = horizontal component of resultant R.
P = any concentrated load.
Δ_L = change in span length due to any cause, + for an increase, − for a decrease.
t = rise or fall in temperature of the arch ring from the mean in degrees Fahrenheit.
c = coefficient of linear expansion or contraction.
f = average unit compression in concrete of arch ring due to thrust.
ϕ = central angle subtended by the axis of the arch.
x,y = coördinates of any point on the axis of the arch ring.

Three-Hinged Arch. The use of the three-hinged arch is discussed on page 539. Since its analysis is simplest and at the same time illustrates important principles of arch design, it is considered first.

Referring to Fig. 157, it is seen that there are two unknown components

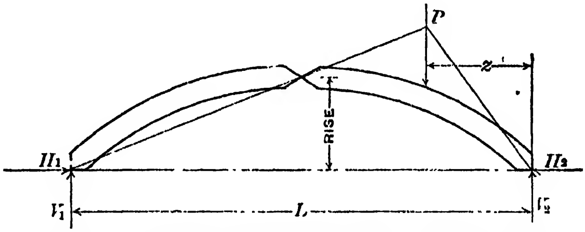


FIG. 157.—Arch with Three Hinges. (*See p.* 546).

of each reaction, making four unknown quantities, H_1, V_1, H_2, V_2, which require four equations to solve them. From statics we have the three equations of equilibrium:

Algebraic sum of vertical components = zero.

Algebraic sum of horizontal components = zero.

Algebraic sum of moments of all forces about any point = zero.

We have here an additional equation from the fact that the bending moment at the crown hinge = 0. Therefore the four components of the reactions can easily be found. Suppose there is only one load, P, on the span. Then

$$V_1 = \frac{Pz}{L} \quad (2) \qquad \text{and } V_2 = \frac{P\,(L - z)}{r} \qquad (3)$$

Since, for equilibrium, the moment at the crown hinge must be 0, the resultant reaction on the left must pass through the left hinge, or

$$V_1\left(\frac{L}{2}\right) - H_1 r = 0. \quad \text{Hence } H_1 = \frac{V_1 L}{2r} \qquad (4)$$

V_1, H_1 = components of left reaction. V_2, H_2 = components of right reaction. L = span. r = rise.

When all loads are vertical, or in any case when the loads are symmetrical about the center, $H_1 = H_2$.

When the loads are not symmetrical and also not vertical, H_2 can be easily found, after H_1 has been determined as above, from the relation that the algebraic sum of all the outer horizontal forces = o. In a three-hinged arch, then, the reactions having been found by means of simple statics as above described, the thrust, shear and bending moment on any section of the arch can be computed and sections designed.*

Two-Hinged Arch. Under the action of the loads on this arch there are produced two components of the reaction at each support, making in all four unknowns, H_1, V_1, H_2, V_2. From statics we have the three fundamental equations of equilibrium, as given above. We must find an additional equation from the theory of elasticity. This additional equation is obtained from the fact that the span does not change its length under the

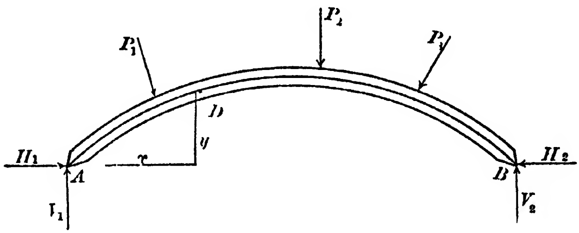


FIG. 158.—Two-Hinged Arch. (See p. 547)

action of the loads. From mechanics† we know that if the arch were fixed at B and free at A, the horizontal motion of A (the origin of coordinates) is given by $\Sigma My\frac{s}{EI}$, where Σ denotes the summation of the products of $My\frac{s}{EI}$ for each section of the arch. Now, since the arch is really prevented by the support from moving horizontally at point A, the above deformation can be placed equal to o, and we have then the fourth equation $\Sigma My\frac{s}{EI} = 0$, which, in addition to the three from statics, enables us to find the reactions H_1, V_1, H_2, V_2. As soon as the reactions are known, the thrust, shear and bending moment at any section of the arch can be found.

*Three Hinged Masonry Arches, Long Spans Especially Considered, by David A. Molitor, Transactions American Society of Civil Engineers, Vol. XL, p. 31.

†"Mechanics of Engineering," by Irving P. Church, 1908, p. 449.

In a similar manner the conditions of equilibrium can be obtained for an arch with only one hinge (at the crown).

"Fixed" or "Continuous" Arches. A method frequently followed with the hingeless arch is to consider the reactions at the ends in the same way as in hinged arches, but the simpler method is to take the forces at a section through the crown. However, in order to better understand the theory and the relation of the external to the internal forces, the arch reactions at the supports will be discussed first and afterward the analysis will consider the forces at the crown.

Let Fig. 159 represent a hingeless arch. The loads having been determined, there are at each support three unknown quantities, namely, the vertical and the horizontal components and the point of application of the reaction. Or, instead of saying that the point of application of the reaction

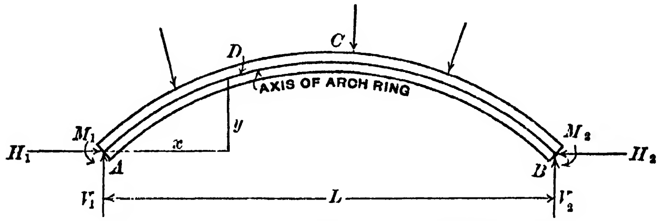


FIG. 159.—Continuous Arch. (*See p.* 548).

is unknown, we can say that there is a bending moment at each support, and that this moment, together with the horizontal and vertical components of the reaction, makes three unknown quantities at each support to be found. There are then six unknown quantities to be determined, namely, H_1, V_1, M_1, H_2, V_2, M_2.

Statics provides the three fundamental equations of equilibrium (see page 546), hence three additional equations must be determined from the theory of elasticity. These three additional equations are given from the three following conditions:

The change in span of the arch $= \Delta x = O$

The vertical deflection at A (the origin of coördinates) $= \Delta y = O$

The change in direction of the tangent at the arch axis at A $= \Delta\phi = O$

These three conditions must be true since the arch is fixed at A and at B, the abutments being assumed immovable

From mechanics,*

$$\Delta x = \Sigma_A^B My \frac{s}{EI} = O \tag{5}$$

$$\Delta y = \Sigma_A^B Mx \frac{s}{EI} = O \tag{6}$$

$$\Delta \phi = \Sigma_A^B M \frac{s}{EI} = O \tag{7}$$

These three equations are general formulas. They are not used directly in arch computations but are necessary in the theoretical derivation of the working formulas given in paragraphs which follow.

These three equations express the conditions that the horizontal, vertical and rotary movements of the left end of the arch ring each equal zero, so far as these motions are caused by the *bending moments only*, acting on the different sections from B to A. The movements due to the *thrust* and *shear* within the ring are not here considered. By means of equations (5), (6), (7) and the three from statics (see p. 546) we can solve for the six unknown quantities at the supports, namely, the horizontal and vertical components of each reaction and the moment at each support, and having thus found the reactions, the stresses within the arch ring can then be computed.

RELATION BETWEEN OUTER FORCES AND THE THRUST, SHEAR AND BENDING MOMENT FOR THE FIXED ARCH

In Fig. 160 let the arch A B be fixed at the two supports. If the loads are known, the horizontal and vertical components of the reactions and also the moment at each support of the arch may be found, as has been shown above. Having these three quantities for each support, the *point of application* of each reaction may then be determined.

Thus in Fig. 160 the point of application at the left support is at *a*, distant y_1 vertically from A, where $y_1 = \frac{M_1}{H}$. Similarly at B, $y_2 = \frac{M_2}{H}$. Having computed y_1 and y_2, thus locating the points of application of the reactions, the force polygon and its equilibrium polygon, *a b c d*, can be drawn, as described more fully on page 577, and the latter will be the true line of pressure for the loading shown. The stresses on any section such as D may

M = moment. s = short length of arch axis. E = modulus of elasticity. I = moment inertia. Δ_x = change of span length. xy = coördinates of a point.

*See "Mechanics of Engineering," by Irving P. Church, 1908, p. 449, or any general treatise on mechanics.

be then studied. The resultant of all outer forces on the left of D is a force acting along the line $a\ b$ of the equilibrium polygon and having a magnitude equal to the force O_0 of the force polygon. This resultant outer force O_0

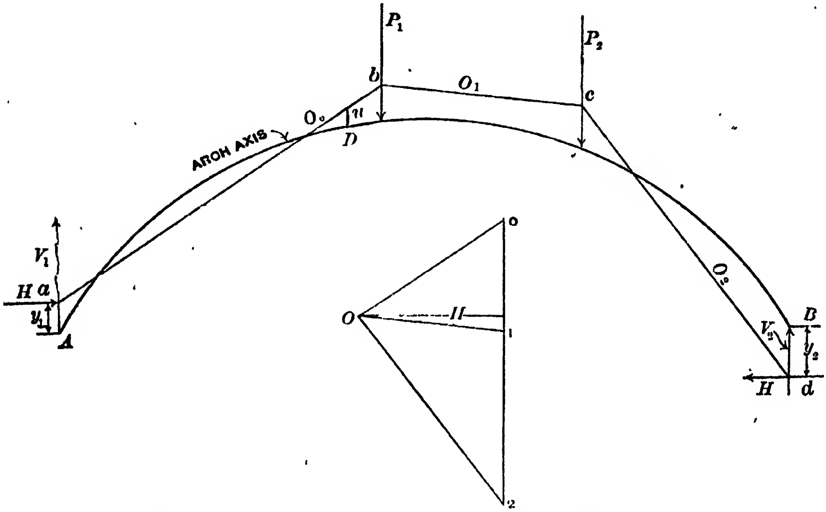


FIG. 160.—Line of Pressure in an Arch. (See p. 549).

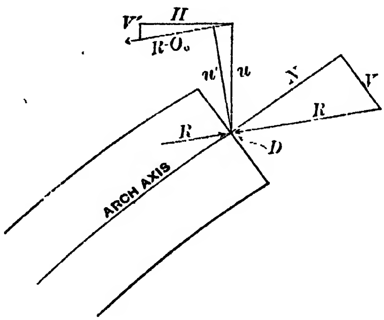


FIG. 161.—Forces Acting upon an Arch Section. (See p. 550.)

acting along ab is resisted by inner forces, i. e., stresses, on the section D which is redrawn in Fig. 161.

The force R is the force opposing the resultant O_0. This force is equiva-

lent to a force R acting at the arch axis and a bending moment $= Ru' = Hu$, where H is the *horizontal* component of R and u is the *vertical* distance from point D on the arch axis to the equilibrium polygon, u' is the *perpendicular* distance from point D to the force $R = O_0$. For vertical loads H is constant throughout the length of the arch ring.

The resultant force R acting at D can be resolved into two components one of which, N, is *tangential* to the axis at D and therefore normal to the section of the arch ring, the other component, V, is *perpendicular* to the axis and parallel to the section.

N is the *thrust*, that is, the tangential component of the resultant force on the section.

V is the *shear*, that is, the radial component of the resultant force on the section.

Hu or Ru' is the *bending moment* about the gravity axis of the section.

Evidently there are sections of the arch where the equilibrium polygon intersects the arch axis. At these sections the bending moment is zero. Furthermore, if the equilibrium polygon is normal to any section there will be no shear on that section. It is possible then to find sections where there is no moment, or no shear, or possibly where there is neither moment nor shear. There is always a *thrust* on every section.

THRUST, SHEAR AND MOMENT AT THE CROWN

Instead of actually finding the components of the reactions and the moments at the supports by the plan indicated on page 549, it is simpler to find the thrust, shear and moment at the crown. Having these, the equilibrium polygon may be drawn and the thrust, shear and moment at any point may be found. The thrust, shear and moment at the crown can be found by use of equations (5), (6), (7), page 549, in which M is the moment of any point D of Fig. 160, page 550, expressed in terms of the values at the crown. Instead, however, of determining these quantities by means of these equations, shorter expressions for the thrust, shear and moment at the crown may be obtained by *taking the origin of coordinates at the crown and studying the motion at that point*.

In Fig. 162, CD represents the vertical section at crown, upon which acts the resultant pressure along the line AB. In the lower part of the figure, for this resultant force is substituted the horizontal thrust, H_c, the shear, V_c, acting at the center of the section CD, and the moment M_c.

Referring to Fig. 163, page 552, and accepting C as origin of coordinates,

Let

$x, y,$ = coördinates of any point D,

M_L = moment at any point D on left half of arch axis of all loads between the point and the crown.

M_R = moment at any point D on right half of arch of all loads between the point and the crown.

m = number of divisions of half of the arch axis.

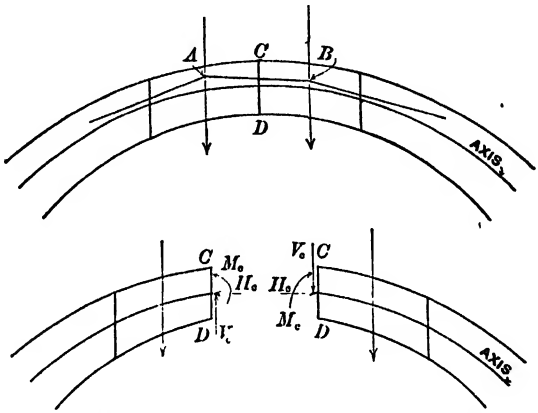


FIG. 162.—Moment and Thrust at the Crown. (*See p.* 551.)

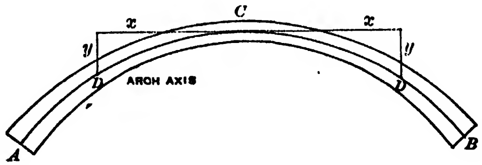


FIG. 163.—Coördinates of Any Point in Arch Axis. (*See p.* 551.)

The formulas given below require that the arch be divided so that the ratio of length of any division to its average moment of inertia is constant. Because of this requirement the end divisions with large moments of inertia may be long, even with comparatively short divisions at the crown. This may cause an inaccuracy which can be largely eliminated by subdividing the load on the end divisions.

The greater the number of divisions the more accurate the results.

For an arch divided in such a way that the ratio of the length of any division to its average moment of inertia is constant (see page 554)

the three unknown quantities, V_c, H_c, and M_c may be found from formulas*

$$H_c = \frac{m\Sigma M_R y + m\Sigma M_L y - \Sigma M_R \Sigma y - \Sigma M_L \Sigma y}{2\,[m\Sigma y^2 - (\Sigma y)^2]} \qquad (16)$$

$$V_c = \frac{\Sigma M_L x - \Sigma M_R x}{2\,\Sigma x^2} \qquad (17)$$

$$M_c = \frac{\Sigma M_R + \Sigma M_L - 2\,H_c\,\Sigma y}{2m} \qquad (18)$$

---

*The horizontal motion of C, Fig. 163, as in preceding analysis, due to bending moments on sections between B and C, is $\Sigma {}_C^B My \frac{s}{EI}$. The horizontal motion of C due to the bending moments on sections between A and C, is $\Sigma {}_C^A My \frac{s}{EI}$. These two motions are equal but opposite in direction, hence,

$$\Sigma {}_C^B My \frac{s}{EI} = -\Sigma {}_C^A My \frac{s}{EI} \qquad (8)$$

Similarly the vertical motions at C are equal,

$$\Sigma {}_C^B Mx \frac{s}{EI} = \Sigma {}_C^A Mx \frac{s}{EI} \qquad (9)$$

Also the changes in direction of the tangent to the axis at C are equal, but opposite in direction, hence,

$$\Sigma {}_C^B M \frac{s}{EI} = -\Sigma {}_C^A M \frac{s}{EI} \qquad (10)$$

If each half of the arch axis be divided into m divisions in such a way as to make $\frac{s}{I}$ constant for all the divisions (See p. 554) the factor $\frac{s}{I}$ and also E may be cancelled. In the equations (8), (9) (10), M, I, x, y, denote respectively the bending moment, moment of inertia of the cross-section, and coördinates at the center point of each division of the arch axis.

At center of any division between A and C the bending moment is

$$M = M_c - V_c x + H_c y - M_R \qquad (11)$$

At center of any division between B and C the bending moment is

$$M = M_c + V_c x + H_c y - M_L \qquad (12)$$

Placing these values of M in equations (8), (9) and (10) and collecting terms, we have

$$2\,M_c\Sigma\, y + 2\,H_c\Sigma\, y^2 - \Sigma\, M_R\, y - \Sigma\, M_L\, y = 0 \qquad (13)$$

$$2\,V_c\Sigma\, x^2 - \Sigma\, M_L x + \Sigma\, M_R x = 0 \qquad (14)$$

$$2\,mM_c + 2\,H_c\Sigma\, y - \Sigma\, M_R - \Sigma\, M_L = 0 \qquad (15)$$

Combining (13) and (15),

$$H_c = \frac{m\Sigma\, M_R\, y + m\Sigma\, M_L\, y - \Sigma\, M_R\,\Sigma y - \Sigma\, M_L\Sigma\, y}{2\,[\,m\Sigma\, y^2 - (\Sigma\, y)^2]} \qquad (16)$$

From (14) $$V_c = \frac{\Sigma\, M_L x - \Sigma\, M_R x}{2\,\Sigma\, x^2} \qquad (17)$$

From (15) $$M_c = \frac{\Sigma\, M_R + \Sigma\, M_L - 2\,H_c\Sigma\, y}{2\,m} \qquad (18)$$

M = moment. H_c = crown thrust. V_c = crown shear. m = number divisions of half axis. x, y = coördinates of a point.

These are fundamental equations in arch analysis. The method of application is illustrated in the example, page 574.

All Σ signs denote summations for *one-half* of the arch axis.

All numerical values of M_L, M_R, x, y, are positive.

A positive value of V_c indicates that the line of pressure at the crown slopes upward toward the left, a negative value, upward towards the right.

A positive value of M_c indicates a positive moment at the crown; a negative value, a negative moment.

The moment at any point between B and C is

$$M = M_c - V_c x + H_c y - M_R \tag{19}$$

while at any point between A and C

$$M = M_c + V_c x + H_c y - M_L \tag{20}$$

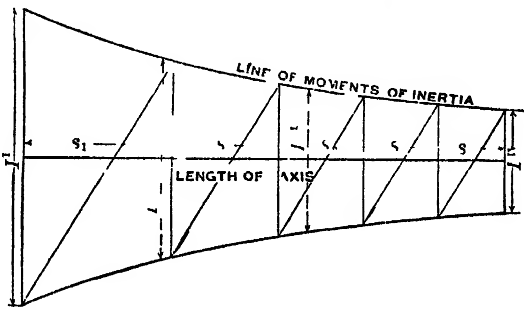


FIG. 164. Diagram for finding Constant $\frac{s}{I}$ (See p. 554)

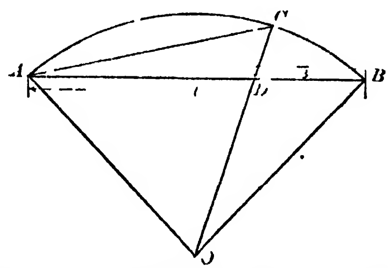


FIG. 165.—Diagram for finding Length of Arc of a Circle (See p. 554)

GRAPHICAL METHOD FOR FINDING CONSTANT $\frac{s}{I}$

Fig. 164 and Fig. 165 give a graphical method of determining the length

M = moment. H_c = crown thrust. V_c = crown shear. x, y = coordinates of a point. s = length of division of axis. I = moment of inertia.

of divisions for a constant $\frac{s}{I}$. If the arch axis is made up of arcs of circles, the length of any arc ACB is equal to three halves of the straight line AC.* The point C is found in Fig. 165 by dividing the chord AB into thirds and drawing a radius through the one-third point. If the arc is an ellipse, a simple method of drawing which is given on page 202, the length may be measured from the drawing. Having found the length of the half axis and drawn it as a horizontal line, the constant $\frac{s}{I}$ is found as shown in Fig. 164 by computing four or more values of I, the moment of inertia, at different points and plotting these to locate the curves as shown. Beginning at the lower left corner of the diagram, trial diagonals (parallel to each other) and vertical lines are drawn, so that the number of spaces between the verticals will represent the number of divisions into which the half arch must be divided. If at the first trial the final diagonal does not come out exactly at the upper right corner which represents the crown of the arch, a new slope is tried for the parallel diagonals.

LINE OF PRESSURE

Having determined the thrust and moment at the crown, the line of pressure may be drawn as shown in folding Fig. 181, opposite page 580, from which the compression and tension at different sections may be found after determining the thrust and eccentricity from the formulas which follow.

It is well to draw the line of pressure before considering the temperature and the effect of the rib shortening, and then afterwards study these, adding or deducting the stresses for the most unfavorable conditions.

EFFECT OF TEMPERATURE AND THRUST

The thrust acting throughout the ring tends to shorten the span. A change of temperature of the ring tends to shorten the span when the temperature falls or to lengthen the span when the temperature rises. The tendency for the span to change its length by a distance Δ_L due to any cause is resisted by a horizontal component H and a moment M_1 acting at each support, and by a thrust and moment in the arch ring. Δ_L is positive for an increase and negative for a decrease in span length.

*Method given in Nouvelles Annales de Mathematiques, Jan. 1907. The error for 40 degrees is less than $\frac{1}{10000}$, for 70 degrees is less than $\frac{1}{1000}$, for 90 degrees is less than $\frac{5}{1000}$.

The thrust and moment at the crown may be found from formulas*

$$H_c = \frac{I}{s} \frac{mE\,\Delta_L}{2[m\Sigma y^2 - (\Sigma y)^2]} \tag{23}$$

and

$$M_c = -\frac{H_c \Sigma y}{m} \tag{24}$$

Rise in Temperature. Under a rise of temperature of the arch ring of t degrees Fahr. the span L would tend to increase in length an amount of ctL, c being the coefficient of linear expansion. Substituting for Δ_L in (23) the value of ctL, the thrust at crown is

$$H_c = \frac{I}{s} \frac{ctLmE}{2[m\Sigma y^2 - (\Sigma y)^2]} \tag{25}$$

The value of the temperature coefficient, c, in equation (25) may be taken for concrete as 0.0000055. Dimensions must all be in same units; if in feet, E *must be in pounds per square foot.* Using a value of E_c of 2,000,000, E is therefore $2{,}000{,}000 \times 144 = 288{,}000{,}000$ pounds per square foot.

Moment at crown is

$$M_c = -\frac{H_c \Sigma y}{m} \tag{26}$$

*The change in total span length, the two halves of the arch being equal, is

$$2\Sigma_C^A My \frac{s}{EI} = \Delta L \tag{21}$$

The change in inclination of tangent to axis at crown is

$$2\Sigma_C^A M \frac{s}{EI} = 0 \tag{22}$$

Replacing the M of equations (21) and (22) by $Mc + Hc\,y$, which is the moment at any point D, Fig. 166, in terms of moment and thrust at the crown, and making $\frac{s}{I}$ constant, there results

$$\frac{s}{EI} Mc\Sigma y + 2\frac{s}{EI} Hc\Sigma y^2 = \Delta L$$

$$mMc + Hc\Sigma y = 0$$

From which

$$Hc = \frac{I}{s} \frac{mE\Delta L}{2[m\Sigma y^2 - (\Sigma y)^2]} \tag{23}$$

and

$$Mc = -\frac{Hc\Sigma y}{m} \tag{24}$$

M = moment. Hc = crown thrust. m = number divisions of half axis. s = length of division of axis. I = moment inertia. L = span. E = modulus of elasticity. Δ_L = change of span length. t = rise or fall of temperature. c = coefficient of expansion. x, y = coördinates of a point.

The moment at any point D may be found as soon as the values of H_c and M_c have been determined by means of the relation

$$M = M_c + H_c y \tag{27}$$

or we can say that the moment at any point equals the thrust H_c* multiplied by the distance from the point in question to the line OO, Fig 166

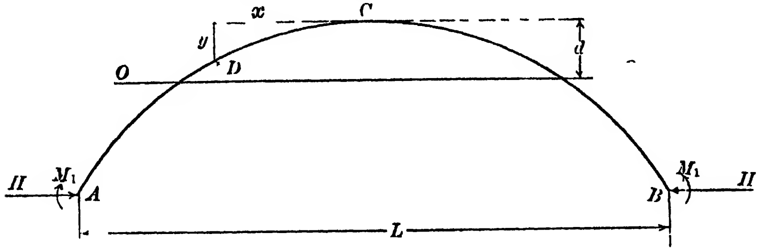


FIG 166—Moments and Thrusts due to Changes of Temperature (*See p* 556.)

Above the line OO, Fig 166, the moments are all negative, being a maximum at the crown, and below OO they are all positive, being maximum at A and B. The line OO is below the crown a distance d Σy. At the two points where OO intersect the arch axis the moments are zero, as is evident from equations (24) and (26).

Fall in Temperature. Here the thrust at crown is

$$H_c = -\frac{I}{s}\;\frac{c\,tLmL}{2\,[m\,\Sigma y^2 - (\Sigma y)^2]} \tag{28}$$

where c is 0.0000055, and moment at crown is

$$M_c = -\;H_c\,\Sigma y \tag{29}$$

and, as above,

$$M = M_c + H_c y \tag{30}$$

In placing a numerical value for H_c in the last two equations, it should be observed that it is a negative quantity If in the equations the values of L and y are in feet, *E is in pounds per square foot* Above OO the moments are all positive, below they are all negative. The thrust at the crown is really a tension in this case.

M = moment Hc = crown thrust m = number divisions of half axis. s = length of division of axis I = moment inertia L = span E — modulus of elasticity t = rise or fall of temperature from mean c = coefficient of expansion x, y = coordinates of a point

*The horizontal thrust is constant throughout the arch, hence Hc at the crown equals H at the support.

An increase and a decrease of 20 degrees Fahr. is probably a sufficient allowance for concrete arches with filled spandrels. For arches with open spandrels the range in temperature of the concrete is somewhat less than that of the surrounding air. For example, in the latter case with a range of temperature of the air from −20 degrees to +100 degrees Fahr., the range for arch computation should be taken at least 40 degrees on each side of the mean temperature.

The methods of combining the temperature moments and thrusts with those due to loads is illustrated in the example, page 579.

EFFECT OF RIB SHORTENING DUE TO THRUST

The thrust acting throughout the arch ring tends to cause a shortening of the span, which, if f is average compression (obtained by averaging values in computation of ring) for unit area, $= \frac{f\,L}{E} = \Delta_L$

Hence

$$H_c = -\frac{I}{s}\;\frac{f\,Lm}{2\,[m\,\Sigma\,y^2 - (\Sigma\,y)^2\,]} \tag{31}$$

and

$$M_c = -\frac{H_c\,\Sigma\,y}{m} \tag{32}$$

and, as in temperature stresses,

$$M = M_c + H_c y \tag{33}$$

The effect of the rib shortening is similar to a fall in temperature.

All the summations above are for one-half the span only. m = number of divisions in one-half of the arch axis.

The effect of rib shortening is slight in many cases but in a flat arch it may be considerable.

DISTRIBUTION OF STRESS OVER CROSS SECTION

The analyses of stress distribution which follow apply not only to an arch but also to any section where there is combined compression and bending.

In an arch, having determined the thrust, shear and bending moment at any given section of the arch ring, the distribution of stress upon the section must be next investigated in order to compute the maximum stresses

M = moment. H_c = crown thrust. m = number divisions of half axis. s = short length of arch axis. I = moment inertia. L = span. y = coordinate of a point. f = compression in concrete.

in the concrete and the steel to see on the one hand that they do not exceed safe working loads, and on the other hand that the design is as economical as possible.

Concrete is strong in resistance to direct shear (See p. 382), and hence the shear is generally negligible in concrete and reinforced concrete arches, although it should be considered in stone masonry arches. Since, also, as will be shown, the bending moment is the thrust times its eccentricity, it follows that the determination of the thrust, which is the normal component of all the forces acting, together with the location of its center of pressure, permit the determination of the stresses required in designing any section of an arch or of any section of any member subjected to eccentric stress. Every section of the arch or of a beam or of a column must be of such dimensions or with such reinforcement that the safe working stresses in the concrete shall not be exceeded.

Plain concrete sections and reinforced concrete sections are considered separately, the same notation being used for both.

Notation

Let

R = resultant of all forces acting on any section.
f_c = maximum unit compression in concrete.
f'_c = maximum unit tension in concrete or minimum compression.
N = thrust, a component of the forces normal to the section.
V = shear, the component of the force R parallel to the section.
b = breadth of rectangular cross section.
h = height of rectangular cross section.
e = eccentricity, that is, the distance from gravity axis to the point of application of the thrust which is the intersection of the line of pressure with the plane of the section.
M = bending moment on the section.
y = perpendicular distance from gravity axis to any point in the section
I = moment of inertia of entire cross section of concrete about the horizontal gravity axis.
I_s = moment of inertia of cross-section of steel about the horizontal gravity axis.
A_c = total area of section of concrete.
A_s = total area of section of steel.
y_1 = perpendicular distance from gravity axis of unsymmetrical section to outside fiber having maximum compression.

y_2 = perpendicular distance from gravity axis of unsymmetrical section to outside fiber having maximum tension or minimum compression.
f'_s = maximum unit compression in the steel.
f_s = maximum unit tension or minimum unit compression in the steel.
p = ratio of steel to total area of section; for rectangular sections p = ratio of steel area to bh.
n = $\frac{E_s}{E_c}$ = ratio of moduli of elasticity of steel and concrete.
k = ratio of depth of neutral axis to depth of beam h.
kh = distance from outside compressive surface to neutral axis.
d' = depth of steel in compression.
d = depth of steel in tension.
a = distance from center of gravity of symmetrical section to steel.
e_0 = value of eccentricity which produces zero stress in concrete at outer edge of rectangular section opposite to that on which thrust acts.
C_a, C_e = constants.

DISTRIBUTION OF STRESSES IN PLAIN CONCRETE OR MASONRY ARCH SECTIONS

In designing plain concrete or stone masonry arches, the maximum compressive stresses must be kept within the safe working compressive strength of the material, and the point of application of the thrust must not lie outside of the middle third of the section. When investigating an existing structure, however, it may be found that the thrust acts outside of the middle third, so that a determination of the stresses in such cases must also be considered.

Plain Arches with Rectangular Cross Section. In plain concrete or stone masonry arches of rectangular cross-section there are five special cases depending upon the point of application of the thrust, as follows:

(*a*) Thrust acting at gravity axis of cross-section.

(*b*) Thrust not acting at gravity axis of cross-section, but within the middle third of the section.

(*c*) Thrust acting at edge of middle third of the section.

(*d*) Thrust acting outside of the middle third of the section and material *able* to carry tension.

(*e*) Thrust acting outside of the middle third of the section and material *not able* to carry tension.

Each of these cases will be considered.

(*a*) When the thrust acts at the gravity axis of the cross-section the stress is compression over the entire section and is uniformly distributed as in Fig. 167.

Maximum compression in concrete is

$$f_c \quad \frac{N}{bh} \tag{34}$$

(*b*) The thrust acts within the middle third but not at the gravity axis, as shown in Fig. 168. When the thrust acts at any other point than the gravity axis there is combined bending moment and direct stress to be considered. If the bending moment is positive, the thrust acts above the gravity axis; if negative, the thrust acts below. In either case the thrust produces a unit compression of $\frac{N}{bh}$ over the entire section. The moment causes compression on the side of the axis where N acts and tension on the

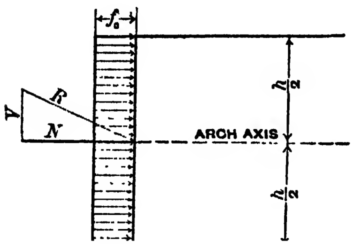


FIG. 167.—Stresses Caused by a Force Acting in the Middle of Plain Concrete Section. (*See p.* 560.)

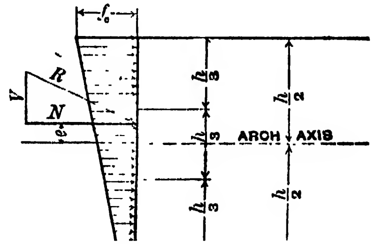


FIG. 168.—Stresses Caused by a Force Within the Middle Third of Plain Concrete Section. (*See p.* 561.)

opposite side. By mechanics, the intensity of stress due to the moment, M, at a distance y from the gravity axis is $\frac{My}{I}$. The actual combined stress at any distance y is then the sum of these two stresses, namely, $\frac{N}{bh} \pm \frac{My}{I}$. The positive sign applies to stresses on the side of the axis where N is applied and the negative sign to stresses on the opposite side.

Since for a rectangular section, $I = \frac{bh^3}{12}$ and $M = Ne$, thrust multiplied by eccentricity, we have:

$$\text{Stress at any point } y \text{ distance from gravity axis} = \frac{N}{bh}\left(1 \pm \frac{12ey}{h^2}\right) \tag{35}$$

The stress at all points of the section is compression and the maximum

M = moment. f_c = compression in concrete. N = thrust. b = breadth. h = height. y = distance from gravity axis. e = eccentricity. I = moment of inertia.

and minimum values are at the top and bottom of the section, respectively, that is, when $y = \frac{h}{2}$,

$$\text{Maximum compression} = f_c = \frac{N}{bh}\left(1 + \frac{6e}{h}\right) \quad (36)$$

$$\text{Minimum compression} = f_c' = \frac{N}{bh}\left(1 - \frac{6e}{h}\right) \quad (37)$$

(*c*) When the thrust acts at the edge of the middle third the eccentricity $e = \frac{h}{6}$ and the maximum and minimum values of the stresses are found by placing $\frac{h}{6}$ for e in the last two formulas of case (*b*) Fig 169 shows the distribution in this case.

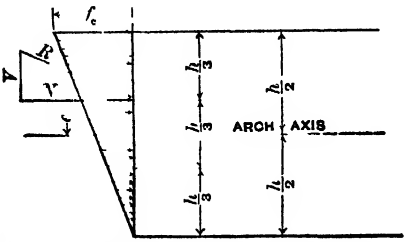


FIG. 169 —Stresses Caused by a Force Acting at the Edge of the Middle Third of Plain Concrete Section. (See p. 562)

$$\text{Maximum compression in concrete} = f_c = \frac{2N}{bh} \quad (38)$$

Minimum compression in concrete = 0.

(*d*) If the thrust acts outside of the middle third and the material is capable of carrying some tension, the distribution of stress is as shown in Fig. 170 There will be compression over a large part and tension over the remainder of the section.

$$\text{Maximum compression} = f_c = \frac{N}{bh}\left(1 + \frac{6e}{h}\right) \quad (39)$$

$$\text{Maximum tension} = f_c' = \frac{N}{bh}\left(1 - \frac{6e}{h}\right) \quad (40)$$

Evidently in each of the above cases (*a*), (*b*), (*c*), (*d*) the maximum compressive and minimum compressive (or maximum tensile) unit stresses are given by the one general formula $\frac{N}{bh}\left(1 \pm \frac{6e}{h}\right)$ This applies for rectangular sections and will hold so long as the safe tensile strength of the concrete is not exceeded.

In arches of plain concrete or of stone. tension should not be allowed to exist.

(*e*) When the thrust acts outside of the middle third and the material

is not able to carry tension, the stress is distributed as compression over a depth less than the entire depth of the section, and cracks may be expected on the "tension" side. The distribution of stress is shown in Fig. 171 below, where (in addition to notation already presented on page 559).

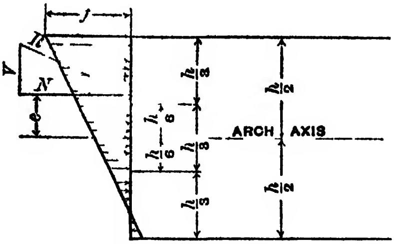


FIG. 170.—Stresses Caused by a Force Acting Outside of the Middle Third of Plain Section (See p. 562.)

g = distance from point of application of thrust to most compressed surface.

Maximum compression =

$$f_c = \frac{2N}{3bg} \qquad (41)$$

Plain Arches with Irregular Cross Section. If the section is not rectangular, the maximum unit compression $= \frac{N}{A_c} + \frac{Ney_1}{I}$, and the minimum unit compression (or maximum unit tension) $= \frac{N}{A_c} - \frac{Ney_2}{I}$. When the second term of this equation is greater than the first, the concrete is in tension.

DISTRIBUTION OF STRESSES IN REINFORCED CONCRETE SECTIONS

Reinforced Concrete Sections of any Shape. The distribution of stress caused by combined thrust and bending moment over a section containing steel reinforcement is shown by the following formulas.

As in column design (page 490) the area of the steel in compression may be replaced by an equal area of concrete by multiplying the steel area by n, the ratio of the modulus of elasticity of steel to the modulus of concrete. Similarly, their moments of inertia may also be compared, and the section treated as if it were of concrete without steel.

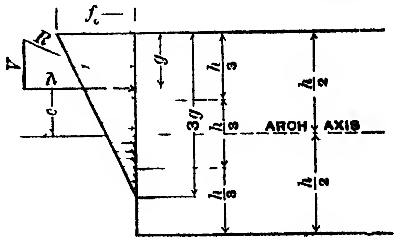


FIG. 171.—Stresses Caused by a Force Acting Outside the Middle Third of Plain Concrete Section. (See p. 563.)

The unit stress then in the concrete at any distance, y, from the gravity

y_1 y_2 = distances respectively from gravity axis to maximum compression of tension.
b = breadth of cross section.

axis is $\dfrac{N}{A_c + nA_s} \pm \dfrac{Ney}{I + nI_s}$. The stress may be compression over the entire section or may be compression over a portion of it and tension over the remainder. The formulas apply in either case so long as the safe tensile stress in the concrete is not exceeded.

Maximum compression in concrete $= f_c = \dfrac{N}{A_c + nA_s} + \dfrac{Ney_1}{I + nI_s}$, where y_1 is the distance from the gravity axis to outermost fiber of concrete on the side of the gravity axis on which the thrust acts.

Maximum compression in steel $= f'_s = n\left[\dfrac{N}{A_c + nA_s} + \dfrac{Ney_3}{I + nI_s}\right]$, where y_3 is the distance from gravity axis to center of gravity of steel on side of gravity axis on which the thrust acts.

Minimum compression in concrete $= f'_c = \dfrac{N}{A_c + nA_s} - \dfrac{Ney_2}{I + nI_s}$, where y_2 is the distance from the gravity axis to the outermost fiber of concrete on

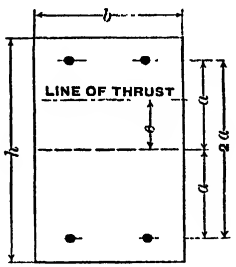


FIG. 172.—Cross Section of an Arch Rib. (See p. 564.)

the side of the gravity axis opposite to that on which the thrust acts. This minimum compression is of course tension when the second term in the last equation is greater than the first.

Minimum compression in steel $= f_s = n\left[\dfrac{N}{A_c + nA_s} - \dfrac{Ney_4}{I + nI_s}\right]$, where y_4 is the distance from the gravity axis to center of gravity of steel on the side of gravity axis opposite to that on which the thrust acts.

Reinforced Concrete Rectangular Sections. For rectangular sections the above general formulas for sections of any shape may be put into slightly simpler forms by substituting the proper terms and assuming equal amounts of steel above and below the center. Special

cases for convenient use in design are also treated below. Since total moment of inertia of combined section, $I + I_s = \frac{bh^3}{12} + npbha^2$; area of section, A, $= bh$ and area of steel in the section, A_s, $= pbh$, we have:

Unit stress in concrete at any point at a distance, y, from gravity axis

$$\text{is } \frac{N}{bh}\left[\frac{1}{1+np} \pm \frac{12\,ye}{h^2 + 12\,npa^2}\right]$$

Maximum unit compression in concrete

$$f_c = \frac{N}{bh}\left[\frac{1}{1+np} + \frac{6\,he}{h^2 + 12\,npa^2}\right] \qquad (42)$$

Maximum unit compression in steel

$$f'_s = \frac{nN}{bh}\left[\frac{1}{1+np} + \frac{12\,ae}{h^2 + 12\,npa^2}\right] \qquad (43)$$

Minimum unit compression (or maximum unit tension) in concrete

$$f'_c = \frac{N}{bh}\left[\frac{1}{1+np} - \frac{6\,he}{h^2 + 12\,npa^2}\right] \qquad (44)$$

M nimum unit compression (or maximum unit tension) in steel

$$f_s = \frac{nN}{bh}\left[\frac{1}{1+np} - \frac{12\,ae}{h^2 + 12\,npa^2}\right] \qquad (45)$$

There are four cases which may occur depending upon value of e.

(1) Thrust applied at gravity axis, as in Fig. 173, where there is no moment on the section and the stress is uniformly distributed. In this case the second term in the brackets of the above formulas becomes zero.

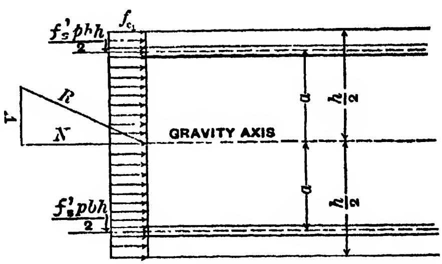


FIG. 173. Stresses Caused by Force Acting at Gravity Axis. (*See p.* 565.)

(2) Thrust applied at such distance from the gravity axis as to cause compression on whole section of the concrete, as in Fig. 174. Here the above formulas are used directly.

n = ratio elasticity. N = thrust. y = distance from gravity axis. f_c = compression in concrete f_s = tension or minimum compression in steel. b = breadth. h = height. e = eccentricity p = ratio of steel. a = distance from center of gravity of section to steel.

(3) Thrust applied at such distance from gravity axis that the compression at one surface becomes zero, as in Fig. 175. In this case the formulas may be simplified as follows:

Maximum unit compression in concrete,

$$f_c = \frac{2N}{bh(1 + np)} \tag{46}$$

Maximum unit compression in steel,

$$f'_s = \frac{nN}{bh(1 + np)}\left[1 + \frac{2a}{h}\right] \tag{47}$$

Minimum compression in concrete = 0.
Minimum compression in steel, which is very small,

$$f_s = \frac{nN}{bh(1 + 2p)}\left[1 - \frac{2a}{h}\right] \tag{48}$$

(4) Thrust applied at such distance from the gravity axis that there is tension at one surface, as in Fig. 176.

The most important question for decision is the compression in the concrete, which must not exceed a safe working stress and is readily found from formula (42), page 565, and the determination of whether the opposite surface is in tension or compression. A simple method for determining this is given in the following paragraphs together with a diagram, further simplifying the process.

If the eccentricity is so great that tensile stress is found in the concrete as determined by methods described in the following paragraphs, a special treatment must be given to determine the stresses, as discussed on page 570.

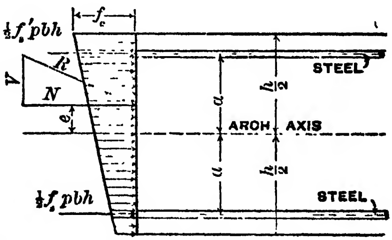


FIG. 174.—Stresses Caused by a Force Producing Compression upon the Whole Reinforced Section. (*See p.* 565.)

In case the result from formula (44) is negative, the stress on this surface of the concrete is tension.

Eccentricity. As in plain concrete arches, the location of the center of thrust determines the distribution of the stress. The stress on one side of

n = ratio elasticity. N = thrust. f_c = compression in concrete. f_s = tension or minimum compression in steel. b = breadth. h = height. p = ratio of steel. R = resultant of forces.

the gravity axis is always compression and if the thrust acts at the gravity axis there is uniform compression over the section. As the center of thrust lies farther and farther from the gravity axis, the compression at the opposite surface decreases until finally it becomes zero and then tension. Equation (44), page 565, gives a means of determining the eccentricity for which there can be neither tension nor compression at the surface opposite to that on which the thrust acts For a reinforced arch this eccentricity is usually more than $\frac{h}{6}$, that is, the line of pressure must not necessarily lie within the middle third.

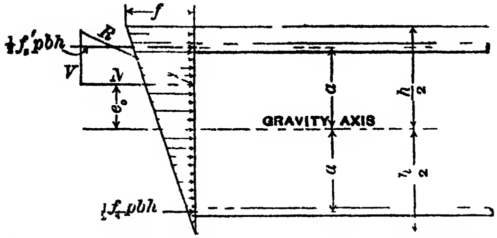


FIG 175 —Stresses Caused by a Force Acting at a Distance e_0 from Center of Gravity of Reinforced Section (*See p* 566)

When the first term in the brackets of this equation is greater than the second, the minimum stress in the concrete will be in compression; when the two terms are equal, the stress is zero in the outer edge of the concrete on the side opposite to that on which the thrust acts; when the second term is greater than the first, this stress will be tension. By equating the two terms the value of the eccentricity, e, may be found for which the stress at the edge is zero.

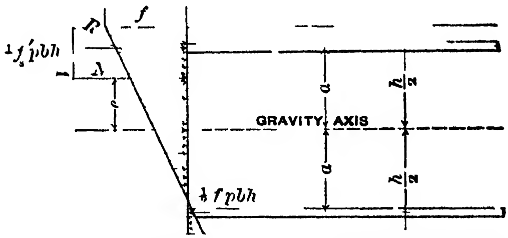


FIG 176 —Stresses Caused by a Force Acting at a Distance Larger than e_0 from the Axis of Gravity of Reinforced Section (*See p* 566)

Using previous notation and also letting e_0 = value of e which makes the stress zero

$$e_0 = \frac{h^2 + 12\,npa^2}{1 + np} \cdot \frac{1}{6h} \qquad (49)$$

If the computed eccentricity, e, is greater than e_0 the concrete is in tension.

Diagram for Determining Compression and Eccentricity. By introducing selected values for some of the letters in formula (49) its solution is simplified. If the ratio of moduli of elasticity of steel to concrete is

e = eccentricity. h = height. n = ratio elasticity. p = ratio of steel.

assumed to be 15, and if the steel is assumed to be imbedded in the concrete $\frac{1}{10}$ of the total depth from each surface so that $2a = \frac{4h}{5}$, which is a close approximation in ordinary design, formula (49) becomes

$$\frac{e_0}{h} = \frac{1 + 28.8\,p}{6 + 90\,p} \qquad (50)$$

where $\frac{e_0}{h}$ is the eccentricity producing zero stress at one surface divided by the total thickness of the arch or beam. The curve in the lower right hand portion of the diagram, Fig. 177, page 569, is plotted and the values of $\frac{e_0}{h}$ can be read for any percentage of reinforcement. For example, if h at any section is 30 inches and the percentage of steel 0.8% (i. e., if $p = 0.008$) $\frac{e_0}{h} = 0.183$ from the diagram, and hence $e_0 = 5.49$. That is, the center of thrust cannot be more than 5.49 inches from the gravity axis without producing tension in the concrete.

If, then, the eccentricity of the thrust on the section in question as previously determined from the line of pressure, or from computation, is greater than the e_0 derived from the curve, there is tension in the concrete, and the percentage of steel may have to be increased or else the depth of section, h, increased. In the latter case it must be remembered that an increase in h with the same area of steel results in a reduction in the percentage of steel.

For determining the maximum compression in the concrete, the curves in the left-hand portion of the diagram, Fig. 177, have been drawn for certain values of n and p. If the same values are selected as are given above, $n = 15$ and $2a = \frac{4}{5}h$, the formula (42) on page 565 becomes

$$f_c = \frac{N}{bh}\left[\frac{1}{1 + 15\,p} + \frac{e}{h}\,\frac{6}{1 + 28.8\,p}\right] \qquad (51)$$

or for definite values of e, h and p

$$f_c = \frac{NC_e}{bh} \qquad (52)$$

In the diagram mentioned, the values of C_e are plotted for values of $\frac{e}{h}$ and different percentages of steel.

To illustrate the use of the diagram, after having found that the eccentricity does not produce tension in the concrete, if the eccentricity is 3 inches

N = thrust. f_c = compression in concrete. b = breadth. h = height. e = eccentricity. p = ratio of steel. C_e = constant.

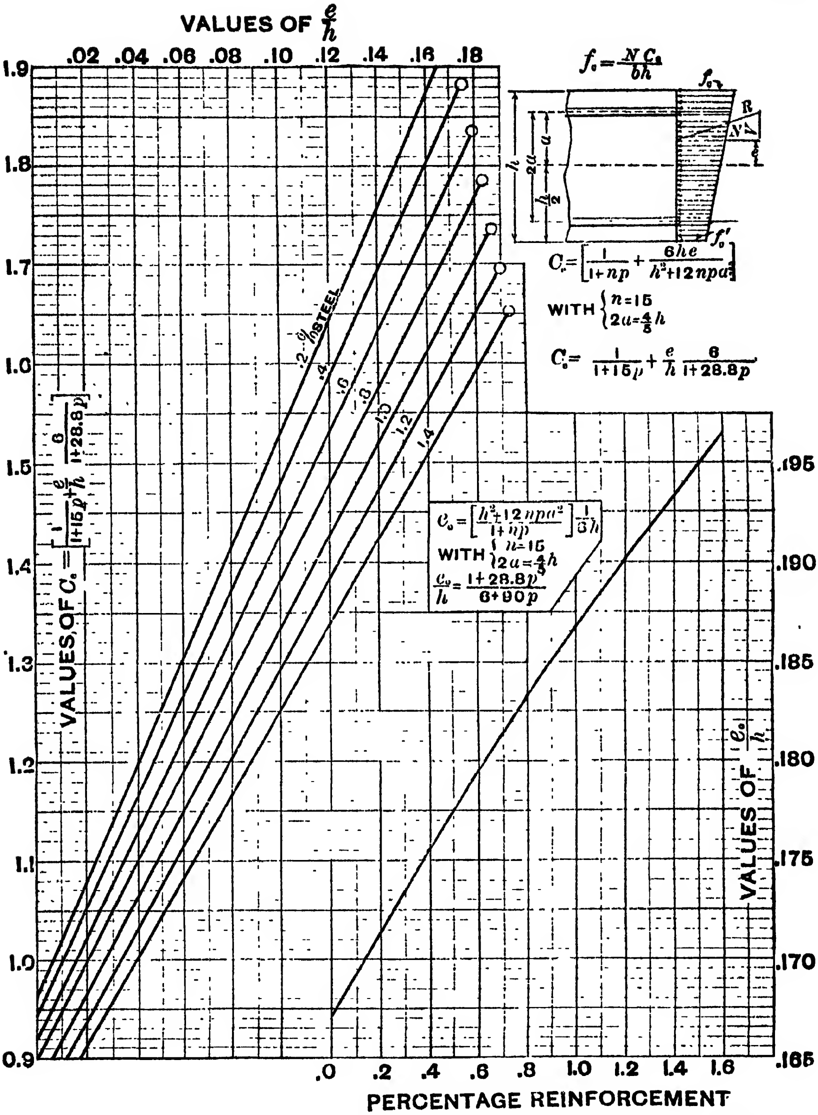


FIG. 177.—Diagram for Determining Compression and Eccentricity. (See p. 567.)

and the thickness of the arch, h, is 30 inches, the value of C_e for 0.8% steel (that is, for $p = 0.008$) is 1.38 and $f_c = \frac{1.38 N}{bh}$.

Distribution of Stress When One Surface is in Tension. When the thrust is applied at a distance from the gravity axis with eccentrictiy, e, greater than that given for e_0 by formula (49), page 567, and the concrete is *assumed unable to carry any tension*, the above general formulas are not easily applied and the following method may be used. Here the steel on the side opposite to that on which the thrust acts is designed to carry all the tensile stresses. In this case having a section with a bending moment and thrust, there are three unit stresses to be determined, namely, maxi-

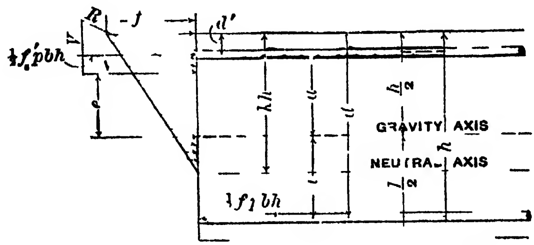


FIG 178.—Stresses Caused by a Force Producing Compression and Tension upon a Reinforced Section Tensile Strength of Concrete Neglected. (See p. 570.)

mum unit compression in concrete, maximum unit compression in steel, and maximum unit tension in steel. The method of procedure is similar to that used for beams in Appendix II, page 757. Referring to Fig. 178, the unit stress in the upper steel, as shown by inspection, is

$$f'_s = nf_c\left(1 - \frac{d'}{kh}\right) \tag{53}$$

The unit tension in the lower steel is

$$f_s = nf_c \frac{d - kh}{kh} \tag{54}$$

Hence, when the compression in concrete, f_c, is known, the stresses in the steel are determined by the above formulas. Since the sum of the stresses

n = ratio elasticity. N = thrust. fc = compression in concrete. f_s = tension in steel. f'_s = compression in steel. b = breadth. h = height. p = ratio of steel. k = ratio depth neutral axis. d = depth tension steel. d' = depth compression steel. R = resultant of forces.

acting on the section must be equal to the thrust, we have, since each steel area is $\frac{pbh}{2}$

$$N = \frac{f'_s\, pbh}{2} + \frac{f_c\, bkh}{2} - \frac{f_s\, pbh}{2} \tag{55}$$

Placing values of f'_s and f_s from (53) and (54) in (55),

$$N = \frac{f_c\, bh}{2}\, \frac{k^2 + 2\, npk - np}{k} \tag{56}$$

The moment of the stresses about the gravity axis, which is obtained by taking the sum of the moments of all the stresses about the gravity axis and eliminating f'_s and f_s by use of equations (53) and (54), is

$$M = f_c\, bh^2 \left[\frac{npa^2}{h^2 k} + \frac{k}{4} - \frac{k^2}{6}\right] \tag{57}$$

Designating by C_a the quantity $\left[\frac{npa^2}{h^2 k} + \frac{k}{4} - \frac{k^2}{6}\right]$ from equation (57) we may write

$$M = C_a f_c bh^2 \tag{58}$$

Hence in investigating a given section of an arch, if M, b, h, C_a are known, the unit compression in the concrete is

$$f_c = \frac{M}{C_a bh^2} \tag{59}$$

To solve this equation easily, values of C_a should be taken from curves. Fig. 180, page 573, gives values of C_a for $n = 15$, $2a = \frac{4}{5}h$ and various values of k.

Evidently before using equations (57) or (59) to find the unit compression in the concrete, the position of the neutral axis must first be determined. To do this we must find the value of k. Since the moment $M = Ne$, that is, the thrust multiplied by the eccentricity, equation (56) may be multiplied by e and equated to (57). From this process, the following equation containing k is obtained

$$k^3 + 3\left(\frac{e}{h} - \frac{1}{2}\right)k^2 + 6\, npk\frac{e}{h} = 3\, np\frac{e}{h} + \frac{6\, npa^2}{h^2} \tag{60}$$

M = moment. n = ratio elasticity. N = thrust. f_c = compression in concrete. f_s = tension in steel. f'_s = compression in steel. b = breadth. h = height. e = eccentricity. p = ratio of steel. k = ratio depth neutral axis. C_a = constant.

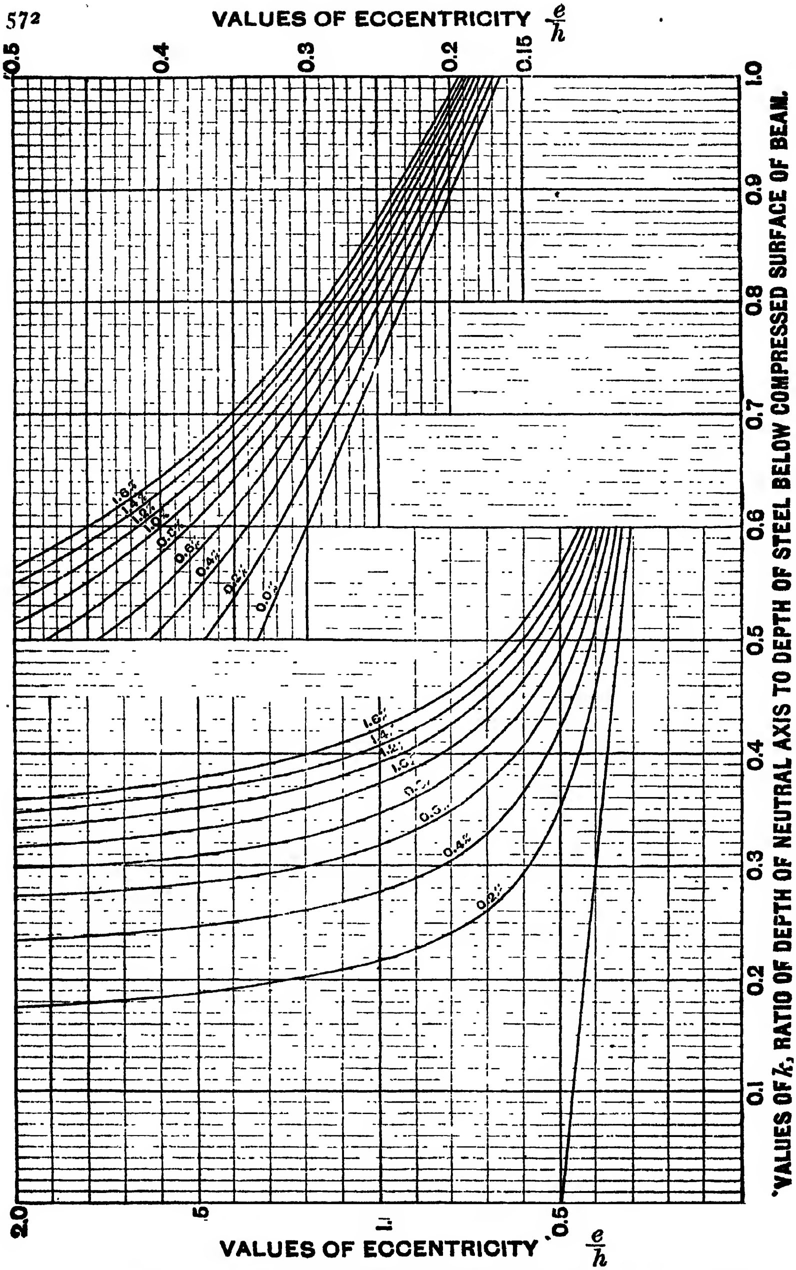


FIG. 179.—Diagram for Determining Depths of Neutral Axis for Different Eccentricities. Based on $n = 15$ and $2a = \frac{4}{5}h$. (See p. 574.)

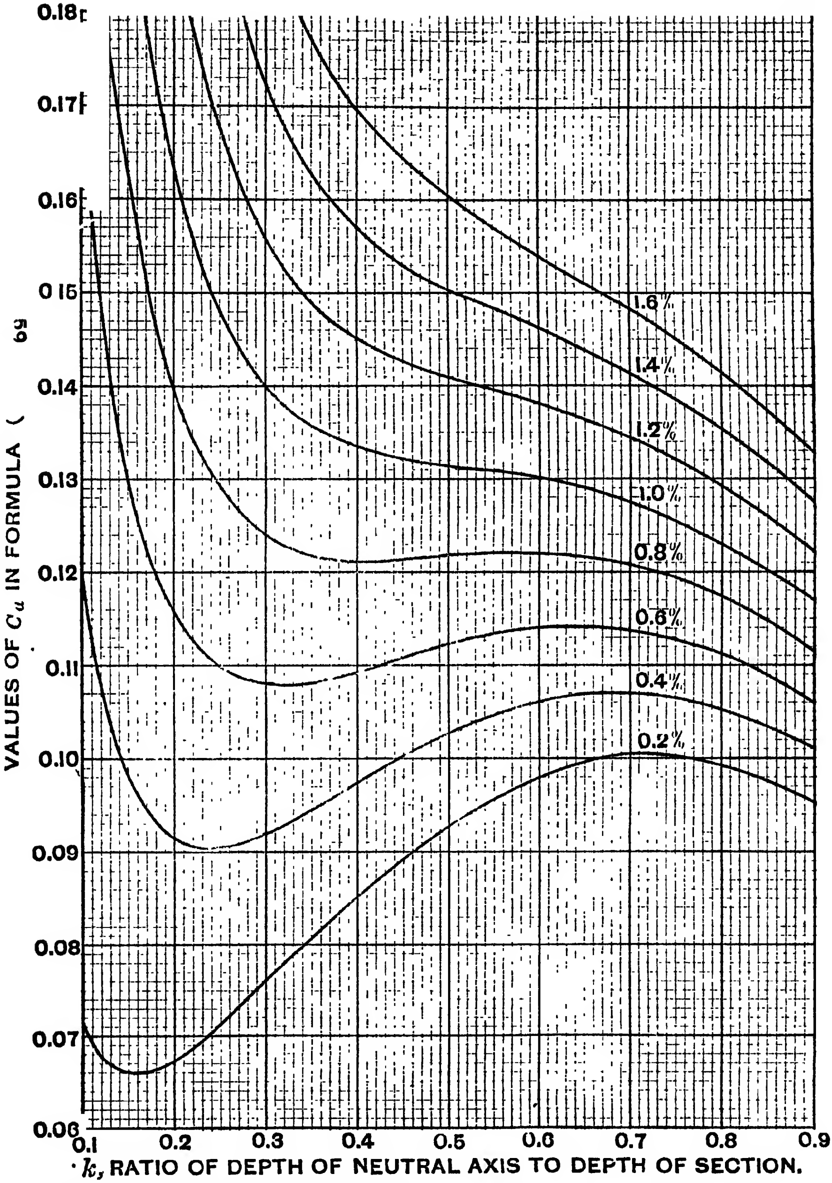


FIG. 180.—Diagram for Determining Constants C_a to be used in Formula (59). Based on $n = 15$ and $2a = \frac{4}{5}h$. (*See p.* 571.)

and k may be obtained from this equation if the size of section, percentage of steel and eccentricity are known.

By solving this formula for $\frac{e}{h}$, using $n = 15$ and $2a = \frac{4}{5}h$, we have

$$\frac{e}{h} = \frac{-k^3 + \frac{3}{2}k^2 + 14.4p}{3k^2 + 90pk - 45p} \tag{61}$$

from which equation, curves for $\frac{e}{h}$ are readily drawn for different percentages of steel. In Fig. 179, page 572, curves are plotted by this formula, using $n = 15$ and $2a = \frac{4}{5}h$, and from these the depth of the neutral axis k, may be found.* This is illustrated in the example, page 580.

In finding the unit compressive stress in the concrete for a given section having an eccentricity greater than e_0 (see page 567) and containing a known quantity of steel, the following quantities would be known: breadth, b; depth, h; ratio of steel, p; ratio of elasticity, n; eccentricity, e; and moment, M. The method of procedure of finding f_c, the maximum compression in the concrete, may then be as follows.

Determine $\frac{e}{h}$. Enter the bottom of Fig. 179, page 572, with this value of $\frac{e}{h}$ and find the k corresponding for the given percentage of steel. Then with this value of k enter Fig. 180, page 573, and find C_a. Apply formula (59), page 571, where $f_c = \frac{M}{C_a bh^2}$.

Having found the unit stress in the concrete, the unit stresses in the steel may be determined from formulas (53) and (54), page 570.

METHOD OF PROCEDURE FOR THE DESIGN OF AN ARCH.

The design of an arch is a trial process; the design being selected and then investigated to see if the sections are of sufficient strength. If the arch

---

*If the value of k must be determined directly, substitute $k = x - \left(\frac{e}{h} - \frac{1}{2}\right)$ when equation (60) takes the form $x^3 + px + q = 0$, and since by Cardan's formula,

$$x = \sqrt[3]{-\tfrac{1}{2}q + \sqrt{(\tfrac{1}{2}q) + \left(\frac{1}{3}p\right)^3}} + \sqrt[3]{-\tfrac{1}{2}q - \sqrt{(\tfrac{1}{2}q)^2 + \left(\frac{1}{3}p\right)^3}}$$

the value of k may be computed. This follows the method suggested by Professor Mörsh in "Der Eisenbetonbau," 1906, p. 111.

h = height. e = eccentricity. p = ratio of steel. k = ratio depth neutral axis.

first chosen is too large or too small it must be revised and the process repeated.

Since the location of the line of pressure and also the stresses are affected by the loading, it is customary either to compute the arch for the dead load plus concentrated loads located at the most unfavorable positions, or else to compute it for the dead load plus a uniform live load covering one-half the arch and also covering the entire arch, to see that the working stresses are not exceeded.

The following steps indicate the method of procedure for the design of a highway bridge shown in folding Fig. 181, opposite page 580. The computations are for the live load over one-half the span. The procedure is similar when the entire span is loaded.

1. Lay out on a drawing the preliminary curve assumed for the intrados. (See p. 540).

2. Assume a crown thickness in accordance with the formula on page 541.

3. Lay out the curve of the extrados and the surface of the roadway. The extrados may be a 3-centered curve, but it is better to use an arc of a circle if possible. It should be so placed as to give a ring thickness at the quarter points of the span of $1\frac{1}{4}$ to $1\frac{1}{3}$ times the crown thickness, and a ring thickness at the springings of 2 or 3 times the crown thickness in this first trial.

4 Draw the arch axis midway between the extrados and the intrados.

5. Divide the arch axis into distances such that the ratio of each distance to the moment of inertia of the cross-section of the ring at the center of the distance is a constant; that is, $\frac{s}{I}$ is a constant. This can be done by trial by beginning at the crown and working towards the springings or by the method described on page 554. The *moment of inertia is of the combined section of concrete and steel about the gravity axis*, hence the size and position of the steel rods must be first assumed, when I may be computed by the formula on page 565. The ratio of area of steel to total area of section at crown may be arbitrarily taken in the first place from 0.007 to 0.0125, that is from 0.7% to $1\frac{1}{4}$%. The divisions are separated by vertical sections.

In the problem here solved the distance, s, next to the crown is 1.14 ft., and that next to the springing is 7.82 ft. The constant ratio, $\frac{s}{I}$ *for this arch* is 11.4* On folding Fig. 181 the centers of the divisions are shown by circles and are numbered 1, 2, 3, etc. All distances are in feet and all quantities

*Greater accuracy may be obtained by using a larger number of divisions than here chosen, and also by subdividing loads P_1 and P_{10}.

involving distance are in foot units. A section of the arch 1 foot wide transversely is considered.

6. Compute the dead and live loads and enter these loads as indicated by P_1 P_2, etc., at the center of gravity of each division. In the accompanying design, a live load of 100 pounds per square foot covers the right half span, while on the left is the dead load alone of the masonry taken at 150 pounds per cubic foot plus the earth fill taken at 100 pounds per cubic foot.

TABLE 1. *Ordinates and Moments in Computation of Example*

Points	x	y	x^2	y^2	M_L	M_R	$M_L x$	$M_R x$	$M_L y$	$M_R y$
10 and 11	0.56	0.01	0.3	0.00	∞	∞	∞	∞	∞	∞
9 and 12	1.71	0.04	2.9	0.00	391	521	668	891	16	21
8 and 13	2.88	0.11	8.3	0.01	1 205	1 603	3 470	4 616	132	176
7 and 14	4.11	0.23	16.9	0.05	2 520	3 346	10 357	13 752	580	770
6 and 15	5.43	0.39	29.5	0.15	4 471	5 923	24 277	32 162	1 743	2 310
5 and 16	6.89	0.63	47.5	0.40	7 327	9 672	50 483	66 640	4 616	6 093
4 and 17	8.57	0.97	73.5	0.94	11 584	15 216	99 275	130 401	11 237	14 759
3 and 18	10.59	1.50	112.2	2.25	18 242	23 791	193 183	251 947	27 363	35 686
2 and 19	13.17	2.39	173.5	5.71	29 480	380 45	388 252	501 053	70 457	90 928
1 and 20	17.94	5.14	321.8	26.41	58 553	74 192	1 052 235	1 331 004	301 476	381 347
Σ	71.85	11.41	786.4	35.92	133 873	172 309	1 822 200	2 332 466	417 620	532 090

All distances in foot-units; all moments in foot-pounds

Values of H_c, V_c and M_c at crown for Live and Dead Loads.

$$H_c = \frac{10\,(417\,620 + 532\,090) - 11.41\,(133\,873 + 172\,309)}{2\,[10 \times 35.92 - (11.41)^2]} + 13\,107 \text{ lb.}$$

$$V_c = \frac{1822200 - 2332466}{1573} = -324 \text{ lb.}$$

$$M_c = \frac{172\,309 + 133\,873 - 2 \times 13{,}107 \times 11.41}{20} = +354 \text{ ft. lb.}$$

Values of H_c and M_c at crown for Rise in Temperature.

$$H_c = \frac{1}{11.4}\,\frac{.0000055 \times 20 \times 41.88 \times 10 \times 2000000 \times 144}{2\,[10 \times 35.92 - (11.41)^2]} = 2545 \text{ lb.}$$

$$M_c = \frac{-2545 \times 11.41}{10} = -2900 \text{ ft. lb.}$$

Values of H_c and M_c at crown for Rib Shortening.

$$H_c = -\frac{1}{11.4}\,\frac{66 \times 41.88 \times 10 \times 144}{2\,[10 \times 35.92 - (11.41)^2]} = -760 \text{ lb.}$$

$$M_c = -\frac{-760 \times 11.41}{10} = +870 \text{ ft. lb}$$

The horizontal components of the earth pressure are so small that they are neglected, except that, for purposes of illustration, they are shown in the case of the load adjoining each springing, where the horizontal components are computed by formulas for earth pressure on page 666. The point of application of the horizontal and vertical components, as shown for P_1, is taken at the arch axis. In practice, earth pressure is negligible

in the design of flat arch rings of the type here selected, and all loads may be taken as vertical. Only where the ratio of rise to span is large need the horizontal components of the earth pressure be considered.

7. Make a table similar to Table 1, page 576. The values of x and y are scaled from the drawing, and are the coördinates of the *center* points of the divisions of the arch axis. The *crown* point of arch axis is here taken as the *origin of coördinates*. The values of M_L and M_R are computed. M_L represents the moment at each of the center points 1 to 10 inclusive of all *loads* lying between the point in question and the crown. Thus M_L for point 10 is 0; for point 9, $M_L = 340 \times 1.15 = 391$ ft. lb.; for point 8, $M_L = 391 + 696 \times 1.17 = 1205$ ft. lb., and so on. The moment at each "center" point being obtained from that at each preceding "center" point. M_R of course represents the moment at each of the center points 11 to 20 inclusive of all *loads* lying between the point in question and the crown. For a symmetrical loading M_L would equal M_R for each pair of center points, such as 1 and 20.

8. Compute H_c, V_c, M_c, that is, the thrust, shear and moment at the crown, as on page 576, by using equations (16), (17), and (18), page 553. If the sign of V_c is plus the line of pressure (equilibrium polygon) at the crown slopes upward towards the left; if minus, as in the present case, upwards toward the right. A plus sign for M_c indicates a positive moment; a minus sign, a negative moment at the crown. For the arch in folding Fig. 181, the crown thrust $H_c = 13107$ pounds, $V_c = -324$ pounds and $M_c = +354$ ft. pounds.

9. Draw a force polygon as shown in folding Fig. 181 by laying off to scale the loads P_1, P_2, etc., as $0-1$, $1-2$, etc. Find the pole by laying off V_c downward (because negative) from the crown point, 10, and then laying off H_c horizontal. The hypothenuse of the triangle having H_c and V_c for sides thus slopes upward to left or upward to right, according as V_c is + or −.

10. Draw the equilibrium polygon as shown on the arch of folding Fig. 181. The resultant pressure acts above the axis at the crown a distance,

$$\frac{M_c}{H_c} = e \text{ if } M_c \text{ is plus, and below by the same amount if } M_c \text{ is minus.}$$

Since here, as is shown later, $e = +0.028$ feet, this distance is laid off vertically above the axis at the crown and through this point the resultant pressure is drawn parallel to the ray O_{10} of the force polygon and so on. It is not really necessary to draw the equilibrium polygon if the moments and eccentricities are computed for the various sections as outlined under item 11, but the polygon, which is the line of pressure, affords a good check on the algebraic work.

11. Determine the moment, thrust, and eccentricity, and if desired the shear at the center points, 1, 2, 3, etc , of the divisions, and enter in a table as shown below The moment is computed from formulas (19) and (20) on page 554, the values of whose terms have already been found by items 7 and 8. The thrust and shear may be scaled from the force polygon. For example, at section 1 on folding Fig 181 the thrust line is drawn parallel to the tangent to the axis at 1, and the shear line at right angles to the thrust line The eccentricities, *e*, of the sections 1, 2, 3, etc , are computed by dividing the moment on the section (see page 561) by the thrust for that section just scaled For positive moments and therefore positive values of *e*, the line of thrust lies above the arch axis

12 Compute the thrust and moment at the crown due to variation in temperature by formulas (25) and (26), page 556, the moments on the

TABLE 2 *Final Moments and Thrusts*

		LIVE AND DEAD				TEMPERATURE		RIB SHORTENING	
Point	$H_c y$	$V_c x$	Mom	Thrust	Ecc	Mom	Thrust	Mom	Thrust
1	67370	−5812	+3259	+14360	+0 23	±10180	±1970	−3030	−610
2	31325	−4267	−2068	+14000	−0 15	± 3180	±2310	− 950	−700
3	19660	−3431	−1659	+13920	−0 1-	± 910	±2430	− 270	−730
4	12713	−2777	−1293	+13600	−0 10	± 440	±2500	+ 130	−740
7	3014	−1331	− 483	+13240	−0 04	∓ 2320	±2530	+ 690	−760
9	524	− 554	− 67	+13160	0 005	∓ 2800	±2545	+ 840	−760
12	524	− 554	+ 911	+13120	+0 07	∓ 2800	±2545	+ 840	−760
14	3014	−1331	+1353	+13200	+0 10	∓ 23-0	±2530	+ 690	−760
17	12713	−2777	+ (27	+13640	+0 05	± 440	±2500	+ 130	−740
18	19660	−3431	− 346	+14040	−0 03	± 910	±-430	− 270	−730
19	31325	−4267	−20 9	+14200	0 15	± 3180	±-,10	950	−700
20	67370	−5812	− 656	+14840	0 04	±10180	±1970	3030	−610

Thrusts in lb Moments in ft lb Shear in arch design is small and need not be computed

various sections by formula (27), page 557, and the thrusts and shears by resolving the crown thrust into tangential and radial components, as shown in the small force polygon in the diagram

A rise in temperature of 20 degrees Fahr , and a fall of the same amount, is sufficient even in the northern part of the United States for arches with filled spandrels.

For the arch shown on folding Fig 181 the crown thrust H_c, due to temperature, is a tension of 2545 lbs , and a *compression* of equal amount. The crown moment M_c is + 2900 ft. lb and − 2900 ft lb

13. The effect of rib shortening due to the thrust is comparatively slight Where necessary to compute it, use formula (31) and (32), page 558. (See p. 576)

For the problem here shown the thrust at crown due to this cause is − 760 lb , and the moment is +870 ft. lb.

14. **Having prepared a table similar to Table 2, page 578, showing**

thrusts and moments on the various sections 1, 2, 3, etc., due to dead and live loads, temperature, and rib shortening, compute the maximum unit compression in the concrete and maximum unit tension, if any, in the steel by use of formulas on pages 565 to 574.

Table 2 shows thrusts and moments for only a few of the sections of this arch, since it is unnecessary to compute all of them. A selection of the more critical sections may be made by inspection of the equilibrium polygon. The following shows the computation of the maximum unit stresses at the crown for the arch in folding Fig. 181, as outlined in items 11 to 13.

LIVE AND DEAD LOADS AND RIB SHORTENING.

Moment	Thrust	
+ 354	+ 13107	Live and dead
+ 870	− 760	Rib shortening
+1224 ft. lb.	+12347 lb.	

$$e = \frac{M}{N} = \frac{1224}{12347} = 0.1 \text{ ft.}$$

p = ratio of steel at crown = 0.0092

Consulting lower right hand part of Fig. 177, page 569, it is seen that the value of $\frac{e_0}{h}$ for 0.92% is *greater* than $\frac{e}{h} = 0.1$. Hence there is *compression over the entire section.*

From formula (42), page 565, max. compression in concrete,

$$f_c = \frac{12347}{1 \times 1}\left[\frac{1}{1 + 15\,(.0092)} + \frac{6\,(1)\,0.1}{(1)^2 + 12\,(15)\,.0092\,(\frac{1}{3})^2}\right]$$

$$= 17100 \text{ lb. per sq. ft.}$$

$$= 119 \text{ lb. per sq. in.}$$

Stresses in steel need not be computed.

The above may be more quickly solved by the use of the curves on the left part of Fig. 177, page 569.

LIVE AND DEAD LOADS AND RIB SHORTENING PLUS TEMPERATURE.

Moment	Thrust	
+ 1224	+ 12347	
+ 2900	− 2545	Temp.
+ 4124 ft. lb.	+ 9802 lb.	

$$e = \frac{M}{N} = \frac{4124}{9802} = .42 \text{ ft.}$$

Consulting lower right hand part of Fig. 177, page 569, it is seen that the value of $\frac{e_0}{h}$ for 0.92% of steel is much *smaller* than $\frac{e}{h} = \frac{0.42}{1} = 0.42$. Hence there is *tension over a part of the section.*

From formula (60), page 571, the value of k is found to be 0.6. From formula (59), page 571, the value of the maximum compression = 35 700 pounds per square foot = 248 pounds per square inch. From formula (54), page 570, maximum tension in steel = 1440 pounds per square inch.

The approximate value of the above compression in concrete may be more quickly found by the use of curves, Fig 179 and 180, and pages 572 and 573 as shown below.

f_c = compression in concrete. e = eccentricity. M = moment. N = thrust. h = height. k = ratio depth neutral axis. a = distance centre of gravity to steel.

The method of computation for other points in the arch is similar, and stresses should be determined at sections where they appear to be the maximum.

From table 2 it is evident that although at point 20 the moment due to dead and live load is very small, its combination with moments due to temperature and rib shortening makes it one of the critical points. The moment and thrust due to live and dead load and rib shortening is $M = -656 - 3030 = -3636$ ft. lb. and $N = 14840 - 610 = 14230$ lb. Hence, $e_0 = \frac{3686}{14230} = 0.26$ ft., for $h = 1.97$, $\frac{e_0}{h} = 0.13$, $p = 0.0037$.

Inspecting the lower part of Fig. 177, page 569, it is seen that the whole section is in compression. From the same diagram for $\frac{e}{h} = 0.13$ and $p = 0.0037$, $C_e = 1.65$. Using formula (52), page 568, $f_c = \frac{14230 \times 1.65}{1.97 \times 12 \times 12} = 83$ lb. per. sq. in.

Combine now the moment and thrust due to live and dead load with those due to temperature and obtain $M = -(10180 + 3686) = -13866$ ft. lb., $N = -1970 + 14230 = 12260$ lb., $e = 1.13$ ft. $\frac{e}{h} = 0.57$.

In Fig. 179, page 572, $k = 0.37$ corresponds to $\frac{e}{h} = 0.57$. By locating this value of k in Fig. 180, the constant $C_a = 0.094$ is obtained, which substituted in formula (59), page 571, gives $f_c = \frac{13866 \times 12}{0.094 \times 12 \times (1.97 \times 12)^2} = 264$ lb. per sq. in. The stress in steel from formula (54) is $f_s = 15 \times 264 \frac{1.80 - 0.37 \times 1.97}{0.37 \times 1.97} = 5800$ lb. per sq. in.

Similar computations should be made for all critical points and when the stresses are either too small or too large, the dimensions or even the shape of the arch must be changed. Small changes may be made without refiguring the whole arch. For larger changes, all computations should be repeated and a new line of pressure determined.

LOADINGS TO USE IN COMPUTATIONS

The usual practice is to make two sets of computations; in the first place, proportion the arch ring for a live load covering the entire span and then for one covering only one-half the span. These two loadings are approximations, more or less exact, to the true loadings which produce the maxi-

mum effects. By computing a table for the thrusts and moments due to a load of unity at different points, or by the use of influence lines, the exact loading to cause maximum stresses may be found.

ALLOWABLE UNIT STRESSES

For highway bridges the maximum compression in the concrete of the ring should not exceed 500 pounds per square inch due to live and dead loads, nor more than 600 pounds per square inch due to live and dead loads, temperature and rib shortening combined. For railroad bridges three-fourths of the above values may be used

DESIGN OF ABUTMENT

The design of the foundation of an arch bridge is as important as that of the arch itself. The arch is designed on the assumption that the foundation is unyielding, and this condition must be approached as nearly as possible in order to insure the stability of the whole structure.

The depth of the foundation as well as the shape is dependent upon the local conditions, and in the more difficult cases these have to be chosen after exhaustive studies A certain shape of abutment is first assumed, and this is then reviewed to see that the load upon the ground does not exceed the allowable load and that it is well distributed Allowable loads are discussed on page 541.

The forces acting on the foundation are

(1) the thrust of the arch, (2) the weight of the foundation, (3) the weight of the earth above it, and (4) the lateral earth pressure The thrust of the arch is the largest when the live loading extends over the whole span of the arch, and for this the line of pressure should be drawn first A line of pressure for the thrust on account of the total dead load and of the live load extending only over one-half the span opposite to the abutment also should be drawn to see whether, because of intersecting the abutment higher up, it does not produce larger pressure on the foundation A good scheme is to design the abutment in such a way that the line of pressure on account of one thrust intersects the base a little way to the left of the center while the other intersects to the right of the center. In some cases a third line for the total dead load, plus live load on the half span nearest the abutment should also be drawn

The line of pressure of the forces should be as near to the center of the base as possible, since the maximum unit pressure is the smallest when the load is distributed uniformly over the entire section. This also prevents uneven settling of the foundation, and thus adds considerably to the stability of the whole structure.

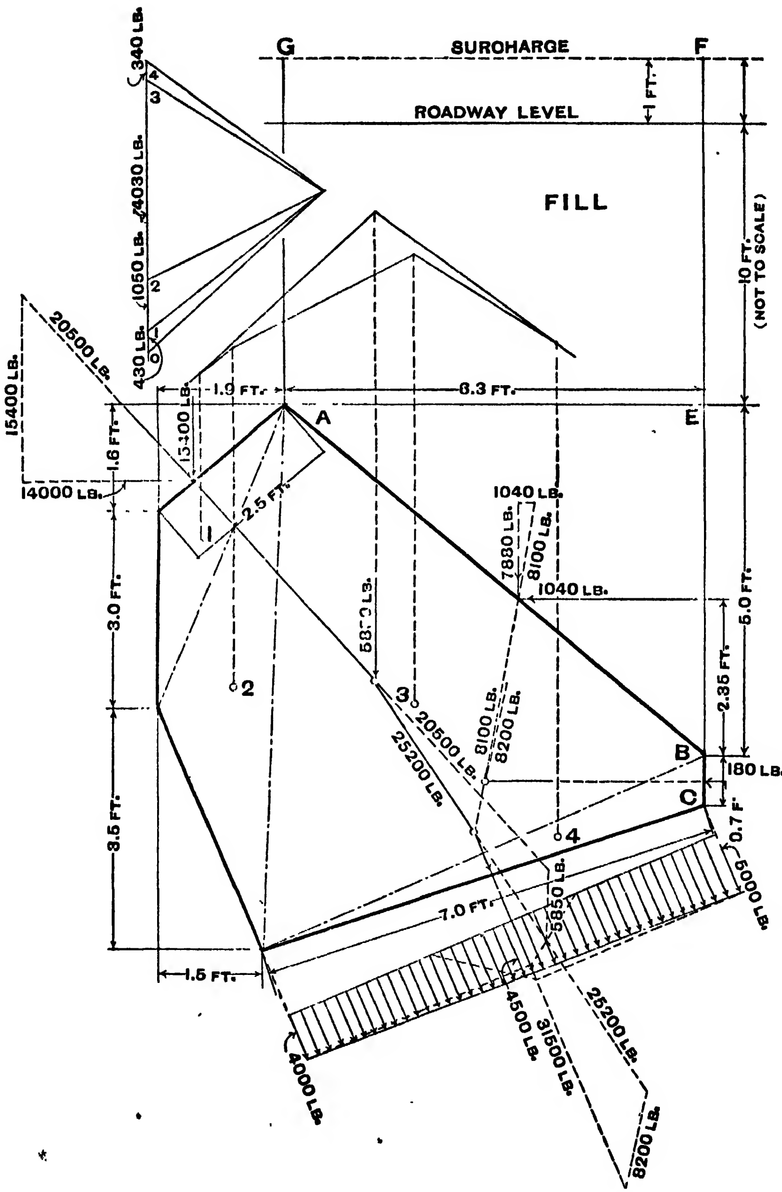


FIG. 182.—Design of a Foundation for an Arch. *(See p. 585).*

(To simplify the drawing only one position of thrust and one line of pressure is drawn.)

Fig. 182, page 584, clearly illustrates the design of an abutment. The outline is assumed, then the location and magnitude of the forces acting upon the abutment are found and the line of pressure determined. If the assumed outline is not satisfactory it should be revised.

For the benefit of those who are not familiar with the common principles of such design, the steps will be considered in detail. The magnitude, 20,500 pounds, and position of the arch thrust is given in the arch example. Since the weight of the masonry acts through its center of gravity, this point must next be found and this is most readily done by dividing the outline of the abutment into triangles and rectangles. The weights of each of these prisms one foot thick are readily computed, and the center of gravity found through which the weight force acts. A force polygon for any pole distance, as shown in the upper left corner of the diagram, is drawn and the equilibrium polygon, by the intersection of the closing lines, locates the resultant of the weight which, by computation, is found to be 5850 pounds.

The pressure on AB consists of the horizontal pressure on BE, and the weight of the prism of earth whose cross-section is $ABFG$* and thickness one foot. Taking the weight of one cubic foot of filling at 100 pounds, the weight of the prism would be $\frac{10 + 15}{2} \times 6.3 \times 100 = 7880$ pounds.

The horizontal pressure on BE is equal to the difference between the pressures on BF and EF.

Let

$$w = \text{weight of one cubic foot of earth,}$$

then, if the weight of earth is assumed at 100 pounds, from formula (2), page 664, pressure on the plane

$$BF = \frac{w}{6} \times BF \times \tfrac{1}{2} BF = \frac{100}{6} \times 15 \times \tfrac{1}{2}\, 15 = 1870 \text{ pounds,}$$

and on the plane

$$EF = \frac{w}{6} \times EF \times \tfrac{1}{2} EF = \frac{100}{6} \times 10 \times \tfrac{1}{2}\, 10 = 830 \text{ pounds.}$$

Hence horizontal pressure on plane BE = 1040 pounds.

The point of application is found from the formula (7), page 666.

In the case under consideration $H = 15$ feet, $h = 10$ feet, where H is the depth of point B and h the depth of A or E below the line of surcharge.

The horizontal pressure on BC is by formula (6), page 666, 180 pounds, and the point of application may be assumed in the middle of BC without appreciable error.

* The live load being 100 pounds per foot is equivalent to a surcharge one foot in height.

Having thus located all forces and found their magnitude, the line of pressure is drawn. This procedure consists simply in finding the resultant of two forces intersecting in one point. The line representing the thrust is prolonged until it intersects the line representing the weight of masonry, 5850 pounds. Beginning at this, the magnitude of the thrust, 20,500 pounds, is laid off to any desired scale and the resultant of this with the weight of the masonry, 5850 pounds, is found to be 25,200 pounds. Combining this new force in turn with the earth pressures of 8100 pounds and 180 pounds completes the line of pressure with a final resultant thrust of 31,500 pounds.

Having found the line of pressure, the thrust is divided by the projection of the base on a line at right angles to the thrust and the maximum pressure on the ground is found by formula (36), page 562, to be 5000 pounds per square foot.

The same result is obtainable by the following simple graphical method:

Find the average unit pressure by dividing the thrust by the area of the projection of the base, drawn perpendicular to the thrust. In this case we have $\frac{31500}{7} = 4500$ pounds per square foot. Plot this, to any convenient scale, perpendicular to the projection to the base at its center; connect the $\frac{1}{3}$ points of the base with the top of this perpendicular, as shown by the dash lines in Fig. 182, and produce one of these lines till it intersects the line representing the direction of the thrust. The perpendicular distance of this point from the projection of the base is the maximum thrust and the distance of the other intersection of a slanting line with the thrust line is the minimum thrust. To draw the trapezoid of pressure, draw, through these two intersections, lines parallel to the projection of the base, as shown, and the extremities of these parallel lines will fix the two corners of the trapezoid. The maximum pressure is always at the end of the base nearest the thrust.

ERECTION

As in other reinforced structures, the erection is as important as the design. Perhaps the first essential is the centering which should be planned out in advance almost as carefully as the arch itself.

Methods of Arch Construction. There are two general methods of laying the concrete in an arch, each of which has strong advocates. By the first, the arch is laid in separate blocks across the bridge, and by the second, in narrow ribs from abutment to abutment. If the block method is followed, the lowest stones at the springing line are laid first, then stones

intermediate between the spring and the key, next the two stones each side of the key, and finally, after filling in the intermediate blocks, the key is placed. This distributes the weight of the concrete uniformly over the arch center, and prevents unequal settlement, which tends to crack the arch near the springing lines. On the other hand, the entire weight falls upon the center, and the latter must be very strongly built. The arch thrust acts at right angles to the joints, and as the blocks extend clear across the bridge, there is no danger of longitudinal splitting, but the radial joints offer planes of weakness in bending

By the other method the work can be readily arranged so that a day's labor consists of the laying of a single rib, thus forming a complete arch of itself, which as soon as it sets bears its own weight This arch section has no joints, so that when subsequently loaded the bending moment is best resisted.

A small arch, where the center can be solidly built, may be laid at one operation, commencing at both abutments and working toward the key so that it is in fact a monolith.

The spandrel or face walls may be carried up at the same time the arch ring is laid, or may be connected with it later by leaving short lengths of steel projecting radially from the concrete of the arch.

If steel is introduced, the consistency of the concrete must be wet enough to thoroughly coat it. This may be accomplished by a quaking or jelly-like mixture, which requires but slight ramming

From an architectural point of view, the treatment of the face is of much importance For a discussion of the different methods reference should be made to page 288.

Railings and ornamental work may be cast in molds if preferred and put in place after hardening

Centering. The falsework for concrete arches is practically the same as for stone arches except that close lagging is necessary. It must be rigid during the construction of the arch and stiff enough to prevent its distortion from the unsupported weight of the concrete before the keying of the arch.

The design of the centering is frequently governed by the character of the ground underneath. In general the framed wood centering made into a truss rests upon pile or trestle bents The spacing of these bents is determined by the foundation and the difficulty of placing them, and by the height and span of the arch. In certain cases it is possible to support the centering in whole or in part by the reinforcement, although this is not usually economical because more carefully framed steel is required than is

necessary for reinforcing the arch. In at least one case* reinforced concrete forms were used.

In connection with the description of arch centers which he has built, Mr. James W. Rollins†, Jr., gives the following notes:

For small arches the simplest center is a circular rib made of three pieces of 2-inch plank, laid with broken joints, all being spiked solidly together, with a tie of plank at the springing. On this, 1-inch lagging is laid close. For a larger arch, the circular rib, as above described, with generally three braces, one at center and one on the quarter at each side, is used, the center of the whole rib having a post under it. We have used such a center up to 30-foot span for both brick and granite arches, carrying a 30-inch arch sheeting.

The design of a center for larger arches depends upon local conditions, also upon the relation of rise to span. In flat arches, with low side walls, it is well to use posts with intermediate bracing, on numerous supports. In a high arch we may use long braces extending directly from a center support to the rib, at intervals of 6 feet to 8 feet.

Mr. Rollins advocated for wedges, seasoned oak, 8 inches wide, 4 inches thick at the thick end, 2 inches at the thin end, and 18 inches long, planed on sliding faces, and thoroughly greased. When setting the center, these wedges, placed between the caps on the bents and the corbels under the lower chord of rib, are tacked together to prevent slipping.

Boxes filled with sand are frequently used between the caps of the bents and the lower chords of the trusses in place of wood wedges. The sand in these must be thoroughly packed to prevent settlement of the concrete before setting. The sand is readily removed by letting it out through a hole in the box. Jack-screws also may answer the same purpose as wedges or sand boxes. By any of these means the centering is easily lowered.

The ribs of the centering are usually made of several pieces of plank spiked or bolted together. Upon the ribs rests the lagging, which usually consists of one or two layers of planking having the top surface smoothed to give a good surface to the soffit of the arch, and laid with tight joints. With thin lagging care must be taken to prevent deflection.

Instead of the ribs forming a part of the truss, they are frequently supported directly upon the wedges resting upon the caps of the bents, the posts of which run up to the soffit of the arch for that purpose.

The centering should be cambered, that is, should be made higher than called for in the arch plans at the center, so that when it is removed, the arch will be in the position assumed for it in the design. Some engineers make

*_Engineering News_, Aug. 30, 1906, p. 215.

†Journal Association of Engineering Societies, July 1901, p. 10. For examples of centers built in various places, see References, Chapter XXXI.

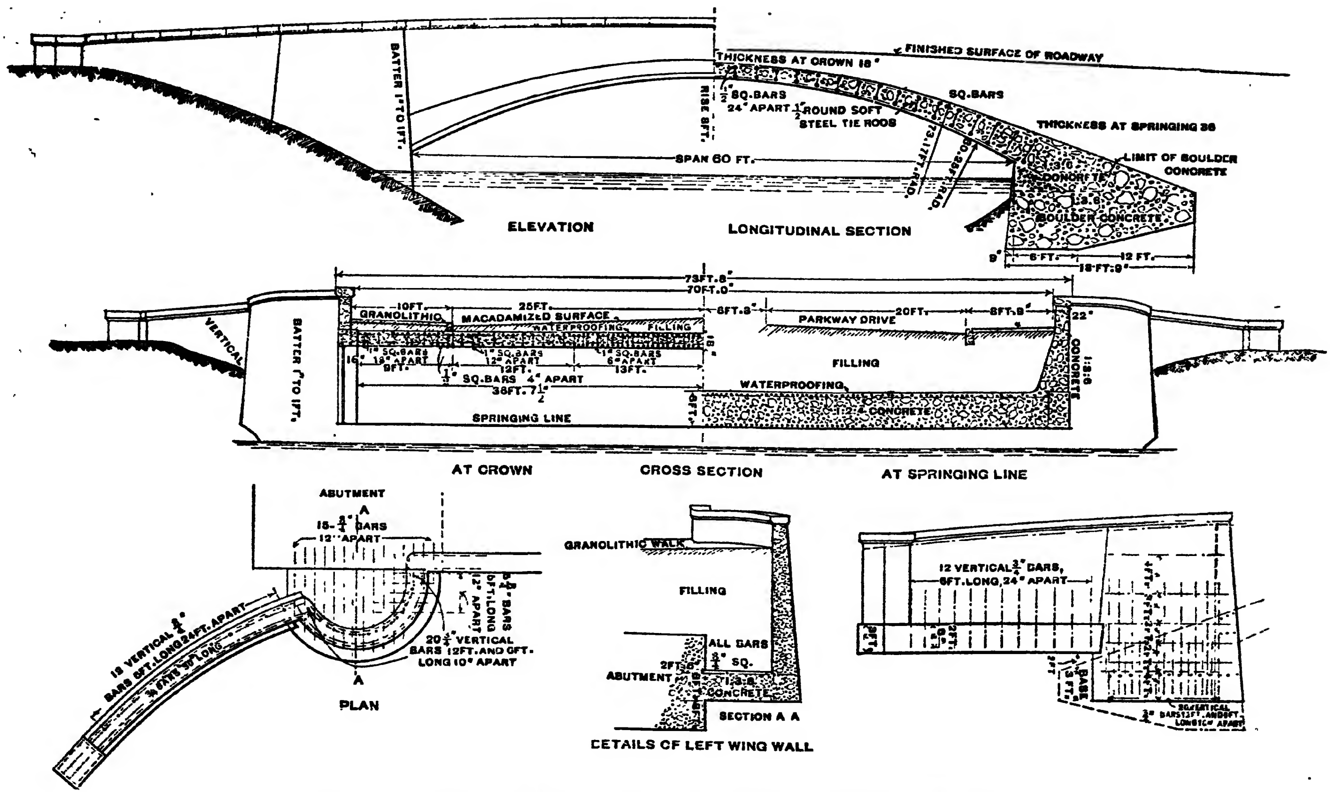


FIG. 183.—Design of Mystic River Bridge, Medford, Mass. (*See p.* 590.)

the camber equal to the deflection of the arch which would be caused by the live and dead loads

In striking the centers sudden settlement must be avoided and the centers must not be removed until the concrete has attained good strength. The time of removal must be determined by the design of the bridge and the weather. For light highway bridges four weeks is usually sufficient, while for a heavy arch of long span eight weeks may be required.

EXAMPLES OF ARCH BRIDGES

Mystic River Bridge, Medford, Mass This arch, illustrated in Fig. 183, page 589, is of the Monier type and carries a parkway over the river It was built in 1906 by the Metropolitan Park Commission, Mr John R. Rablin, Chief Engineer

The arch has a span of 60 feet, a rise of 8 feet, and a crown thickness of 18 inches Both the intrados and the extrados are segmental The side walls are of concrete with a vertical expansion joint at each abutment. The retaining wall for the earth fill over the abutments is of reinforced design and curved as shown in the details in the drawing

Granite Branch Railroad Bridge. A railroad bridge of similar design to the Mystic River Bridge was built by the Metropolitan Park Commission of only 4 feet longer span than the highway bridge described The heavier loading necessitated a thickness of crown of 24 inches instead of 18 inches with a thickness at springing still greater in proportion

3-Hinged Ribbed Arch on Ross Drive, District of Columbia. A different type of structure and one which illustrates the combination of arch ribs with a reinforced concrete floor system is illustrated in Fig 184, page 591. This was built in 1907 by the Engineering Commissioner, Washington, D C, Mr. W J Douglas, Engineer of Bridges

The central arch is 100 feet clear span and 15 feet rise, and the roadway, which is 16 feet wide and macadamized, is laid upon a 6 inch reinforced concrete floor slab supported by longitudinal concrete girders which in turn rest upon columns supported directly by the concrete ribs The three arch ribs, which are reinforced as shown, are 2 feet wide throughout their length with a thickness of 2 feet 6 inches at the crown

Each hinge consists of two steel castings, shown in detail, with a pin 4 inches in diameter, and these hinges are imbedded in the concrete. An expansion joint is provided in the roadway deck over each springing. The floor of the arch was computed for a 6-ton wagon, and the ribs for a live load of 100 pounds per square foot of roadway The maximum compression on the concrete of the ribs under live and dead loads is 500 pounds per

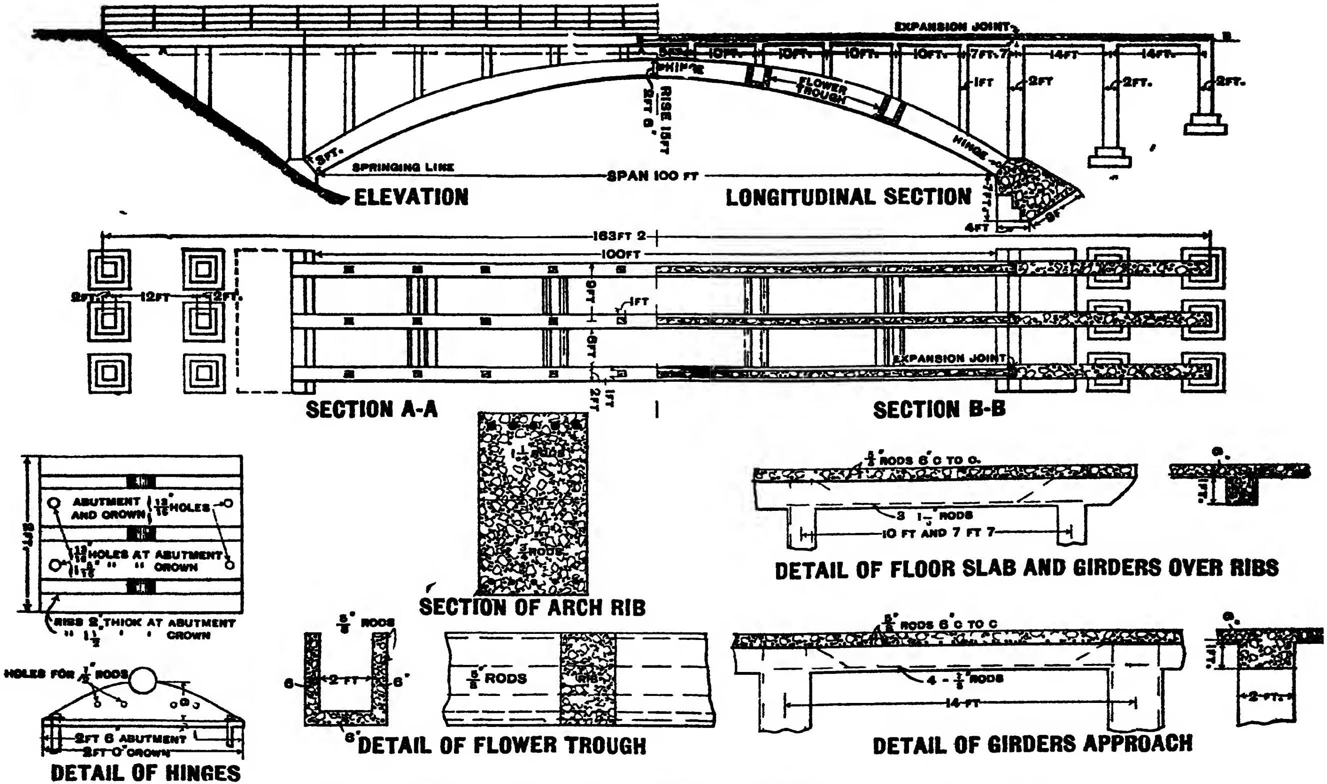


Fig 184 —Three-Hinged Ribbed Arch, Ross Drive, District of Columbia (See p 590)

square inch, and there is no tension. The cost of the structure was $8000, which is equivalent to about $3.00 per square foot of the roadway.

Walnut Lane Bridge, Philadelphia. A notable structure in concrete is the Walnut Lane Bridge built as it is with a clear span of 233 feet. The arch was completed in 1908 under the direction of the Bureau of Surveys, Mr. George S. Webster, Chief Engineer and Mr. Henry H. Quimby, Assistant Engineer. The principal arch consists of two ribs, upon which rest cross walls connected by small longitudinal arches of 20 feet span carrying the spandrel wall supporting the I-beams of the floor.

A fine photograph of the arch is shown in Fig. 156, page 532, and cross sections illustrating the design in Fig. 185, page 592. The balustrade is entirely of concrete, the posts being molded on the ground and the surface washed off with water to reveal the aggregate.

Other Notable Bridges. For references to other bridges built in recent years, see Chapter XXXI.

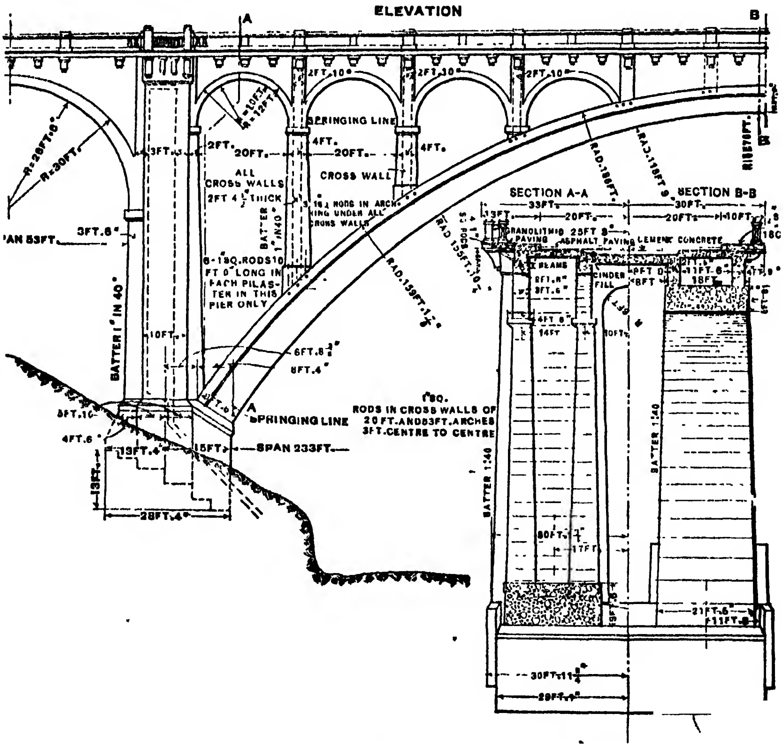


Fig. 185 Walnut Lane Bridge, Philadelphia. (*See p.* 592.)

CHAPTER XXIII

SIDEWALKS, BASEMENT FLOORS AND PAVEMENTS

The introduction of reliable American Portland cements has rendered concrete available for sidewalks and other similar purposes at a price not more than two-thirds of that previous to 1890, when German and English cements were used. Portland cement being thus commercially within reach of builders, masons have become familiar with its use, and concrete sidewalks, because of their economy and durability, are supplanting those of other materials.

Street pavements are also being made of concrete, and with apparent success,* by methods similar to those which obtain in sidewalk construction.

The essentials for a good concrete sidewalk are an artificial foundation of firm but porous material, through which the rain water may percolate, a base of good strong concrete, and a wearing surface of rich mortar, troweled to a smooth, dense surface. The walk must be divided into blocks, with the joints between them forming lines of weakness, so that if any cracks occur through shrinkage, settlement, or frost, they will occur at the joints and thus not be noticeable.

Vault light construction in concrete requires even greater skill than ordinary walks, and should never be attempted by inexperienced constructors.

The construction of basement floors is similar to sidewalk work except that in dry ground an artificial foundation is not always necessary, and, there being less danger of settlement and frost, the blocks of such a floor may be of larger size, having occasional joints to provide for contraction from changes in temperature.

Floors above the ground level in buildings whose design is considered in Chapter XXIV, page 609, may be surfaced with mortar in a manner similar to the wearing surface of walks, or the concrete may be floated without the extra coating of mortar.

MATERIALS FOR CONCRETE SIDEWALKS

The selection of a first-class Portland cement is an absolute necessity.† Natural cements will not stand the wear, and Puzzolan cements are liable

Engineering News, Jan. 28, 1904, p. 84.

†See Cement Specifications, p. 29.

to surface deterioration from the action of the weather. Walks have been built with a Natural cement concrete base, and a wearing surface of Portland cement mortar, but the results have been unsatisfactory, for even if the surface coat is laid before the Natural cement concrete base has set the Portland cement does not adhere strongly and is likely to scale off.

Mr. Harry T. Buttolph* suggests that the breaking up of the surface appears to be due to the difference in expansion of Natural and Portland cement. He has noticed that the surface of such slabs sometimes curls up like a sheet of paper.

For the foundation, by which is meant the prepared surface underneath the concrete, any porous material such as broken stone, gravel (preferably with sand screened out), or cinders may be employed.

For the base, which consists of a layer of concrete from 3 to 5 inches thick, ordinary materials, such as broken stone and sand, screened gravel and sand, or gravel as it comes from the bank without screening, may be used for the aggregate Unscreened gravel is not generally advisable, however, because a more uniform mixture can be obtained by screening the gravel and remixing the sand with it in definite proportions (See p. 112.) The proportions frequently used in our large cities for the concrete base are 1 part Portland cement to 2 parts sand to 5 parts stone, based in some localities upon the volume of cement as packed in the barrel, and in others upon the volume loose, although the resulting proportions obtained in the two cases are very different. (See p 218) In many cases these proportions are richer than is necessary. In Germany,† proportions 1:3:6 are recommended for heavy duty, and 1 5 10 for light work, while for ordinary requirements 1:4 8 are specified. The last two proportions appear rather lean for ordinary conditions, but 1:3:6, if the relative volumes are based on a unit of 3 8 cu. ft. to the barrel, should be satisfactory for ordinary conditions, with 1:2½ 5 for more important construction, or for pavements to be subjected to severe usage, such as teaming. If the proportions are based upon the volume of cement measured loose, the required parts of sand and stone must be decreased by about 10%; thus 1:3:6 would become about 1:2¾:5½.

The wearing surface, whose thickness varies in different specifications from ½ to 1 inch, should be laid with the same first-class Portland cement as is the base. Customary proportions are equal parts of cement and aggregate. Either sand, or fine crushed rock, or a mixture of the two,

*Personal correspondence.

†"How to Use Portland Cement," translated from the German of L. Golinelli by Spencer B Newberry, p. 26.

may be used to form the mortar. If crushed rock is used, — and good crushed rock is usually preferable to sand, — it should be of a texture such as granite or trap, which will break into cubical, rather than flat or laminated fragments. The size of crushed stone specified by the majority of engineers is that which will pass a ¼-inch sieve, although a few cities require finer material, Chicago, for example, specifying* torpedo sand ranging from ⅛-inch down. Such sand is too fine to give a strong mortar. On the other hand, some cities, including Omaha, Neb.,† require crushed stone which will pass a ½-inch mesh sieve.

The requirements in various cities throughout the United States in 1900 are shown in the following table:

Requirements in Various Cities.‡ (*See p.* 595.)

City.	Foundation		Base.		Wearing Surface		Dry Coating.		Size of Blocks	Guarantee.
	Thickness.	Material.	Thickness.	Proportions.	Thickness	Proportions Cement Sand	Proportions Cement.	Sand.		
	Inch		Inch		Inch					Yr.
Boston	12	Broken stone, gravel or cinders	3	1:2:5	1	1:1	...		3½ to 6 ft. sq.	10
Rochester ..	6	Sand, gravel, broken stone or cinders	||	1:5	1	2:3	...		....	3
Philadelphia	3	Sand, gravel, broken brick, stone or cinders	3	.....	2	1:2	1:1		....	..
Washington	0		4	1:2:5	1	2:3	1:1		....	5
Chicago ...	12§	Cinders	4¼	1:2:5	¾	1:1	...		5 ft. x 6 ft.	10
Milwaukee .	4	Cinders or broken stone	2½	1:3.5	1	1:1	...		24 to 36 sq. ft.	...
St. Louis...	8	Cinders	3½	1:3	½	1:1	...		....	1
Omaha	4	Gravel, slag or stone...	3	1:2:4	1	1:2	3:1		....	5

Coloring Matter. The appearance of a walk is improved by being slightly colored. The following formulas are recommended by Mr. L. C Sabin:¶

*1899 Specifications.

†1898 Specifications.

‡From Typical Concrete Sidewalk Specifications, by Sanford E. Thompson, in *Cement*, July 1900, p. 85.

§No foundation required where the soil is clean sand.

||Specified for each contract.

¶Sabin's "Cement and Concrete", 2nd Edition, p. 382.

Colors for 1 : 2 Mortar. By Louis C. Sabin (see p. 595)

MATERIAL.	½ LB. PER 100 LB. CEMENT.	4 LB. PER 100 LB. CEMENT.	COST PER LB.
Lamp Black	Light slate	Dark blue slate	15 cents
Prussian Blue	Light green slate	Bright blue slate	50 "
Ultra Marine Blue		Bright blue slate	20 "
Yellow Ochre	Light green	Light buff	3 "
Burnt Umber	Light pinkish slate	Chocolate	10 "
Venetian Red	Slate, pink tinge	Dull pink	2½ "
Red Iron Ore	Pinkish slate	Light brick red	2½ "

NOTE: Colors vary with quantity of material added. Cost is per lb. of coloring matter. Colors are apt to fade unless formed by color of crushed rock.

Quantity of Materials Required. The volumes of materials required to cover a certain area of surface are determined by the thickness of the walk or floor, the proportions in which the materials are mixed, and the character of the materials.

The following table gives the approximate quantity of materials necessary for 100 square feet of surface for walks of various thicknesses of base and wearing surface. It is assumed in compiling the table that the coarse aggregate of the base contains about 45% voids, and that the stone and

Materials for 100 Square Feet of Concrete Sidewalks. (See p. 596.)
Proportions based on a barrel unit of 3.8 cubic feet.

Base.							Wearing Surface.						
	Proportions. 1:2½:5			Proportions. 1:3:6				Proportions. 1:1		Proportions. 1:1½		Proportions. 1:2	
Thickness.	Cement.	Sand.	Stone.	Cement.	Sand.	Stone.	Thickness.	Cement.	Sand.	Cement.	Sand.	Cement.	Sand.
in.	bbl.	cu. yd.	cu. yd	bbl.	cu. yd.	cu. yd	in.	bbl.	cu. yd.	bbl.	cu. yd.	bbl.	cu. yd.
2½	1.10	0.39	0.78	0.94	0.40	0.80	½	0.85	0.12	0.68	0.14	0.56	0.16
3	1.33	0.47	0.94	1.13	0.48	0.96	¾	1.28	0.18	1.02	0.21	0.85	0.24
3½	1.55	0.55	1.10	1.32	0.56	1.12	1	1.70	0.24	1.36	0.29	1.13	0.32
4	1.77	0.63	1.25	1.51	0.64	1.28	1¼	2.13	0.30	1.70	0.36	1.41	0.40
4½	1.99	0.70	1.41	1.70	0.72	1.44	1½	2.56	0.36	2.04	0.43	1.69	0.47
5	2.21	0.78	1.56	1.89	0.80	1.60	2	3.41	0.48	2.72	0.57	2.26	0.63

NOTE.—Select and add together the quantities of each material corresponding to the required thickness and proportions of base and wearing surface.

sand are measured loose by shoveling into boxes or barrels, on the basis of the volume of a cement barrel of 3.8 cubic feet. For example, proportions 1: 3: 6 are equivalent to 1 barrel Portland cement, 11.4 cu. ft. of sand and 22.8 cu. ft. of broken stone or gravel, while proportions 1: 2 are equivalent to 1 barrel of Portland cement to 7.6 cu. ft., or one bag of Portland cement to 1. 9 cu. ft. of sand or crushed stone. The variation in volume of mortar produced with sand and crushed stone of different fineness may affect the quantities for wearing surface by at least 10%, but to provide for such variation, and to allow for waste, 10% has been added, in computing the values, to the quantities in the table on page 231.

Since the volumes are given separately for the base and wearing surface, the quantities required for walks of other thicknesses may be readily estimated, as illustrated in the following example:

Example: — What materials will be required for a walk 8 ft. in width and 150 ft. long, the base to be 3 in. thick, of concrete in proportions 1: 3: 6, and the wearing surface one inch thick, in proportions 1 part cement to 1 part sand?

Solution: — Referring to the table we find directly that for 100 sq. ft. of base 3 in. thick, 1.13 bbl. Portland cement, 0.48 cu. yd. sand, and 0.96 cu. yd. broken stone or gravel are required. Similarly, for 100 sq. ft. of the wearing surface one inch thick we should require 1.70 bbl. cement and 0.24 cu. yd. sand. For each 100 sq. ft. of completed walk there would therefore be needed 2.83 bbl. cement, 0.72 cu. yd. sand, and 0.96 cu. yd. broken stone or gravel; and since there are 1 200 sq. ft. in an area of 150 by 8 ft., for both base and wearing surface we should require 34 bbl. Portland cement, 9 cu. yd. sand, and 12 cu. yd. broken stone or gravel.

TOOLS

The following implements are required in ordinary concrete walk construction:

Mortar box for mixing the materials for wearing surface.

Platform about 12 ft. square for mixing concrete* (see Fig. 7, p. 22).

One or more iron wheelbarrows for handling the materials and the concrete (see Fig. 4, p. 18).

Square-pointed shovels (see Fig. 3, p. 18).

Hoe.

2-inch scantling of a width corresponding to the thickness of the walk.

⅜-inch stuff of same width as scantling, for curved forms.

Steel square.

*Sometimes unnecessary.

Spirit level.
Straight-edge long enough to extend across the walk.
Two rammers about 5 inches square, with handles about 4 feet long (see Fig. 99, p. 281)
Wooden stakes
Iron pins and twine for stretching line.
Mason's trowel
Pointing trowel
Plasterer's steel trowel (see Fig 186, p 601).
Plasterer's wood float
Groover (see Fig 187, p 601)
Edging trowel (see Fig 188, p 602).
Dot roller (see Fig. 189, p 602)

METHOD OF LAYING SIDEWALKS

Successful sidewalk construction is as dependent upon careful attention to small details which have been proved essential to good workmanship, as upon adherence to the more general directions given in any set of specifications. The full description of methods to be employed in laying a walk are given for the benefit of those who are unable to take advantage of the experience of specialists in this line Experienced contractors often can perform such work better and cheaper than it can be done by day labor.

Thickness of Walk. A total thickness of 4 inches of concrete and mortar laid upon a 10 inch foundation of porous material gives excellent results for ordinary sidewalks, although 5 inches is often required for public works. In locations subject to wide changes in temperature, as Boston and vicinity, a thickness of 4 inches has proved satisfactory, while in some cities 3½ inches only is required. For a 4 inch walk it is advisable to make the base 3 or 3½ inches and the wearing surface 1 or ½ inch thick. The slope of surface often adopted is ¼ or ⅜ inches to the foot.

Driveways or walks which are subjected to excessive wear may be 5 or 6 inches thick, the upper 1 or 1½ inches constituting the wearing surface.

Foundation. The construction of the foundation is as important as the laying of the concrete. For out-of-door construction the foundation should generally be from 6 to 12 inches thick, depending upon the character of the soil. In localities unaffected by frost and having soil sufficiently porous to carry off surface water, the foundation may be omitted entirely, and the concrete laid upon natural ground excavated to the required depth.

In Washington, D. C.,* no foundation is specified, and even in Chicago* it is not required where the soil is clean, porous sand. For basement or cellar floors which are not to be subjected to frost, the concrete may usually be placed directly upon the soil; but in compact ground, or where surface water is troublesome, blind drains of pipe or of cobble stones, carefully rammed, should be laid at various points.

The materials for a foundation, where such is required, may be broken stone, gravel, cinders, or coarse sand. In order to make it more porous, broken stone or gravel should be screened. Whatever material is employed it must be thoroughly rammed so as to present a firm and unyielding surface. Cinders or sand should be thoroughly wet when being rammed.

Concrete Base of Walk. The coarse concrete constituting the main body of the walk is generally called the base. Before this coarse concrete of the base is placed, the surface must be carefully laid off into squares or blocks. Such divisions are absolutely essential, since the joints furnish lines of weakness along which cracks will occur if the concrete is affected by the freezing of the soil beneath tree roots, unequal settlement, or temperature changes, and also facilitates the replacing of a block if one is injured from any cause.

There are three distinct methods of forming separate blocks: (*a*) laying the blocks alternately, and then filling in between them; (*b*) allowing the scantling of the forms to remain in place until after the concrete is laid, and then filling the spaces they occupied with lean mortar or sand; (*c*) placing tarred paper between the blocks. The first method is usually preferable.

The size of the blocks depends upon the width and shape of the walk or floor. Blocks nearly but not quite square have a better appearance than those which are distinctly oblong. The limit of size for a 4-inch walk is generally placed at 6 feet square. In 5 inch work this may be safely increased to 8 feet square. Joints should be placed around trees and about 6 inches from buildings, manholes, or other adjacent structures.

After ramming and leveling the foundation, if there is no curb to be formed, strips of scantling 2 inches thick, and of a width corresponding to the thickness of the walk, are placed on edge along the back and front lines of the walk, and held in place by stakes driven behind them. These strips should have notches cut in them to designate the location of the dividing line between the blocks. Other strips, located by these notches, are placed across the walk, which is now ready for the concrete.

The concrete materials in the specified proportions are mixed as de

*Specifications for 1899.

scribed on page 20. If the surface of the road is hard and smooth, the mixing may be done upon it without any platform. In any case, it must be very thorough, some contractors employing a man to rake each shovelful as it is turned by the two shovelers. Enough water should be added to produce a jelly-like consistency, the mortar rising to the surface when lightly rammed. The surface of the coarse concrete must be below the level of the top of the forms so as to give room for the finishing coat, or wearing surface.

If the walk or floor is laid in alternate blocks by the first method (*a*), described above, the forms around each block are left in until after the top coat or wearing surface has been placed, and has slightly stiffened, when they may be removed and the alternate blocks laid. The latter must be placed on the same day, however, to avoid difficulty in forming the surface joints between the stones. If a filler is placed between the blocks, the forms are lifted soon after the concrete of the base is laid, and before the wearing surface is spread, and the joints filled with sand or, in some cases, by a "separator" of lean mortar mixed, say, 1 part cement to 4 or 5 parts sand. Whatever the material used, it must be weaker than the concrete.

Wearing Surface. As soon as a few of the blocks of concrete base have been laid, and before they have set, the mortar for the wearing surface must be placed. This surface, as described on page 594, consists of a mixture of cement and sand, cement and fine crushed stone, or cement and a mixture of sand and stone. The materials should be very exactly proportioned, so as to give a uniform color. The cement must not be mixed with the sand long in advance of its use because the natural moisture in the sand will cake the cement. If the work is progressing so slowly that the cement must be measured by pailfuls, a determination must first be made of the number of pails of loose cement in a bag or barrel of packed cement, and the number of pails of sand in a barrel of loose sand, then the relative volumes calculated to allow for the increase in bulk of the loose over the packed cement. Each pail must be filled in exactly the same way, so that one measure will not be more densely packed than the next. The sand and cement must be mixed dry until the color is absolutely uniform, when, if coloring matter is used, it is added to this dry material. Water is added to give about the consistency employed by a mason in laying brick, so that it can be readily leveled off with a straight-edge. This mortar is carried from the mortar box to the walk in pails, and smoothed off with a straight-edge guided by the tops of the forms.

The surface is roughly floated with a plasterer's trowel, shown in Fig. 186.

soon after leveling with the straight-edge, but the final floating is not performed until the mortar has been in place from two to five hours and has partially set. The final floating is done first with a wooden float and afterwards with a metal float or plasterer's trowel. Just before the floating, a very thin layer of "dryer," consisting of dry cement and sand, mixed in proportions 1:1 or even richer, is frequently spread over the surface, but this is generally undesirable as it tends to make a glassy walk.

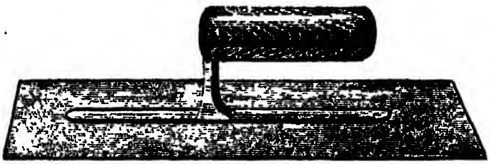


FIG. 186.—Plasterer's Trowel, or Metal Float. (*See p.*600.)

The surface is now ready to groove, for by this time the intermediate stones should be in place. As has been stated, the cross joints are in line with notches in the outside forms. The mason can thus locate the joints between the blocks of base concrete. To find the line exactly, he runs his small pointing-trowel down through the upper layer, and feels for the joint below. With the ends of the joints thus marked, he lays a straight-edge flat across the walk against these marks, and, walking across on the straight-edge, marks the line and also cuts through the partially set mortar and concrete by running his small pointing-trowel to the full length of the blade. Moving the straight-edge back a fraction of an inch, he runs his groover (see Fig. 187) along the line cut by the trowel, using the straight-edge for a rule. Both edges of the walk are rounded off by the edging trowel (see Fig. 188), which is a small float with one of its edges curved. The entire surface is finally gone over once more with the metal float to erase any marks or scratches which may have been made. A dot roller (see Fig. 189) or grooved roller may be employed to relieve the smoothness.

FIG. 187.—Groover. (*See p.*601.)

The exact time at which the surface should be floated depends upon the setting of the cement, and must be determined by the mason. Considerable skill is required in this troweling to prevent the formation of hair cracks by over-troweling, and to insure a surface which will not wear rough as a result of insufficient troweling.

If the walk is exposed to the hot sun it may be necessary to cover it with a wood or canvas frame, or with moist sand, for several days

after its completion, as it is absolutely necessary that it shall not dry out too quickly

Effect of Frost upon New Concrete Sidewalks. If concrete sidewalks are exposed to frost before thoroughly hard and dry, the surface is likely to blister and scale off in patches about $\frac{1}{8}$ inch thick. It is best, therefore, to avoid sidewalk construction in freezing weather.

Concrete Curbing. Concrete curbing for artificial sidewalks is largely displacing stone curbing. The curb is built just in advance of the walk. It is divided into blocks and is separated from the walk by joints similar to the joints between the blocks. The soil is excavated, and a foundation of porous materials of the same thickness as that employed under the walk proper is placed and rammed. In Boston* a layer of ordinary concrete 12 inches wide and 8 inches deep is placed upon this foundation to underlie the curb. The curb proper is 12 inches deep and 8 inches wide at the bottom, tapering on the outside to a width of 7 inches at the top, with its inside face vertical. At least one inch of the face and of the surface consists of mortar or granolithic, like the wearing surface of the walk. A typical sidewalk and curb is shown in Fig. 190. The back of the curb is formed against a temporary plank. For the face mold, a 12-inch planed plank is set on edge to the proper batter and may be held in place by driving stakes about 4 inches out from it, and nailing strips from the top of these stakes to the top edge of the plank, so that they can be knocked up and the plank loosened without disturbing the face of the curb. When ready to place the concrete for the curb, which should be laid before the layer of concrete underlying it has set, a 1-inch board is placed on edge just inside of the 12-inch plank, with occasional thin strips or wedges between it and the plank. The coarse concrete of the curb is then placed back of this board, and thoroughly rammed so that its surface is one inch

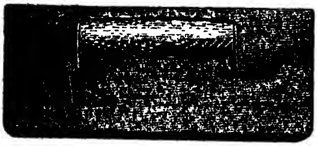


FIG. 188.—Edging Trowel. (*See p.* 601.)

FIG. 189.—Dot Roller. (*See p.* 601.)

*Specifications for 1899.

below the top of the forms, and when sufficiently hard, the 1-inch board is drawn up from the face, and with the aid of a trowel its place is filled with wearing surface material. The outside form is generally allowed to remain over night, and in the morning the outside surface is floated. A ruled joint like that between the blocks is formed between the curb and the remainder of the walk

A metal corner is sometimes laid in the exposed edge of the curb to protect it from wear

Combined Curb and Gutter. One of the advantages of a concrete walk lies in the ease with which it is adapted to special construction A gutter 5 or 6 inches thick, with a pitch corresponding to the crown of the street, is often laid in combination with the curb. It is underlaid with a porous

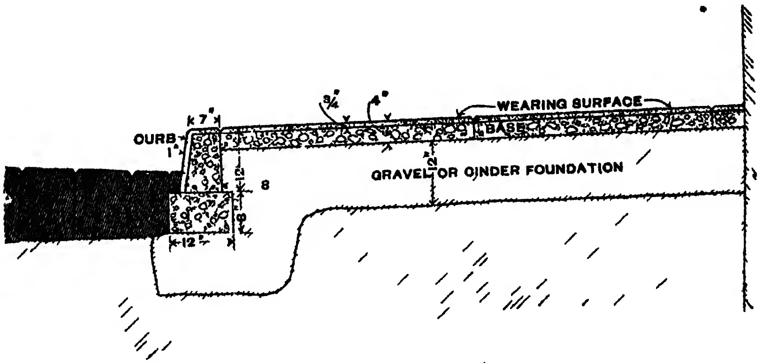


FIG 100 Typical Concrete Sidewalk and Curb (*See p* 602)

foundation, and in some cases by a sub soil tile drain The blocks forming the combined gutter and curb are made about 6 feet in length, and are in alternate sections so as to form definite cross joints, but each section of the curb and gutter must be built together, with no longitudinal joint between them

Vault Light Construction. Sidewalk lights over basement areas or subways are formed of circular lights of plate glass, set in reinforced concrete slabs, supported by steel or reinforced concrete beams Steel rods about $\frac{3}{16}$ inch diameter are interlaced in both directions between all of the rows of glass discs The width of the slab between beams is governed by the thickness of the slab, a customary width being 3 to 4 feet The dimensions of the beams and girders, whether of steel or reinforced concrete, depend upon their loading and span (See table, p. 508.) A typical vault

light construction supported by steel girders and stiffened by concrete ribs as designed by Mr. Ross F. Tucker, is illustrated in Fig. 191.

If concrete beams or stiffeners are used, they must be laid at the same time as the slabs are placed, so as to be in the same piece with them, but contraction joints must be provided as shown. In laying the slabs, the position of the glass discs may be located by an iron plate with holes of the size of the glass discs. On top of this iron form, a layer of oiled paper is

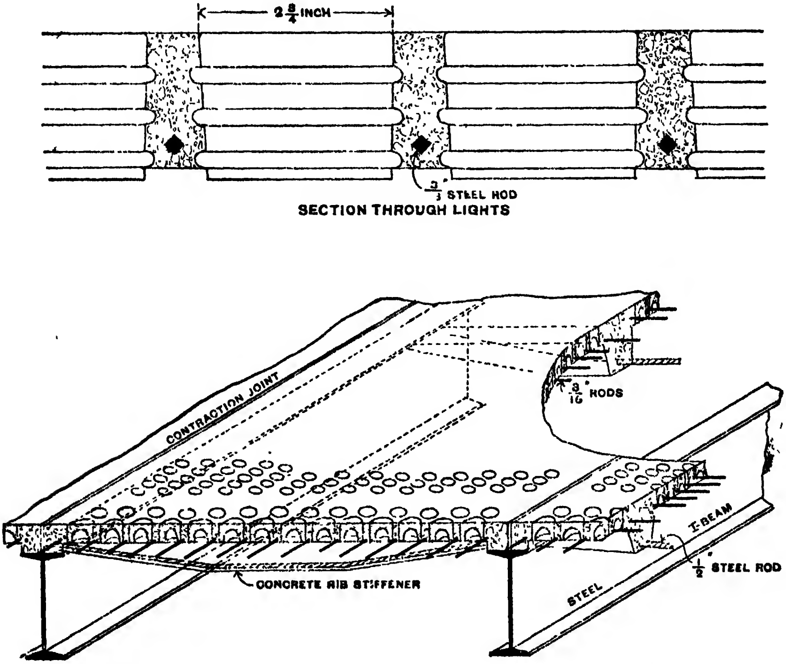


FIG. 191.—Typical Vault Light Construction. (See p. 602.)

spread to prevent the cement sticking to it, the lenses are set upon the paper over the holes, the reinforcing rods placed, and the mortar poured around the glass, and its surface troweled after partially setting, same as the surface of a granolithic walk. After the mortar has become thoroughly hard, the metal plate and the paper may be removed.

COST AND TIME OF SIDEWALK CONSTRUCTION

The cost of concrete sidewalk or basement floor construction is extremely variable. The job at any one location is likely to be small, not occupying

more than a few days, so that the time and expense of transporting men and materials, and the time getting started upon the work, constitute an important item. The skill of the men employed in placing and finishing the concrete affects the cost still more, since an experienced gang may easily lay three times as much surface of walk in a day as inexperienced men, even if the latter are accustomed to ordinary concrete work. Excavation is another variable item, depending upon the quantity of earth to be removed and the character of the material.

A gang of convenient size consists of —

One mason.

One man to assist the mason in placing forms, and to level and ram the concrete.

Three men mixing and placing coarse concrete for base.

One man mixing top dressing for wearing surface.

If excavation is included in the work, more laborers may be needed. The amount of walk covered by a gang is limited by the surface which can be floated and troweled by the mason. Unless he works overtime, the laying of concrete must stop about the middle of the afternoon in order that the wearing surface may have opportunity to set. Meanwhile, the concrete gang may prepare and ram the foundation and get everything in readiness to begin concreting promptly the next morning. With a gang of the size suggested a foreman adds considerable to the expense, and it is often advantageous to so arrange the work as to make the mason responsible for its quantity and quality. A bonus paid for an excess over a certain area of surface covered is an effective incentive for a good day's work. In order to properly fix such a bonus the employer must know the relative times required for plain sidewalk and curb. The size of the blocks must also be considered, since the labor upon the joints forms a prominent division of the work.

Under average conditions a mason skilled in this class of work should float and trowel a surface of 600 to 700 square feet in eight hours, if no allowance is made for time which is necessarily lost between jobs and in commencing work. This lost time will lower the average by an amount varying with the size of the job. If the excavation is ready, five men working with the mason should prepare the foundation and place the base concrete and the mortar for the wearing surface for a walk 4 to 4½ inches thick. For a thicker walk, one more man may be required in the gang to keep up with the mason, since a thick walk requires more concrete or mortar.

The contract price for a granolithic or artificial walk from 4 to 5 inches

in thickness, with Portland cement at about $2 00 per barrel, varies from $0.22 to $0.30 per square foot. The cost of curbing runs about $0.75 to $1.00 per linear foot without a metal strip, and 25 to 50 cents higher with it.

DRIVEWAYS

For driveways the concrete is laid similarly to that in sidewalk construction. The total thickness may be 5 inches for light travel, or 6 to 7 inches for heavy teaming. Grooving the surface in 6-inch squares affords foothold for the horses.

CONCRETE STREET PAVEMENTS

Concrete pavements in alley ways, constructed like sidewalks except marked off into small blocks, have been in successful use in Boston and elsewhere, since 1894. In 1896 a street pavement built in Richmond, Ind., by Mr. H. L. Weber, proved so successful that many other concrete pavements have been laid there, and the use has been extended to other cities. Results have been satisfactory where traffic is not too heavy, and where the very best of materials and workmanship have been employed.

The construction of concrete street pavements is similar to sidewalk construction but even greater care must be used to be sure that it is monolithic from top to bottom so that there can be no separation of layers. Unless the soil is very porous so as to drain off the water and at the same time form a non-compressible foundation, a porous material like broken stone or screened gravel thoroughly compacted and rolled should be laid for a depth of about 5 to 6 inches. Sometimes a 6-inch concrete foundation is also advisable. After laying the foundation a concrete base 4 inches to 6 inches thick is laid in proportions of about 1 : 2½ : 5 and the wearing surface must be placed at the same time or immediately following it so as to make one solid layer. The construction of the wearing surface is the most critical part of the work, for upon it depends the durability of the pavement. The best aggregate is crushed granite or trap or a mixture of this and sand. It must be free from dust and a considerable proportion of it should be as large as ¼ in. in size. Instead of using 1 : 1½ or 1 : 2 mortar, it is still better to form a true concrete, using proportions of about one part cement to one and one half parts sand to two parts of crushed screenings. This is laid wet and may be troweled and divided into small blocks, or may be given a rough finish to afford a good foothold for horses. Expansion joints should be made along the curbs and across the street about every 30 feet apart.

CHAPTER XXIV

CONCRETE BUILDING CONSTRUCTION

The rapid development of the use of concrete both in the United States and Europe is the best evidence of its adaptability for a building material. This is exemplified in numerous structures which, not only from an engineering standpoint but architecturally as well, are models of the builder's art.

In work above ground, concrete is most extensively employed in the building of floors and roofs. Its especial availability for this class of construction has been made possible by the introduction of numerous systems of metal reinforcement, the application of which has resulted in the reduction of the thickness and brittleness of the slabs.

The fire-resisting qualities of Portland cement concrete when composed of first-class materials, such as sand, and gravel, hard broken stone, or cinders, appear both from experimental and actual fire tests to be equal or superior to those by any other material. (See Chapter XVIII, p. 327.) Moreover, its strength and permanence, when it is carefully laid and properly reinforced, are unquestioned, and by employing a wet mixture the mortar in the concrete surrounds and effectually prevents the corrosion of the metal with which it is reinforced.

Its fire-resisting quality has led to the adoption of reinforced concrete for stairways, for columns and girders, and finally for entire buildings. The growing confidence in its utility for office buildings seems to promise for it successful competition with steel fireproof construction and a wide use in this class of structures. The cost of the reinforced concrete for an office building built of this material in 1904, based on actual construction records, with cement at $2.00 per barrel delivered on the work, was about 20% less than the estimated cost of the steel and tile of ordinary fireproof construction. As the concrete portion constituted about one-fifth of the total cost of the building, the net saving is reduced to about 4%, a very considerable sum, however, when figured on a fifteen-story office building. There is also an additional saving in other materials due to the reduction in height of the building because of the thin concrete floors, and to the fewer coats of plaster, with omission of furring, on walls and ceilings.

The Ingalls Building, designed by the Ferro-Concrete Construction Company and erected in Cincinnati, O., in 1903, was the first notable

example of a concrete office building in the United States. Sixteen stories high, it is entirely of concrete, with the exception of the facing of the exterior walls.

For factory building reinforced concrete is gradually superseding "slow-burning" mill construction with its brick walls and timber beams and columns. In certain cases the concrete has been found actually cheaper than the wood, three 6 story factory buildings in Cambridge, Mass., for example, being erected in 1908 at a lower cost than competitive estimates for wood and brick construction. Even if the cost for reinforced concrete runs from 8% to 10% higher than the estimate for brick walls, timber columns and girders, and plank floors since the concrete portion is only about one-half the total contract, the increased cost of the entire building is only 4% to 5%. The concrete building has greater durability and is fireproof, thus reducing running expenses and affording lower insurance rates.

For dwellings and other small buildings the cost of the forms alone may exceed that of the materials and labor on the concrete. In estimating the labor, allowance must be made for the time which is often necessarily lost in waiting for the cement to harden or the forms to be removed. For these reasons it may be more economical to work with a small gang, taking an entire day to lay the concrete to the height of one section of forms.

For the cellar and foundation walls of frame or brick houses (see p. 619), concrete is usually cheaper than rubble masonry.

A method of construction of light curtain or division walls consists in plastering Portland cement mortar upon metal lathing. A 2-inch wall thus made forms a permanent and fire-resisting partition. (See p. 627.)

Molded blocks of mortar or concrete (see p. 629), or concrete tile (see p. 629), are adapted to certain classes of structures. Under favorable conditions the cost may be less than that of a brick wall of equivalent thickness.

CONCRETE FLOORS

Concrete floor slabs are supported by steel or sometimes by timber girders, or are formed in combination with reinforced concrete girders. The metal reinforcement which is universally adopted for the slab not only reduces the thickness and weight of the floor, but prevents sudden failure, an extremely important consideration in this class of structures.

Concrete floor panels between steel girders must compete chiefly with porous tiling and brick arches. The relative cost of these three materials, while dependent upon the location of the work and market prices, is usually, all things considered, in favor of concrete. The encasing of the steel I-beams with fine concrete or mortar affords fire protection to the girders and, if desired, a continuous surface for plastering.

Design of Concrete Floors. The design of a complete floor system with reinforced concrete beams, girders and slabs is illustrated in the example, pages 468 to 474. The details of the design are also treated in Chapter XXI and the tables in the same chapter, pages 507 to 526, give means to determine very quickly the dimensions and reinforcement for different spans and loadings. Reinforced floors are strongest when made continuous over several bays provided they are properly reinforced at the top over the supports as well as in the bottom at the center. It is essential that the beam and slab shall be laid at the same operation. Slabs laid between steel I-beams, as in Fig. 193, page 616, are not so strong as when built in with reinforced concrete beams.

The arrangement of the floor beams and girders in a building of reinforced concrete depends upon so many considerations that special study is required in each case.

The smallest quantity of material is required with floor panels of short span and frequent floor beams to support them. However, very thin slabs and beams of concrete are not easy to construct properly, and there is difficulty in imbedding the metal, so that we may, in general, limit the thickness of both to not less than 3 inches. For the slabs this minimum should be raised where a floor is liable to sudden strains, such as the falling of a load, which tend to punch a hole through the floor. For beams a more practical minimum width is usually 5 or 6 inches, since the cost of the form, which is but slightly more for a large than for a small beam, is a considerable item, and a deep, thin beam is in danger of buckling and requires frequent cross beams or stiffeners.

The spacing of the beams may, therefore, be governed in some cases by the required thickness of the floor slabs and in others by their own economical construction. Similar considerations, applied to column and foundation construction, govern the design of the principal girders.

The Ingalls Building* presents an example of slabs of long span supported by heavy girders, and the factory of the Pacific Coast Borax Company† an example of thin floor slabs with frequent deep but narrow concrete beams.

In simple cases the dimensions and reinforcements of concrete floor girders may be obtained directly from the tables, pp. 509–511. More difficult problems require mathematical calculation as treated in Chapter XXI. Not only must the size of the tension rods in the bottom of the beam be considered, but also the size and location of the U-bars, the reinforcement

*See page 611.

†See page 621, also *Engineering Record*, July 30, 1898.

in the top of the beam, if required, and the proper connection with the columns. The girder illustrated in Fig. 192, page 613, is a typical design for a concrete beam supporting a heavy load, although the dimensions and reinforcement apply, of course, to a particular piece of construction.*

There are several methods of laying floors supported by steel girders, one of the most common of which is illustrated in Fig. 193, page 616. The haunches of the slab are carried down to the lower flange of the I-beam, the under surface of which may be covered with metal lathing for fire protection and plastering. The I-beam may be entirely enclosed in the concrete, but it is difficult to place the material under the lower flange. Where head room is very valuable, the top of the slab is laid flush with the top of the beams and the metal is placed between the beams instead of running over them In either case the outline of the concrete may form the ceiling, the plastering being placed directly upon it so as to form panels, or the ceiling may be suspended from the I-beams on metal lathing.

Floors are sometimes laid as continuous slabs, imbedding simply the upper flange of the I-beams in the concrete. The forms are cheaper to construct, but the strength is less than with the haunches, and the web of the I beam is not protected from fire. For ceilings, separate slabs may be formed resting upon the lower flanges of the I-beams Still another type of floor consists of concrete arches sprung between the lower flanges of the I-beams, just as brick arches are formed, and filled to the floor level with cinders. They do not necessarily require reinforcement.

The metal reinforcement in a floor slab should be as near to the under surface as is consistent with durability and fire resistance. For a strictly fireproof building it is safest to allow at least an inch of concrete below the metal, but under ordinary conditions this may be reduced to ¾ inch or ½ inch, provided the concrete is mixed wet and carefully placed around and under it. If plain rods are used, they must be prevented from slipping by selecting very long lengths or by anchoring the ends, or both If the ends are bent for this purpose, there must be a considerable thickness of concrete beyond the bend to prevent the tendency under load to straighten out and thrust through the concrete.

Safe Floor Loads. The following loading for floors, suggested for the Boston building laws by a committee of the Boston Society of Civil Engineers in 1904, represents first-class modern practice:

All new or renewed floors shall be so constructed as to carry safely the weight to which the proposed use of the building will subject them, and every permit granted shall state for what purpose the building is designed

*See also designs suggested, page 453.

to be used; but the least capacity per superficial square foot, exclusive of materials, shall be:

For floors of dwellings and for apartment floors of apartment and public hotels, fifty pounds.

For office floors and for public rooms of apartment and public hotels, one hundred pounds.

For floors of retail stores and public buildings, except schoolhouses, one hundred and twenty-five pounds.

For floors of schoolhouses, other than floors of assembly rooms, eighty pounds, and for floors of assembly rooms, one hundred and twenty-five pounds.

For floors of drill rooms, dance halls and riding schools, two hundred pounds.

For floors of warehouses and mercantile buildings, at least two hundred and fifty pounds.

The loads for floors not included in this classification shall be determined by the Commissioner, subject to appeal, as provided by law.

The full floor load specified in this section shall be included in proportioning all parts of buildings designed for dwellings, hotels, schoolhouses, warehouses, or for heavy mercantile and manufacturing purposes. In other buildings, however, certain reductions may be allowed, as follows: In girders carrying more than 100 square feet of floor, the live load may be reduced by 10 per cent. In columns, piers, walls, and other parts carrying two floors, a reduction of 15 per cent of the total live load may be made; where three floors are carried, the total live load may be reduced by 20 per cent; four floors, 25 per cent; five floors, 30 per cent; six floors, 35 per cent; seven floors, 40 per cent; eight floors, 45 per cent; nine or more floors, 50 per cent.

Weight of Concrete in Floors and Girders. The following table is based on an average weight of broken stone or gravel concrete of 150 lb. per cubic foot, and of cinder concrete of 112 lb. per cubic foot, to each of which has been added the weight of 4 lb. per cubic foot to provide for maximum weight of about 1% of reinforcing steel

The weight of stone concrete varies not only with the proportions of the mixture (see p. 361) but also with the specific gravity of the aggregate, and for particular cases, the weights on page 3, which are based on tests made at the Watertown Arsenal and Washington University and checked by calculation from the specific gravity of different materials, may be used instead of the table. The table, however, is sufficiently exact for ordinary practical purposes.

Floors in the Ingalls Building. In the Ingalls Building at Cincinnati, Ohio, whose floors above the second floor were designed for a live loading of 60 pounds per square foot, the principal panels, which are about 16 feet square, are 5 inches in thickness, and reinforced with $\frac{1}{2}$-inch rods. Smaller

panels of 3 to 6 feet in length are about 3 inches thick with ½-inch bars. The spacing of the rods varies with the length of the span. Where the panels are approximately square, the tension rods run in two directions, and where the panels are long and narrow, the tension rods run across the panel, with ¼-inch rods about 3 feet apart running lengthwise of the panel, to prevent contraction cracks. The principal girders are 32 feet long between centers of columns, and 27 inches in depth (measured to surface of concrete floor), and of width varying from 20 inches at the lower floors to 16 inches at the upper floors. Cross girders about 16 feet in length and 18 inches deep, of widths varying from 12 to 9 inches, are placed in the center of the span of the main girder, thus dividing the floor into slabs

Weight of Reinforced Concrete in Slabs and Beams. (*See p.* 611.)

Weight of Reinforced Slabs per Square Foot.			Weight of Reinforced Beam one inch wide per foot of length.	
Thickness in	Stone Concrete lb.	Cinder Concrete lb.	Depth of Beam in.	Stone Concrete* lb.
2	26	19	6	6.4
2½	32	24	7	7.5
3	38	29	8	8.6
3½	45	34	9	9.6
4	51	39	10	10.7
4½	58	43	12	12.8
5	64	48	14	15.0
5½	70	53	16	17.1
6	77	58	18	19.2
7	90	68	20	21.4
8	103	77	25	26.8
9	115	87	30	32.1
10	128	97	35	37.4

* Multiply by the length of beam in feet times its width in inches.

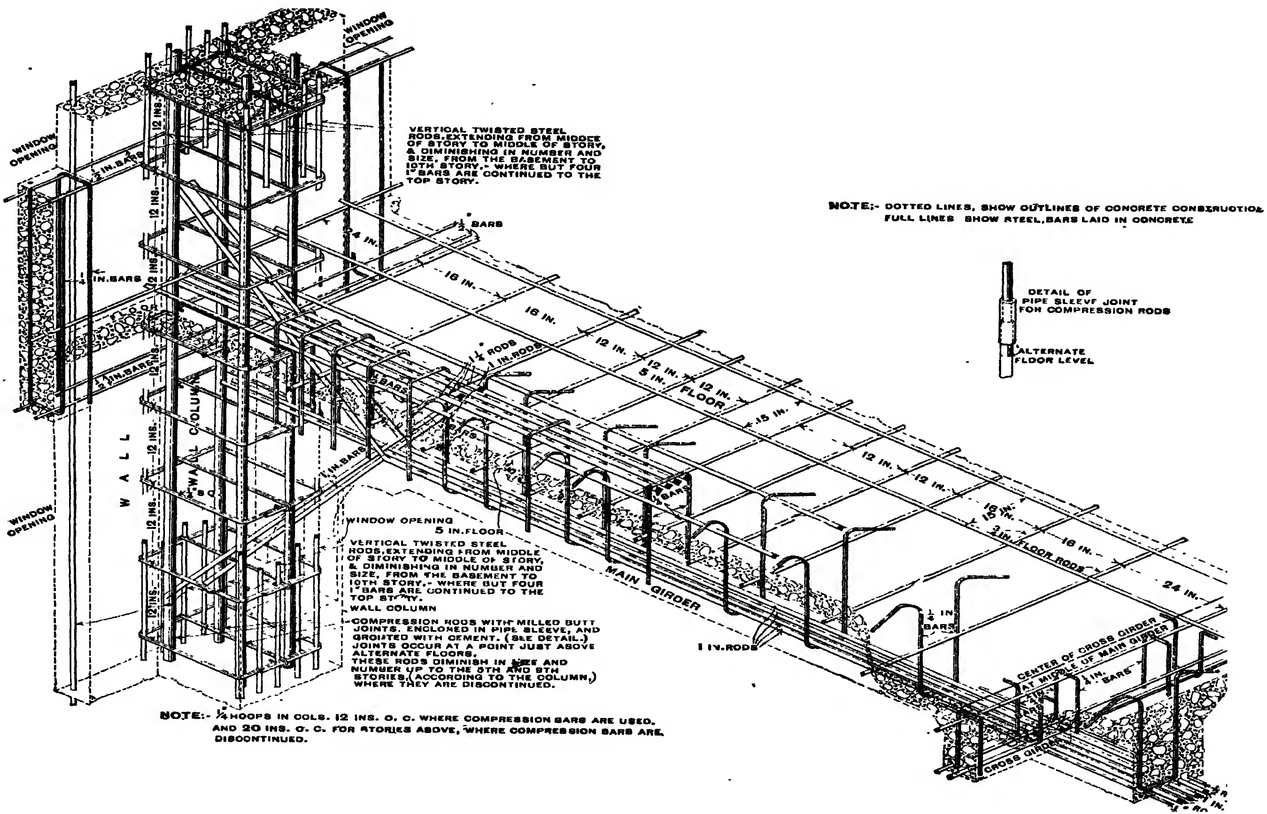
about 16 feet square. Fig. 192, page 613, is an isometric view showing the dimensions and reinforcement of the floor, main girder, cross girder, wall column, and wall in the fourth and fifth floors. The total distributed loading on the main girder is about 15 tons live load in addition to the weight of the reinforced concrete.

Materials for Floors. A first-class Portland cement which will meet the standard specifications given on page 29 must be selected. The rules for the selection of the aggregate are the same as for other classes of concrete. The size of the coarsest aggregate is often limited to one inch. but if well graded, so that the larger particles will not collect and prevent the flow of the mortar around the steel, the limit of size for beams, say,

FIG. 192 TYPICAL REINFORCING IN BUILDING CONSTRUCTION

(See p. 612)

CONCRETE BUILDING CONSTRUCTION



5 inches in width and floors not less than 4 inches thick may be as high as $1\frac{1}{2}$ inches.

Cinders for concrete should contain but little unburned coal and be free from soot. A clean cinder will not discolor the palm when held in it and rubbed with the fingers. Usually a better mixture can be obtained by screening the fine stuff from the cinders, and then, if gritty, mixing it with sand, than by using unscreened material, although if the fine stuff is found by tests to be uniformly distributed through the mass, it may be used without screening and a smaller proportion of sand added.

Usual proportions for floor concrete are $1: 2\frac{1}{2}: 5$, that is, one barrel packed Portland cement, 9.5 cu. ft. sand, and 19.0 cu. ft. of screened stone or screened cinders. If the thickness of the floor is such as to provide a wide margin of safety, the proportions may be $1: 3: 6$ (based on a barrel of 3.8 cu. ft.), while for extra strong work $1: 2: 4$ may be specified. For beams and girders $1: 2: 4$ and $1: 2\frac{1}{2}: 5$ are common proportions. Cinder concrete should not be used for girders, but under certain conditions may be employed for floor slabs. While it is lighter in weight, generally cheaper, and equal in fireproof qualities to first-class stone concrete (see p. 329), it is not so strong. Hence, for the same loading a greater thickness is required, and it is not usually economical even for floor slabs except the span and the loading are so small that the thickness of the floor is governed, not by required strength, but simply by the practical conditions of laying which limit it to a thickness of not less than 3 inches. In carefully designed reinforced buildings stone concrete is generally preferred.

The quantity of cement, sand, and stone or cinders required for any structure may be calculated from the table on page 231, or, for slabs, taken directly from the table on page 596.

Laying Floors. The general directions for mixing and placing concrete, given in Chapter II, p. 20, and Chapters XIV, and XV, are applicable to building construction.

The concrete must be mixed wetter than in sidewalk or basement floor construction, as described in the preceding chapter, so that the mortar may flow around the metal and thoroughly coat and protect it from rust and fire. The criterion of wetness may be that unless handled quickly it will flow off the shovel.

If the concrete floor is to provide a wearing surface, a granolithic finish may be given to it by laying a mortar wearing surface before the lower portion has set, as described for sidewalks in the preceding chapter, or the concrete may be troweled without the coating of mortar. The latter plan is amply sufficient for floors which are not subjected to excessive wear.

For a board floor, nailing strips are laid upon the concrete, or imbedded in it at right angles to the supporting beams. With cinder concrete the plan is sometimes followed of nailing the floor boards directly into the concrete. The objection to this is that the surface of the concrete must be leveled with great care, and it is difficult to relay the boards if a new floor is required because the concrete becomes so hard with age.

The cost of the labor of laying a concrete floor is dependent upon the character of the building. In a case under the observation of the authors where the floors consisted of cinder concrete resting upon steel I-beams, a gang of nine laborers, with a foreman (in addition to the engineman, who run the elevator,) mixed concrete in the basement to supply a gang of eleven men, with foreman, who, on one of the upper floors, were placing

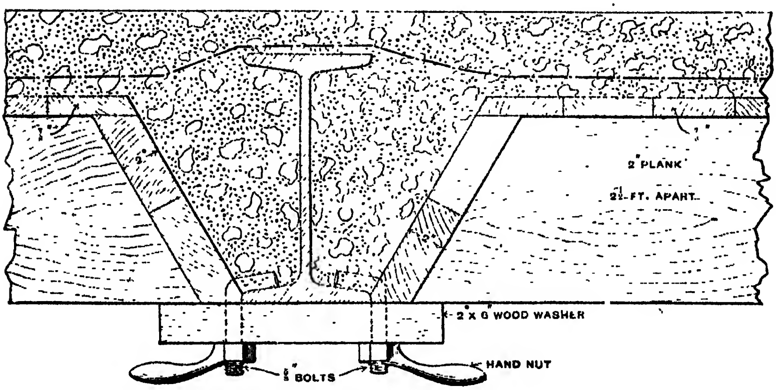


FIG. 193.—Form for Concrete Floor between Steel I-Beams. (*See p.* 616.)

metal, wheeling concrete, leveling it, and cleaning forms. Six carpenters, with foremen, were employed building the forms, which were supported from the girders, in advance of the concreters. This gang averaged 22 to 25 batches (corresponding to 17 to 19 cu. yd.) of 1 : 2 : 5½ cinder concrete in nine hours.

Floor Forms. In a large building the floor panels should if possible be so designed that the same forms may be used more than once, although they must not be removed until the concrete has attained sufficient strength to sustain its own weight and any loading which will come upon it.

If the floor slabs are supported by steel I-beams, the forms may be attached to the lower flanges, as shown in Fig. 193 a design of Mr. William F. Kearns. The steel, however, must be bent up further from the support than is shown in the drawing and carried nearer to the top of the slab to prevent cracking near the I-beam.

If the girders are also of concrete, the supports for the form must be heavy enough to carry the weight of the beam of concrete, as well as the floor slab and the men and materials upon it. The forms must be so tight as to prevent the water and thin mortar running away from the concrete and carrying off the cement. This may best be accomplished by tongued-and-grooved or bevel-edged boards, but it is often possible to use square-edged lumber if it is thoroughly wet to swell it before placing the concrete. Joints in the beam forms may be covered with cleats.

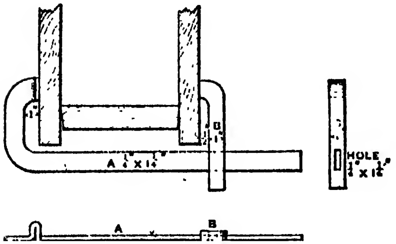


FIG. 194.— Clamp for Beam or Small Column Form. (*See p.* 617.)

A simple form of clamp for beam or small column forms, used originally in Europe, is shown in Fig. 194. The hook, *A*, is a plain piece of flat iron ¼ inch by 1½ inches, with one end bent and curved as shown. The dog, *B*, is a square piece of iron, with the end slightly turned and a hole slightly larger than the flat iron, *A*, punched through it. This is tightened by hammering on its lower end. The outward pressure of the form boards upon its upper end causes it to bind, and prevents it from slipping back. If it fails to hold, in any case, a wooden wedge is readily driven in to assist in tightening.

CONCRETE STAIRS

The design of concrete stairs is a simple problem in reinforced concrete construction. A stairway may consist (1) of an inclined slab of reinforced concrete with the steps molded upon its upper surface, or (2) of two or, for a wide stairway, three inclined girders to form the stringers, with the stairs between them. The first method is suitable for short flights not over 8 or 10 feet in length measured on the slope, and the thickness and reinforcement are calculated as for a slab supported at the ends. (See pp. 512 to 515.) The principal reinforcement is of course in the direction of the length with occasional cross metal for stiffening. A slab 5 inches thick measured at the foot of the risers is suitable for a stairway half a story high.

When built with side girders, the dimensions of each of the latter may be calculated as a concrete beam with a longitudinal rod near the lower surface. A small rod also runs across from girder to girder at the foot of each riser so that the risers are practically reinforced beams. It is usually cheaper to construct the under side of the stairs as a slab than to build

forms for each stair. The forms for the stringers may consist of planks notched for treads and risers, with boards nailed across as molds for the faces. If a fine finish is desired, the method of surfacing described for curbing may be followed. (See p. 602.)

CONCRETE ROOFS

Concrete roofs are designed and laid in much the same manner as are floors. The forms also are similarly constructed. As the weight of the roof itself forms a large proportion of the total load upon the girders, cinder concrete, because of its light weight, is especially adapted to this class of construction. The strength of the concrete may also play a smaller part in roofs than in floors, because the length of span may be governed by other conditions, and the concrete may often be laid as thin as is practicable to lay it and properly imbed the metal.

The wetness of the concrete is limited by the slope of the roof, although for a steep slope it may be necessary to confine the surface of the concrete by forms.

The proper thicknesses and reinforcement for different spans may be obtained from tables on page 512 or 515, selecting the weights from the data in the paragraphs which follow.

Roof Loads. A roof load is made up of the weights of the roof itself, the roof covering, the snow load, and the wind load.

The weight of the concrete may be obtained from the tables mentioned.

Prof. Mansfield Merriman* gives the following estimates for the weight of roof covering:

Tin, 1 lb. per square foot of roof surface.
Iron, 1 to 3 lb. per square foot of roof surface.
Slate, 10 lb. per square foot of roof surface.
Tiles, 12 to 25 lb. per square foot of roof surface.

Average may be taken at 12 lb. per square foot.

The snow load varies with the slope of the roof and the locality. Prof. Merriman allows for an approximate average 15 lb. per square foot of horizontal area.

The wind load, which acts horizontally, varies with the velocity of the wind, a usual pressure being assumed as 40 lb. per square foot of vertical surface. This pressure multiplied by the sine of the angle of slope of the roof gives the pressure normal to the surface.

In practice it is common to specify a minimum value for the roof load to

* Merriman's "Roofs and Bridges," p. 4.

include the weight of the roof covering, snow, wind and any moving loads which may come upon it. A usual value for this total is 30 pounds per square foot.

It is seldom advisable to build concrete roofs without an external covering, such as tar and gravel. However, small surfaces laid by expert workmen at one operation to avoid joints and designed with special reinforcement have given satisfaction.

Concrete is adapted to roofs of special design. One form is the dome, which is discussed and illustrated on page 626.

CONCRETE WALLS

If Portland cement concrete could be laid in thin walls as cheaply as in mass work it would be one of the most inexpensive materials for permanent construction. As a matter of fact, an experienced contractor can build a 6-inch wall of concrete which will be stronger, more durable, and no more expensive than a 12-inch wall of brick.

The chief cost in concrete wall construction is in the labor of building and raising the forms and of hoisting the concrete. The former varies with the method of construction and the number of angles in the wall. In the case of a large structure the concrete may be hoisted in elevator buckets* by power. If the building is small and the concrete is hauled up by hand in buckets to a height of, say, 15 feet, at least twice as many men will be required to fill pails, haul up, and carry to place as are needed for measuring and mixing the concrete on the platform below.

Methods of surfacing concrete walls are described on page 288. Plastering is unsatisfactory.

Cellar Walls. Cellar or basement walls adapted to withstand earth pressure may be thinner when of concrete than when built of stone, because laid as a continuous vertical slab supported at top and bottom.

For a wall of 1 : 2½ : 5 Portland cement concrete with a spreading base imbedded in the earth, a thickness of 10 inches will withstand without reinforcing metal a pressure of 6 feet of earth. If the top of the wall is strengthened by a wooden sill imbedded in or dogged to the concrete, and the sill is stiffened by floor joists, the wall becomes a slab supported at its bottom by the earth and at its top by the sill. A 6-inch wall 8 feet high will thus withstand the pressure against it of 6 feet of earth. However, ¼-inch rods, spaced about 2 feet apart in both directions, will greatly stiffen so thin a wall, and prevent cracks before the concrete is thoroughly hard. If desired, a coping of concrete wider than the wall itself may be formed at the top and a ½-inch rod placed horizontally in its inner face.

*Method used at the Ingalls Building is illustrated in *Engineering News*, July 30, 1903. p. 95

The earth must not be filled in against the back of the wall until three or four weeks after placing, unless portions of the interior forms are left in place and carefully braced.

Designs for reinforced concrete retaining walls are illustrated on page 666.

A simple form for a cellar or foundation wall is illustrated in Fig. 195 A ranger, *AA*, is lined, and lightly spiked to occasional studs whose pointed ends are driven into the ground, and kept in line by strips of wood running from it to stakes in the bank. In some cases it may be advisable also to set a lower ranger between the studs and the bank. Occasional stakes, *BB*, are driven in the ground, and a ranger, *CC*, for the inside row of studs,

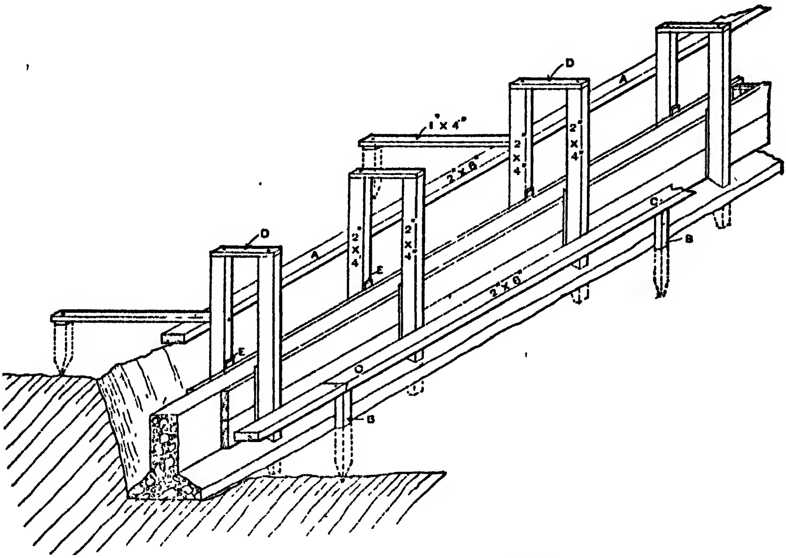


FIG. 195.—Form for Cellar Wall. (*See p.* 620.)

is laid on top of them, lined, and lightly spiked to them, while the upper ends of these studs are held by cleats, *DD*, run across to the inner row of studs. Vertical strips, *EE*, about ⅞ inch square, are placed inside of each stud for the form planks to rest against, and after a section of concrete is laid are easily knocked out, and the form planks raised to another level. The first layer of concrete is allowed to flow out under the lower plank to form a footing, above which the cellar floor is laid. The number of the laborers and the height of the forms should be such that the planks may be raised each morning, provided the concrete is hard enough to withstand the pressure of the thumb without indenting.

Walls for Buildings. Concrete walls are either of single thickness, or double with an air space between. The double wall has greater stability, and the air space renders the interior of the building less subject to changes in temperature and more completely moisture-proof. Moisture is likely to collect on the inside of a single wall.

A single concrete wall 4 inches thick with its base spread to provide a footing is at least equivalent to an 8-inch brick wall, and a 6-inch concrete is at least equivalent to 12 inches of brick. It is advisable to place small reinforcing rods, about ½ inch in diameter, 12 inches to 2 feet apart in walls 6 inches thick or under, not only to increase their permanent strength, but to guard against accidents during or immediately after construction. Occasional projections or pilasters improve the appearance and add to the strength of a single wall.

Each face of a hollow wall is usually 3 to 4 inches thick, 3 or 3½ inches being the minimum thickness at which concrete can conveniently be placed.

The four-story factory building of the Pacific Coast Borax Company at Bayonne, N. J., designed by Mr. E. L. Ransome, is an excellent example of hollow wall construction. The thickness of both faces of the walls is 3½ inches. The walls of the first story are 16 inches from surface to surface, that is, the space between is 9 inches, while the walls of the upper stories are made thinner by reducing the width of the hollow space. The general construction of a hollow wall is illustrated in Fig. 197, page 623.

The walls of the Ingalls Building consist of concrete 8 inches in thickness, faced with brick or marble. They are supported by reinforced columns spaced about 16 feet on centers, and the portions of the wall at the floor lines, that is, between the top of the window of one story and the window-sill of the story above, are, in reality, concrete beams reinforced by two ½-inch bars placed 2 inches above the top of each tier of windows, with ½-inch horizontal bars 2 feet apart over the remainder of the wall. In addition to the column reinforcement vertical bars are placed 2 inches from each window opening.

The marble facing is supported at each floor line by triangular projections in the concrete, and the brickwork in the stories above by square projections 3½ inches wide. The marble is also held at each horizontal joint by anchor bolts imbedded in the concrete, and the brickwork by ties of round, straight rods about 8 or 9 inches long and ½ inch in diameter, placed through small holes in the outer forms before concreting so as to extend 5 inches into the concrete.

Wall Forms. A simple form for a cellar wall is illustrated and described

on page 620. A form for a wall of single thickness is illustrated in Fig. 196 The concrete is first laid to the full height of the ribs, then the bolts are loosened, the ribs raised one-half their length, so that one-half of each still laps over the concrete to keep the wall true and straight, and the forms are again filled with concrete to the top. Two bolts to each pair of ribs are all that are required after the concreting is commenced. These are removed before the wall is hard, so that they need be simply greased and the holes filled solid full with mortar mixed in the same proportions as the

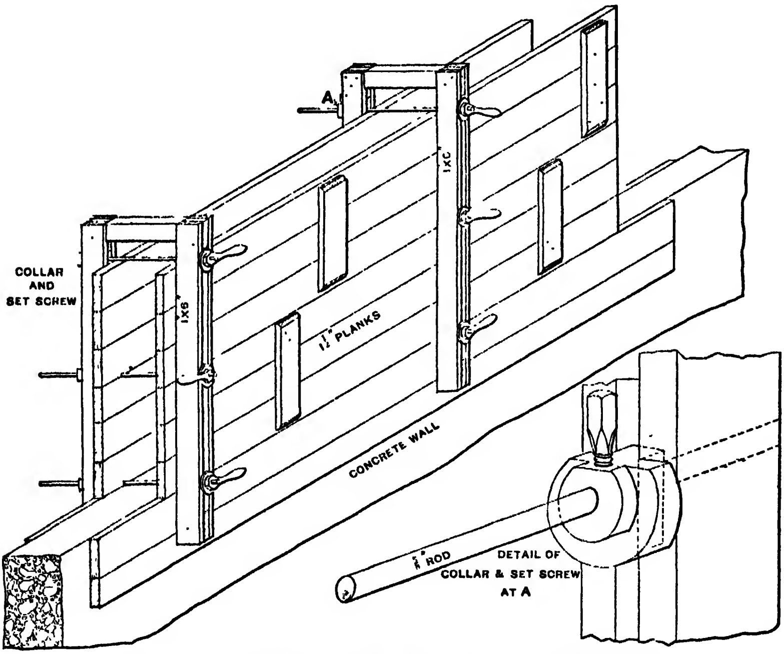


FIG. 196 —Ribs for Holding Form Plank. (See p. 622.)

mortar in the concrete. The collar and set screw shown in detail is convenient where the walls or columns are of various dimensions, although usually an ordinary threaded bolt with nut and washer may be used.

A design for a form for a hollow wall is shown in Fig. 197. The ribs and bolts are so arranged that the latter do not pass through the concrete, the form being raised when the concrete reaches their level. In the same figure is shown a style of tongued and grooved molding with edges slightly beveled, which may be used to form the horizontal joint instead of nailing

a triangular strip upon the planks. If the surface is finished as a monolith of course no moldings are required. The forms must be nearly watertight, to prevent the mortar running away from the stones.

Placing Concrete in Walls. For thin walls it is necessary to use mushy concrete, so soft that it must be handled quickly or it will run off the shovel. It should not, however, be so wet that the mortar is watery, or it will run away from the stones and leave pockets in the finished work. The concrete should be joggled rather than rammed, the chief object being to prevent collections of stones in one place, which will cause notice-

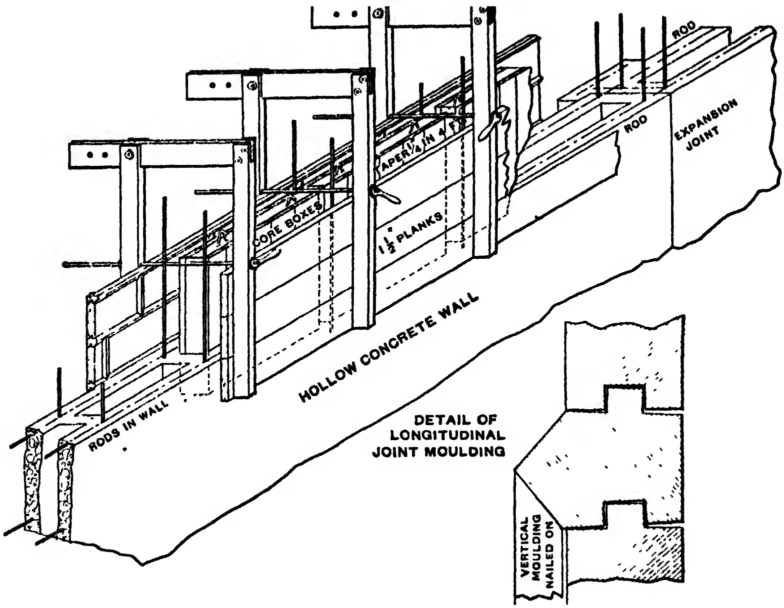


FIG. 197. — Forms for Hollow Walls. (*See p.* 622.)

able voids on the surface. The ramming of concrete is discussed on page 281, and methods of surfacing are described on page 288.

The size of stone for walls is sometimes limited to $\frac{3}{4}$ inch or one inch. However, a larger sized material, even up to 2 inches, has been used by Mr. Thompson in 4 and 6-inch walls with satisfactory results.

CONCRETE COLUMNS

Methods of design and allowable working stresses are recommended in Chapter XXI, page 488. Unless of very large diameter in proportion to

the length, columns should be always reinforced, not only to strengthen them but to guard against possible emergencies. If the steel is not actually figured to take stress, $\frac{3}{4}$ or $\frac{7}{8}$ inch rods, one in each corner, are customary reinforcement. For wall columns or others where there is slight eccentricity, extra rods may be inserted on the side where there is the greatest stress. If the loading is appreciably eccentric, allowance must be made for it in the design, and the stresses and reinforcement may be computed from the analyses presented on pages 558 to 574.

The columns of the Harvard Stadium,* illustrated in our frontispiece in process of construction, range in size from 14 inches square to 24 by 33 inches, and contain $\frac{3}{8}$ and $\frac{1}{2}$-inch rods in the corners with square loops of $\frac{1}{4}$-inch rods placed around them horizontally at intervals of about fifty times the diameters of the loop rods. The allowable compressive stress for 1 : 3: 6 concrete in columns was taken at 350 lb. per square inch. The outer wall is supported by hollow piers, 66 by 36 inches over all, 4 inches thick on the longer faces, and 6 to 8 inches thick on the ends.

The 1904 specifications of the Prussian Public Works place the horizontal rods at distances apart of not more than thirty times their diameters.

A typical section of column in the Ingalls Building is shown in Fig. 192, page 613. The rods designed to assist in bearing the compressive stress are 4 inches in diameter in the lower portion of the column, and are gradually reduced to one inch diameter at the upper stories. They are connected at the ends with pipe couplings and the joints grouted. The outer rods on each edge of the column are designed to resist the wind stresses. To avoid complication in the drawing, these are not shown at the floor level.

The construction of the molds for a concrete column is illustrated in Fig. 198, which shows a column of the Harvard Stadium under construction.

COST OF CONCRETE BUILDING CONSTRUCTION

So many factors enter into the cost of concrete buildings that it is impossible to give data which will apply to all conditions without specifying the character of the design, the size, height and shape of the building and the unit cost of materials and labor. Any structure must be accurately estimated, paying special attention to the cost of forms. A few general rules are given on page 26.

Mr. Emil Perrot† gives the following approximate average values per

* Described by Lewis J. Johnson in Journal Association Engineering Societies, June, 1904, p. 293.
† Proceedings National Cement Users' Association, 1909.

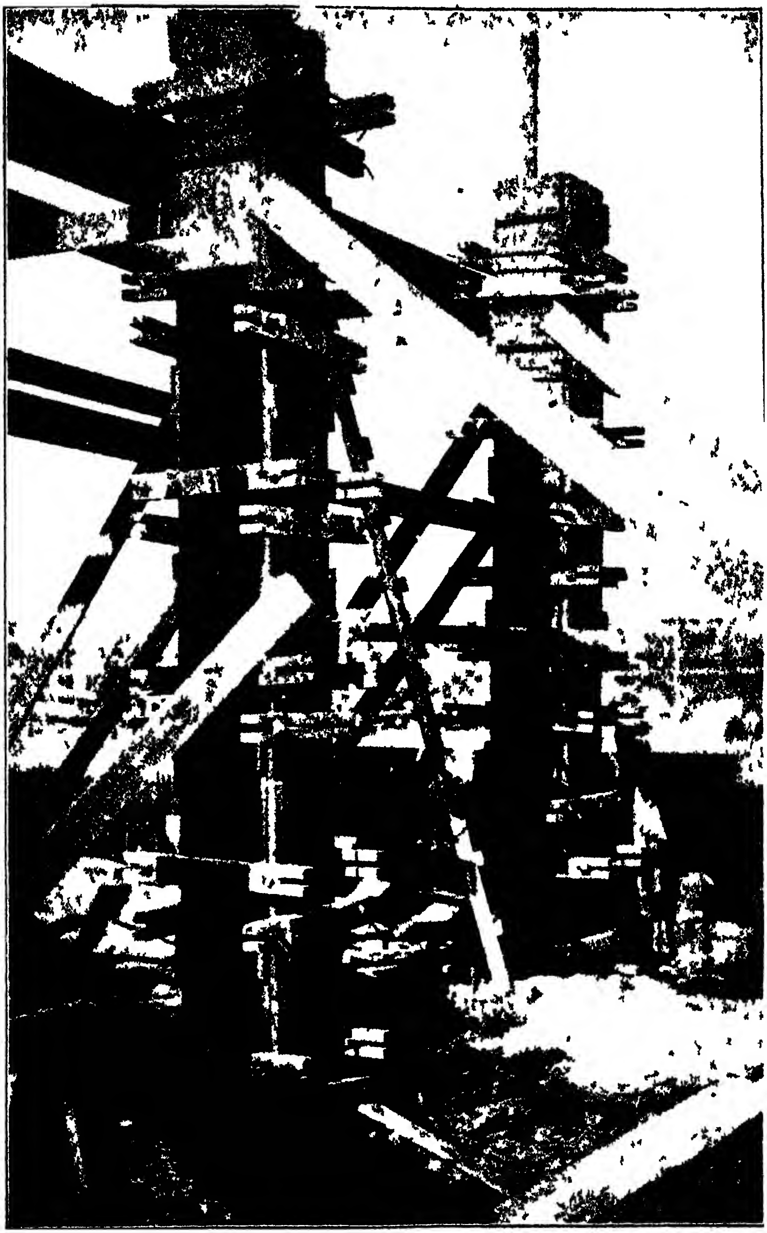


FIG 198 — Molds for Columns at Harvard Stadium (See p 624)

cubic foot for different types of buildings, which are useful for rough approximate estimates by the prospective builder:

1. Warehouses and manufactories. Cost, 8 to 11 cents per cubic foot.

2. Stores and loft buildings. Cost, 11 to 17 cents per cubic foot.

3. Miscellaneous, such as schools and hospitals. Cost, 15 to 20 cents per cubic foot.

These costs include the building complete, omitting power, heat, light, elevators and decorations or furnishings.

DOMES

Reinforced concrete is admirably adapted to the construction of domes, since the concrete can take all the compressive stresses, and the steel the tensile stresses developed in the lower curves of the dome and in the arch ring.

While a number of domes have been constructed entirely of reinforced concrete, in Europe up to over 70 foot spans, the more common practice in America has been to carry a concrete shell on a framework of structural steel.

Yale University Dome. An example of the latter type is the dome of one of the bi-centennial buildings at Yale University, New Haven, Conn., for example, 55 feet in diameter at the bottom and 34 feet high, consists of a skeleton of 24 8-inch I-beam ribs, supported at the top against a circular steel rim, with reinforcing metal imbedded in the 3½-inch thickness of concrete between them. The surface of the concrete was formed by "screeding" it with a curved templet whose length was the entire height of the arch.

Dome of Temple Adath Israel. A dome entirely of reinforced concrete is represented in cross section in Fig. 199, page 627. This is the main dome of the Temple Adath Israel at Boston, Mass., designed and built by Mr. O. W. Norcross, under the supervision of Mr. C. H. Blackall, Architect.

The dome proper, which has a span of 52 feet 9 inches, is 5 inches thick at the haunch and 3 inches thick at the crown, and is composed of 1 : 2 : 4 broken stone concrete. The reinforcement consists of expanded metal, 3-inch mesh No. 10 gage, from the tension ring to the angle of rupture, and 2-inch mesh No. 12 gage for the remainder of the section. The 5 by 4 by ½-inch angle tension ring is supported by 4 by 3 by ⅜ inch angle struts, one on each side of all the haunch windows, which in turn carry the weight of the dome to the steel girders of the roof below.

In designing the dome, the stresses were computed by Prof. William Cain's

analytical method,* the essential features being somewhat similar to the Habrich Construction as applied to domes in Europe.

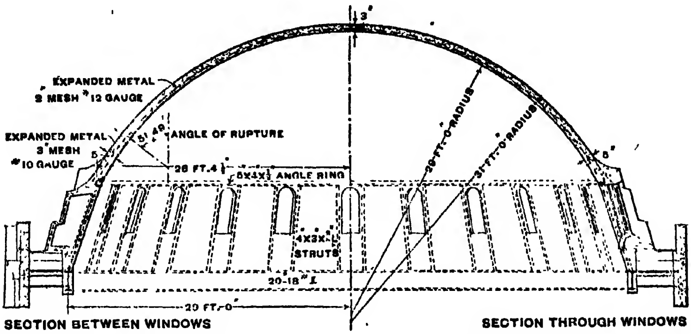


FIG. 199. Dome of Temple Adath Israel, Boston. (See p. 616.)

WALLS OF MORTAR PLASTERED UPON METAL LATH

Partitions of plaster from metal lathing are used extensively for fire-proof office buildings and hotels, and are also adapted, when made with Portland cement mortar, to certain classes of outside walls.

For a one-story building, timber or steel posts may be set upon concrete foundations, and the walls constructed by using ¾-inch or 1-inch channel irons for studding, to which the metal lathing is attached, and then covered (on both sides) with Portland cement mortar about 2 inches thick, the studding being generally set from 12 to 16 inches on centers, the spacing depending on the height of wall. Such walls are also adapted for high buildings where steel frames are used, as the studding can be securely bolted to the steel work, and the metal lathing and cement applied in the same manner as for one-story buildings.

For curtain walls the first coat of mortar is usually mixed with one barrel of first-class Portland cement to three barrels of coarse sand, and one cask of lime putty, or paste, into which is mixed a small quantity of long cattle hair. The second coat, which is applied before the first coat is thoroughly dry, consists of one barrel of Portland cement to three barrels of sand with about a bucketful of lime putty, without hair. The finish coat is generally mixed in the proportions of one part Portland cement to two parts sand

* Transactions American Society Civil Engineers, Vol. LV, p. 201.

This finish coat may be troweled or floated to a smooth or rough surface, as may be desired, or it may be given what is known as a "slap-dash" finish by throwing the mortar on with a brush or twig broom.

Ornamental Construction. Concrete or mortar may be cast by special molds into blocks of any desired size or shape, or molded for ornamental decoration in designs which vie both architecturally and in durability with finely carved sandstone, limestone, and granite. The color may be slightly

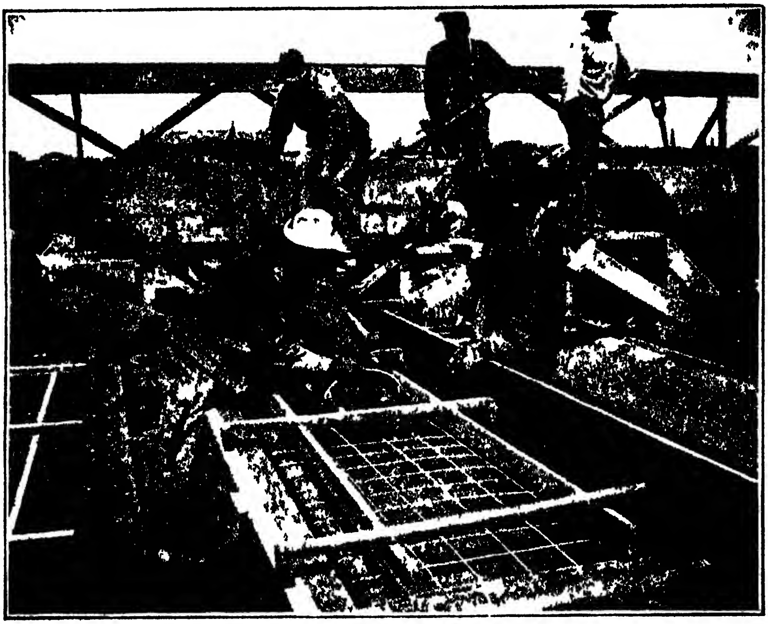


FIG. 200 – Pouring Seat Slab of Harvard Stadium (*See p* 628.)

varied by mixing different kinds of crushed stone Artificial coloring matter is apt to fade.

Ornaments are run whole in a mold which is made in halves, or are molded in two or three pieces and cemented together. Molds of plaster-of-Paris, shellacked within, are commonly employed.

Another method of molding, similar to that employed for iron castings, is with fine, damp sand, which is sometimes treated by a patented process. A wooden core is made and sand packed around it, then the core is removed, and the mortar is poured in The surfaces, after setting, may be rubbed down and floated. Fig. 200 illustrates the pouring of a seat slab

at the Harvard Stadium.* The wooden core, which was of the form of an L, for riser and tread, has been removed from the sand, reinforcing wire placed, and thick grout of the consistency of cream is being run in from a box car. The proportions of material were about one part Portland cement to 2½ parts fine crushed trap rock under ⅜-inch diameter.

Surfacing is treated on page 288.

CONCRETE BUILDING BLOCKS

Numerous machines and patented methods are on the market for forming building blocks of Portland cement, mortar or concrete to compete with brick and stone for house fronts. Some of the machines form the blocks from concrete mixed rather dry and pressed into the mold, while other methods employ a semi-liquid consistency, and the material is merely poured into the molds. The blocks may be hollow so as to extend clear through the wall, or each face of the wall may be laid with separate blocks. If care is exercised in molding and the sizes and surface appearance of the blocks are varied, a wall of pleasing architectural effect is possible.

The material for building blocks should be first-class Portland cement and fine crushed rock, or fine gravel and sand ranging in size from ½ inch in diameter to dust. Fine sand or fine dust alone makes with Portland cement a very porous stone, and must therefore never be used.

CONCRETE TILE

Concrete hollow tile is being made for the same uses as terra cotta tiling for partitions and floors, and also for dwelling houses in the construction of outside walls as well as of interior partitions. The sizes and shapes of the blocks are varied for the different purposes.

One of the best patented processes for making concrete tile consists in pouring wet concrete of the consistency of grout, into a mold and then, by application of a steam jacket, which forms a part of the mold, evaporating enough of the water from the concrete to permit the withdrawal of the tile from the mold within a few minutes. The product thus has the density and uniformity of wet mixed concrete, and is very true and uniform in shape and size and in thickness of walls. Plastering appears to adhere to it better than to most other forms of concrete.

* Lewis J. Johnson in Journal Association Engineering Societies, June, 1904, p. 305.

REINFORCED CONCRETE CHIMNEYS

High factory chimneys of reinforced concrete are being built in this country and abroad. The cost, especially of those over 100 feet high, is usually much less than brick. If designed and built upon the same principles and by the same methods which have proved essential in other types of reinforced concrete construction, they can be depended upon to give permanent satisfaction.

Reports* from a large number of chimneys have shown that concrete is unaffected by the heat from an ordinary steam boiler plant. The temperature in such chimneys seldom exceeds 700° Fahr. while 400° to 500° Fahr. is more usual. Experimental tests also indicate that concrete is not appreciably injured at temperatures of 600° to 700° Fahr.†

To provide for extremes, it is advisable, however, to build an independent inner shell of concrete or firebrick for at least a portion of the height. Concrete should not be used for a chimney in connection with special high temperature furnaces.

Since concrete and steel have substantially the same coefficient of expansion‡ there is no danger of heat causing a separation of the reinforcement from the concrete.

The expansive effect of heat is a more serious question. Stresses are set up in the shell of any masonry chimney because of the hot interior and cold exterior surfaces. A concrete chimney, however, has thinner walls so that the stress is less than in one of brick or tile and it is also better reinforced. Provision for temperature stresses are discussed in paragraphs on design which follow.

Construction. A reinforced concrete chimney is more difficult to construct than many other kinds of concrete construction because of its height and shape, and it therefore should be handled by experienced builders.

It is essential in chimney construction that the materials be very carefully selected. The sand as well as the cement should be tested by determining the actual tensile strength of mortar made from it. The stone preferably should be of the nature of a hard trap rock ½ inch maximum size. Proportions 1 : 2 : 3 have been found to give good results. A dry mix should not be used, since insufficient water will produce a porous concrete which does not adhere to the steel. The consistency must be wet enough to quake and form jelly-like mass when lightly rammed, so as to properly imbed and

* A special investigation of reinforced concrete chimneys was made by Sanford E. Thompson in 1907 for the Association of American Portland Cement Manufacturers. Many of the points here discussed are summarized from the report, which is printed as Bulletin No. 18 of the Association.

† Tests of Metals, U. S. A.

‡ See page 287.

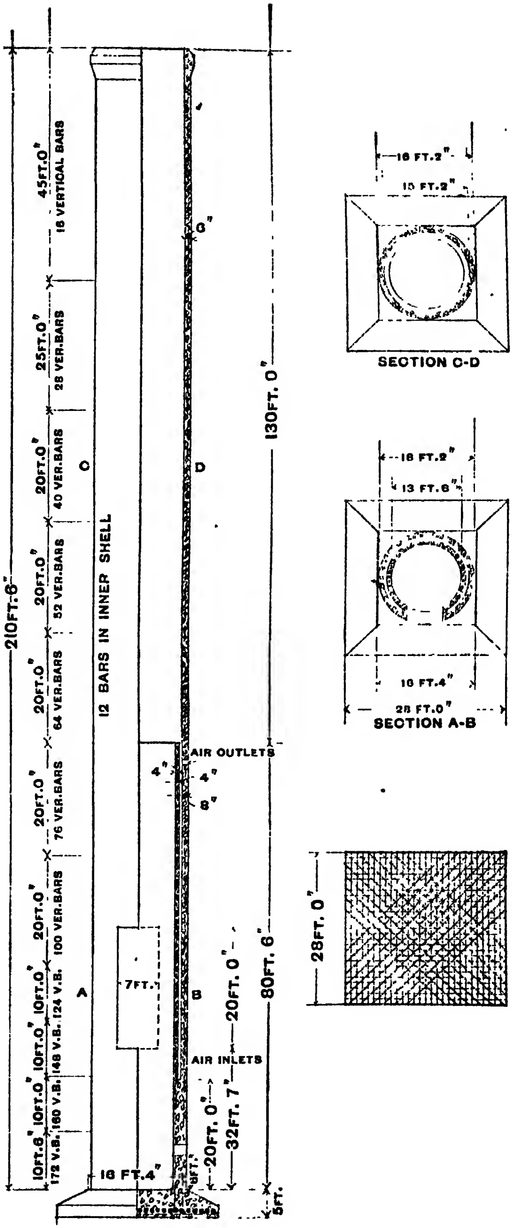


FIG. 201. Design of Chimney of the Edison Electric Illuminating Co., Brooklyn, N. Y. (*See p. 632.*)

bond the reinforcement. No exterior plastering should be permitted because it is liable to check and scale. The steel should be good quality round or deformed bars. Bars with flat surfaces like T-bars are inferior because the flat surfaces give a poor bond and the angles make the placing of the concrete difficult. Deformed bars of small size quite closely spaced are specially good for the horizontal steel to distribute the temperature stresses and high carbon steel of first-class quality also has advantages for the horizontal reinforcement.

Design. The design of a chimney built in Brooklyn, N. Y., in 1907 is illustrated in Fig. 201.

Design of Reinforced Concrete Chimneys. A reinforced concrete chimney consists primarily of a concrete shell with vertical steel bars imbedded in it all around the chimney. The shell must be of proper thickness and the steel bars sufficient in size and number to withstand the stresses due to the weight of the chimney and to the action of the wind. A chimney of this type differs essentially from one of brick in that the diameter at the base is so small as compared to the height that it would overturn under a heavy wind were it not for the vertical bars of steel which serve as anchors and hold it on the windward side.

Wind, in blowing against a chimney, causes compression on the side opposite to the wind and tension on the side against which the wind is acting. This compression is resisted by the concrete and steel on the leeward side, while the tension or pull is taken by the steel on the windward side.

In addition to the vertical reinforcement, a reinforced concrete chimney should be provided with horizontal hoops of steel, the object of which is to stiffen the vertical steel, to distribute cracks in the concrete due to a difference in temperature between the interior and exterior and to resist the diagonal tension.

In designing a reinforced concrete chimney the problem then is primarily to determine at various horizontal sections the necessary thickness of the concrete shell and the required amount of vertical reinforcement, so that the allowable working stresses in the concrete and in the steel shall not be exceeded under the action of the forces to which the structure may be subjected. The problem is one in mechanics, involving the equilibrium of a system of forces, and, with certain reasonable assumptions, the laws of mechanics may therefore be applied to these forces, producing thereby certain rational formulas from which the necessary proportions of the chimney may be determined. The complete analysis and development of the most useful formulas are given in Appendix III, page 765, of this treatise, the formulas themselves being reproduced below.

The problem of the determination of stresses due to the difference in temperature between the interior and the exterior of the shell involves many uncertainties. The heat tends to expand the inner surfaces, producing tension in the outside surface of the shell and compression in the interior surface. Although the distribution of the stress is not clearly known, the variation of the heat through the shell not being uniform, tentative computations indicate high stresses so that it is a question whether vertical temperature cracks can be entirely prevented any more than they can be prevented in brick or tile chimneys. The function of the horizontal steel may therefore be to distribute these cracks and to resist the vertical shear or diagonal tension. This horizontal steel should be distributed therefore by using small diameter bars closely spaced rather than large bars spaced further apart. Because of the possibility of vertical temperature cracks, the concrete should never be relied upon to carry tension or vertical shear, and the amount of horizontal reinforcement to resist this may be obtained in a similar fashion to the determination of vertical stirrups in a beam. In Appendix III, page 772, the analysis for the shearing stresses is indicated, and the final formula is presented below together with suggestions for adapting the horizontal reinforcement to temperature stresses.

The amount of vertical reinforcement, the thickness of the shell, and the percentage of horizontal reinforcement may be obtained from the following formulas, the derivation of which is given in Appendix III, page 765.

Let

W = weight in pounds of the chimney above the section under consideration.

M = moment in inch-pounds of the wind about that section.

f_s = maximum tension in the steel in pounds per square inch.

f_c = maximum compression in the concrete in pounds per square inch (measured at the mean circumference).

$n = \dfrac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete.

D = mean diameter of shell in inches (i. e., diameter of center of ring).

r = mean radius of shell in inches.

t = total thickness of shell in inches.

A_s = total cross-sectional area, in square inches, of reinforcing bars in the section under consideration.

k = ratio of distance of neutral axis. from mean circumference on compression side, to the mean diameter D.

z, C_P, C_T = constants for any given value of k, Tables 1 and 2, pages 635, 636.

p_0 = ratio of area of steel hoop to area of concrete.
h_1 = height in feet of chimney above section under consideration.
F = effective wind pressure against chimney in pounds per square foot.

Then

$$A_s = \frac{8\ (M - W\ z\ D)}{C_T f_s D} \tag{1}$$

$$t = \frac{2\ W + (C_T f_s - C_P f_c n)\ \dfrac{A_s}{\pi}}{C_P f_c D} + \frac{A_s}{\pi D} \tag{2}$$

$$p_0 = \frac{h_1\ F}{18.8\ f_s\ t} + 0.0025 \tag{3}$$

Formulas (1), (2), and (3) correspond to formulas (7), (8), and (9) in Appendix III.

In the formula for p_0, the first term gives the ratio of steel to resist vertical shear or diagonal tension, and the second term is an arbitrary ratio designed to distribute the temperature strains. To best distribute the temperature strains, a maximum spacing of the horizontal bars is recommended as 6 inches to 10 inches.

In the formulas the terms z, C_P and C_T are constants, the values of which are fixed for any given position of the neutral axis. By means of tables 1 and 2 (pp. 635-6) these constants may be easily and quickly determined so that the solution of formulas (1) and (2) is rendered quite simple after the selecting of the diameter and height of the chimney and computing the bending moments due to the wind at the various sections considered. The thickness of shell must be assumed in formula (1) in order to determine the average diameter D and to compute the weight W. A new computation may be made to correct this if necessary. For economical distribution of concrete and steel, computation must be made for several sections in the height. It is advisable to make the thickness of exterior shell never less than 5 inches but the number of steel rods may be gradually reduced toward the top.

Summary of Essentials in Design and Construction. In the investigation* referred to, the essential requirements are summarized as follows:

(1) Design the foundations according to the best engineering practice.

(2) Compute the dimensions and reinforcement in the chimney with conservative units of stress, providing a factor of safety in the concrete of not less than 4 or 5.

* See footnote, p. 630.

(3) Provide enough vertical steel to take all of the pull without exceeding 14,000 or at most 16,000 pounds per square inch.

(4) Provide enough horizontal or circular steel to take all the vertical shear and to resist the tendency to expansion due to the interior heat.

(5) Distribute the horizontal steel by numerous small rods in preference to larger rods spaced farther apart.

(6) Specially reinforce sections where the thickness in the wall of the chimney is changed or which are liable to marked changes of temperature.

(7) Select first-class materials and thoroughly test them before and during the progress of the work.

(8) Mix the concrete thoroughly and provide enough water to produce a quaking concrete.

(9) Bond the layers of concrete together.

(10) Accurately place the steel.

(11) Place the concrete around the steel carefully, ramming it so thoroughly that it will slush against the steel and adhere at every point.

(12) Keep the forms rigid.

The fulfillment of these requirements will increase the cost of the structure, but if the recommendations are followed, there should be no difficulty in erecting concrete chimneys which will give thorough satisfaction and will endure.

Table 1. Values of Constants C_P, C_T, z and j for Different Positions of the Neutral Axis, (i. e., for various values of k)

For use with equations (1), (2) and (3), page 634, and (7), (8) and (9), pages 771 to 773. k is ratio of distance of neutral axis from mean circumference on compression side to the mean diameter D. Value of k to suit the condition of the problem is obtained from Table 2, page 636.

k	C_P	C_T	z	j
0.050	0.600	3.008	0.490	0.760
0.100	0.852	2.887	0.480	0.766
0.150	1.049	2.772	0.469	0.771
0.200	1.218	2.661	0.459	0.776
0.250	1.370	2.551	0.448	0.779
0.300	1.510	2.442	0.438	0.781
0.350	1.640	2.333	0.427	0.783
0.400	1.765	2.224	0.416	0.784
0.450	1.884	2.113	0.404	0.785
0.500	2.000	2.000	0.393	0.786
0.550	2.113	1.884	0.381	0.785
0.600	2.224	1.765	0.369	0.784

***Table 2.** Location of Neutral Axis for various combinations of compressive stress, f_c, tensile stress, f_s and ratio of moduli, n, (see p. 633.)*

MAXIMUM TENSILE STRESS IN STEEL f_s	k RATIO OF DEPTH OF NEUTRAL AXIS TO DEPTH OF STEEL BELOW MOST COMPRESSED SURFACE OF BEAM														
	$n = 10$ Maximum compressive stress in concrete, f_c					$n = 12$ Maximum compressive stress in concrete, f_c					$n = 15$ Maximum compressive stress in concrete, f_c				
	300	400	500	600	700	300	400	500	600	700	300	400	500	600	700
8000	.272	.334	.384	.428	.466	.310	.375	.428	.474	.512	.360	.428	.484	.530	.568
9000	.250	.308	.357	.400	.438	.285	.348	.400	.444	.483	.334	.400	.454	.500	.538
10000	.231	.286	.334	.375	.412	.264	.324	.375	.418	.456	.310	.375	.428	.474	.512
11000	.214	.266	.312	.353	.389	.246	.304	.353	.395	.433	.290	.353	.405	.450	.488
12000	.200	.250	.294	.334	.368	.231	.285	.334	.375	.412	.272	.334	.384	.428	.466
13000	.188	.236	.278	.316	.350	.217	.270	.316	.356	.392	.257	.316	.366	.409	.447
14000	.176	.222	.263	.300	.334	.204	.255	.300	.340	.375	.243	.300	.349	.391	.428
15000	.166	.210	.250	.285	.318	.198	.242	.286	.324	.360	.231	.286	.334	.375	.412
16000	.158	.200	.238	.272	.304	.184	.231	.272	.310	.344	.220	.272	.319	.360	.396
17000	.150	.190	.228	.261	.291	.175	.220	.261	.298	.330	.210	.261	.306	.346	.382
18000	.143	.182	.218	.250	.280	.166	.210	.250	.285	.318	.200	.250	.294	.334	.368
19000	.136	.174	.208	.240	.270	.160	.201	.240	.275	.306	.192	.240	.283	.322	.356
20000	.130	.166	.200	.231	.260	.152	.194	.231	.264	.296	.184	.231	.272	.310	.344

In connection with reinforced concrete chimneys, the problems which arise are of two general kinds:

(1) A problem in design, involving the determination of the necessary thickness of shell and required amount of reinforcement at the various sections of a chimney of given height and diameter.

(2) A problem in the review or investigation of a chimney of given height and diameter having a certain thickness of shell and a given amount of reinforcement to determine the stresses in the concrete and the steel under the action of certain forces.

The application of the foregoing formulas to such problems and the use of the accompanying tables may best be illustrated by the following numerical examples, although the designer is advised also to refer to Appendix III, pp. 765–773 for a thorough understanding of the subject.

DESIGN OF A CHIMNEY. *Example 1.* Given a chimney with height above section considered, 110 ft.; mean diameter at section considered, 10 ft.; allowable pressure in concrete (f_c), 500 lb. per sq. in.; allowable tension in steel (f_s), 14 000 lb. per sq. in.; ratio of moduli n, 15; wind pressure (on normal plane) 50 lb., per sq. ft., weight of concrete taken as 150 lb. per cu. ft. What is the necessary thickness of shell and amount of reinforcement at the given section?

Solution. As in all chimney designs, it is necessary here to make a trial assumption of the thickness of shell in order to estimate the weight. Suppose

we assume a 6-inch shell for the entire height above the section. Assuming that a wind pressure of 50 lbs. per square foot on a normal plane corresponds to $\frac{6}{10}$ of 50 pounds or 30 pounds per square foot on the projected diameter of a cylindrical surface we have the bending moment due to the wind,

$$M = [10.5 \times 110 \times 30] \times \frac{110}{2} \times 12 = 22\ 869\ 000 \text{ in. lb.}$$

and the total weight of the chimney above the section,

$$W = 3.1416 \times 10 \times 0.5 \times 110 \times 150 = 259\ 180 \text{ lb.}$$

For $f_c = 500$, $f_s = 14\ 000$, and $n = 15$, table 1 gives $k = .349$

For $k = .349$ table 2 gives $C_p = 1.637$, $C_T = 2.335$, $z = .427$

Substituting in equation (1),

$$A_s = \frac{8\ (22\ 869\ 000 - 259\ 180 \times .427 \times 120)}{2.335 \times 14\ 000 \times 120} = 19.6$$

Therefore 19.6 square inches of steel are required.

If $\frac{3}{4}$ inch round rods are selected, 45 of them would be required.

Substituting in equation (2), we have

$$t = \frac{2 \times 259180 + [(2.335 \times 14000) - (1.637 \times 500 \times 15)]\ \frac{19.6}{3.1416}}{1.637 \times 500 \times 120}$$

$$+ \frac{19.6}{3.1416 \times 120} = 6.6 \text{ inches}$$

Therefore a 6.6 inch shell would be used.

In general the values of A_s and t as thus obtained should be readjusted by computing W on the basis of the computed thickness of shell. In the case at hand, however, the original assumption of a 6-inch thickness corresponds, for all practical purposes, with the computed thickness of 6.6 inches, so that recomputation is, in this case, unnecessary. If the walls of the chimney taper in thickness the value of W must be altered accordingly.*

Having determined the required thickness of shell and amount of vertical reinforcement there remains the question of the necessary horizontal or circular reinforcement. Substituting in formula (3) for f_s say 14000 lb., we have

$$p_0 = \frac{110 \times 30}{18.8 \times 14000 \times 6.6} \times 0.0025 = 0.0044$$

Area of steel, $A_s = 6.6 \times 12 \times 0.0044 = 0.35$ sq. in. Thus $\frac{1}{2}$ inch round rods should be spaced $6\frac{3}{4}$ inches on centers.

In a similar manner any other section of the chimney may be proportioned.

Review of a Chimney. *Example 2.* Given a chimney with height above section considered, 90 ft; mean diameter at section considered, 8 ft.; thickness of shell at section considered, 6 in.; vertical steel at section condsidered, 60 — $\frac{5}{8}$ in. round rods; wind pressure (on normal plane, 50 lb. per sq. ft.); weight of concrete taken as 150 lb. per sq. ft.; ratio of moduli, n, 15.

What are the maximum stresses in the concrete and in the vertical steel at the section under consideration?

*In relatively high chimneys steel cannot be stressed to 14,000 lbs. per sq. in. (see p. 774).

Solution. A problem of this kind must necessarily be solved by a method of successive trials, since the position of the neutral axis is not known. The location of the neutral axis is determined by the values of f_c, f_s and n, two of which, in this case, are unknown. The method of procedure, therefore, is to assume outright a trial position of the neutral axis, select the constants accordingly, substitute in equations (1) and (2) and solve them for f_s and f_c.

Then see if the position of the neutral axis, as fixed by these values of f_s and f_c and the given n, is the same as the position assumed at the start. If the two positions agree, then f_s and f_c as found are the actual stresses; if not, a new position of the neutral axis must be assumed, new constants selected, and new values of f_s and f_c computed from equations (1) and (2). Thus a series of trials must be made until the location of the neutral axis as assumed is consistent with the computed values of f_c and f_s together with the given n.

In this problem, assuming 30 pounds pressure on the projected area, we have the bending moment due to the wind,

$$M = [8.5 \times 90 \times 30] \times \frac{90}{2} \times 12 = 12\ 393\ 000 \text{ in lb.}$$

and the total weight of the chimney above the section,

$$W = 3.1416 \times 8 \times 0.5 \times 90 \times 150 = 169\ 646 \text{ lb.}$$

$$A_s = 60 \times .3068 = 18.41 \text{ sq. in.}$$

Now suppose we assume the neutral axis at, say, $k = .400$

For $k = .400$, table 1 gives $C_p = 1.765$, $C_T = 2.224$, $z = .416$

Substituting in equation (1) we have

$$18.41 = \frac{8\ (12\ 393\ 000 - 169\ 646 \times .416 \times 96)}{2.224 \times f_s \times 96}$$

whence $f_s = 11400$

Substituting in equation (2) we have

$$6 = \frac{2 \times 169646 + (2.224 \times 11400 - 1.765 \cdot f_c \cdot 15)\ \frac{18.41}{3.1416}}{1.765 \times f_c \times 96} + \frac{18.41}{3.1416 \times 96}$$

whence $f_c = 416$

Now $f_s = 11\ 400$, $f_c = 416$, and $r = 15$ gives $k = .354$ which does not correspond with our original assumption of $z = .400$. Evidently the true k must lie somewhere between the assumed and determined values, hence if we now assume, say, $k = .375$ and recompute, we obtain $f_s = 11\ 000$ and $f_c = 435$, the values of which together with $n = 15$ gives $k = .371$ which checks fairly well with the assumption of $k = .375$. For all practical purposes we may therefore say that the maximum stress in the steel is 11 000 pounds per square inch, while the maximum stress in the concrete is 435 pounds per square inch. The results indicate that both the thickness of shell and the amount of steel are greater than are necessary for safe stresses.

CHAPTER XXV

FOUNDATIONS AND PIERS

Concrete excels as a material for foundations, and here finds its widest and most important field of usefulness. It is pre-eminently adapted to such construction, because the stresses are chiefly compressive, the forms are easily built, and the surface appearance need not be considered.

Concrete is peculiarly suited to under-water foundations because, although it requires careful handling, it can be placed with great facility. It is now used even in piling. (See p. 650.)

Within recent years concrete has been adopted for foundations above ground, such as bridge piers, and is standing the test of durability even when subjected to excessive wear and impact. (See p. 654.)

Since the design of a foundation or sub-structure is governed almost as much by the character of the underlying rock or soil as by the superstructure, brief reference is made to the standard practice in estimating loads, although the treatment of engineering principles, as such, is not within the province of this treatise.

Reinforced concrete footings are treated in detail (see p. 644).

BEARING POWER OF SOILS AND ROCK

Sound hard ledge will support the weight of any foundation and superstructure, but if the rock is seamy or rotten it may require thorough examination and special treatment. If its surface is weathered, it must be removed. A sloping surface must be stepped or the foundation designed with sufficient toe to prevent sliding.

The sustaining power of earths depends upon their composition, the amount of water which they contain or are likely to receive, and the degree to which they are confined. An approximate idea of the loads which may be safely placed upon uniform strata of considerable thickness is given by Mr. George B. Francis*:

There are several classes of strata that are readily definable, such as ledge rock, hard pan, gravel, clean sand, dry clay, wet clay, and loam, and when these strata are of considerable thickness and uniform for considerable areas, they may be loaded with safety (provided the material

*Journal Association Engineering Societies, June 1903, p. 340.

placed thereon is not of less density than the natural material upon which it is placed, viz, concrete or brick work on ledge rock) as follows

Ledge rock, 36 tons per square foot.
Hard pan, 8 tons per square foot.
Gravel, 5 tons per square foot
Clean sand, 4 tons per square foot.
Dry clay, 3 tons per square foot.
Wet clay, 2 tons per square foot.
Loam, 1 ton per square foot.

Mr Francis, however, calls attention to the many kinds and mixtures of materials, and to the consequent impossibility of applying such specific rules as the above to all cases He also emphasizes the necessity for varied and ample experience when fixing safe allowable pressures

If the piles are driven to firm strata, such as rock or hard pan, the loading which a pile will stand is determined by the crushing strength of the timber If supported wholly or in part by friction, it is customary to calculate the safe loading by a formula based upon factors obtained by experiment, or by one based upon the penetration of the pile from the blow of the pile driver

An engineer experienced in pile driving in a particular locality can often determine by judgment whether the piles have reached a firm bearing, but it is usually safer to formulate exact specifications Mr Joseph R Worcester* advises for piles which meet a hard resistance, a penetration of one inch under a 2 000 lb hammer falling 10 feet, and for piles which hold by friction, a penetration of 3 inches under a 2 000 lb hammer falling 15 feet He prefers heavier hammers if they are available

A mean of the various formulas† gives for approximate average values, after applying a factor of safety of 3, a safe load of 16 tons for bearing piles and 9 tons for friction piles * These loads apply to ordinary piles of spruce and Norway pine

A commonly used formula for determining safe loading on piles with reference to the penetration under blows of the hammer is the *Engineering News* formula, which is as follows

Let

P = safe load in tons upon a pile.
W = weight of hammer in tons
h = height of fall in feet
p = penetration in inches under last blow.

*Journal Association Engineering Societies, June, 1903, p 285

†The various pile formulas are tabulated and discussed by Ernest P Goodrich, in Transactions American Society of Civil Engineers, Vol XLVIII, p. 180.

Then

$$P = \frac{2Wh}{p+1}$$

Mr. Worcester states with reference to spacing piles:

The minimum distance between centers of piles depends upon two factors: the hardness of the soil and the size of the butts. Ordinary spruce piles may be well driven 24 inches on centers, while large and long piles cannot be driven to advantage closer than 30 inches. Another governing condition must be taken into account, however, and that is the supporting power of the soil as a whole. Where the piles reach a real hard pan, the soil will generally resist all the pressure that the piles can bring on it, unless it consists of a thin crust overlying a soft material; but when the soil is so soft that the piles hold by friction only, and there is enough friction to carry all the soil between the piles down with them, in case they go together, the spacing becomes a question of how much the underlying soil will support per square foot. For example, if the soil can only support 2 tons per square foot, and the piles could each carry 18 tons, it is useless to place them closer than 3 feet on centers.

CONCRETE CAPPING FOR PILES

Although some authorities advocate stone capping for piles, even if the cost is more, it is generally considered good practise to lay the concrete directly upon the head of the pile. The ground is excavated to a depth of one or two feet around the piles, and if very soft, a layer of broken stone or chips may be spread and rammed hard upon it before laying the concrete. The load is distributed by the concrete, and the supporting power of the soil between the piles is utilized.

The thickness of the concrete above the piles must be sufficient to distribute the superimposed weight, and the reactionary load of the pile head acting upwards. If the layer is very thin there may be danger of the pile head shearing through the concrete. The objection sometimes raised to concrete capping is that the upward crushing stress upon the concrete by the head of the pile may be excessive, especially if loaded before the concrete is thoroughly hard. In considering this tendency, it must be borne in mind that under concentrated loading the concrete will sustain a higher stress per unit of area of contact than if the load is distributed. (See p. 249.)

DESIGN OF CONCRETE FOUNDATIONS AND FOOTINGS

The load upon a building foundation need not always be taken as the dead load plus the entire live load for which the superstructure is de-

signed, because in most structures the full live load will never be imposed upon all the floors at the same time. A conservative suggestion for reduction in the live load is given on page 611.

To prevent cracks in a structure, it is not only necessary to select a proper unit pressure on the soil but also to see that this pressure is uniform, so that if there is settlement it will be the same throughout. To satisfy this condition, the center of the loads from the columns or other portions of the structure should coincide with the center of gravity of the base. The area of the footing should be proportional to its load. When such an arrange-

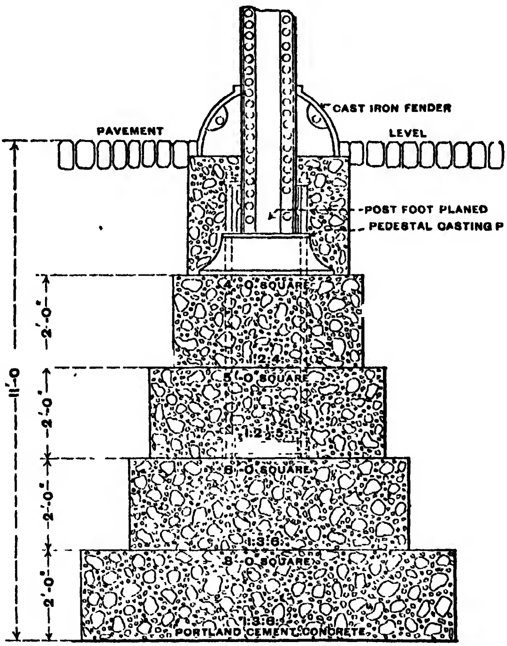


FIG. 202.—Typical Column Foundations of Boston Elevated Railway. (See p. 643.)

ment is difficult, the foundations under different columns should be separated and the area of base of each be made proportional to the superimposed load.

Frequently the building line nearly coincides with the property line and the foundation must be placed entirely inside the building. In such cases, to prevent eccentric pressure, either cantilever construction may be used for transmitting the exterior column loads centrally to the footing, or a combined footing designed as explained on page 647.

In structures such as chimneys or narrow buildings which are subject to wind pressure, the foundation should be designed with due consideration of the eccentricity caused by the wind.

With vertical loading upon rock or soil whose sustaining power per square foot is equal to or greater than the unit load, the dimensions of the foundation are fixed by the size of the structure, the safe load which can be sustained by the concrete, or by resistance to overturning. If the load is greater than an equivalent area of soil can sustain, the area of the base of the concrete must be enlarged, and the concrete battered or stepped or reinforced. It is a common engineering practice to make the length of the projections or steps of plain concrete one-half the height of the block, and this usually gives good results in buried foundations where the surrounding earth assists to prevent splitting.

The effect of concentrated loading must be considered when designing a footing. (See p. 367.) The pedestal bases for the Boston Elevated Railway were designed, when covering one-half the area of the concrete, with 25 % higher unit stresses for the concrete in actual contact than when covering the entire area. Fig. 202, page 642, shows a typical foundation for the columns.*

The following figures are suggested as conservative safe loads, when the surface of the concrete is larger than the loaded area. Lower stresses should be used with moving loads or when the area of the foundations is no greater than that of a column which it supports. The figures are based on ordinary concrete with a factor of safety of 4 at one month and a factor of $5\frac{1}{2}$ at six months.

Safe Loads on Foundations.

Proportions of Concrete by volume†	Safe Loading Lb. per sq. in.	Safe Loading Tons per sq. ft.
1:1:3	700	50
1:2:4	650	47
1:2½:5	575	41
1:3:6	500	36

For a vibrating or pounding load these values should be reduced from $\frac{1}{4}$ to $\frac{1}{2}$, depending upon the nature of the loading.

I-Beam Footings. Formerly, footings were made by imbedding steel I-beams, or in some cases old rails, in concrete for column footings. The concrete serves to distribute the loads and protect the steel. A typical footing, designed by Mr. John S. Branne,‡ is illustrated in Fig. 203, page 644. In this particular case the situation required a cantilever girder connecting this foundation with the next, but the footing shown is itself designed for a total load of 173 tons, of which 120 tons are dead load and 53 tons live load.

* George A. Kimball in Journal Association Engineering Societies, June, 1903, p. 351.

† Based on a barrel of packed cement of 3.8 cu. ft., weighing 376 lb. net.

‡ Journal Association Engineering Societies, Feb., 1901, p. 142.

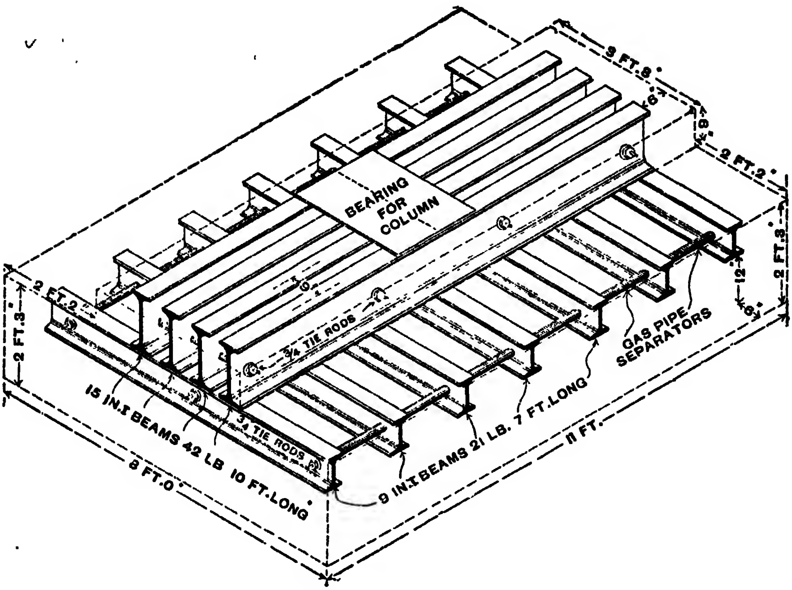


FIG. 203.—Concrete and I-beam Footing. (*See p.* 643.)

REINFORCED CONCRETE FOOTINGS

To distribute the load over a large area of soil without carrying the foundations, by successive steps, to a considerable depth and using a large mass of concrete, a single slab may be employed and reinforced with steel to prevent the projection breaking.

This in almost all cases permits a very great reduction in the quantity of material and reduces the cost. A footing reinforced with rods is designed to utilize the strength of the concrete, and is therefore more economical than the I-beam type of design just referred to, and is always to be preferred to it.

A reinforced concrete footing is really a flat slab and should be designed as such. The theory of the flat plate, explained on pages 483 to 488, therefore applies to it and the formulas on page 485 may be used directly for determining the bending moment. The principal formula for the maximum bending moment is as follows:

Let

M = maximum moment causing radial fiber stress

w = uniform distributed load on surface of the plate in pounds per square foot

r_0 = radius of base of column in feet
r_1 = radius of footing in feet
C_1, C_2 = constants to use in formula
Then

$$M = wr_0^2\,(0.2 + C_1 + C_2)$$

Values of the constants C_1, C_2 are found in the table page 518.

The application of the formula and principles is best illustrated in the example which follows.

Example 1. Find the dimensions of a footing for a column 28 inches square carrying 392000 pounds, when the allowable pressure on the soil is 2 tons per square foot?

Solution. The necessary area of footing is found by dividing the total superimposed load by the allowable unit pressure on the soil, or is $\frac{392\,000}{4000} = 98$ square feet, thus requiring an area 10 feet square. The footing may be considered as a flat slab loaded by the uniformly distributed upward pressure of the soil and fixed rigidly to the column. The formulas given above were deduced for a circular plate, but may be applied without appreciable error to a square footing. The radii to be used in the formulas are the averages of the radii of the circumscribed and inscribed circles.

$$r_1 = \frac{5 + 7.00}{2} = 6.00 \text{ ft.} \qquad r_0 = \frac{1.17 + 1.63}{2} = 1.4 \qquad \frac{r_1}{r_0} = 4.3$$

Using the formula and substituting for the constants values found from the table, page 518, corresponding to $\frac{r_1}{r_0} = 4.3$ and using as Poisson's ratio, $g = 0.1$ we have

$M = 4000 \times 1.4^2\,(0.2 + 6.7 + 3.49) = 81600$ ft. lb. per foot of width, which is equivalent to in. lb. per inch of width. For tension in steel, $f_s = 16000$; compression in concrete, $f_c = 650$; ratio of elasticity, $n = 15$; ratio of steel, $p. = 0.0077$; the constant from page 519, C is 0.096 and the depth of steel, (p. 418), $d = 0.096\sqrt{81600} = 27\ 2$ in.

The amount of steel will be found in the following manner. Find the area of steel required for the whole circumference of the inner circle of the plate, the radius of which at present is 1.4. Divide this amount by four and place it in two directions, at right angles, distributing it over an area slightly larger than the base of the column. Double the spacing of rods outside of the column, as the bending moment decreases very rapidly as shown in Fig. 204.

Circumference is $2 \times 1.4 \times 12 \times 3\ 1416 = 105.5$ in. Area of concrete. $A = 105.5 \times 27.2 = 2870$ sq. in. Area of steel, $A_s = 2870 \times 0.0077 = 22.1$ sq. in. Area of steel to be placed in one direction, $A = \frac{22.1}{4} = 5.53$ sq. in. The width of column is 28 in., hence six 1 in. square rods 5 in. on centers may be used. The spacing of the rods on the remaining area of the footing will be made 10 in. Deformed bars are advantageous because of increased bond strength.

Another method of arranging reinforcement is to place the bars in 4 layers, 2 of them diagonally.

The thickness of the footing may be decreased by judgment toward the edges without reducing its effective strength.

Shear Reinforcement. A footing to resist diagonal tension may require shear reinforcement of vertical stirrups or bent bars, placed near the column, where maximum shear occurs and diagonal cracks may be expected.

While the action of the internal shearing stresses are somewhat complicated, the following plan may be adopted.

Find the unit shear at the edge of the column, dividing the loading tributary to the area of the footing outside of the column by the moment arm jd (which may be taken at 27×0.87) times the circumference 4×40 in. and we have

$$v = \frac{(100 - 5.5)\,4000}{4 \times 40 \times 27 \times 0.87} = 100$$

lb. per sq. in. Assuming that one-third of it is taken by concrete, 66 lb. per sq. in. must be provided for by steel.

Next find by trial the circumference where the unit shear is 40 lb. per sq. in., the amount which may be safely resisted by concrete, so that the shear to be provided for by the steel is zero. In this case this circumference has been found by trial to be distant 39 in. from the center of column, and is shown on the diagram by dot and dash line.

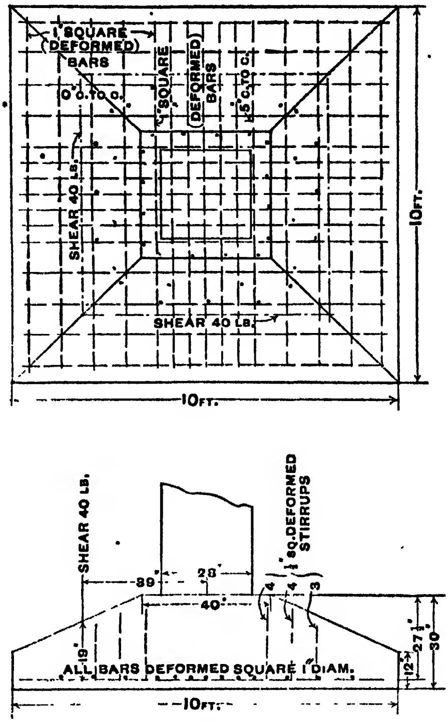


FIG. 204.—Square Footing. (*See p.* 645.)

Now multiply the horizontal area enclosed between the circumference of the column and this newly found circumference by half of the previously determined unit shear to be taken by the steel (66 lb.) and obtain the total amount of shear to be taken by the stirrups, Select the diameter of stirrups in accordance with the discussion on page 453, divide the amount of shear by the area of one stirrup and by the unit tensile strength and obtain the number of stirrups. In this case the area requiring stirrups is $78^2 - 28^2 = 5300$ sq. in., and the total amount of shear to be provided for, $\frac{66}{2} \times 5300 = 174\,900$ lb. If $\frac{1}{2}$-inch square stirrups are selected (area 0.25 sq. in.) with a unit tensile strength,

$f_s = 16\ 000$ lb. per sq. in., $\dfrac{174\ 900}{16000 \times 0.25} = 44$ stirrups in all, or 11 stirrups each side are necessary. Their spacing is evident from the drawing.

Bond Stress. The bond stresses must also receive attention. It will often be necessary to increase the depth of the footing or the amount of rods to provide for the excessive bond stress. The discussion, page 456, and formulas given there find here a direct application. Reference may be made also to the similar treatment in the design of a retaining wall footing, page 670.

Combined Footings. Sometimes it is necessary to connect the footings of two or more columns. The design of such combined footing differs from that of a single one. When the loads carried by the columns are different, the footing to distribute the loading uniformly should have the shape of a trapezoid. The following example will illustrate method of figuring:

Example. Let P_1 and P_2 be respective loadings of colums I and II, 30 and 24 inches square; $P_1 = 400000$ lb., $P_2 = 580000$ lb. The distance between centers of columns is 15 ft. and the allowable unit pressure on the soil is 4 tons, 8000 lb., per square ft. Find the dimensions of footing. (See Fig. 205.)

Solution. The total superimposed load is 980 000 lb., then the necessary area of footing, $\dfrac{980\ 000}{8000} = 123$ sq. ft. The magnitude of the parallel sides is unknown, and two equations are necessary for the determination. First equation may be obtained from the formula that the area of trapezoid equals the average of the sum of the parallel sides multiplied by its length. The length of the trapezoid is $15 + 1.75 + 1.50 = 18.25$ ft., and the area $= 123 = \dfrac{a+b}{2} \times 18.25$. Hence $a + b = 13.5$ ft. The second equation may be found from the requirement, that the center of gravity of the trapezoid coincide with the center of gravity of the combined column loading. The distance from A of the center of gravity of column loadings O, found by taking moments of loads, is 6.1 ft. and $l = 6.1 + 1.75 = 7.85$. Using the common equation for the center of gravity in a trapezoid gives $l = 7.85 = \dfrac{18.25}{3}\,\dfrac{a + 2b}{a+b}$. Solving the two equations for a and b, $a = 9.6$ ft., $b = 3.9$ ft.

To facilitate the finding of bending moments, the length of b, the width of the footing on the center of gravity line, may be computed from the relation $\dfrac{a-b}{18.25} = \dfrac{a-b_1}{7.85}$ and the length l_1, from the common formula for the distance of the center of gravity $b_1 = 7.15$ ft. $l_1 = 3.74$ ft. and $l_2 = 7.85 - 3.74 = 4.11$ ft.

Assuming the maximum moment* at center of gravity, $M = 580000 \times 6.1 \times 12 - \left(\dfrac{9.6 + 7.15}{2} \times 7.85 \times 8\ 000 \times 4.11 \times 12\right) = 16\ 550\ 000$ in. lb. for the width of beam equal to b_1. The moment for one inch of width, $M = \dfrac{16\ 550\ 000}{7.15 \times 12} = 193\ 000$ in. lb.

*Maximum moment is actually at section of zero shear but the error is inappreciable.

For tension in steel, $f_s = 16\,000$, compression in concrete, $f_c = 650$, ratio of steel, $p = 0.0077$, and $C = 0.096$ (see p. 421) then depth to steel, $d = 0.096\sqrt{193\,000} = 42.2$ in. As $A_s = 42.2 \times 12 \times 0.0077 \times 7.15 = 27.9$ sq. in. for the whole width, 18, $1\frac{1}{4}$ inch deformed bars will be used.

To prevent bending of the projections of the footing, transverse reinforcement will be introduced. The projections are assumed to act as cantilevers, loaded by half of the column loading multiplied by a ratio of the difference between the width of the footing, a, and the diameter of the column to the

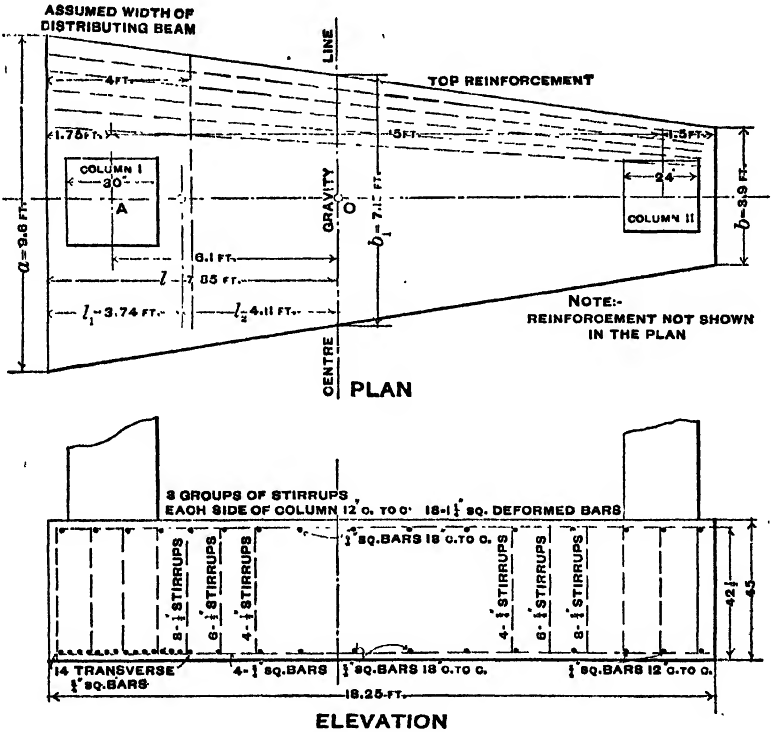


FIG. 205.—Combined footing. (See p. 647).

width of footing, or $\frac{9.6 - 2.5}{9.6}$. The moment arm equals half of the length of the projection, and the moment, $M = \frac{580\,000}{2} \times \frac{9.6 - 2.5}{9.6} \times \frac{3.55}{2} \times 12 = 4\,570\,000$ in. lb.

Assuming a width of the distributing beam equal to 4 ft., the depth will be $d = 0.096\sqrt{\frac{4\,570\,000}{4 \times 12}} = 29.6$ in. The depth is smaller than the depth of the whole slab. Since a larger depth is used, the percentage of steel will be found as follows:

$C = \sqrt{\dfrac{42.2^2 \times 48}{4\,570\,000}} = .137$ from formula (1) (p. 418) and $p = 0.0033$ from table on page 520; $A_s = 42.2 \times 48 \times 0.0038 = 7.70$ sq. in., hence $13\frac{1}{4}$ in. square bars will be used.

NOTE.—The required depth of the distributing beam may be sometimes larger than the depth of the whole slab. In such case the footing may be either thickened under the column or steel introduced at the top and bottom. The latter scheme should be adopted only when additional excavation for the beam cannot be made readily.

In a similar way the distributing reinforcement for column II is found.

$M = \dfrac{400\,000}{2} \times \dfrac{3.9 - 2}{3.9} \times \dfrac{0.95}{2} \times 12 = 555\,000$ in. lb. Assume a width of imaginary beam equal to 3 ft., then $d = 0.096\sqrt{\dfrac{555\,000}{36}} = 12$ in. As larger depth is used, the percentage of steel will be found as in previous case.

$C = \sqrt{\dfrac{42.2^2 \times 36}{555000}} = .334$. For this value of C less than 0.1% of steel is needed, and will be taken arbitrarily.

SPREAD FOOTINGS

When the allowable pressure on the soil is very small or when the building is supported by piles sustained by friction, it may be necessary to spread the foundation over the whole area of the building, either using a thick mass of plain concrete or a thinner slab of reinforced concrete design as a flat plate, or a beam and slab system.

Flat Slab Foundations. A flat slab may be designed by the method of flat plates explained on pages 483 to 487. The slab is considered as an inverted flat plate loaded by the reaction of the ground and supported by the columns.

Special provision should be made in the design where there is unequal loading.

Since the distributed pressure acts upward, the bottom of the plate under the columns is in tension and the top of the plate between the columns; hence the steel must be in the bottom of the slab under the columns, and should be bent up to the top of the slab between columns. The column base must be large enough to prevent excess loading or too great moments and shears in the concrete.

Beam and Slab Foundation. For a combination of beams and slabs the principles of floor design are followed except that the distributed load acts upward. The beams or ribs may be built either above or below the slab, the former method permitting a T-beam design, but, on the other hand, requiring an extra fill and separate floor surface in the basement. The formulas and discussion relating to floor design in Chapter XXI apply.

FOUNDATION BOLTS

It is often difficult to locate bolts in concrete with sufficient exactness for setting a machine. To obviate this difficulty, the head of the bolt should be provided with a large washer* to give a good bearing surface, the bolt placed in its approximate position, with washer down, and an iron pipe or a light wooden box placed around the bolt resting upon the washer. When the machine is set, to prevent the bolt from rusting, the iron tube or box should be filled with mortar. In any case the tube or box should be filled with sand before the machine is poured up with sulphur or cement grout, in order to keep these materials from running down the bolt holes.

CONCRETE PILES

Concrete piles may be employed in place of wood where the loading is excessive, and where the durability of timber piles is questioned either because of probable worm action or the rotting of the timber. If the bearing is frictional and the piles are driven through ground which is continually wet, there is usually no advantage in concrete over timber piles unless in certain instances where the low level of the ground water or the tide water is so far beneath the structure that the concrete piles permit the commencement of the foundation at a considerably higher level and thus save excavation and material.

Concrete piles are formed (1) in place, or (2) are molded above ground and driven with a pile driver.

Various methods have been suggested for forming the hole into which the concrete is to be placed. One of the patented processes consists in driving a double shell of metal into the ground, removing the inner one, and leaving the outer to form a mold for the concrete. The two shells and pile driver are shown in Fig. 207, page 652. The inner shell or pile core, which is of heavy sheet steel and constructed so that it can be made to collapse for removal from the ground, is placed within the other thinner shell, and driven like an ordinary pile. The core is then collapsed and withdrawn, leaving the outer shell, which is closed at the bottom, to be filled with concrete. By providing considerable taper, additional support is obtained from the soil.

* The washers, which are used for transmitting the pressure of large bolts to the concrete or other foundations, should be carefully designed with heavy ribs so as to transmit a uniform pressure per square inch of area. Neither wrought nor cast iron plates should be used for washers under large bolts.

Another system, illustrated in Fig. 206, consists in driving a single shell with either a concrete or a steel point, then slowly withdrawing it, and filling the space which it occupied with concrete whose surface is kept far enough above the lower end of the tube to maintain the head necessary to resist the pressure of the ground.

In still another method, which is especially adapted for underpinning, the tube is washed down with a water jet to firm strata, and the bottom of the excavation is enlarged by an expanding arrangement to form a base, as shown in Fig. 208.

Piles made in situ may be reinforced if desired.

Cast Piles. Reinforced piles which are formed above ground are designed like columns with vertical reinforcement connected at intervals with horizontal wire rods.

The pile* used in a foundation for the Boston Woven Hose & Rubber Company, Cambridge, Mass., is illustrated in Fig. 209. These piles averaged about 30 feet long. The hammer weighed 4700 pounds and the blows were cushioned by a head consisting of a plate iron collar 16 inches square on the inside and 3 feet in height, which incased an oak block 16 by 16 by 18 inches, to the bottom of which six thicknesses of rope and four layers of rubber belting were nailed. The piles were

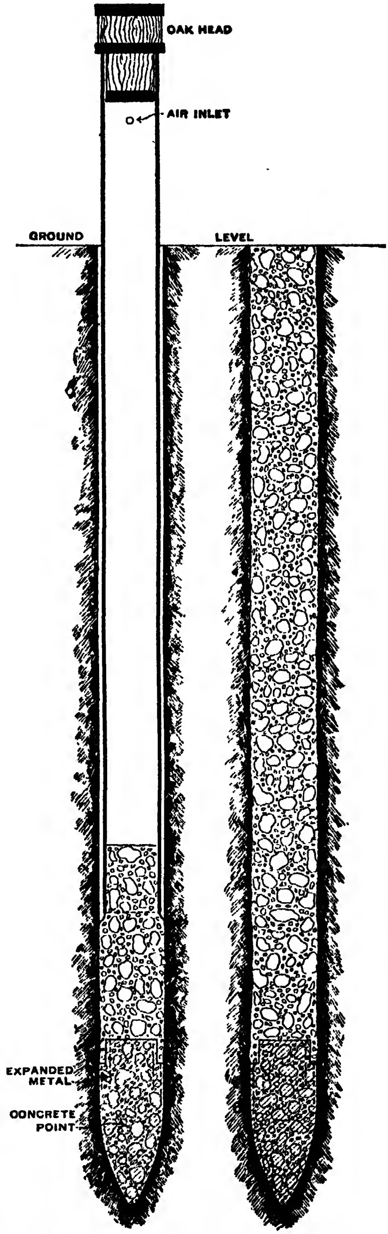


FIG. 206. — Concrete Piles. (*See p.* 651.)

* For full description of piles and driving see "Cast Reinforced Concrete Piles," by Sanford E. Thompson and Benjamin Fox, Journal Association of Engineering Societies, Vol. XLII, 1909.

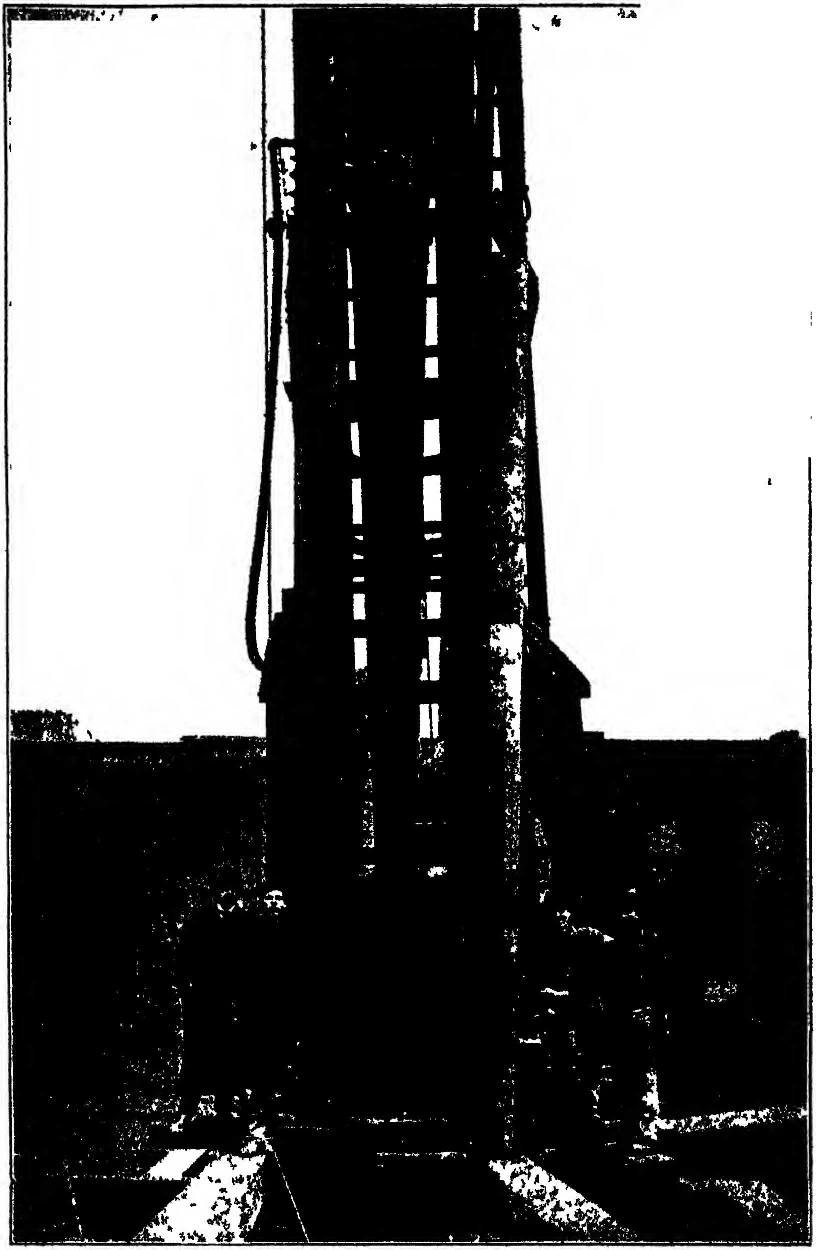


Fig. 207. — Cores for Concrete Piles. (See p. 650.)

driven at the age of thirty to forty days. The usual drop was 3 feet, but in some cases this was increased to 10 feet without injuring the pile.

The designs drawn up in 1903 for the Pennsylvania Railroad Tunnel* under the Hudson River call for a shell of cast iron surrounded by concrete and supported at intervals by steel screw piles filled with concrete.

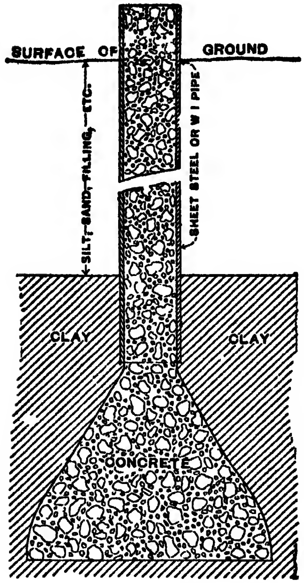


Fig 208.—Concrete Pile with Enlarged Footing. (*See p.* 651.)

Sheet Piling. Poling boards of concrete were employed by Mr. Howard A. Carson, Chief Engineer in the construction of the approaches to the East Boston Tunnel. These are described† as follows:

The excavation was through gravel and clay, and through sand containing some water. Trenches 16 feet long and 16 feet apart were dug to about the level of the bottom of the building foundation. Below the foundation one-half of each trench, or 8 feet in length, was carried down to grade. The bank below the foundation was held in place by means of concrete slabs used as sheet piling, as illustrated in Fig. 210. These slabs were from 6 to 8 feet long, 6 inches wide, and 2 inches thick, and each was reinforced with six square steel rods running the entire length of the slab and shown in Fig. 211. If wooden sheeting had been used, it would have been necessary either to have concreted directly against it and left it in place, or to have pulled the planks as the concrete was filled in. If the first method had been used, the planks would in time have become rotten, leaving a vacant space. If the planks had been pulled, there would have been danger that some of the earth under the building would run and a settlement of the building follow. In order to guard against any slight voids which might have been left in driving, grout was poured in behind the sheeting. This sheeting served not only to hold the bank in place, but was used, in place of a back wall, to waterproof against. The sheeting was not disturbed, and the wall of the Tunnel was built directly against it.

*Engineering News, Oct. 15, 1904, p. 331.

†Ninth Annual Report, Boston Transit Commission, p. 41.

BRIDGE PIERS

Concrete is employed for bridge piers either as filling for ashlar or cut masonry or for the entire pier. In the latter case, in which the face is also of concrete, the chief question is as to its ability to withstand the wear of the water, the ice, and floating debris. Mr. Martin Murphy* stated as early as 1893 that concrete was generally adopted in Nova Scotia, and with successful results, for abutments and piers "in the most exposed positions, in the midst of strong currents, without any external protection, where exposed to heavy ice floes, to blows from timber rafts, and, in many instances, to undermining by scour." In Nova Scotia it is the common practise to construct the body of the pier of rubble concrete with a 6 to 9-inch facing of richer concrete. In answer to inquiries, Mr. Murphy wrote the authors in 1904: "The concrete piers erected in this Province for the last eighteen or twenty years have withstood the action of the weather, and fulfilled all that was claimed for them in my paper, read before the International Congress in 1893. The erection of such piers and abutments is now in almost universal application in Canada."

In the Kansas City flood of 1903, the piers of solid concrete, although located where they were struck by all the heavy debris which totally destroyed many of the stone masonry structures of the same size, remained practically uninjured.

In 1900 a Committee of the Association of Railway Superintendents of Bridges and Buildings† made the following inquiry: "For what classes of structures do you use Portland ce-

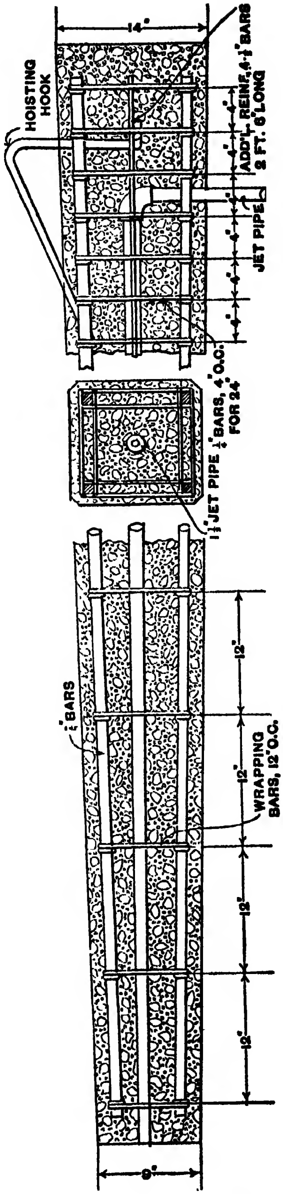


FIG. 209.—Piles used at Cambridge, Mass. (*See p.* 651.)

*Bridge Substructure and Foundations in Nova Scotia, Transactions American Society of Civil Engineers, Vol. XXIX, p. 620.

ment concrete?" Out of thirty-three replies received, seventeen were in favor of employing this material for both the foundation and neat work of bridges, piers, and abutments.

Plastering of concrete piers and abutments should be prohibited. If a mortar surface is required, an excellent facing, to be placed next to the form as the concrete is laid, is a mixture of one part cement to $2\frac{1}{2}$ parts hard broken stone screenings $\frac{1}{2}$ inch in size and under. Ordinarily, however, no surface finish is required unless superficial treatment is given for the sake of appearance. (See p. 288.)

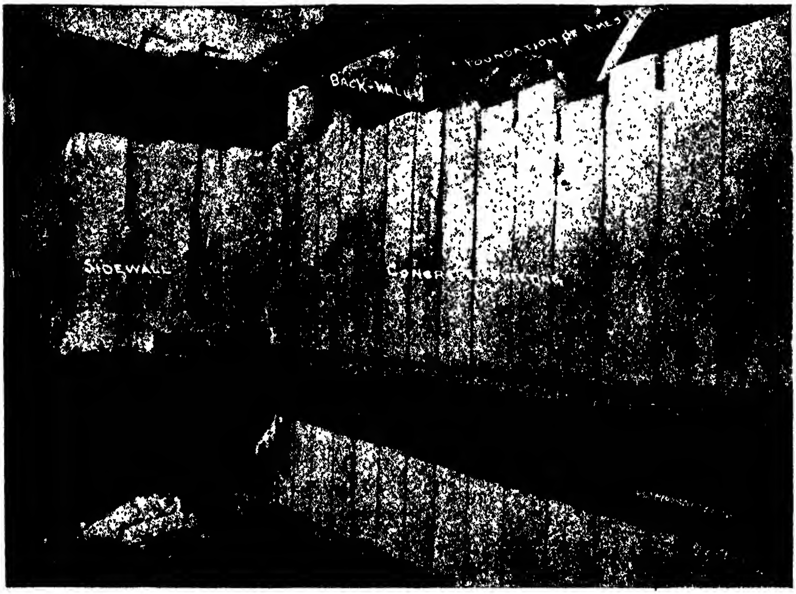


FIG. 210. - Concrete Sheet Piling in Approaches to East Boston Tunnel. (*See p.* 653.)

Pier Design. Most railroads are substituting concrete for ashlar masonry in bridge piers.

The standard pier of the N. Y. Central R. R., adapted to any height up to 40 feet, is shown in Fig. 212, page 657.* The width, which depends upon the length of span, is as follows:

*Arranged from original drawing, for which the authors are indebted to Mr. Wilgus.

Spans up to 40 feet width, A, = 4 ft. 0 in.
Spans 40 to 60 feet width, A, = 4 ft. 6 in.
Spans 60 to 80 feet width, A, = 5 ft. 0 in.
Spans 80 to 100 feet width, A, = 5 ft. 6 in.
Spans 100 to 125 feet width, A, = 6 ft. 0 in.
Spans 125 to 150 feet width, A, = 6 ft. 6 in.
Spans 150 to 200 feet width, A, = 7 ft. 0 in.
Spans 200 to 250 feet width, A, = 7 ft. 6 in.
For skew crossings, increase width, A, if necessary.

Foundation is varied to suit local conditions. Concrete 1:3:6 is employed for it unless stone masonry is cheaper. The starkweather is carried 2 feet above high water, and its cap is of 1.1:2 concrete. The coping of the pier is reinforced with galvanized wire netting or wire cloth, a somewhat unusual requirement.

The Illinois Central R. R., in their 1904 design, reinforce the surface of piers with vertical and horizontal steel rods, and imbed a single I-beam in the pointed nose at each end of the pier.*

The Chicago, Milwaukee & St Paul Railway Company takes the extra precaution to strengthen the noses or starlings of its concrete piers only at points where there is considerable ice and driftwood.† They build a 7 inch street car rail into the nose of the pier, with the head projecting slightly from the concrete. Other roads also show no reinforcement in their standard design.

It would appear that reinforcement is probably unnecessary except in situations where the piers are subjected to unusual impact.

All of the roads named above have piers in streams which subject them to considerable wear from ice and drift, and the concrete has proved satisfactory.

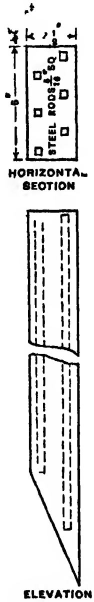


FIG. 211. - Sheet Piling (See p. 653.)

FOUNDATIONS UNDER WATER

The best and most durable concrete foundations, especially in work in sea water, are laid within cofferdams from which the water has been pumped, or in pneumatic caissons. However, because of the

*From drawing kindly furnished by H. W. Parkhurst, Engineer.
†Authority of C. F. Loweth, Engineer.

difficulty and expense of these methods, they cannot usually be followed. If the bottom is prepared by dredging, and, if necessary, driving piles, good practise permits the use of a single line of sheeting, suitably supported with rangers, to prevent the wash of the water and keep the concrete from spreading.* Permanent metal cylinders are sometimes sunk in place of the sheeting.

Methods of laying concrete under water are described in Chapter XV, page 301, and the effect of sea water upon concrete is discussed by Mr. R. Feret in Chapter XVI.

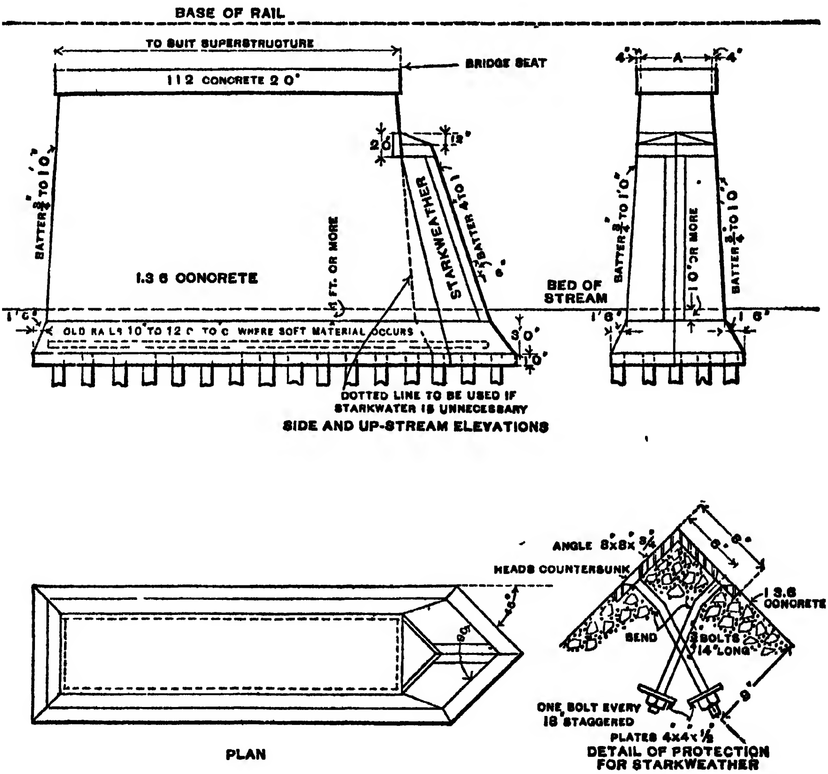


FIG. 212.— Standard Concrete Bridge Pier, N. Y. C. & H. R. R. R., W. J. Wilgus, Chief Engineer. (See p. 655.)

*See Foundations for New Cambridge Bridge, Boston, by Sanford E. Thompson, *Engineering News*, Oct. 17, 1901, p. 283.

For under-water work, a larger factor of safety should be employed than for work above ground, the concrete should be slightly richer in carefully selected cement, and the aggregate so proportioned as to give a dense and impervious mixture.

Concrete for the foundations of walls and piers for high office buildings is usually laid in oblong or circular caissons of steel or wood,* after excavating under air pressure. Steel pipes are sometimes sunk with the aid of the water jet, and afterwards filled with concrete.†

*_Engineering News_, Sept. 26, 1901, p. 222.

†Jules Breuchaud, Transactions American Society of Civil Engineers, Vol. XXXVII, p. 31.

CHAPTER XXVI

DAMS AND RETAINING WALLS

For walls to resist the pressure of earth or water, concrete frequently possesses marked advantages over other classes of masonry. With proper management, in most localities its cost may be brought below that of rubble masonry. Its adaptability for thin walls and for certain classes of face work often make it a suitable substitute in complicated designs for first-class masonry, with a consequent large saving in cost. In combination with steel its possibilities for special designs are almost unlimited, and the future will see marvelous advances in its use for ordinary engineering and hydraulic construction.

Water-tightness, often an essential element for this class of structures, has received general treatment in Chapter XIX, page 338. Portland cement concrete may be made water-tight more readily than stone masonry laid in mortar of similar proportions to the cement and sand in the concrete, since large voids or stone pockets in the concrete are more easily prevented than the "rat-holes" so frequently found in the bedding of stones in mortar. Moreover, skill in laying combined with special treatment of the surface or the addition of certain ingredients permits construction in concrete—strengthened with steel reinforcement—of thinner walls for resisting the flow of water than is possible in stone masonry.

Reinforced concrete retaining walls cannot be designed by "rule of thumb," and therefore a careful consideration of the forces acting and of the stresses in the concrete is presented in this chapter. Since the earth pressure is the controlling factor, it has been necessary to introduce a practical discussion of this before taking up the details of the design and examples of the two principal types

RETAINING WALLS

Retaining walls to support the pressure of earth may be designed:

(1) of gravity section with plain concrete or stone masonry;

(2) of thin reinforced concrete section of the inverted T type with spreading base or footing;

(3) of thin section, reinforced and supported by buttresses or counterforts.

Another plan sometimes adapted to cellar wall construction (see p. 619) consists in embedding the base and supporting the top of the wall with tim-

ber, steel or reinforced concrete beams, so that the concrete forms a vertical slab supported at top and bottom.

Reinforced concrete retaining walls are almost always more economical than a gravity section of either plain concrete or masonry. In walls of gravity section the materials cannot be fully utilized because the section must be made heavy enough to prevent overturning by its own weight, counterforts or buttresses being of comparatively little advantage because, in stone masonry, the wall is liable to break away from them. In reinforced concrete retaining walls, on the other hand, a part of the sustained material is used to prevent overturning, and the section need be made only strong enough to withstand the moments and shears due to the earth pressure. Since the wall is lighter, exerts smaller pressure on the soil, and may be made if necessary with a very broad base, the special foundations or piling which are often necessary for a gravity wall frequently may be avoided. Reinforced concrete properly designed can be depended upon as absolutely reliable.

The economy of a reinforced concrete wall over one of gravity section for either stone masonry or plain concrete is obvious because of the saving in material. The cost of forms is practically the same for gravity section and reinforced designs.

Whether the T-section of reinforced wall or the wall with counterforts is the more economical depends upon certain conditions. The principal condition is the height of the wall, but the intensity of the earth pressure and the relative cost of concrete and steel and forms also enter into the consideration. The construction of the T-section is simpler and the placing of steel easier, so that it is preferable where skilled labor is scarce. The form construction in the counterforted wall is considerably more expensive. Comparative studies of the two types indicate that the counterfort type is scarcely ever economical when the height is less than 18 feet. Rules for designing walls of gravity section are first given and then, after the discussion of earth pressure, the designs of both a T-type and a counterforted section are treated.

FOUNDATIONS

A firm foundation is essential whatever the type of the design. Piles may be necessary, or to avoid sliding, a stepped base may be required. Unequal settling is more dangerous for a retaining wall than for many other structures, because if it is thrown out of plumb, the earth will move and produce forces much in excess of the calculated ones. Allowable pressures on different soils are referred to on page 640.

The depth of foundation must be sufficient to prevent heaving of the material in front of the wall, and to protect it from frost. A depth of 3 feet may be given as a minimum, while 4 or 5 feet is necessary in temperate or very cold climates.

DESIGN OF RETAINING WALLS OF GRAVITY SECTION

The thickness of base of a retaining wall of gravity section, that is, one in which the earth pressure is resisted by the weight of the masonry, is generally taken without mathematical calculation as a certain ratio of the height of the wall. An easily remembered rule is to make the base $\frac{4}{7}$ of the height. The table of empirical values adopted by Mr. Trautwine* for thickness of base of wall to resist earth pressure under average conditions is in accordance with good engineering practice. While he gives no values for concrete, they may safely be assumed equivalent to those for cut stone laid in mortar, which are as given in the following table. The earth is assumed to slope up from the top of the wall till it reaches a level at the height indicated by the ratio in the first column.

Thickness of Retaining Walls of Gravity Section with Earth Surcharge
BY JOHN C. TRAUTWINE. (See p. 661.)

Ratio of Height of Earth to Height of Wall	Thickness of Base as ratio to Height of Wall	Ratio of Height of Earth to Height of Wall	Thickness of Base as ratio to Height of Wall.
1	0.35	2	0.58
1.1	0.42	2.5	0.60
1.2	0.46	3	0.62
1.3	0.49	4	0.63
1.4	0.51	6	0.64
1.5	0.52	9	0.65
1.6	0.54	14	0.66
1.7	0.55	25	
1.8	0.56	or more	0.68

The height of the wall is assumed to be measured above the surface of the ground in front of it.

The batter of the face of a retaining wall is customarily limited to $1\frac{1}{2}$ inches to the foot, and the back is usually vertical. This fixes the width on top.

The values in the table may be employed for long walls of concrete with no reinforcement. In many cases, because of the monolithic character of concrete, a ratio of thickness to height from 10% to 20% less may be adopted with safety, if the character of the filling back of the wall precludes

* Trautwine's "Civil Engineer's Pocket-Book", 1902, p. 606.

excessive pressure, and if the base is slightly spread. For more accurate determinations of gravity sections, the principles which follow relating to reinforced designs are applicable.

Angle of Internal Friction. The selection of the angle of internal friction is of much importance as it affects largely the magnitude of the earth pressure. For ordinary cases the values given on page 665 may be used, but for very important structures, where the additional cost is warranted, special experiments may be advisable.

WEIGHT OF EARTH

In the calculation of retaining walls, and many other structures, the weight of earth in place is a prime factor. The weights of dry material, based upon experiments by the authors, are represented in the following table. Most of the figures for weights of earth give the weights per cubic foot after excavation in a loose or a compacted condition. In the authors' experiments the excavation was measured, so that the weights represent the material in place. As fills will eventually assume much the same characteristics as earth in original excavation, the figures may be employed for either natural earth or filled material. The weight of earth containing water varies with the character of the material and with the conditions. Gravel containing ordinary moisture weighs about 2% more than dry gravel and sand may weigh from 3% to 10% more, depending upon its fineness, since fine sands absorb the most water. Wet muck weighs about 75 lb. per cubic foot. These percentages assume that the bank is provided with natural drainage; if the earth is literally filled with water which cannot run off, its weight will be increased by a quantity of water nearly equal in volume to the voids in the material, which vary with the character of the material from 20% to 50% of the bulk of the earth in the bank.

Many of the values appear high, but they are the result of careful tests.

Average Weight of Ordinary Earth before Excavation.

	Pounds per cu. ft.
Sand	105
Gravel	135
Gravelly clay	130
Loam	90
Hard pan	130
Dry muck	40

BACKING

Since the weight of soil saturated with water is much larger than when it is dry, the pressure increasing with the amount of water so that it may even

exceed the hydrostatic pressure, the backing should be provided with adequate drainage. For this, a filling of gravel or crushed stone may be placed directly against the wall with weep holes at suitable distances apart.

EARTH PRESSURE

The principal force governing the dimensions of any retaining wall is the earth pressure. Its magnitude varies largely with the character and wetness of the soil, the inclination of the back of the wall, and the slope of earth above it.

Of the numerous theories, all of which are based on some assumptions not always met with in practice, Rankine's theory seems to be the most reliable yet developed, and although it does not always represent the true conditions, it gives safe results. It is based upon the assumptions that the earth is composed of granular homogeneous particles without cohesion, held only by friction developed between them, and that the mass of earth extends indefinitely. On a vertical plane the resultant pressure always acts parallel to the slope of the earth and at a point one-third of the height from the base, when the surface of the earth is level with the top of the wall or slopes back from it.

The following table of pressures determined by Rankine's formula gives horizontal earth pressures for different heights of wall, based on an angle of repose of earth of 35°—a fair assumption under average conditions—and also average unit pressures for the same assumptions. For other heights of wall, the horizontal unit pressures with the same angle of repose are directly proportional to the heights, and the total pressures are proportional to the squares of the height.

Total Earth Pressure and Average Unit Pressure upon Vertical Walls of Different Heights (*See p.* 663.)

	Height of Wall in Feet.							
	5	10	15	20	25	30	35	40
Total pressure P, in lb.	350	1400	3150	5600	8750	12600	17150	22400
Average unit pressure in lb. per sq. ft.	70	140	210	280	350	420	490	560

The table assumes (*a*) horizontal surface of earth, (*b*) vertical back of wall, (*c*) weight of earth per cubic foot, 100 pounds, (*d*) angle of repose, 35°. For other weights of earth the values in the table are proportional to the weight per cubic foot.

Passive pressure, that is, the resistance of a mass of earth against moving, is many times as great as the active pressure but because of the shrinkage of filling as ordinarily placed it cannot be counted on for its full value unless the earth is in its natural state

The general formulas evolved by Mr Rankine from the assumptions given above and which apply both to gravity walls and to reinforced walls, are presented below.

Wall with Vertical Back. Let

P = resultant earth pressure in pounds on a vertical surface for a length of wall equal to one foot
H = total height of wall in feet
H_1 = depth below top of wall of any point in feet.
h = height of surcharge in feet
w = weight of earth per cubic foot
δ = angle of inclination of earth behind the wall
φ = angle of internal friction of the earth
C_p = constant depending upon δ and φ (See table on page 665)

Then*

$$P = \frac{1}{2} wH^2 \cos\delta \, \frac{\cos\delta - \sqrt{\cos^2\delta - \cos^2\varphi}}{\cos\delta + \sqrt{\cos^2\delta - \cos^2\varphi}} \tag{1}$$

For known values of the angle of inclination and internal friction, the terms embracing them become constant and

$$P = C_p \, wH^2 \tag{2}$$

The intensity of pressure at any point the depth of which is H is

$$\text{Unit pressure} = 2\, C_p w H_1 \tag{3}$$

and its direction is parallel to the direction of the total pressure *

* For walls with horizontal filling, $\delta = 0$, hence

$$P = \frac{1}{2} wH^2 \frac{1 - \sin\varphi}{1 + \sin\varphi} \tag{4}$$

Unit pressure at any depth, H_1 is $wH_1 \frac{1 - \sin\varphi}{1 + \sin\varphi}$ and acts horizontally.

If angle of slope equals angle of internal friction, i e , if $\delta = \varphi$,

$$P = \frac{1}{2} wH^2 \cos\delta \text{ and Unit pressure is } wH_1 \cos\delta \tag{5}$$

Formulas (2) and (3), however, apply to these cases by using the proper value of C_p given in the table.

The values of the constant C_p are given in the table below.

Data for Determining the Earth Pressure.

Rule: To find the earth pressure on a vertical wall without surcharge, H ft high, multiply the proper value of C_p by the square of H in feet and by the weight of the filling per cu ft $P = C_p wH^2$ (see p 664) For formulas for inclined walls and walls with surcharge, see pp 665 and 666

ANGLE OF INTERNAL FRICTION ϕ	VALUES OF CONSTANT C_p IN RANKINE'S FORMULA (2) p 664							
	Slope with horizontal							
	1 to 1	1 to 1½	1 to 2	1 to 2½	1 to 3	1 to 4	Level	
	Corresponding angle of slope δ							
	45°	33° 40′	26° 30′	21° 50′	18° 30′	14° 0′	0	ϕ
55°	0 09	0 07	0 06	0 06	0 05	0 05	0 05	0 29
50°	0 15	0 09	0 08	0 07	0 07	0 07	0 07	0 32
45°		0 13	0 11	0 10	0 09	0 09	0 09	0 35
40°		0 18	0 14	0 13	0 12	0 12	0 11	0 38
35°		0 29	0 19	0 17	0 16	0 15	0 14	0 41
30°			0 27	0 22	0 20	0 18	0 17	0 43
25°				0 30	0 26	0 23	0 20	0 45
20°					0 36	0 29	0 25	0 47

NOTE If the angle of internal friction of the earth is unknown, the following average values may be used Coal, shingle and broken stone, 50°, earth, 35°, clay, 30°, sand dry 30° sand moist 35°, sand wet, 20°

As stated above, the pressure is assumed to act parallel to the slope of the surface of the earth, and for walls without surcharge acts at one third of the height of the wall from the base The maximum unit pressure is at the base, and is equal to twice the average, while the minimum at the top equals zero, so that the variation of the unit pressures may be represented by a triangle

Wall with Inclined Back. The earth pressure, R, on an inclined plane ab (Fig. 213) is the resultant of P, the horizontal pressure on the vertical plane ac, and W, the weight of the prism of earth abc, and acts at one-third the height from the bottom

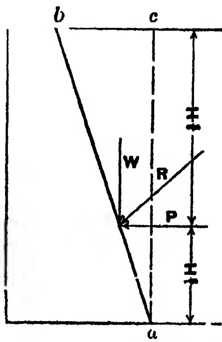


FIG 213.—Earth Pressure on Inclined Back of a Wall (See p. 665)

Surcharge. When the earth behind the wall is loaded in any way, for example, when a highway or a railway track runs along the wall, or when the embankment is used as a storage for material—then this loading causes additional pressure on the wall, which may be provided for by replacing the load by an equivalent surcharge of earth. The height of this surcharge, h, is the extra load per square foot divided by the weight of a cubic foot of earth. Thus a load of 500 pounds per square foot is equivalent to a surcharge of 5 feet if the earth weighs 100 pounds per cubic foot.

Vertical Back of Wall with Surcharge. The earth pressure on a retaining wall with surcharge equals the pressure on the surface *ab* less the pressure on *bd*. Using a constant from the table, page 665,

$$P = wH^2\,C_p - wh^2\,C_p = w\,(H^2 - h^2)\,C_p \qquad (6)$$

and this may be represented by the trapezoid *aced* (see Fig. 214). The distance of the point of application of this force from below the middle point in the height of the wall,

$$x = \frac{(H - h)^2}{6\,(H + h)} \qquad (7)$$

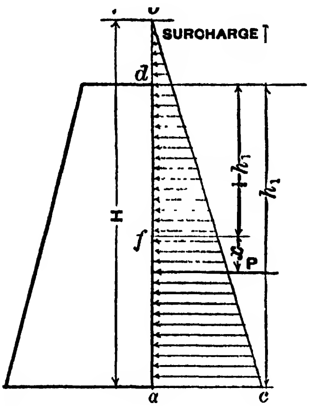


FIG. 214.—Earth Pressure on Vertical Back of Wall with Surcharge. (*See p.* 666.)

Wall with Inclined Back with Surcharge. For an inclined back, the pressure, as in the case of a wall with inclined back without surcharge, is the resultant of P, the pressure on the vertical projection of the wall found by formula (2) and W, the weight of the prism of earth one foot of length, the cross-section of which is a trapezoid. Equation (7) gives the vertical distance of the point of application of the resultant below the middle point in the height of the wall.

DESIGN OF REINFORCED RETAINING WALLS

A properly designed retaining wall, whether of reinforced concrete or of plain masonry, must fulfil the following conditions: It must be stable (1) against overturning, (2) against sliding, (3) against settling, (4) against crushing or overstressing of the material.

To prevent failure by overturning, the moment of downward forces about the outer edge of the base, $M_1 = W_1 l_1 + W_2 l_2$, must be greater than that of the overturning moment, $M_2 = Pl_3$ (see Fig. 215). The ratio of those two moments, $\frac{M_1}{M_2}$, is called the factor of safety. For reinforced concrete walls, the factor of 1.5 to 2 may be considered as ample, because the stability of wall is increased by the resistance of earth to shear along the line ab, Fig. 215, and the passive pressure of the filling in front of the wall, which two items are not considered in figuring the factor of safety.

The horizontal component of the resultant pressure on the foundation causes the tendency of the wall to slide. This force is opposed by the resistance to compression of the earth on the plane dc (see Fig. 215) and by the friction F. The friction is equal to the vertical pressure multiplied by the tangent of friction between concrete and earth, or, if

F = total friction,
$W_1 + W_2$ = weight of concrete and earth,
ϕ = angle of friction between earth and concrete

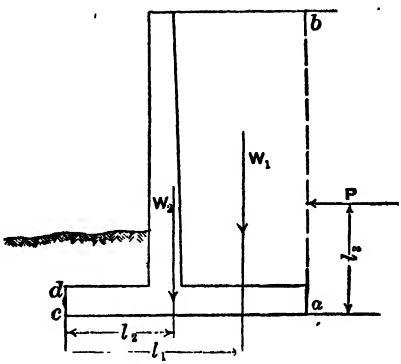


FIG. 215.—Forces Acting upon a Retaining Wall and their Moment Arms. (See p. 667.)

Then

$$F = (W_1 + W_2) \tan \phi$$

If the wall slides, the cohesion of the earth along the line ab (Fig. 215) must be destroyed, which item increases the stability against sliding. The tangent of the inclination of the resultant pressure, that is, the ratio of its horizontal to vertical component, should not be larger than the tangent of the angle of friction.

Sometimes a vertical projection of the base may be needed, which may be placed in the middle of the base or at either end.

Having determined the earth pressure as explained in preceding pages, the design of a reinforced concrete retaining wall resolves itself primarily into the determination of the thickness and reinforcement of concrete slabs to be obtained by the principles outlined in Chapter XXI on Reinforced Concrete Design. The methods to follow can be illustrated best by practical examples, which are given in full below.

A retaining wall is especially subject to temperature stresses. To locate the stresses at specially prepared joints, contraction joints may be placed at stated intervals. In an unreinforced wall, a spacing of 20 to 30 feet between joints is necessary to prevent intermediate cracks. By introducing steel to prevent the formation of visible cracks, no joints are necessary. Steel reinforcement for shrinkage and temperature contraction is treated on page 499.

EXAMPLE OF T-SHAPED RETAINING WALL

Example 1. Design a retaining wall 12 ft. high above ground to support a sand filling. Angle of internal friction of sand, which weighs 100 lb. per cu. ft. is 35°, and the fill slopes back at the same angle. Working stresses for the 1 : 2½ : 5 concrete in compression, $f_c = 500$ lb per sq. in.; steel in tension, $f_s = 16\,000$ lb. per sq. in.; ratio of moduli of elasticity, $n = 15$; allowable shear involving diagonal tension, $v = 32$ lb. per sq. in; bond of steel to concrete, $u = 80$ lb. per sq. in.

Solution. If base is imbedded 4 ft to protect from frost, and if the footing is assumed 18 inches thick, total height of wall is 16 ft and height of stem 14 ft. 6 in. The design is shown in Fig 216, page 669.

Upright Slab. Earth pressure on stem from formula (2), page 664, taking value of C_p from the table, $P_1 = 0.41 \times 100 \times 14.5^2 = 8600$ lb. This acts at ⅓ the height. Horizontal component, $H_1 = P_1 \cos 35° = 7040$ lb., and since the weight of wall and vertical component of earth pressure do not affect the vertical slab, the moment, $M = 7040 \times \frac{1}{3} \times 14.5 \times 12 = 408\,000$ in lb.

Thickness of vertical slab at bottom, using formula (9), page 421, and table of constants, page 519, and adding 1.7 in to the depth to steel to properly imbed it, is $d + 1.7 = 0.29 \times 0.118\sqrt{408000} + 1.7 = 23.5$. Ratio of steel is $p = 0.005$ (to correspond to working stresses), hence area of steel is $A_s = 1.31$ sq. in. per foot of length of wall. This is satisfied by ⅞ in. round bars placed vertically 5.5 in. on centers. (See table, p 507) The thickness of wall at top may be selected as 12 in. The moment decreases from the bottom upwards so the steel may be reduced as shown in Fig. 216, page 669

Since total shear, $V = 7040$ lb., unit shear involving diagonal tension, is

$$v = \frac{7040}{12 \times 21.8 \times 0.894} = 30 \text{ lb. per sq. in.}$$ (See p. 447.) As this does not exceed working stress, no stirrups are needed

Bond stress is $u = \dfrac{7040}{21.8 \times .894 \times 2.18 \times 2.75} = 60$ lb. per sq. in. (see p. 457).

Length of bar to imbed in footing to prevent pulling out is $50 \times \frac{7}{8} = 43.8$ in. (see Table on page 454), hence the vertical bars must extend into the base this distance, or else be provided with bent ends (see page 466).

* A table of dimensions and reinforcement for *T*-shaped and for counterfort retaining walls of different heights, compiled by Sanford E. Thompson, is given in "Concrete in Railroad Construction," published by The Atlas Portland Cement Co.

To obtain this bond, the vertical rods frequently are bent into the right cantilever of the footing. If instead they are bent to run into the left cantilever, they may form the horizontal reinforcement there, as shown in Fig. 216.

Footing. In a correctly designed wall the resultant force should intersect the base within the middle third of its length. This determines the ratio of length of footing to height of wall, and can be obtained only by trial for any particular case. A study of different conditions shows that this ratio is generally 0.4 to 0.6, depending upon the inclination of earth pressure, the weight of the fill, and finally upon the ratio between the length of the projecting toe and the total length of the base. The length of base best suited for our example was found after several trials to be 8 ft. 9 in.

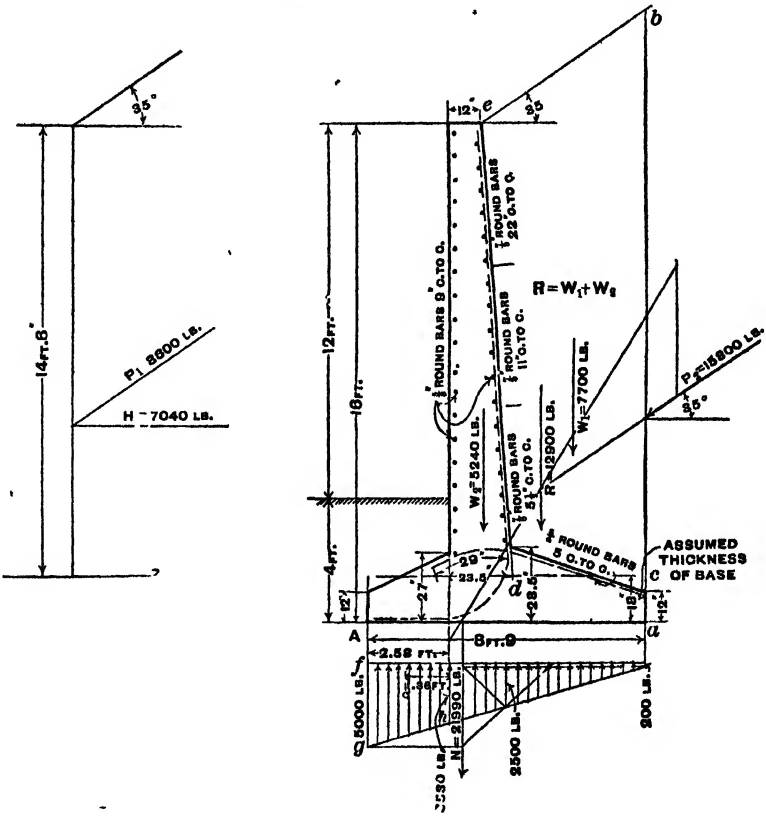


FIG. 216.—Design of T-shaped Retaining Wall. (*See p. 669.*)

The forces acting on the footing are P_2, the earth pressure on the plane *ab*, W_1, the weight of prism of sand, *bcde*, and W_2, the weight of the retaining wall itself. The distance from the toe to the line of action of the resultant R

of W_1 and W_2 may be obtained as follows: Find center of gravity of earth and center of gravity of concrete; multiply the distance from A to these centers of gravity by the respective weight, and thus obtain the statical moment. Divide the sum of these moments by the sum of the weights, $W_1 + W_2$, and the location of the center of gravity of the combined weight is obtained. The line of pressure drawn for P and R intersects the base just inside of the middle third.

Normal component of resultant, $N = 21\,990$ lb. and horizontal component, $H = 12\,900$ lb. Hence, ratio $\frac{H}{N} = 0.587$, which is smaller than the tangent of the angle of friction, hence there is no danger of the wall sliding.

Maximum unit pressure on soil (from formula (36), p. 562) is 5000 lb. per sq. ft., while the minimum equals nearly zero.

The graphical method of finding the distribution of forces on the base is explained on page 586.

Left Cantilever. Omitting weight of slab and of earth above it as negligible, the forces acting on this part of the footing are represented by trapezoid $fghi$. Total force is $\frac{5000 + 3530}{2} \times 2.58 = 11\,000$ lb. and moment arm from the diagram is 1.36 ft.; hence bending moment, $M = 11\,000 \times 1.36 \times 12 = 179\,500$ in. lb. per ft. of width.

The minimum depth to steel from formula (1), p. 418, using Table 10, page 519, is $d = 14.5$ in., and the area of steel, $A_s = 0.868$ sq. in. However, this depth may be too small to satisfy the bond stress, which is below considered.

Further, if vertical steel in the vertical wall is all bent and carried into the left cantilever of the footing, we should have 1.30 sq. in. of steel per foot of width or $\frac{7}{8}$ in. round bars spaced $5\frac{1}{2}$ in. cc., which for a depth of 14.5 in. gives a ratio $p = 0.0075$, or greater than is necessary. If desired, therefore, a part of this steel may be carried only far enough into the footing to prevent its pulling out, or if bond stress were not excessive, the depth, d, might be reduced below $14\frac{1}{2}$ in. The bond for the suggested depth must be considered.

Unit bond, $u = \frac{11000}{14.5 \times 0.9 \times 2.75 \times 2.18} = 140$ lb. per sq. in. (see p. 457).

The bond is excessive unless deformed bars of known worth are used, when the depth, d, of $14\frac{1}{2}$ in., or to properly protect the steel a total depth of 16 in., may be permitted. To decrease the bond stress, for round bars the depth of the cantilever must be increased as follows: Assume the decreased ratio, p, for the increased section of concrete at $p = 0.0045$. Then the corresponding values from Table 12, page 521, $k = .300$, $j = .900$.

From page 457 $u = \frac{V}{jd\Sigma o}$ hence $d = \frac{V}{uj\Sigma o}$. Substituting values,

$$d = \frac{11000}{80 \times 0.9 \times 218 \times 2.75} = 25.5 \text{ in., and total depth } 27 \text{ in.}$$

The depth of beam must be increased to 27 in. in order to decrease the bond stress to 80 lb. per sq. in.

Right Cantilever. It is evident from Fig. 216, page 669, that three forces act on the right cantilever: the upward pressure of the soil, the downward weight of the earth filling, and the vertical component of the earth pressure. The resultant of these forces acts downward, hence the moment is negative.

The computations for amount of steel and the shear and bond stresses are similar to that for the left cantilever.

The length of imbedment necessary to prevent slipping is not treated in the previous case, so it may be given here in detail.

Area of concrete, $A = 12 \times 27 = 324$ sq. in.; area of steel, $A_s = 1.07$ sq. in. and ratio of steel, $p = \frac{1.07}{324} = 0.0033$. From table 10, p. 519 find the corresponding k and j, $k = .268$, $j = .911$. From formula (8), p. 420, since $M = 329\,000$ inch pounds, $f_s = \frac{329\,000}{27 \times .91 \times 1.07} = 12\,500$ pounds. For this stress in steel, the length of imbedment from table on page 454 is $39 \times \frac{3}{4} = 29$ in.

Both cantilevers may be tapered toward the end to a minimum practicable depth, since the moments decrease from the support to zero at the end.

Horizontal Reinforcement for Temperature. Temperature reinforcement is treated on page 499.

EXAMPLE OF RETAINING WALL WITH COUNTERFORTS

Example 2. Design a reinforced concrete wall with counterforts to support a sand filling 20 ft. high above ground, using same assumptions as in Example 1, page 668.

Solution. In this type of wall the vertical slab acts as a slab supported by the counterforts, the principal steel being horizontal. The projecting toe of the footing is a cantilever and the footing below the earth is a slab supported by the counterforts. The counterforts tie the imbedded footing to the vertical slab and act as cantilevers fixed to the footing. Design is shown in Fig. 217, p. 672.

The slabs may be considered as partly continuous, using the moment $M = \frac{wl^2}{10}$. If carefully designed for negative moment $M = \frac{wl^2}{12}$ might be permissible. (See p. 428.)

Instead of forming a projecting toe as a cantilever, it is sometimes more economical when the projection is large to introduce small buttresses and construct this part of the footing also as a partly continuous slab.

The first step in the operation of design is to determine the length of base and the relation between the projecting toe and the base by trial, the allowable pressure on the soil and the minimum angle of inclination of the resultant earth pressure being the determining factors. The method is the same as for a T-type wall, as outlined on page 670.

Spacing of Counterforts. The spacing of counterforts or ribs may be found on the basis of minimum material*, from which 8 feet may be adopted.

Vertical Wall. The vertical wall must be considered in narrow horizontal strips as slabs supported by the counterforts, partly continuous, and loaded uniformly. The earth pressure changes with the height, so that the pressure upon the different strips decreases from the bottom up. The pressure against the bottom strip as given on page 672 is 1480 lb. per sq. ft., or 123 lb. per ft. of width for 1-inch of height. Using $M = \frac{wl^2}{10}$, $M = \frac{123 \times 64 \times 12}{10} = 9500$ inch pounds per inch of width. Hence (p. 418) $d = .118\sqrt{9500} = 11.5$ in.; thickness of wall is thus 13 in., and area of steel, $A_s = 0.005 \times 11.5 \times 12 = 0.69$ sq. in. per ft. of height. Round bars $\frac{5}{8}$ in. diameter spaced $5\frac{1}{2}$ inches on centers may be used.

For convenience in construction the thickness of the wall may be made uniform, and the spacing of rods increased with the decreasing earth pressure, as shown on the drawing. The negative bending moment may be provided for by introducing short rods in front of buttresses, or by bending the rods. (p. 428.)

* For full discussion, see "The Design of Retaining Walls," by H. A. Petterson, Engineering Record, Vol. LVII, 1908, p. 777; for practical purposes the following demonstration illustrates the necessary steps. Use notation page 529, also let x = spacing of buttresses in feet; Q = the maximum horizontal unit pressure on vertical wall, which occurs at the bottom of the wall. Q, from formula (3), page 664, is 1480 lb. per sq. ft. Taking a strip of the vertical slab one ft. in height, whose span is the spacing of the counterforts, the bending moment is then $M = \frac{1480 \times x^2 \times 12}{10} = 1780x^2$; the depth to steel, (p. 421), $d = .29 \times .118\sqrt{1780}\,x = 1.43x$, and the volume per foot of length of wall is $\frac{1.43x}{12} \times 1 \times 22 = 2.6x$ cu. ft. Maximum unit weight acting on horizontal footing slab is 5 325 pounds per sq. ft. Hence $M = \frac{5\,325 \times 12 \times x^2}{10}$, $d = .29 \times .118\sqrt{5325 \times 1.2x^2} = 2.72x$, and volume per foot of length of wall is $\frac{2.72x}{12} \times 1 \times 8.25 = 1.9x$

The thickness below steel is a constant for any spacing and therefore need not be considered in fixing the volume.

Assume the thickness of counterfort as 16 in., and volume will be $\frac{22 \times 8.25 \times 16}{2 \times 12} = 121$ cu. ft., and for one foot of length of wall, $\frac{121}{x}$. Because of the greater cost, per unit of volume, of the counterforts over that of the slab work in a wall of this type, the quantity representing the counterfort volume may be increased by, say, 100%. The expression for this quantity then becomes $\frac{121}{x} \times 2$. Hence total volume, $Q = 2.6x + 1.9x + \frac{121}{x} \times 2$

or $Q = 4.5x + \frac{242}{x}$ and $\frac{dQ}{dx} = 4.5 - \frac{242}{x^2} = 0$ (for minimum, first derivative equals zero).

$$= \sqrt{\frac{242}{4.5}} = 7.3 \text{ ft.}$$ For practical purposes, say 8 ft.

Horizontal Footing Slab. This slab may be considered as composed of narrow strips uniformly loaded and supported by the counterforts. The loading is the difference between the weight of the earth above it plus the vertical component of the earth pressure and the upward pressure of the soil. As indicated in the drawing, this difference is a maximum at *a* and decreases toward *b*. In this case the maximum unit loading is 5566 − 241 = 5325 lb per sq ft. The maximum bending moment in this slab considering it as partly continuous is

$$M = \frac{5325 \times 64 \times 12}{10}$$

= 40 800 in lb. Depth of steel, $d = 0.29 \times 0.118\sqrt{40800} = 21.75$ in, hence thickness may be taken as 23.25 in. The area of concrete is then 261 sq in, hence area of steel required is $A_s = 1.31$ sq in, which is satisfied by $\frac{7}{8}$-in bars spaced $5\frac{1}{2}$ in on centers. The thickness of this foundation slab may be made uniform, and the spacing of the rods increased as the loading decreases.

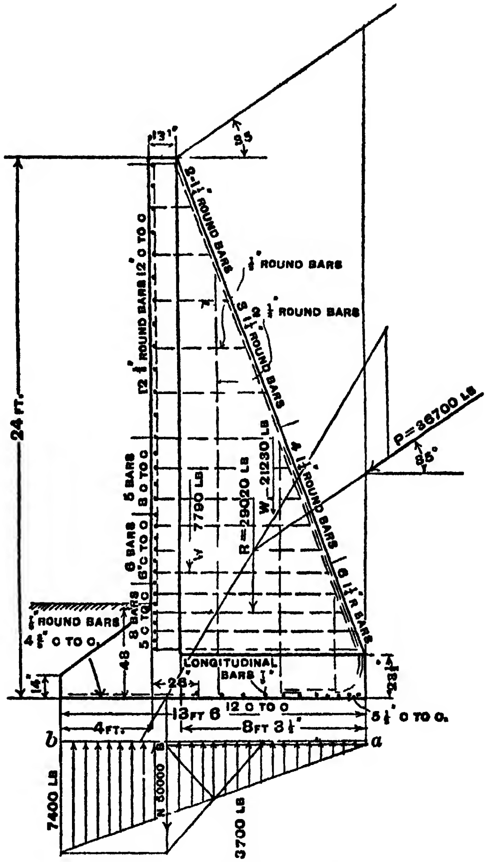


FIG. 217.—Design of Retaining Wall with Counterforts. (See p. 671.)

The negative bending moment must be provided for by introducing at the top of the slab, under the counterforts, short rods of equal size and spacing to the bottom ones or else these bottom rods must be bent down at each counterfort. (See p. 428.)

Counterforts. A counterfort is really an upright cantilever beam supported by the horizontal foundation slab and carrying as its load the vertical slab of the wall, which, in turn, takes the earth pressure. The thickness of the counterfort, which must be sufficient to insure rigidity and resist unequal pressures during construction, may be selected by judgment.

To determine the quantity of steel required in the counterfort, we find the horizontal component of the earth pressure per foot of wall to be (from formula (2), p. 664) $.41 \times 22 \times 22 \times 100 \times .819 = 16\,200$ lb.; hence, the total force transmitted to the counterfort, since they are spaced 8 ft., is $8 \times 16\,200 = 129\,600$ lb. The bending moment, since the force acts at one-third the height, is then $M = 129\,600 \times \frac{22}{3} \times 12 = 11\,400\,000$ in. lb. The thickness of the counterfort is taken at 16 in., the depth to steel, $d = 110$ in. From formula (1), p. 418, $C = \sqrt{\frac{bd^2}{M}} = \sqrt{\frac{16 \times 110^2}{11\,400\,000}} = 130$. By interpolation in the Table 11 on page 520 between items 3 and 4, the ratio of steel, $p = 0.00416$ and area of steel $A_s = 110 \times 16 \times .00416 = 7.36$ sq. in. Six 1¼-in. round bars will satisfy this.

The portion of the counterfort receiving the greatest tension is the inclined edge, so these bars are placed near to this surface. Besides these bars, horizontal and vertical bars are necessary to tie the vertical and horizontal slabs to the counterfort, to transfer the forces and provide for diagonal tension. These bars should be bent into the slabs to obtain as good a bond as possible. The principal tension bars in the counterforts also must be well imbedded in the horizontal foundation slab, and bent so as to attain their full strength in tension. The value of hooking is discussed on page 466.

COPINGS

A coping may be formed on a concrete retaining wall, which will shed water and look nearly as well as cut stone, by sloping the top back from the face and treating surfaces by methods described on pages 288 to 293.*

DAMS

Concrete is a suitable substitute for stone masonry (*a*) in gravity dams, where the masonry is laid in large masses, whenever the cost per cubic yard of concrete rubble is cheaper than stone masonry of equal quality, and (*b*) in curtain or arch dams of thin section reinforced with steel.

Concrete of cement, sand, and crushed stone cannot always compete in price with rubble masonry laid in cement mortar, because, although the labor cost of laying concrete is less, more cement is required per cubic yard; but by introducing large stones into the concrete, the percentage of cement per cubic yard may be reduced to the same quantity or even less than in water-tight rubble masonry. Therefore, the concrete rubble is apt to be the cheaper, since the cost of crushing the stone for the concrete is small

* See illustration of form construction in Engineering News, July 9, 1903, p. 37.

compared with the difference in expense of employing skilled masons or unskilled labor.

Methods of laying rubble concrete and the calculation of the quantity of cement per cubic yard are discussed in Chapter XV, pages 300 and 298.† As is there stated, the concrete must be of soft, mushy consistency so that the large stone may be properly imbedded.

The relative cost of rubble concrete and stone masonry depends upon the price of cement at the work and local conditions. The dam at Boonton, N. J., a section of which is shown in Fig. 219, p. 676, contains 240,000 cubic yards of concrete rubble, and was built at a contract price, not including the cement, of $1.98 per cubic yard. Only 0.6 barrels Portland cement were used per cubic yard, although the proportions of the concrete matrix were 1: 2¾: 6½. This small quantity of cement was due to the large proportion of stones which averaged from one yard to 2½ yards each and occupied 55% of the total volume. The contract price mentioned includes the preparation of the large stones and the crushed stone, and their transportation from a quarry three miles away. It is believed by the authors that the price and also the quantity of cement per cubic yard represent minimum figures in first-class construction, but the force account showed that the contractor was making a fair profit, and inspection of the work and its water-tightness prove that there was no skimping in the use of cement. On this particular job the quotation of the highest bidder was nearly double the accepted price.

With reinforced concrete the engineer is able to branch out into special types whose design may be applicable to local conditions.

Design of Gravity Dams. A foundation must be secured which will resist the pressure upon it and prevent percolation of water under the masonry. The end connections with the adjacent soil or rock must also be carefully considered. The section of the dam must be of such thickness and design as to prevent (1) leakage, (2) overturning, and (3) sliding.

Leakage through a concrete dam of gravity section need only be considered to the extent that no careless work be allowed.

To avoid tension in the foundation it is necessary that the resultant of all the forces of pressure and weight shall pass through the middle third of the base. Dangerous sliding need not usually be feared if the dam is designed to resist overturning. In considering the resistance of friction, Mr. Joseph P. Frizell* states that smooth stone slides on smooth stone

* Frizell's "Water Power", p. 19.

†Tables of Quantities are given on pp. 236, 237.

under a horizontal force of two-thirds its weight, and to slide on gravel or clay, stone requires a force nearly equal to its weight.

The pressure of the water upon any submerged surface is equal to the area of the surface in square feet times the weight of a cubic foot of water times the depth of the center of gravity of the surface below the water level. This pressure tends to overturn the dam, and is resisted by the weight of the dam, and in some cases, where the up-stream face slopes, by the weight of the water upon the dam.

The treatment in Frizell's Water Power of the location of the center of pressure, and the moment produced by it, is especially clear and practical.

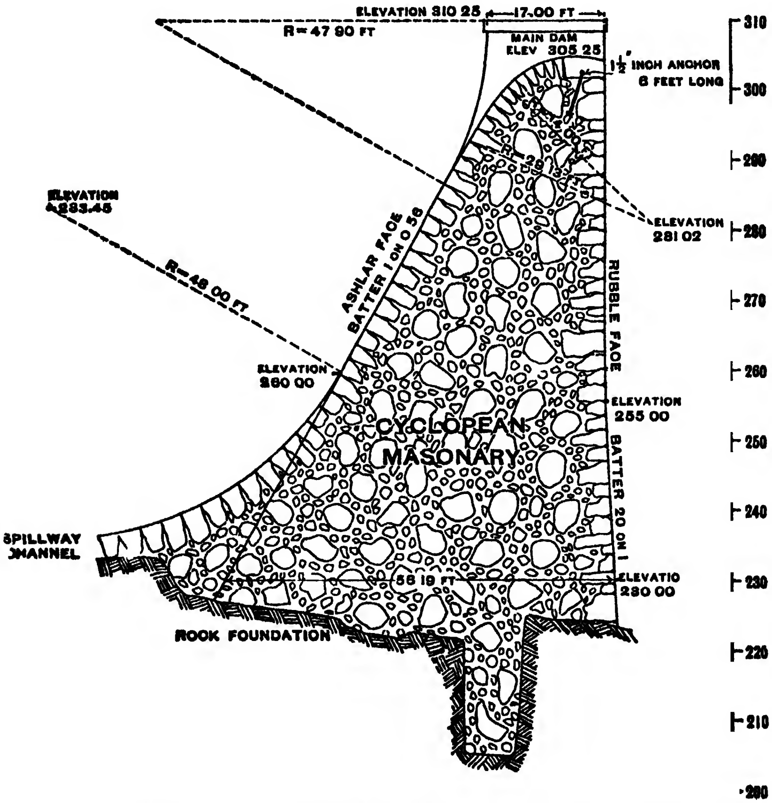


FIG. 219.—Section through Overflow of Boonton, N. J., Dam. (See p. 676)

Fig. 219 represents a section through the overflow of the concrete dam at Boonton, N. J., the construction of which is described on page 300.

The extreme height of the dam at the highest point above the foundations is 110 feet. An interesting practical test of the water-tightness of concrete occurred when the reservoir was filled. A vertical well was left in the dam in order to provide access to two drainage gates, and although the water in the reservoir is 100 feet deep, and is separated from the well by only 5 feet 6 inches of concrete mixed in the proportions 1: 2¾: 6½, the well remains entirely dry.

Reinforced Dams. The aim in reinforced dams is to reduce the quantity and cost of materials, and at the same time to permit a much broader base, and a sloping water-tight deck for the up stream face. The water pressure is thus made to increase instead of oppose stability.

A section of such a dam at Schuylerville, N. Y., 250 feet long and 25 feet high, is shown in Fig. 220 The buttresses are on 10-foot centers, and support a deck tapering from 8 inches to 12 inches thick, while the overfall apron is 8 inches thick. A foot-bridge lighted by electric lights passes through under the crest, giving access from the mill to the railway platform on the other bank.

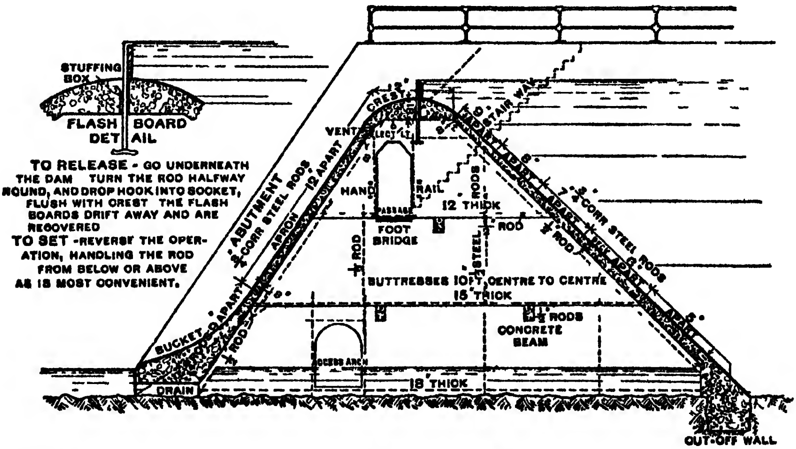


FIG. 220. —Section of Reinforced Concrete Dam at Schuylerville, N. Y. (See p. 677.)

Arched Dams. Curved dams, designed in plan as a single arch, convex up-stream, are considered by foremost authorities to be of doubtful economy, as the extra length requires more material than is saved by the reduced cross-section.

Recently, a type of dams consisting of a series of arches supported by piers or steel lattice work has been suggested, and this idea may receive further development through the introduction of reinforced concrete.

A dam in the form of a buttressed wall with a vertical up-stream surface has been suggested by Mr. George L. Dillman,* the dam in plan consisting of parabolic arches.

The design for a dam at Ogden, Utah,† consists of a number of piers, triangular in vertical section, forming buttresses to support an up-stream sloping face composed of circular concrete arches from 6 to 8 feet thick. The arches are designed to be covered on their upper surface with ¼-inch steel facing. The top of the dam, which is also formed by arches between the piers, carries a roadway.

CORE WALLS

Concrete is largely superseding rubble masonry for core walls in earth dams and dikes. The forms can be roughly made without reference to the appearance of the faces, while a thin wall of concrete may be built water-tight more easily than one of rubble masonry. Unless reinforced, core walls are generally of the same thickness as those of rubble masonry. The Natural cement concrete core wall of the Sudbury Dam, built by the Boston Water Commissioner and his successor upon the work, the Metropolitan Water Board of Massachusetts, is 2 feet thick at the top, with a batter of one in fifteen on both faces, until it reaches a maximum width of 10 feet. At Spot Pond Reservoir, several dikes with core walls of Portland cement concrete, of 15 to 18 feet average height, are 2½ feet in thickness throughout.

The dike for the Jersey City Water Supply Company at Boonton, N. J., is designed for a total height of 54 feet. The lower 30 feet is 4 feet 8 inches thick, and at this height it begins to batter, so as to reach a width of 3 feet at the top.

Although core walls may often be economically built of rubble concrete, the stones must be of smaller size, and cannot occupy so large a volume of the mass as in gravity dams, since the sections are thinner. In the construction of the Boonton Dike, mentioned above, one contractor was placing rubble to the extent of 20% of the total mass, while another was placing 33%. In the former case the stones were loaded on to derrick skips and unloaded by hand; in the latter case, they were hooked by the derrick. This 33% probably represents a maximum for a wall 5 feet thick or less.

Since a thin wall of reinforced concrete may be made equally strong, and more elastic than a thick wall of plain concrete, reinforcement may eventually be employed to reduce the section, and therefore the quantity of material.

*Transactions American Society of Civil Engineers, Vol. XLIX, p. 94.

†Henry Goldmark in Transactions American Society of Civil Engineers, Vol. XXXVIII, p. 290

CHAPTER XXVII

CONDUITS AND TUNNELS

Since the principal stresses in arches are compressive, concrete is peculiarly suitable for all classes of arched structures. Eccentric loading may be provided for by increasing the thickness of the concrete at the points of greatest stress, by steel reinforcement, or by both. The steel may also prevent failure of thin sections of the arch from excessive stresses due to suddenly applied loads or to settlement of the foundation.

Concrete is supplanting cut stone in arch bridges because of its relative cheapness. Although not entirely acceptable from an architectural standpoint because of the difficulty in obtaining a satisfactory surfacing, several methods of treating the face have been used with fair success. (See p. 288.) This objection may also be met by facing the arch with cut stone. Methods of arch design are treated in Chap. XXII.

Concrete arches and conduits are likely to be cheaper than brick even at the same price per cubic yard, because the greater strength of the concrete makes a thinner section possible.

Tunnels (see p. 689) and subways (see p. 692) are now built almost exclusively of concrete, or of combinations of concrete and steel.

CONDUITS

Sewer and water conduits of almost any size or shape may be built of concrete. In the larger sizes, and in conduits under pressure, steel reinforcement occasionally may be advisable from the standpoint of safety and economy.

Concrete was first used in conduits to form in bad ground a foundation for a brick invert. Later it was adopted instead of brick for the entire arch, and finally, in many instances, the brick invert lining has also been replaced by concrete.

While concrete may not be preferable to brick in all localities and under all conditions, its advantages are sufficient to always warrant a very careful investigation of its adaptability to the work in question.

As far back as 1850 sewers and aqueducts of béton or béton-coignet (see p. 1) 8 feet in diameter were constructed in France. The materials consisted of ¼ part heavy Paris cement, one part hydraulic lime, and 5

parts sand.* Some of these structures, notably the viaduct of La Vanne, are said to have cracked and flaked.† Not until the beginning of this century, however, was concrete extensively used for conduit construction, although in the extreme western part of the United States for a number of years it had been employed to a certain extent upon irrigation works for lining both canals and tunnels, a thickness of 4 or 6 inches corresponding to 8 inches or two rings of brickwork.‡

Comparison of Brick and Concrete Conduits. Even with no reinforcement Portland cement concrete is unquestionably stronger, when properly proportioned and laid, than brickwork of equal thickness. Therefore, even if the cost per cubic yard of the two materials, including centering, is practically the same, the concrete is made more economical than brick by the adoption of a thinner ring, or a ring of varying thickness proportioned to suit the actual stresses.

A comparison of data shows that concrete conduits can be built at one-fifth to one-third less cost than brick conduits of equal diameter. Williamsport,§ Pennsylvania, furnishes an example where bids were obtained for brick, plain concrete, and reinforced concrete. The contract bids on the plain concrete section averaged considerably less than the brick, and the bids on reinforced construction the lowest of the three.

Referring to the reconstruction of sewers necessitated by the New York Subway, Mr. William Barclay Parsons, Chief Engineer, makes the following statement in his report to the Board of Rapid Transit Commissioners: ||

> During the year 1901 an experiment was made to construct sewers *in situ* in concrete. The first experiment gave such satisfactory results that the principle has been extended to other sewers in a similar manner during the year, except that instead of building the arch of brick, as was done at first, the whole sewer in many cases has been built of concrete. The advantages of this form of construction are that a perfectly smooth surface is obtained without joints, with all connections, curves, cut-waters and other details molded to perfect lines, and that construction can be carried on more rapidly.

*Leonard F. Beckwith in Transactions American Society of Civil Engineers, Vol. I, p. 108. Mr. Beckwith also gives a table of strength of béton from Michelot.

†O. Chanute in Transactions American Society of Civil Engineers, Vol. X, p. 307.

‡William Barclay Parsons in Transactions American Society of Civil Engineers, Vol. XXXI, p. 314. See also description of the lining of a water works tunnel with concrete in Massachusetts, by Desmond Fitzgerald, Transactions American Society of Civil Engineers, Vol. XXXI, p. 394. See also References, Chapter XXXI.

§*Engineering News* Supplement, Sept. 11, 1902, p. 92.

||Report for 1902, p. 271

It is reported that these concrete sewers have cost one-third less than brick sewers of the same size.*

Concrete, especially if reinforced, has another great advantage over brick, in that it is able to withstand internal water pressure.

Water-Tightness of Conduits. Water-tightness is to a certain extent dependent upon the proportion of cement to sand. If for a concrete conduit the sand and cement are mixed in the same proportions employed for the mortar between the joints in a brick sewer, the structures ought to be equally impervious. For example — a 1: 2½: 5 concrete should be as water-tight as brick laid in 1: 2½ mortar.

If the concrete invert is laid in separate sections, these may be connected by a stepped joint similar to one of the many joints between the different courses in brickwork. A conduit to resist water pressure without leakage must be longitudinally reinforced.

The best proof, however, of the practicability of laying concrete conduits which will prevent the percolation of water, is the fact that sections 4 inches and 6 inches in thickness, which satisfactorily withstand water pressure, have been and are still being built.†

Lime thoroughly hydrated or slaked, or Puzzolan cement, may eventually prove to be the most satisfactory ingredient to mix with Portland cement concrete as a substitute for a portion of the cement, its extreme fineness assisting in filling the minute voids and thus increasing the imperviousness.

The general subject of water-tightness is discussed in Chapter XIX.

Durability of Concrete Inverts. Concrete inverts have proved in practise to be equal, if not superior, in durability to the best hard-burned brick.

The hardness and smoothness of surface obtainable with concrete reduce the friction to a minimum and render it less liable to erosion than are other materials. Concrete sewers built at Duluth, Minnesota, furnish a practical example of the ability of Portland cement mortar to resist erosion. After twenty years of wear, they show no appreciable deterioration or enlargement in diameter, while brick sewers laid at the same time required rebuilding after six or seven years. A section of the Duluth drains, about 2 000 feet long and 4 feet in diameter, was built on a 13 per cent. grade where the velocity of the water was 42 feet per second, with an invert of flat granite flags laid with 1: 1 Portland cement joints. The flow of water during heavy storms was tremendous, carrying down with it quantities of sand and boulders, but after two years of wear the invert

**Engineering News*, March 6, 1902, p. 201.

†See Sewers and Conduits in References, Chapter XXXI.

showed ridges of mortar between the granite flags, indicating that the Portland cement mortar was more durable than the granite.

Experiments by Mr. Eliot C. Clarke indicate that Portland cement mortar in proportions 1: 2 will withstand erosion better than either richer or leaner mortar. (See p. 125.)

Design of Concrete Conduits.* The selection of shapes and sizes of conduits suitable for different flows of water and sewage is treated in literature on hydraulics and sewerage. If the material adopted is concrete, it should be of a minimum thickness consistent with good workmanship, strength, and durability. Steel reinforcement reduces the quantity of concrete required for the larger sizes, but for a diameter of 3 feet or less there is no practical advantage in its use unless the conduit is under pressure, because the minimum thicknesses which can be advantageously placed in a sewer trench are sufficient to withstand all strains. Even in larger conduits the use of steel reinforcement is not usually advisable under ordinary conditions, because of the cost and the difficulty of properly placing the metal.

In preference to an entire concrete section, many engineers advocate an invert of one or sometimes two rings of brick laid in a concrete foundation and surmounted with an arch of either brick or concrete. Others favor a concrete invert paved with a granolithic wearing surface, — thoroughly troweled, — from one-half to one inch thick.

The design of a conduit is dependent upon the depth and character of the material through which it passes, but a few typical illustrations may afford hints for special cases. The proportions of the concrete should be carefully determined by an examination of the aggregate at hand. (See Chapter XI, page 183.) A mixture of one part packed cement, 2 parts sand, 4 parts stone or gravel, is rich enough for important work, while proportions as lean as 1: 4: 8 may sometimes be employed for sub-foundations or backing. In most cases the selection will lie between these two extremes. Natural cement, because cheaper than Portland, is especially adapted for foundations and filling which are not subject to stress or to wear. Puzzolan cement is also suitable in many instances.

The Weston Aqueduct of the Metropolitan Water Works, Massachusetts, built on a gradient of one in 5 000, has in loose earth a typical section shown in Fig. 221. In compact earth the excavation is narrower, and the width of base is reduced as shown by one or the other of the dotted lines, AB or CB. In embankment, the foundation is carried lower and horizontal reinforcing rods are sometimes placed at intervals just below the brick invert lining.

*Earth pressure on conduits is discussed on page 693.

In the Chicago Clearing Yards* drainage is accomplished by concrete sewers. The 36-inch and 42-inch diameter mains are 8 inches thick, the 48-inch diameter are 10 inches thick, and the 84 and 90-inch mains,

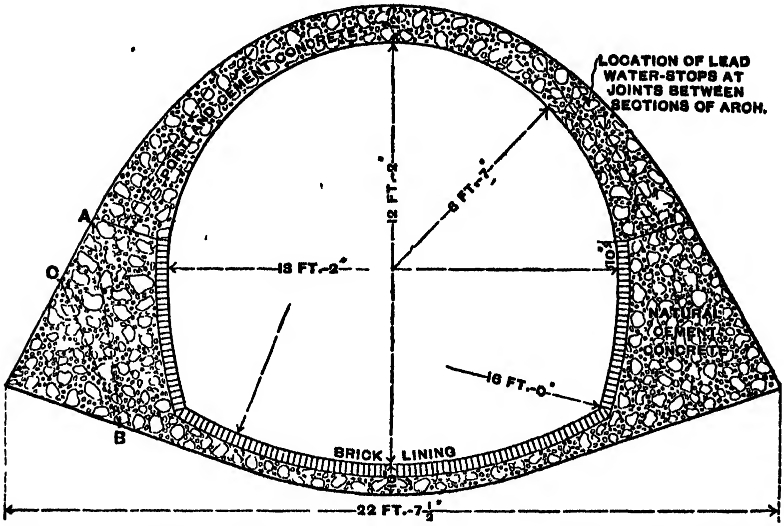


FIG. 221 —Typical Section of Weston Aqueduct in Loose Earth. (*See p.* 682.)

12 inches thick. The ring in each size is of uniform thickness, and the lower portions of the interior surface are covered with a ½-inch coat of plaster.

In large concrete conduits, even when of circular shape, and passing through material which needs no foundation, it is good practice, whether or not reinforcement is employed, to thicken the walls at the spring of the arch. At Williamsport, Pennsylvania, a 11-foot concrete sewer, suggested as a possible substitute for a 4-ringed brick sewer, was designed 13 inches thick at the crown and invert, and 19½ inches thick at the haunches with no reinforcement.

The Jersey City Water Supply Company constructed in 1903 a conduit reinforced with twisted steel. A typical section, taken through a manhole, is shown in Fig. 222, as designed by Mr. William B. Fuller. Longitudinal reinforcement consists of $\frac{3}{16}$-inch rods spaced about 18 inches apart, and circumferential reinforcement is formed by rings of ⅜-inch rods about 12 inches apart. Through rock open cut the metal was placed only in the

*See article by E. J. McCaustland, *Cement*, Sept., 1902, p. 265.

arch, and as far down on each side as the filling would extend. The open-cut conduit is shown in process of construction in Fig. 97, page 278.

At Kalamazoo, Michigan, Mr. George S. Pierson adopted for a creek culvert* a section shown in Fig. 223.

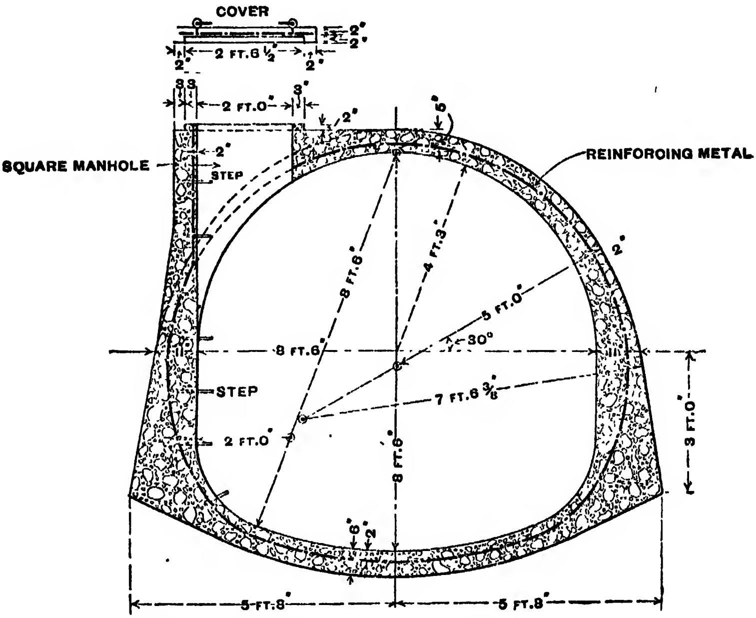


FIG. 222.—Typical Section of Jersey City Water Supply Conduit in Loose Earth. (See p. 683.)

At Grenoble, France,† in 1902, a concrete-steel penstock was built to withstand a pressure of 65 feet head of water. The thickness of wall is from 8 to 10 inches, reinforced with longitudinal bars $\frac{1}{4}$ to $\frac{1}{2}$ inch diameter and circular hoops $\frac{3}{8}$ to $\frac{7}{8}$ inch diameter, forming a mesh about 4 inches square.

Thickness of Conduits. Mr. Fuller's general rule‡ for determining the thickness of concrete in conduits is as follows:

If concrete is not reinforced and ground is good, — able to stand without sheeting, — make crown thickness a minimum of 4 inches, and then one

*Described in *Engineering News*, Feb. 12, 1903, p. 163.
†*Engineering Record*, Mar. 7, 1903, p. 249.
‡Personal correspondence.

inch thicker than diameter of sewer in feet. Make thickness of invert same as crown plus one inch except never less than 5 inches. Make thickness at haunches two and a half times thickness of crown, but never less than 6 inches. This rule is expressed in the following table:

Thickness of Conduits.

Diameter of Conduit.	Thickness of Crown, inches.	Thickness of Haunch, inches.	Thickness of Invert, inches.
2	4	6	5
6	7	18	8
12	13	33	14

If ground is soft or trench is unusually deep, these thicknesses must be increased according to experienced judgment.

If reinforcement is used, the thickness for conduits of ordinary sizes is usually determined by the minimum thickness of concrete which can be laid so as to properly imbed the metal. This minimum for the large diameters where steel is advisable may be taken as 6 inches.

Methods of Conduit Construction. There are four general methods of construction of concrete conduits: (1) The lower portion of the invert is laid by template and the remainder of the circle by centering. (2) The

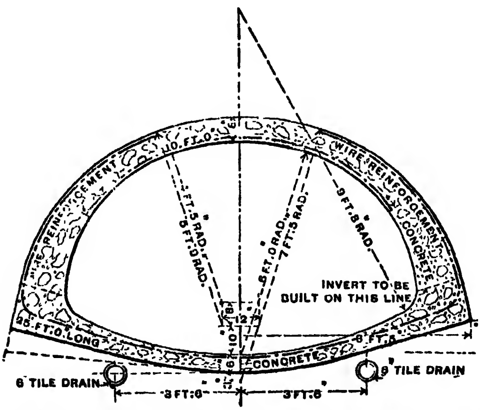


FIG. 223.—Creek Culvert at Kalamazoo, Mich. (*See p.* 684.)

invert is formed by an inverted center, and the arch by an upright center. (3) A center the size of the entire sewer, but with a removable bottom, is placed, the sides and arch are built, and then the bottom of the center is removed, and the invert is laid. (4) The entire sewer is formed as a monolith. The size of the sewer and the character of the work influences the choice of method.

If the invert is to have a brick lining or a granolithic finish, after excavating the material to the required grade and shape, profiles or templets are placed in advance of the finished concrete, and the surface is formed with the aid of a straight edge placed longitudinally from the finished concrete to the nearest template. If the sides run up sharply, as in a small sewer, the concrete may be held in place by strips of lagging, 2-inch by 2 inch for a very small sewer, or wider for a larger size. This lagging rests at one end on the finished concrete, and at the other end on the template, and is placed as the work progresses. In horseshoe sewers the invert may be shaped with templates and straight edge, and the side walls laid back of plank forms.

One of the simplest methods of constructing a small sewer whose invert is to be entirely of concrete, without reinforcement, is that adopted by the New York Transit Commission.* The process is described as follows:

Previous to setting the invert form in place for constructing a length of invert, concrete was placed on the bottom of the trench in a layer thick enough to bring its top surface up to within from ½ inch to ¼ inch of flow-line grade. To insure the accuracy of this work and also to insure the accurate alignment of the form a template was suspended from the trench timbering and adjusted to line and grade. After placing the bottom layer of concrete the form (a center 12 feet in length) was accurately set in position by resting its rear end on the end of the last completed invert and supporting its forward end on a foundation accurately set in grade. The flow line was then accurately formed by filling the space between the bottom of the form and the concrete foundation layer with a mortar of one part Portland cement to one part sand. The form was then firmly braced in position by struts nailed to the trench sheeting, and vertical planking was set up to form the outside of the spandrel. The concrete was then placed and carefully rammed against the form so as to insure a smooth surface. The invert concrete was composed of one part Portland cement, two parts sand and four parts broken stone to pass a 1 inch ring. This mixture was placed (not dropped) into position and carefully rammed. The ends of each successive section of invert were mortised to insure a firm and intimate connection with the next section, and 2 by 4 inch strips, laid longitudinally along the center of the tops of the side walls of the invert section, formed mortises for bonding the arch ring to the invert. The forms were left in place at least 24 hours to allow the concrete to set. After the invert was set and the form withdrawn a thin cement wash was brushed over its surface to smooth any slight roughness. This work gave a surface almost polished in comparison with the best brickwork.

This method of procedure affords no opportunity of troweling the surface, but in a sharply curved invert it is difficult to use a trowel. The plan

*Engineering News, Mar 6, 1902, p 199.

described is not suitable for a large reinforced sewer because so much time is required to set the center and the steel that the layer of concrete in the bottom sets too hard to unite with the mortar finishing coat.

In a large conduit the smoothest and best wearing surface is obtained by laying a comparatively narrow strip of invert by means of profiles or templets and straight edge, and troweling it. If desired, a granolithic (or mortar) finish may be given, but with thorough troweling, excellent results are secured with concrete. The arch center, which in such cases must be nearly a complete cylinder, is placed after the strip of invert concrete has set, mortar is spread on the edges of the invert strip already laid, and the circle is completed with fresh concrete. A longitudinal groove also assists in forming a tight joint.

To avoid this joint, a similar plan has been followed to that just described, except that the form, which is a complete cylinder open at the bottom, is placed, before laying any concrete, upon concrete blocks previously prepared in molds and then laid in the bottom of the trench. The lowest strip of invert is not laid until after the sides and arch are in place, the concrete for it being let down through holes left in the crown for the purpose, and troweled as thoroughly as the obstructions of the forms will permit

It would at first appear that the sewer could more readily be made monolithic by placing a complete cylinder and pouring concrete around it for the invert arch. The objection to this, however, is the great difficulty in placing the concrete in the extreme bottom, and also the tendency of the center to "float" from the upward pressure of the concrete This difficulty is also encountered to a less extent in the method described in the preceding paragraph.

In a sewer whose invert and arch are constructed separately, the arch centers are made and placed as for brick, except that a smoother and tighter surface is necessary, and the forms are oiled to prevent adhesion. A covering of sheet metal has often been successfully used In order to lay the concrete of the arch sufficiently wet to obtain a smooth surface, an outside set of forms, open at the crown, is usually essential.

The laying of a large water conduit for the Jersey City Water Supply Company is illustrated in Fig. 97, page 278.

If a plaster finish is required by the specifications, the mortar may be spread upon the arch center before placing the concrete, or troweled on to the intrados after the completion of the work. In the aqueduct of the Metropolitan Water Works, Massachusetts* (see Fig. 221, p 683), a

*Third Annual Report, Metropolitan Water Board, 1898, p. 56.

Portland cement wash was first used on the Portland concrete arch, but it was afterwards found that thin plastering gave better results. The plastering was put on to increase the water-tightness and to make a smoother surface. As a rule, the authors do not consider it necessary or advisable to plaster the arch.

Conduit Forms. The construction of forms* so that they may be readily "struck" and removed requires considerable ingenuity and design. Invert centers for a small sewer, designed by Mr. William G. Taylor and employed in the Medford, Massachusetts, sewers, are illustrated in Fig. 224.

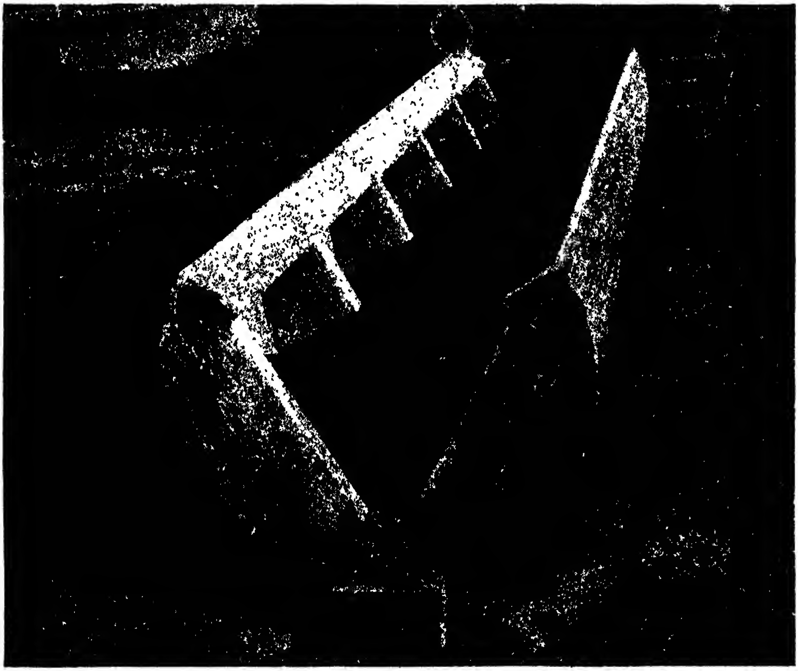


FIG. 224. — Center for Invert of 30-inch Sewer at Medford, Mass. (*See p.* 688.)

Conduits in Tunnel. The methods of construction, except as regards the handling of the concrete, are substantially the same in tunnel as in open-cut. It is generally necessary, however, to provide loose longitudinal lagging for the arch, and place it stick by stick as the concrete is laid. The extreme crown or key for a width, say, of 2 feet, is most easily laid

*Various styles are referred to under "Forms" in References, Chapter XXXI.

upon cross strips or short segments in the same way that a brick arch in tunnel is keyed. The concrete for the key must be mixed fairly dry, and rammed lengthwise of the tunnel.

The tunnel section of the conduit of the Jersey City Water Supply Company is similar in inside dimensions to the open-cut section. (Fig. 222, p. 684.) It is plain concrete with no reinforcement. The thickness of the arch and sides is 8 inches and of the invert 6 inches, but points of rock are allowed to jut into this section "provided a minimum thickness of 6 inches is maintained in the arch, and of 3 inches in the sides and bottom."

TUNNELS

The general principles of design and methods of construction for large railway tunnels are similar to those for sewer and water conduits. The external strains are of course greater and must be provided for according to local conditions. In some cases water-tightness is essential; in others, which compose the large majority, the drift is through dry material, and the ballast may be laid directly upon the bottom.

Tunnel Design. The standard section of a double-track tunnel of the Pittsburgh, Carnegie & Western R. R.* has an arch 26 inches thick and side wall laid on a batter, inside, of one inch to the foot, and of such thickness as to reduce to 26 inches at the springing line.

The standard section of single arch† in the New York Subway for a tunnel 25 feet wide is 18 inches at the crown. In rock drift this thickness is carried down to the springing line, from which point the inside face is battered inward. In deep open-cut construction the arch is thickened at the haunches to about 4 feet, and the outside of the wall is waterproofed.

The East Boston Tunnel, completed in 1904, is shown in section in Fig. 225. The sketch also illustrates the general construction of steel framework and lagging which, after completion, were entirely removed. The invert between A and B was laid after the rest of the section was complete. The method of carrying on the work is described on page 691.

The approaches to the Harlem River Tunnel‡ of the New York Subway were excavated in open-cut, then roofed over, and the tube thus formed pumped out. The section of this tunnel under the river is lined with cast-iron segments.

The single-track tubes of the Pennsylvania R. R. tunnels§ under the

*_Engineering News_, May 21, 1903, p. 447.

†Contract Drawing No. C 9.

‡George S. Rice in Journal Association of Engineering Societies, Dec., 1902, p. 224.

§_Engineering News_, Oct. 8, 1903, p. 327.

channel of the Hudson River at New York City are designed with a cast iron shell made in segments bolted together and lined on the inside with concrete 2 feet thick.

Methods of Tunnel Construction. Concrete side walls and arches in tunnels constructed without the use of compressed air are laid by means of forms and centers, whose design varies with the character of the excavation

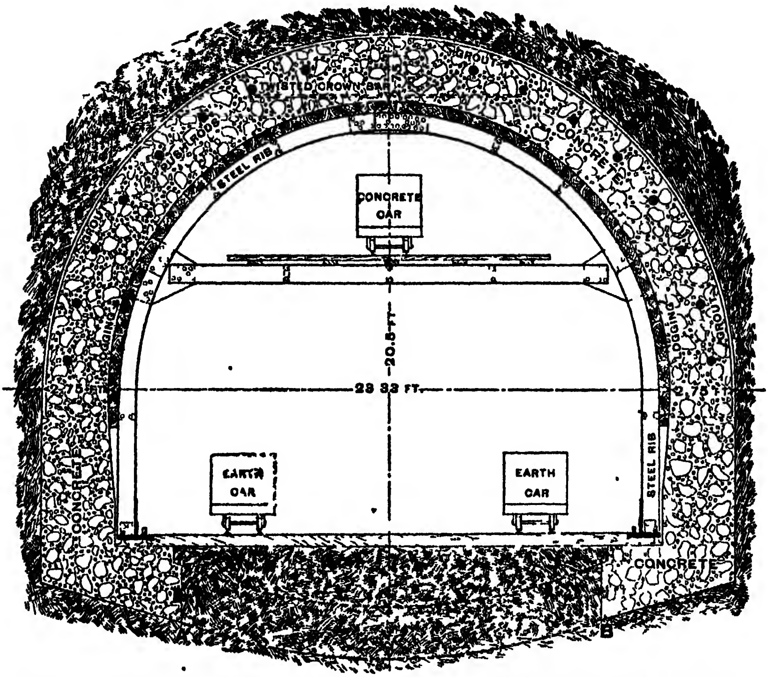


FIG. 225.—Section of East Boston Tunnel during Construction. *(See p.691.)*

and the general arrangement of the structural machinery.* To provide clearance so that the arch center may be lowered and moved ahead, the side walls may be carried up above the springing line. For supporting the center, a temporary frame consisting of a timber resting on posts is set up close to each side wall, and the center is jacked up to line and supported by wedges. By placing the side timbers in advance, the arch may be hauled ahead on rollers by hand tackle or hoisting engine.

*In the serial on The New York Rapid Transit Railway, *Engineering News*, Sept. 18 and Oct. 8, 1902, are excellent descriptions with sketches and illustrations of the methods of construction on one of the sections of the New York Subway and in the Harlem Tunnel. See References for further examples.

The East Boston Tunnel, shown in Fig. 225, is an interesting illustration of a tunnel entirely of concrete built with the aid of compressed air.* Two side drifts, solidly timbered, were kept from 60 to 150 feet in advance of the shield, so that the concrete side walls, which were built in these to a height of about 16 inches below the springing line of the arch, had an opportunity to set for about ten days before the shield reached them. The shield, resting on rollers, moved along on these side walls, and the main excavation was made under it. The concrete arch was built under the tail end of the shield, in lengths of 30 inches, as soon as the earth was removed. The shield was forced ahead by 16 hydraulic jacks, acting against the cast-iron cruciform push rods, 3 inches in diameter, shown in the drawing, which were placed in the concrete in 30-inch lengths, so as to form continuous rods the entire length of the tunnel. The supports for the centering consisted of steel ribs,† also shown in the figure, placed 2½ feet apart, and supporting 4-inch lagging, against which the concrete was laid. Portland cement grout, usually 1 cement to 2 fine sand, was forced in on top of the arch so as to form a film about 1½ inches thick. The invert was laid as the shield progressed. The progress of excavation and lining in May, 1901, was about 6 feet in twenty-four hours, about 60 men being then employed on each of the two shifts.

The specifications for the East Boston Tunnel‡ limited the sizes of the gravel to 2 inches, and stated that 5% only should be less than ¼ inch. The proportions required that "to each 123 pounds of dry Portland cement there shall be 2½ cubic feet of sand and 4 cubic feet of gravel, and such a proportion of water as the engineer shall from time to time determine. The sand and gravel shall not be packed more closely for the above measurements than is done by shoveling in a dry state into a measuring box." Compensation was awarded the contractor when these proportions were varied. Crushed stone screenings were largely used instead of sand.

Closing Leaks. In the East Boston Tunnel a layer of neat cement mortar was spread upon a surface of old concrete before laying a new section, but even this did not prevent slight percolation of water at these joints after the removal of the air pressure. Although the leakage through these was almost inappreciable, they gave the walls a somewhat unsightly appearance, and to stop them holes 6 inches or less in depth were drilled in the concrete, and ⅜-inch pipes inserted, through which neat cement grout was forced by a power pump. The leakage in September, 1904, in 1.4

*Howard A. Carson in Journal Association of Engineering Societies, Dec., 1902, p. 205.

†Ribs were of wood on one of the sections.

‡Construction Contract, Boston Transit Commission, Section B, East Boston Tunnel, 1900.

miles of tunnel,—over one-half mile being directly under the harbor,—was not more than 7 to 8 gallons per minute.

SUBWAYS

Subways are technically distinguished from tunnels as constructions in open-cut instead of drift, although portions of a subway are often really of tunnel construction. The term *subway* is applied to accessible conduits for water mains, electric cables, etc., as well as to underground passages for traffic, but it will be considered here in the latter sense only.

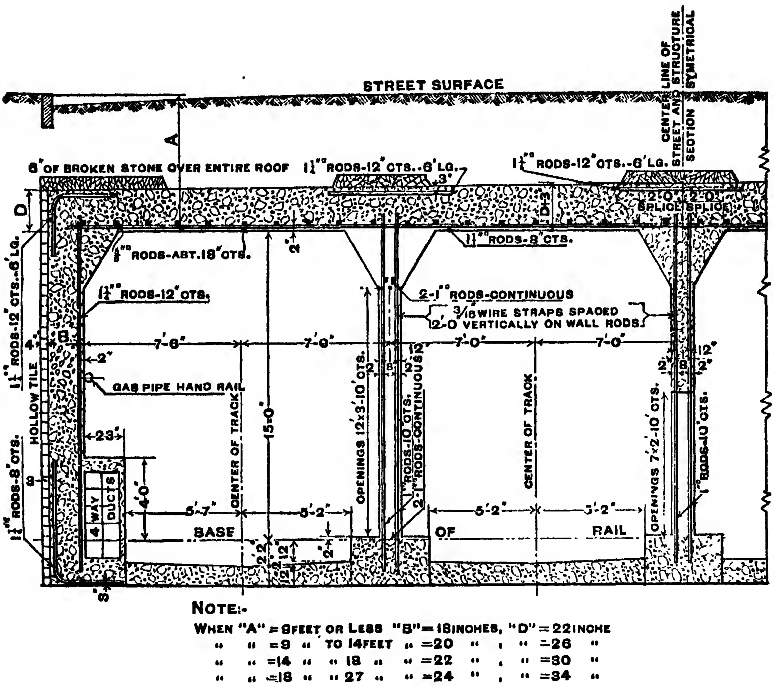


FIG. 226.—Typical Section of Reinforced Concrete Construction in New York Subway. (*See p.* 692.)

Subway Design. To save the headroom required by a circular arch, the roof of the subway is usually made flat. The older portion of the New York subway is built with a framework of steel I-beams, the bents being spaced about 5 feet apart and the roof formed by arches of concrete* sprung

* Concrete has superseded brick for such arches.

between the lower flanges of the cross girders, which are also completely imbedded in concrete. The walls are of concrete, 15 inches thick, forming arches between and imbedding the posts.

The typical design of the Philadelphia subway is reinforced concrete throughout, except that steel columns incased in concrete are used for the supports between the tracks. The walls are longitudinally reinforced to prevent shrinkage or temperature cracks with about $\frac{3}{10}$ of 1% of steel,* and this was found sufficient to prevent all except very small cracks, so that the structure is practically dry even although the backfilling may retain considerable moisture above the level of the underdrains.

The more recent portions of the New York subway also are entirely of reinforced concrete, the typical design† in 1909 being shown in Fig. 226, page 692.

During the course of construction in New York it was decided to widen one of the portions already complete. The contractors moved the concrete side walls and roof, 275 feet long, bodily, without injury‡.

DESIGN OF CONDUITS

The external pressure on structures buried in the ground is very indefinite, depending not only upon the character of the fill, but also upon the method of excavating and filling the trenches and tamping the filling.¶

For small depths up to 3 feet the sum of the weight of the earth and the live load may be taken as acting on the structure. For larger depths, however, the sum of these two forces would be excessive, and may be decreased. According to Mr. Frühling§ the effect of the live load decreases as a parabola until it is zero at 16½ feet, and may be represented by formula (1) using notation below.‖

$$q_1 = w\ (h - 0.06\,h^2 + 0.0012h^3) \quad (1) \quad \text{and } q_2 = Q\,\frac{(16.5 - h)^2}{269} \quad (2)$$

The weight of the earth increases only to a depth of about 16½ feet according to formula (2) and is constant for larger depths.

The sum of the force q_1 and q_2 thus found gives the working load per square foot. Allowance should be made for impact when necessary.

* Personal correspondence with Mr. Charles M. Mills, Principal Assistant Engineer.

† Presented by courtesy of Mr. Henry B. Seaman, Chief Engineer.

‡ See descriptions and illustrations in *Engineering News*, June 11, 1903, p. 515.

¶ For an excellent treatment of this subject with formulas for moments, see "Tests of Cast-Iron and Reinforced Concrete Culvert Pipe," by Arthur N. Talbot, University of Illinois, Bulletin No. 22, 1908.

§ Handbuch für Eisenbetonbau, Band III, p. 510.

‖ Notation. q_1 = pressure per sq. ft. due to dead load; q_2 = pressure per sq. ft. due to live load; w = weight of earth per cu. ft.; Q = unit live load; h = depth in ft.

Conduits with Arch Top Only. The computation of the arch is similar to that for an arch bridge, and is given in Chapter XXII. The loads are carried to the sides of the arch conduit, which act as abutments. Experience indicates that it is not safe to count to a large extent upon the filling at the sides of the conduit to prevent them from cracking.

Longitudinal bars should be introduced to assist in providing for unequal settlement as well as to resist temperature stresses.

Circular Pipes. Under vertical forces the maximum positive moment acts at the top and bottom of the pipe and produces tension on the inside surface, and the maximum negative moment acts on the sides, causing tension on the outside surface* Double reinforcement however is usually introduced.

Rectangular Conduits Square and rectangular conduits† are designed as rigid frames loaded by weight of earth and live load acting on upper horizontal slab, reaction acting on lower horizontal slab, and earth pressure acting on sides of conduits. The stresses may be computed as in ordinary slabs (see page 421) after determining the moment by formulas given below.

Let

M_1 = negative moment at the four corners and at the center of vertical slabs, caused by vertical loads

M_2 = positive moment in the center of the lower or upper slab, caused by vertical loads

I_l, I_h = moment of inertia of horizontal and of vertical slabs, respectively.

l, h = span of horizontal and of vertical slabs, respectively.

w = uniformly distributed load.

Then

$$M_1 = \frac{wl^2}{12} \frac{lI_h}{lI_h + hI_l} \quad (3) \qquad \text{and } M_2 = \frac{wl^2}{8} - M_1 \quad (4)$$

The formulas apply to vertical loads as indicated above

For earth pressure, assuming it as uniformly distributed, these same formulas may be used, but the earth pressure, which acts at right angles to the vertical load, causes positive moment, M_2, in center of vertical slabs and negative moment, M_1 at corners and also at center of horizontal slabs. For the earth pressure moments l and h must be transposed. The moments, M_1 and M_2, due to earth pressure must be computed separately and then may be combined with M_2 and M_1, respectively, due to vertical loads. The moments to be combined are of opposite signs and their sum may not represent the most unfavorable condition, which, of course, must be selected.

* See footnote ¶ page 693.

† A table of dimensions and reinforcement for square and for rectangular conduits under different conditions is given by Sanford E Thompson in "Concrete in Railroad Construction," published by the Atlas Portland Cement Co.

CHAPTER XXVIII

RESERVOIRS AND TANKS

A new field has been developed for concrete design in the building of covered reservoirs and filtration plants for water purification works. Plain or reinforced concrete is now commonly employed for the floors, columns, vaulted roofs, tanks, and filter basins. The Filtration Works at Little Falls, N. J.,* furnish a modern example of such construction. For open reservoirs, concrete is frequently substituted for stone masonry both in the retaining walls and core walls, and also is used for lining the bottom.

Concrete tanks are used not only for water but for chemicals.

OPEN RESERVOIRS

The principles of design and construction of retaining walls have already been discussed in Chapter XXVI. The contraction cracks, which are almost certain to occur in long walls of any class of masonry, may be provided for by some form of expansion joint. Cut off walls of clay† may be placed to prevent the passage of water through these vertical joints, or open wells‡ may be left at intervals in the walls, and after setting for a month or more filled with concrete. This concrete filling is placed preferably upon a cold day, when the contraction in the wall is greatest.

The lining for the bottom depends upon the character of the underlying soil or rock. Usually a layer of 1 : 2½ : 5 concrete 4 to 8 inches thick, if properly laid and troweled, will provide a lining sufficiently impervious for practical purposes §

In small reservoirs, where earth and rock meet so as to present danger of unequal settlement and consequent serious leakage, a strip of reinforcing metal may be placed over the line of division.

COVERED RESERVOIRS

A common type of design for covered reservoirs consists of a concrete bottom, underlaid, where necessary, with 12 to 16 inches of clay puddle

*Transactions American Society of Civil Engineers, Vol L, p 394.

†See paper by Chas W. Paine in Journal Association of Engineering Societies, October, 1902, p. 151.

‡Transactions American Society of Civil Engineers, Vol L, p. 406.

§For other methods of lining see Chapter XIX on water-tightness.

and laid in the form of inverted groined arches. Piers of concrete or brick rest upon the thick haunches of the arches, and the roof is formed of groined arches supported by the piers and covered with a layer of earth. For the prevention of leakage, the principles already discussed in Chapter XIX, on Water-tightness, are applicable. The contraction of the concrete is a common source of cracks, but when comparing concrete with other kinds of masonry, it must be noted that concrete is no more liable to temperature contraction than brick and stone, the brick division walls, for instance, of the Albany Filtration Plant,* showing cracks similar in number and appearance to the cracks in the outside concrete walls.

Reservoir Walls.† Since the walls are supported at the top by a roof, there is less danger of overturning, and thinner sections may be used than for open reservoirs. This class of structure also presents opportunity for thin walls reinforced with steel.

Walls of plain concrete for shallow reservoirs or filter beds are frequently 2 feet to 2 feet 6 inches at the top, with a batter on the outside of 1 in 10.

The wall of a circular reservoir supporting a dome-shaped roof should be reinforced at the top with one or more rings of steel to resist the thrust.

Methods of forming expansion joints for open reservoir walls, described on page 695, are also applicable to covered reservoirs.

Reservoir Piers. The dimensions of the piers are readily calculated after designing the roof and determining its weight, and the weight of the earth covering. In concrete piers of dimensions suitable for reservoirs, a working pressure of 400 pounds per square inch may be safely allowed when the proportions of the concrete are 1 : 2½ : 5.

A floor of inverted groined arches will distribute the pressure of the piers if the soil is unstable. In some cases it may be necessary to place reinforcing steel in the footing (see design of column footings on page 644) to prevent unequal settlement.

In ordinary cases no reinforcing steel is needed in the piers. However, if the load upon them is extra heavy and the reduction of their dimensions is of importance, steel may be introduced to assist in carrying the compression. (See p. 489.) Also, if the columns are of considerable height, say, over 12 feet, a small rod near each corner, with occasional horizontal hoops, may be placed as described on page 624.

Reservoir Floors. The floor should be smooth, fairly impervious, and

*Transactions American Society of Civil Engineers, Vol. XLIII, p. 282.

†Methods of calculating the wall pressure, the amount of reinforcement required, as well as other tables and data relating to covered reservoir construction, are given in a paper on Covered Reservoirs and Their Design, by Freeman C. Coffin in Journal Association Engineering Societies, July, 1899, p. 1.

strong enough to resist the upward water pressure from the underlying soil when the reservoir is emptied. Mr. Coffin* considers a thickness of 3 or 4 inches sufficient when the soil is so compact that there is no danger, when empty, of pressure from without. In pervious earth he suggests 6 inches of concrete for heads as great as 20 feet.

Inverted groined arches for the floor not only distribute the pressure of the piers, but also present increased thickness of concrete around the piers where there is most danger of unequal settlement, give a minimum volume of concrete, and afford channels for the passage of the water when the reservoir is emptied.

The groined arches are laid in alternate diamonds before the piers are built, so that each pier will rest upon the corners of four diamonds. The method of laying the floor arches at the Albany Filtration Works† is illustrated in Fig. 227.

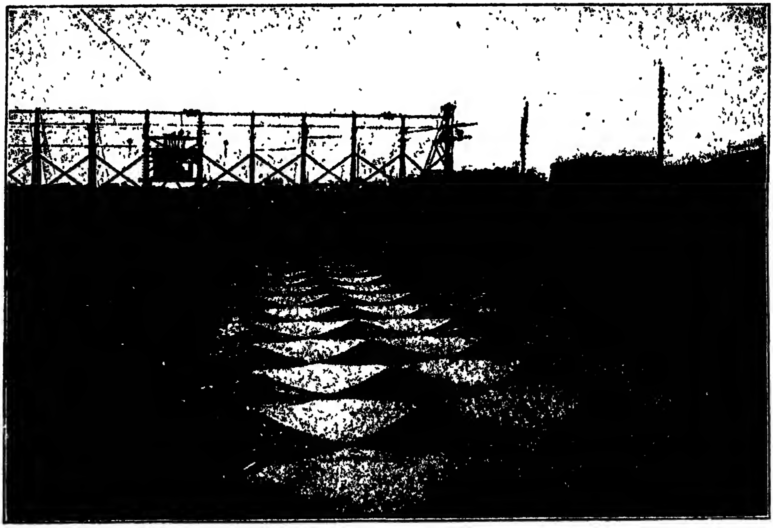


FIG. 227.— Reservoir Floor. *(See p. 697.)*

Before the concrete has set, the surface may be covered with a granolithic or mortar finish, as in sidewalk construction (see p. 600), or it may be simply troweled. Methods of treating joints between blocks and other means of waterproofing are discussed on page 346.

*See second footnote on p. 696.

†Allen Hazen in Transactions American Society of Civil Engineers, Vol XLIII, p. 262

Reservoir Roofs. Groined elliptic arches* are especially suited to reservoir roofs because requiring the minimum volume of concrete to support their own weight and the weight of the earth above them.

Mr. Coffin† says that the cost per cubic yard of groined arches of concrete is about one-half that of brick masonry. Although the centering costs more than brick because a tight surface is necessary, the brickwork is more expensive on account of the great amount of cutting required. He further states that "the cost of the centering, their supports, placing and removing them, is from 15 to 20 cents per square foot for the interior surface of the reservoir if it is all centered at once."‡

Mr. Leonard Metcalf has compiled a table§ of data relating to reservoirs in the United States covered with groined arches, which shows a range in span of arch from 10 feet 6 inches to 16 feet, a rise varying from one foot 6 inches to 4 feet, and a thickness at crown, in all cases but one, of 6 inches. The proportions of the concrete range from 1 : 2½ : 4 to 1 : 3 : 5.

TANKS

Reinforced concrete is cheaper for tanks than sheet steel, and more durable than wood. It is especially adapted for tanks used in paper and pulp mills to hold chemicals. When made of wood or other material which is affected by acid and bleach liquor, such tanks require constant repairs. Concrete not only furnishes a durable material, but one into which outlet castings may be readily built, and to which, if properly flanged so that the concrete cannot shrink away from the metal, the cement will adhere and form a tight joint. The gates and other connections, which are usually of brass or bronze, must be so heavy that the corrosion and wear upon them will not necessitate removal and therefore repairs to the concrete, since it is impossible to form a satisfactory joint between old and new concrete in a thin wall.

There are two distinct methods of concrete and mortar tank construction. In one, forms are built and the concrete is laid with metal reinforcement in the usual manner, and in the other, a framework of metal lathing, the shape of the tank, is constructed, and Portland cement mortar plastered upon it, as described on page 627.

*Methods of centering and placing the concrete of the vaulting are described in detail and illustrated in Mr. Hazen's paper in Transactions.

†See second footnote on p. 696.

‡Mr. Coffin also gives interesting diagrams showing quantities and costs of materials and labor for covered reservoirs.

§See Report of Annual Convention of the New England Water Works Association, 1903, *Engineering News*, September, 1903, p. 238.

Methods of Construction. The materials for the concrete must be very carefully proportioned so as to give a water-tight wall (see p. 339), and the stone should be of such size that a good surface can be readily obtained. The concrete should be mixed so wet that it will completely cover the metal reinforcement and flow against the form, and it is absolutely essential that the entire tank be built in one operation.

Mr William B. Fuller's methods of constructing a thin wall require that the concrete be mixed very wet, so that after wheeling 25 feet it will settle down to a level in a wheelbarrow. The laborer shovels it from the barrow, throwing one shovelful in a place, and goes the entire length of the section or around the circumference, thus forming a very thin layer and preventing the separation of the ingredients

The forms for the Little Falls tank described and illustrated on page 700 consisted of 2½ by ⅞-inch tongued and grooved boards, planed one side and placed vertically. Around the outside of the top of this cylinder of boards was placed a horizontal rib consisting of two sets of boards, 8 in each set, cut to a circle and laid in two thicknesses so as to break joints. Below this rib, a wire rope was wrapped around the forms spirally, so that the separate spirals were about one foot apart The lower ends of the staves were held by the bottom portion already built, otherwise another rib would have been required at the bottom. The inside form consisted of three cylindrical centers built like ordinary sewer centers and placed upright one above the other, each about one foot 3 inches high. These were suspended so that the bottom of the lowest allowed for the 3-inch thickness of the concrete bottom. They were held temporarily in place sideways by pieces of board 3 inches long placed between them and the outside forms. As soon as the centers were fixed in position the concrete for the bottom was poured down through the middle of them and immediately afterward the walls were poured. This concrete flowed out slightly under the bottom center, but was easily removed after setting There were no reinforcing angles between the bottom and the sides. The rods of the bottom extended very nearly to the outside lagging, and the side rods extended down almost to the lower surface of the concrete bottom. Two tanks were built at once, and the contract price of each was $100.

Examples of Tanks. The Filtration Plant at Little Falls, N. J., whose structural features were designed by Mr. Fuller, has a tank or well 41 feet high and 10 feet in diameter, which sustains the pressure of water either from within or from without. The walls are 15 inches thick at the bottom and 10 inches thick at the top. Rings of ½-inch Ransome twisted steel rods were placed about every 2 feet in the center of the wall, and vertical

rods $\frac{3}{4}$ inch in diameter and about 5 feet apart were also set in the center of the wall, thus forming a series of hoops and posts.

On a platform in the same building is a tank 4 feet high and 4 feet in diameter. The walls are 3 inches thick, and contain rings of $\frac{1}{4}$-inch twisted rods placed about 6 inches apart, and $\frac{1}{2}$-inch vertical rods about 2 feet apart. The floor of the tank is also 3 inches thick, with $\frac{1}{8}$-inch rods spaced so as to make a 6-inch square mesh. This tank is shown in section in Fig. 228.

The Illinois Steel Company, South Chicago, employ circular concrete tanks* for storing cement. These are 25 feet in diameter and 50 feet high, with walls 7 inches thick at the bottom and 5 inches thick at the top. The concrete is reinforced by rings spaced 4 inches apart and varying in diameter from one inch at the bottom to $\frac{3}{8}$ inch at the top.

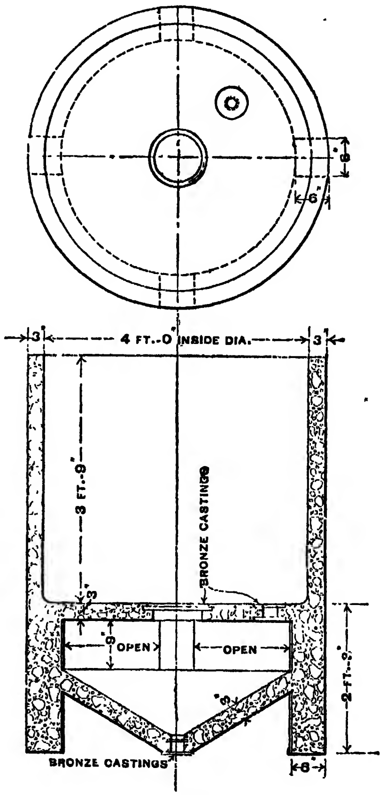


FIG. 228.—Concrete Feed Tank for Mechanical Filter at Little Falls, N. J. (*See p.* 700.)

At Milford, Ohio, is a stand-pipe† of reinforced mortar 80 feet high and $15\frac{1}{2}$ feet outside diameter. The thickness of the shell for the lower 30 feet is 9 inches, for the next 25 feet, 7 inches, and for the remaining 25 feet, 5 inches. The outside face is vertical. The concrete foundation is 20 feet in diameter and 6 feet thick. On top of this, T-bars, 1 by 1 by $\frac{1}{8}$ inch, were placed radially from the center to within 6 inches of the outer edge, and the shell was started directly from these. The horizontal base around and within the shell was then strengthened by a layer of 1:3 mortar 6 inches thick in the interior of the tank and 16 inches thick around the outside of it. The shell is of 1:3 mortar reinforced with T-bars 1 by 1 by $\frac{1}{8}$ inch, spaced 18 inches apart vertically and in horizontal rings varying from 2 inches

Engineering News, August, 1902, p. 148.

†See *Engineering News*, Feb. 1904, p. 184.

apart at the base to 3 inches at the top. T-shaped steel is not so suitable as round for reinforcement because of the lower adhesion. Stone with the sand would have produced a denser and cheaper mix.

STORAGE RESERVOIRS

Storage reservoirs for waterworks and other purposes are being built of reinforced concrete. The design of square or rectangular reservoirs involves problems similar to those met with in the design of retaining walls (see p. 659). In circular reservoirs, the thickness of the walls is usually based upon judgment to insure the proper placing of the concrete for water-tightness, while the horizontal reinforcement is designed to resist all the tension due to water pressure. The amount of horizontal reinforcement at various sections will vary with the water pressure, being zero at the top and increasing toward the bottom, and may be determined thus:

Let

H = height of reservoir in feet above section considered.

D = diameter of reservoir in feet.

A_h = area in square inches of horizontal steel per foot of height at section considered.

f_s = allowable unit stress in steel in pounds per square inch.

At any horizontal section the total tensile force, per foot of height, tending to rupture the reservoir on any diameter is $62.5\ HD$. Since the area of steel resisting this force is $2A_h$, we have $2A_h f_s = 62.5\ HD$, or

$$A_h = \frac{31.3\ HD}{f_s}$$

A comparatively low unit stress in the steel should be adopted, preferably not over 10 000 or 12 000 pounds per square inch, to prevent the formation of cracks in the concrete as it stretches.

Joints require special treatment to prevent leakage. (See page 284.)

In a high circular reservoir, the thickness of wall and vertical reinforcement should be considered as in chimney design (see p. 630).

Waltham Reservoir. The reservoir in Waltham, Mass., is 100 feet in diameter and 37 feet high, and the walls are 18 inches thick at the bottom and 12 inches at the top, the inside surface being vertical. The wall reinforcement consists of $1\frac{1}{8}$ inch round bars, simply lapped at the ends and varying in spacing from the bottom to the top, so as not to stress the steel beyond 12 000 pounds per square inch. The aggregates were especially graded according to the recommendation of one of the authors, and 5 per cent of hydrated lime, based on the weight of the cement, was added to increase the water-tightness.

CHAPTER XXIX

MISCELLANEOUS STRUCTURES.

The more important structures are treated with considerable detail in preceding chapters. The uses of concrete and reinforced concrete are now so numerous and are increasing so rapidly that only brief reference can be made to a few of the smaller and of the less common structures.

In railroad work, not only for the more important structures like piers, abutments and arches, but for the numberless smaller details like telegraph poles, ties, bumping posts, and signal posts, is reinforced concrete being employed. Roundhouses, stations and terminal warehouses are being designed either exclusively or in part of this material.

In power development, not only the dams are of concrete, but the canals, penstocks, flumes, and the power stations themselves

In water-works construction the use of concrete has extended to reservoirs, filter basins, tanks and conduits, and, in some of the recent works, concrete with its imbedded steel for reinforcement is almost the only structural material.

Even the farmer and the householder are utilizing concrete in various ways for barns, garages, chicken houses, floors, fences, silos, tanks, troughs, drains and many other of the small details which make for economy, durability and convenience. By mixing and placing the concrete according to the directions laid down in Chapter II and using sufficient reinforcement (in some cases ordinary fence wire is suitable), many an inexperienced man has built permanent structures of pleasing appearance. For reinforced concrete work such as floors, roofs and stairs, an engineer should be called upon to design the dimensions and reinforcement.

Telegraph Poles. Wooden poles are being replaced in many localities by poles of reinforced concrete because of their greater durability. The Pennsylvania lines west of Pittsburg* have installed poles from 20 to 28 feet high, 8 inches square at the bottom, tapering to 6 inches square at the top, with corners chamfered 2 inches. Holes are left in the pole for the brace and cross-arm bolts and also for the climber steps The reinforcement may be greatest at the bottom and reduced above to allow for the lessening stress.

* Concrete Engineering, July 1908, p. 189.

In 1907 Mr. Robert A. Cummings* made comparative tests of reinforced concrete and white cedar poles. The former were 13 inches square at the butt and 7 inches at the top, reinforced to withstand the weight of 50 wires all coated with ice to a diameter of one inch. These were stronger than the wooden poles of substantially the same size. After breaking, the ends of the concrete poles were held in a slightly inclined position by the reinforcement, while the wooden poles broke square off and fell to the ground

Ties. Concrete ties of varied designs† have proved satisfactory for slow speed traffic, especially in yards and on turnouts. They also have been used to a certain extent on high speed track. One of the most important features is the connection with the rail which is generally made through a cushion block of wood. If the tie supports both rails, it must be reinforced in the center at the top to resist the negative bending moment. The ends of the ties should also be well reinforced to prevent breakage in case of derailment

Road Beds. For tunnels, concrete roadbeds have been found economical because of the very great saving in maintenance expense

Roundhouses. Reinforced concrete affords a durable and inflammable material for the structural portions and the roofs of roundhouses, while the walls may be built either of concrete or of brick.

Cinder and Ash Pits. Concrete will stand as high temperature as will be given to it by hot ashes and cinders.

Grain Elevators. By building of reinforced concrete the danger from fire is avoided as well as the necessity for constant repairs.

Coal Pockets. For coal storage the strength and fireproofness of reinforced concrete is bringing about its general adoption.

Boiler Settings. Reinforced concrete boiler settings have been in successful use in several plants for a number of years The initial cost is probably not less than brick but greater durability and freedom from repairs is claimed by the users of concrete settings.

Double walls are required with an air space between. The inner wall may be about 5 inches thick and the outer about 6 inches, both thoroughly reinforced to prevent as far as possible the development of cracks Bars ⅜-inch diameter, spaced 6 inches apart both ways, afford effective reinforcement The walls may be tied together at intervals with bars The reinforcement permits building the setting to any shape over the boiler, although wherever it comes in contact with the boiler, a 3 inch layer of mineral wool should be introduced to allow for variation in expansion.

* *Cement Age*, Aug 1907, p. 841

† *Concrete Review*, 1908, published by the Association of American Portland Cement Manufacturers.

A fire-brick lining must be used. A thickness of 8 or 9 inches is more economical than a 4½-inch lining because it can be replaced without disturbing the concrete. Spaces must be left at the ends of the fire-brick lining to allow for expansion.

The concrete should be as rich as 1 : 2 :4 and the best aggregates are quartz sand and trap rock about ¾ inch maximum size. For high temperatures gravel and limestone aggregates should be avoided. Cinders of first-class quality should make durable walls when mixed with sand and cement in rich proportions.

Fences. Fences have been built of solid concrete, of mortar plastered on wire lath, of concrete rails set in concrete posts, and of concrete posts with galvanized fence wire between them. The last plan is the most common. For farm or division fences the length of posts may be 7 feet, allowing 3 feet of this to set into the ground, and the size may be 5 or 6 inches square at the bottom and 4 or 5 inches square at the top with ¼-inch rods in each corner. Forms are easily made singly or so as to mold several posts at once.

Silos. Silos of solid monolithic concrete built in circular forms may have walls 6 inches thick reinforced with ½-inch bars bent to circles and placed 12 inches apart. Occasional vertical bars are also necessary. The concrete must be mixed wet and placed very carefully so as to give a perfectly smooth interior surface, so solid and dense that the ensilage will not be dried out next to the wall.

Greenhouses. Greenhouses themselves, as well as the floors, tables, water troughs, hotbeds, and minor appurtenances, are being built of concrete. The directions throughout the various chapters in this treatise for structures of different classes will be found to apply to these details.

House Chimneys. Chimneys for residences may be of concrete if heavily reinforced, but the expense of forms usually will make them more costly than brick.

Chimney caps of concrete should be well reinforced to prevent cracking.

Residences. Residences are built of solid reinforced concrete; concrete blocks (see p. 629); concrete tile, plastered (see p. 629); and mortar plastered on metal lath (see p. 627).

Solid or monolithic concrete is especially adapted to fine residences and permits unique architectural treatment. Eventually with the development and consequent reduction in cost of form construction, reinforced concrete may be more generally employed for dwellings of small and moderate size.

CHAPTER XXX

CEMENT MANUFACTURE

This chapter contains a short historical sketch followed by a brief outline of the processes of modern cement manufacture, illustrated with views of typical machinery.

HISTORICAL

Lime must have been used by the Egyptians thousands of years before Christ, as the stones in the pyramids apparently were laid in mortar of common lime and sand. It is even thought by some that these ancients understood the principle of mixing lime and clay together to make a real cement.

Concrete was made by the Romans as early as several centuries before Christ. For most of their work, they used lime mixed with sand and stone, but understanding the value of puzzolana or volcanic ashes to render lime hydraulic, they employed these two materials in combination with the sand and stone for marine construction. For less important work, they often mixed lime and coarsely powdered brick with the aggregate. Vitruvius, writing in the first century, describes methods of making concrete with lime alone, and also gives as the formula for making it of slaked lime and Italian puzzolana:

12 parts of puzzolana, well pulverized.
6 parts of quartz sand, well washed.
9 parts of rich lime, recently slaked; to which is added
6 parts or fragments of broken stone, porous and angular, when intended for a "pise" or a filling in.

In the Middle Ages concrete was employed, after the Roman fashion, for both walls and foundations. In the former it was generally laid as a core faced with stone masonry. Large stones were often imbedded in the mass.

The fact that clay contained in certain limes rendered them hydraulic was discovered by John Smeaton, when studying the designs for the third Eddystone Lighthouse, about 1750. Early in the following century, Vicat, by his extended scientific researches in France, earned for himself the name of the founder of hydraulic chemistry.

In England, in 1796, James Parker made from nodules of argillaceous limestone, calcined and ground, what he called Roman cement. This process he patented, and from it the Natural cement industry was developed. It was Joseph Aspdin, of Leeds, England, who really invented Portland cement by discovering in 1824 that an artificial mixture of slaked lime and clay, highly calcined, formed a hydraulic product. On account of its resemblance in color and hardness to the Portland stone which was much used in England at that time, he called his invention Portland cement. Two patents had been granted in England a few years before his time, but as in these the materials were not heated to vitrification, hydraulic lime instead of cement was produced.

The Portland cement industry was not developed to any great extent until about twenty years after Aspdin's discovery, when J. B. White & Sons in Kent, England, commenced its manufacture. Later, Mr. John Grant gave a great impetus to Portland cement manufacture by experimental studies upon the practical action of cements, mortars and concretes under varied conditions. The results of his tests he presented to the Institution of Civil Engineers in 1866, 1871, and 1880.

The first manufactory for producing Portland cement in France was established toward the middle of the last century at Boulogne-sur-Mer. In Germany the first factory was erected soon after this, for the production of the Stettin Portland cement, and with such successful results that in 1900 Germany produced more Portland cement than any other country.

The discovery in the United States of a rock suitable for Natural cement was made in 1818 by Canvass White, an engineer connected with the construction of the Erie Canal, and Natural cement was made in Madison and Onondaga Co., N. Y., in that year. The first Natural cement in the Rosendale district was made at Rosendale, Ulster Co., N. Y., about 1823. Mr. D. O. Saylor was the founder of the Portland cement industry in the United States. His discoveries were made in the Lehigh Valley. He experimented from 1871 to 1875 and marketed cement in 1875.

PRODUCTION OF CEMENT

The total production* of hydraulic cement in the United States for 1908 was 52 910 925 barrels, of which 51 072 612 barrels were Portland cement, 1 686 862 barrels were Natural cement, and 151 451 barrels were Puzzolan or Slag cement. The average values per barrel were, for Portland cement $0.85, for Natural, $0.49 and for Puzzolan, $0.63.

The superior quality of Portland over Natural cement and the increasing

* Edwin C. Eckel in The Cement Industry in the United States in 1908.

economy of its manufacture is evinced by a comparison of these figures with those of 1890, when only 335 500 barrels of Portland cement were produced against 7 082 204 barrels of Natural cement. The imports of cement in 1890 were 1 940 186 barrels, and in 1908, 842 121 barrels.

The production of Portland cement in the United States by individual States is represented in the following table.

Production of Portland Cement in the United States in 1900 and 1908 by States

State	1900 Producing Plants	1900 Quantity barrels	1900 Value	1908 Producing Plants	1908 Quantity barrels	1908 Value
Pennsylvania	14	4 984 417	4 984 417	17	18 254 806	13 899 807
Indiana	1	30 000	37 500	7	6 478 165	5 386 563
Kansas	1	80 000	100 000	7	3 854 603	2 874 457
Illinois	3	240 442	300 552	5	3 211 168	2 707 044
New Jersey	2	1 169 212	1 169 212	3	3 208 446	2 416 009
Michigan	6	664 750	830 940	15	2 892 576	2 556 215
Missouri				4	2 929 504	2 571 236
California	1	44 565	89 130	4	2 480 100	3 268 196
Washington				2		
New York	8	465 832	582 290	7	1 988 874	1 813 623
Ohio	6	534 215	667 769	8	1 521 764	1 305 210
Iowa				1		
Kentucky				1	1 205 251	1 176 499
Tennessee				1		
Texas	2	26 000	52 000	2	917 977	924 039
Oklahoma				2		
South Dakota	1	38 000	76 000	1	809 306	1 057 433
Colorado	1	35 708	71 416	2		
Arizona				1	507 603	805 235
Utah	1	70 000	175 000	2		
Maryland				1		
Virginia	1	58 479	73 099	1	502 225	511 118
Massachusetts				1		
Alabama*				2	310 244	274 995
Georgia*				1		
Arkansas†	1	40 000	70 000			
North Dakota	1	400	1 200			
	50	8 482 020	9 280 525	98	51 072 612	43 547 679

* Product in 1900 combined with Virginia. †Product in 1900 combined with Missouri.

About 40% of the total production in 1908 was in the Lehigh Valley of Pennsylvania and New Jersey. In 1900, 73% came from that district.

PORTLAND CEMENT MANUFACTURE

Portland cement is made from a mixture of calcium carbonate and silicate of alumina.

The processes of manufacture differ with the natural state in which

these materials are found, but the operation consists essentially of (1) pulverizing and mixing the two ingredients, (2) heating to a temperature which is near the melting point, i. e., calcining, (3) grinding to a fine powder.

If either of the raw materials occurs in a moist state it is generally customary to mix them wet, and after a preliminary grinding introduce them into the kilns. Dry raw materials for calcining or burning in the old style stationary kilns must be formed into plastic bricks with the aid of water, but the rotary kiln, invented in 1885 by Mr. Frederick Ransome, has revolutionized the manufacture of Portland cement by making it possible to introduce the mixed substances into the furnace, in either a dry or wet state, without hand labor.

After calcination, the methods of grinding the clinker are independent of the character of the raw materials or the type of kiln.

The Association of German Cement Manufacturers, to protect the good name of German Portland cement, requires that its members shall sign the following:*

The members of this Association are permitted to bring into the market under the term of "Portland Cement" only such material as is prepared from an intimate mixture of lime and clay materials as essential ingredients, burning to sintering and subsequent grinding to the finest of flour. They obligate themselves not to recognize as Portland cement any material which is prepared otherwise than above stated, or which during or after the burning has been mixed with foreign bodies, and to look upon the sale of other material under the name of Portland cement as deceiving the purchaser. These requirements are not to forbid the addition of not more than three per cent of other material to the Portland cement for the purpose of regulating the setting time.

The members of the Association further obligate themselves to furnish Portland cement which will in all respects meet the requirements of the Prussian Minister of Public Works.

When a consumer requires cement for a particular purpose, coarser grounp than the requirements, or colored, its preparation is allowable.

If a member of the Association offends the above given obligation, he shall be expelled from the Association His expulsion is made known publicly.

The manufactured product of each member of the Association is tested yearly in the laboratory of the Association at Karlshorst near Berlin; and the results are given out at the General Meeting of the Association.

Raw Materials for Portland Cement Manufacture. The raw materials, as stated above, consist essentially of calcium carbonate and silicate of alumina. Their exact proportions are determined by their chemical composition. A usual ratio is about 75% carbonate to 25% silicate. The two substances occur in nature in so many forms that we have a

* Quoted in *Cement Age*, January 1909, p. 24.

large range of choice in raw materials. The following combinations are actually used in different cement manufacturing plants in the United States:

Cement rock and limestone
Limestone and clay.
Limestone and shale.
Marl and clay.
Chalk and clay.
Limestone and slag.
Alkali waste and clay.

Cement rock is an argillaceous limestone, rather soft in texture, which in the Lehigh Valley usually requires from 10% to 20% of limestone to give it the correct Portland cement composition. Occasional deposits are found which are suitable to use with no admixtures, or from which the desired proportions may be obtained by mixing two different strata in the same quarry. Several other States, among them the Virginias, Alabama, Colorado, and Utah, have a geological formation similar to that in the Lehigh Valley from which Portland cement is made.

In the Hudson River Valley, near Catskill, New York, are situated large manufactories employing a hard limestone which is nearly pure carbonate of lime, requiring 20% to 25% clay or shale and producing a fine quality of cement. A somewhat similar mixture is used in California and in scattered localities in the Central States.

The marl used for cement is a wet, calcareous earth, in some localities of organic origin from shell deposits, and in other places of chemical formation. There are large cement plants using marl and clay in western New York, Ohio, Indiana, and Michigan.

Chalk and clay deposits resembling those in England are worked in South Dakota, Texas, and Arkansas.

Certain blast furnace slags similar to those used in the manufacture of Puzzolan cement, when combined with a suitable admixture of limestone, produce, after calcination, a true Portland cement.

The waste from the manufacture of soda, when employing the ammonia soda process with suitable raw materials, is substantially a precipitated chalk, and is burned with clay to produce Portland cement.*

In Germany the Alsen and Stettin brands are made from chalk and clay, the Dyckerhoff and Mannheimer brands from limestone and clay, while the Germania and Hanover works use marl and clay. In Fngland

*B. B. Lathbury, *Engineering News*, June 7, 1900, p. 372.

raw materials consist principally of chalk and clay. Belgium manufacturers use chalk and clay, and a Portland cement from natural rock is also manufactured in that country. In France, marl and clay, and chalk and clay, are the chief raw materials for true Portland cements.

The character and proportioning of the raw materials and the processes of chemical combination are discussed by Mr. Spencer B. Newberry in Chapter VI.

The following table illustrates the composition of various classes of materials which are used for Portland cement, and also the resulting analysis of the cement in each case:

Comparative Analyses of Raw Materials and Portland Cements.

		Cement Rock and Limestone.			Limestone and Clay.[4]			Marl and Clay.			Chalk and Clay.*		
		Cement Rock.[1]	Limestone.[2]	Cement.[3]	Limestone.	Clay.	Cement.	Marl.[5]	Clay[6]	Cement.[7]	Chalk.[8]	Clay.[9]	Cement.[10]
Silica	SiO_2	19.06	1.98	19.92	3.30	55.27	21.50	1.75	62.10	22.52	0.35	60.30	22.10
Alumina	Al_2O_3	4.44	0.70	9.83	1.30	28.15	10.50	1.57	20.09	6.69	0.75	11.07	11.32
Iron Oxide	Fe_2O_3	1.14		2.63					7.81	3.54		8.13	
Calcium Oxide	CaO	38.78	53.31	60.32	52.15	5.84	63.50	49.24	0.65	63.82	54.95	4.40	60.76
Magnesian Oxide	MgO	2.01	0.97	3.12	1.58	22.5	1.80	0.44	0.96	0.60		1.27	1.10
Sulphuric Acid	SO_3			1.13	0.30	0.12	1.50	0.15	0.49	0.98		2.50	1.40
Carbonic Oxide	CO_2	32.66	42.94		40.98			39.16			43.17	7.47	1.94
Water	H_2O				8.37				8.00				
Organic Matter								7.50				4.06	
Other Constituents							0.40			1.08	0.85	0.45	1.38

NOTE.—Carbonates in raw materials, given in some of the analyses, have been transformed into oxide.
[1] Cement Rock. Lehigh Valley District, Penn. 21st Annual Report, U. S. Geological Survey. Pt. 6, p. 404.
[2] Pure Limestone, Lehigh Valley District. W. E. Snyder, Analyst.
[3] Lehigh Valley Cement. Booth, Garrett & Blair, Analysts.
[4] Hudson River Valley. Mineral Industry, Vol. 6, p. 97.
[5] W. H. Simmons, Analyst, 22d Annual Report, U. S. Geological Survey, Pt. 3, p. 650.
[6] Shale. Mineral Industry, Vol. 6, p. 99.
[7] Michigan. W. H. Simmons, Analyst, 22d Annual Report, U. S. Geological Survey, Pt. 3, p. 680.
[8] Water, 23%. Analysis from David B. Butler, England.
[9] Estuary Mud. Roughly dried, lost 33⅓%. Analysis from David B. Butler, England.
[10] English Portland Cement. Analysis from David B. Butler, England.

Processes in Portland Cement Manufacture. The method of mixing the materials in preparation for their introduction into the kilns has led to

*The authors are indebted for these analyses of chalk and clay to David B. Butler, of England, who prepared them for this Treatise.

a classification of processes into (1) wet process, and (2) dry process. The former is often subdivided into wet and semi-wet, depending upon the quantity of water added at the time of the mixing.

The *wet process* is employed with soft or wet materials, such as chalk and clay, or marl and clay. The carbonate of lime and the clay are mixed in a vat or wash-mill with a large excess of water. Agitators break up the lumps and so finely reduce the particles that they are held in suspension in the water and flow off over the top of the vat. In another basin the stuff is allowed to settle, the water is drawn off, and the "slurry" becomes hard enough to handle in barrows and then form into bricks to be dried, and finally calcined in stationary kilns.

By using a smaller quantity of water, say 40 or 45%, the settling process and consequent hand-labor is avoided, and the material is made only fluid enough to handle in pumps. After grinding, it may be pumped directly into the rotaries, or, if stationary kilns are used, the pumps throw it to the drying room to be made into bricks. This process is called in England the semi-wet process, but as it is practically the only wet process used in the United States, it is here simply termed the wet process.

The *dry process* was first used in Germany as a result of the substitution of limestone for the chalk of England. The two ingredients are ground and mixed in a dry state. If the kilns are stationary, the mixed material must be moistened with sufficient water to form plastic bricks, which are then dried, but for rotary kilns no water is added, the mixture of dry materials passing, after being ground, directly into the kiln.

Dry Process with Rotary Kilns. The introduction of rotary kilns into new cement plants is universal, while many of the older mills are substituting them for their stationary kilns. Where rock, or rock and clay, form the raw materials, they are mixed and ground, and introduced into the rotary in the form of a dry powder. If marl or chalk furnish the carbonate of lime, the wet process of mixing and grinding is usually employed, as described on page 720, although in a few plants each of these materials is dried when entering the mill, and the operations are similar to those described below for rock mixtures, except that driers and disintegrators are substituted for stone crushers.

The process of manufacturing Portland cement from rock, or rock and clay mixtures, in plants equipped with rotary kilns, consists essentially of crushing the materials, — either separately or after mixing them, — drying, grinding, calcining in the rotaries, cooling, grinding to powder, and packing.

The details of the process will be best understood by briefly describing

the typical machinery shown in the illustrations. Various types and makes of grinding machinery will produce similar results, those selected being merely representative.

If two stones of fairly similar texture and each of uniform composition form the raw materials, they may be carefully weighed and thrown together into the breaker. Otherwise, they are treated separately, and mixed just before the grinding which precedes the calcination. A common type of breaker is the gyratory crusher shown in Fig. 78 on page 244, No. 5 or No. 6 being the usual size employed. This reduces the stone to a size varying from dust to about 2½ inch diameter. A further reduction in size to about ½-inch is accomplished in plants of modern design by crackers of the coffee mill type (see Fig. 229), or similar machinery.

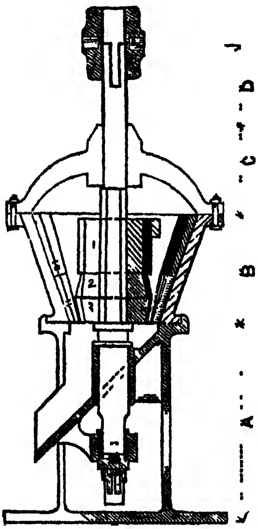


FIG. 229.—Coffee Mill Cracker. (*See p.* 712.)

Clay, if used, is dried in broken lumps, and then may be pulverized by passing it through a disintegrator consisting of two horizontal rolls, one corrugated or toothed and the other smooth.

An economical form of dryer for clay or stone consists of a long revolving steel tube about 4 feet in diameter, provided with shelves on its interior surface, formed by horizontal Z-bars. The hot gases from the kiln may be made to pass through the tube and meet the raw material.

By treating the two materials separately up to this point, an extremely accurate mixture is obtained by weighing the ingredients in a pair of automatic weighing machines (see Fig. 230), so arranged that one of the pair will not dump until both are charged.

Samples of the two materials are taken, just before mixing, at definite periods throughout the day, and analyzed to determine the correct proportions. A partial analysis showing the quantities of the principal constituents may be all that is necessary except at occasional intervals. The maintaining of correct proportions is one of the most essential elements in the manufacture.

Another grinding of the mixed materials in tube mills, Kent Mills, Griffin Mills, Fuller Mills (pp. 716, 717), or similar machines, to a fineness which will pass a screen having 20 to 30 meshes per linear inch, completes the preparation for the rotary kilns. The actual fineness of the

ground stone at this point is such that 90% to 95% or even a higher percentage will pass a screen having 100 meshes to the linear inch. Fine grinding before burning is one of the secrets of successful manufacture.

The best type of rotary kiln (see Fig. 231) used for calcining dry materials, consists of an inclined steel tube from 60 to 200 feet long. The diameter is generally 6 to 12 feet, though occasionally smaller than this t the upper end and tapering to the larger size at a point about one-

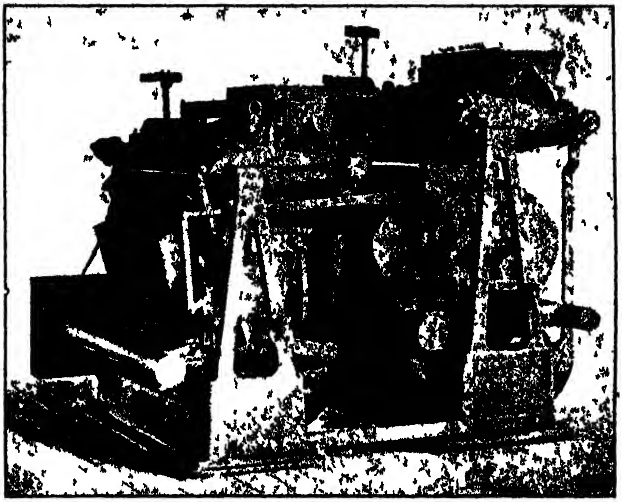


FIG. 230 —Tandem Automatic Weighing Machine (See p 712.)

third of its length from the upper end The lining may be of U-shaped fire-brick in order to present, as a non-conductor of heat, a hollow surface against the shell of the rotary. The lower end of the rotary is closed by a stationary brick wall, and through the center of this passes a pipe which feeds the petroleum, or more frequently the powdered coal which in a separate building is crushed to pea size and pulverized in tube mills, or other pulverizing machines, so that about 90% passes a 100-mesh screen, the finer the coal the greater its efficiency.

The ground stone may be fed into the upper end of the rotary by a spiral conveyor enclosed in a pipe which is water jacketed so that the material will not cake. The degree of calcination is governed by the supply of raw material, the speed of rotation of the rotary, which rests on rollers geared to a speed-changing device, and the quantity of fuel. If coal is used for fuel, it is fed by a blast from a fan, and the quantity is regulated by a spiral

FIG. 231.—Rotary Kiln. 'S

conveyor running at changeable speed. The heat in the kiln is so intense that the coal burns as a gas without apparent smoke or cinder. The proper temperature, which is said to be 2700° to 3000° Fahr., is determined by the appearance of the burning stone. At a certain point in its descent the material becomes semi-vitrified and forms into irregular balls or clinkers, which roll around and finally fall out red-hot at the lower end in particles, most of which range in size from sand to 1-inch diameter. The clinker, when properly burned, is of a greenish black color with a faint glisten, and contains but few large pieces. It slightly resembles in appearance the clinker often found among the ashes of hard coal.

The output of a rotary varies with the length and diameter from 150 to 200 barrels per 24 hours for a 60 foot kiln to 1000 to 1200 barrels, for a 158 to 200 foot kiln with a smaller coal consumption per bbl.

The clinker, after being cooled in some form of cooler, is crushed by passing between horizontal rolls

or through some other form of crusher, and is then ready for the fine grinding, or, if desired, it may be stored either out of doors or under cover until needed. Strangely enough, wetting the cinder does not injure it provided it is dry when it enters the fine grinders.

The fine grinding is generally accomplished by passing the clinker through ball mills and then through tube mills, or by a single operation in such machines as the Griffin, Kent or the Fuller Mill. A section of a ball

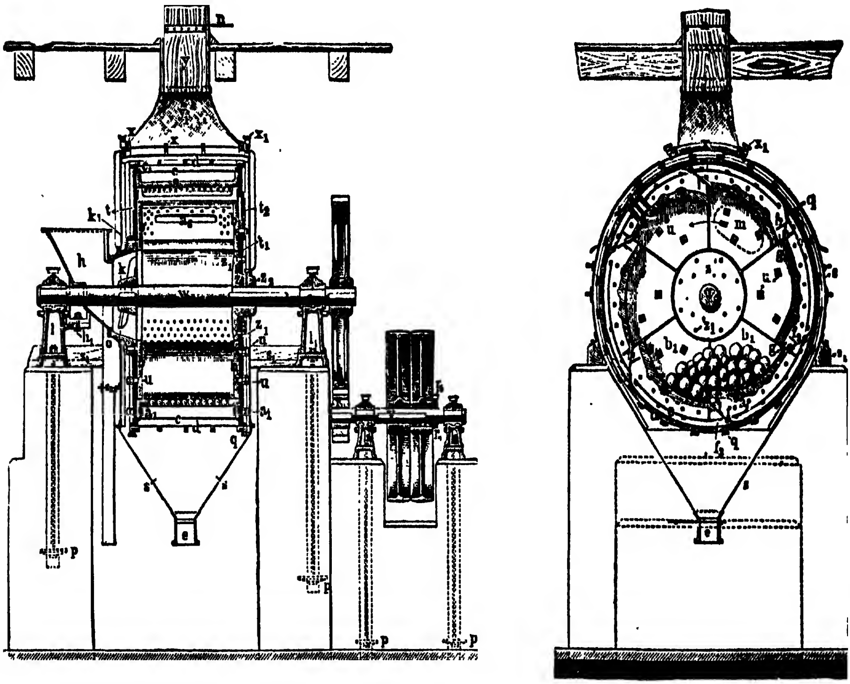


FIG. 232.—Ball Mill. (*See p.* 715.)

mill is shown in Fig. 232. It consists essentially of a cylindrical drum, lined with castings of hard, tough steel, and containing forged steel balls 8 or 10 inches in diameter. Rotation of the drum grinds the stone or clinker between the balls and the plates, and the powder passes through sections of screens — which for clinker have usually 20 to 28 meshes to the linear inch — into the hopper below. A single ball mill, such as is shown in sketch, running on clinker, should give an output of, say, 5 500 to 7 500 pounds per hour.

A tube mill (see Fig. 233) consists of a long horizontal cylinder filled

nearly to its axle with flint pebbles imported from Europe, which average about 2 to 3 inches in diameter The cement is ground by rolling around with the flints. It is then thrown by centrifugal force against the screen, which regulates the fineness of grinding and prevents the passing of pieces of flint. A tube mill which passes, say, 250 barrels of cement per day,

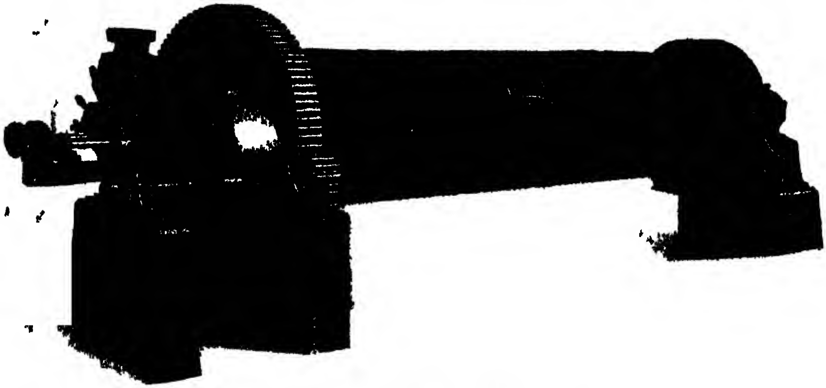


FIG. 233 Tube Mill (*See p* 715)

will require the renewal of the flint pebbles at the rate of about 600 lb. per week More tube mills than ball mills, usually twice as many, are required for the finish grinding

The Griffin mill (see Fig 234) is used by many manufacturers in preference to ball and tube mills The mill is driven by a horizontal pulley, from the center of which, by a universal joint, is suspended a vertical shaft having fixed at its lower extremity a crushing roll, which revolves on its axis at a speed of about 200 revolutions per minute, and also rotates by centrifugal force against the ring or die where the pulverizing is accomplished The material to be ground passes first into the pan below the crushing roll, upon the under side of which are shoes or plows which stir it up and force it up between the roll and the die.

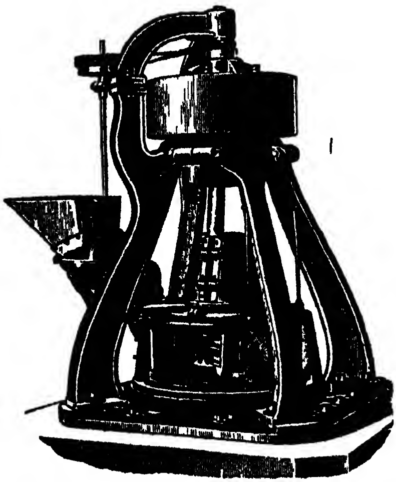


FIG. 234.—Griffin Mill. (*See p.* 716)

The cement or stone is so finely powdered that, held in suspension by the moving air, it passes through a cylindrical screen above the roll, and falls through slots in the circumference of the pan into the hopper below, to be carried off by a conveyor. The screen in mills for grinding clinker is 30 to 32 mesh to the linear inch but as it is placed vertically, it lets through only cement of such fineness that 75 to 80% of it will pass a 200-mesh sieve.

The Kent pulverizer, shown in Fig 235, which is used in a few plants,

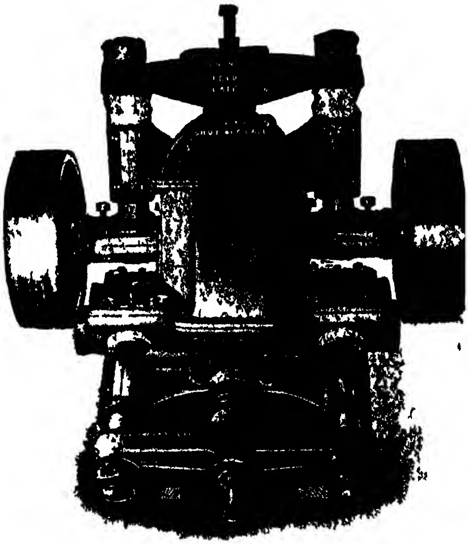


Fig 235 Kent Mill (See p 717)

consists essentially of an upright circular case containing within it three rolls surrounded by a revolving ring The material is ground by passing between the internal circumference of this ring and the rolls, which are pressed against it by springs

The Fuller-Lehigh mill, illustrated in Fig 236, has come to the front during the past few years as a fine grinder for grinding coal, raw material and clinker. The material to be reduced is fed to the mill from an overhead bin by means of a feeder mounted on top of the mill This feeder is driven direct from the mill shaft by a belt running on a pair of three-step cones, which permits the operator to accommodate the amount of material entering the mill to the nature of the material being pulverized

The grinding is done by means of four unattached steel balls 12 inches in diameter, which are propelled by four equidistant horizontal arms or

pushers radiating from a vertical central shaft. The material discharged by the feeder falls between the balls and the die and is reduced to a finished product in one operation. Above the die and the balls and attached to the yoke propelling the balls, is a fan with two rows of fan blades, one above the other. The lower set of blades lifts the finished product from the pulverizing zone into the chamber above the die, where it is held in suspension until it is floated out through a screen by the fanning action of the upper row of blades. The finished product is then discharged through a spout which may be placed at any one of four quarters of the mill. When the mill is in operation, it is continually handling only a limited amount of material at any one time. As soon as the material is reduced to the desired fineness, it is lifted out of the pulverizing zone and discharged from the machine.

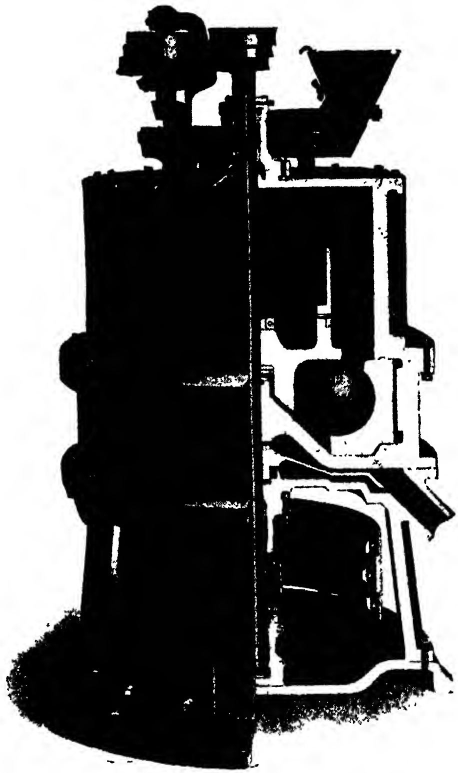


FIG. 236.—Lehigh-Fuller Mill. (*See p. 717.*)

It is customary to store the cement in bulk and weigh it out into bags or barrels as required for shipment. An automatic weighing machine similar to that shown in Fig. 230, page 713 (except that it is single instead of double), is a convenient apparatus for bagging. With this machine a weighing gang consists of three men. The nominal capacity of a single machine is 3 000 bags in ten hours, and the authors have known as many as 3 900 bags to be filled in this time.

In outlining the cement machinery, no reference has been made to the methods for conveying the material from one machine to another. Bucket

conveyors, belts and spiral conveyors are all more or less used. A spiral conveyor is a helical blade on a revolving shaft, set in a square or circular trough or tube of larger size than the spiral, so that the material packs around the circumference, and the blade comes in contact only with the powdered material.

Plaster of Paris (calcium sulphate $CaSO_4$), or gypsum ($CaSO_4 + 2H_2O$) the same substance in crystalline form, is an important addition to cement as a regulator of its setting, and from 1 to 2% is used in nearly all Portland cement manufactories. The gypsum must be added after the calcination and before the final grinding, in order to insure the proper result

The laboratory of a cement plant is an important feature. Not only must the chemical composition of the raw materials and the finished product be analyzed (see Appendix I) at frequent periods, but the cement must be mechanically tested for fineness, time of setting, tensile strength at seven and twenty-eight days, and, perhaps most important of all, for soundness. Most manufacturers use some form of the accelerated or hot test. This is not only due to the fact that many engineers require the cement to pass an accelerated test for reception, but because the chemists in the cement factories consider this test of great value in checking up the quality of cement.

Wet Process with Rotary Kilns. The rotary or Ransome kiln was first used in England on wet materials Rotaries have been widely, in fact almost universally, adopted in the United States for calcining dry materials, and more recently this field has been extended to use with slurry containing as much as 40% of water, which is pumped into the end of the rotary and dried by the same flame used for calcination. With kilns of ordinary length, Mr. Henry S Spackman states* that at least 25% more fuel is required for burning than with dry materials, and the temperature of the gases in the chimney is about 400° Fahr., one-third to one-half that from dry kilns. The product per kiln, according to Mr. Spackman, is not much more than 100 barrels per kiln, or about one-half the output with dry materials.

Higher production than this has been attained by lengthening the kilns so as to utilize more thoroughly the heat of the flame. Lengths of 70 to 100 feet are used, or a cylindrical kiln about 60 feet in length and 6 feet in diameter, lined with firebrick, is connected at its upper end with an independent drying tube 40 to 50 feet long of slightly smaller diameter and with no lining. A kiln 6 feet in diameter by 60 feet long, with a 54-inch by 50-foot dryer extension, working on wet materials, has been known in certain cases to give an average capacity of from 135 to 140 barrels per day.†

*Proceedings Philadelphia Engineers' Club, April, 1903.

†Statement of Allis-Chalmers Co. to the authors.

In the United States the raw materials most commonly employed in the wet process are marl and clay. The marl as it comes to the mill is broken up in some form of a disintegrator. The clay is dried and pulverized and is then mixed with the marl, which is about of the consistency of thick cream, in a pug mill, or an edge-runner. (See Fig. 237.)

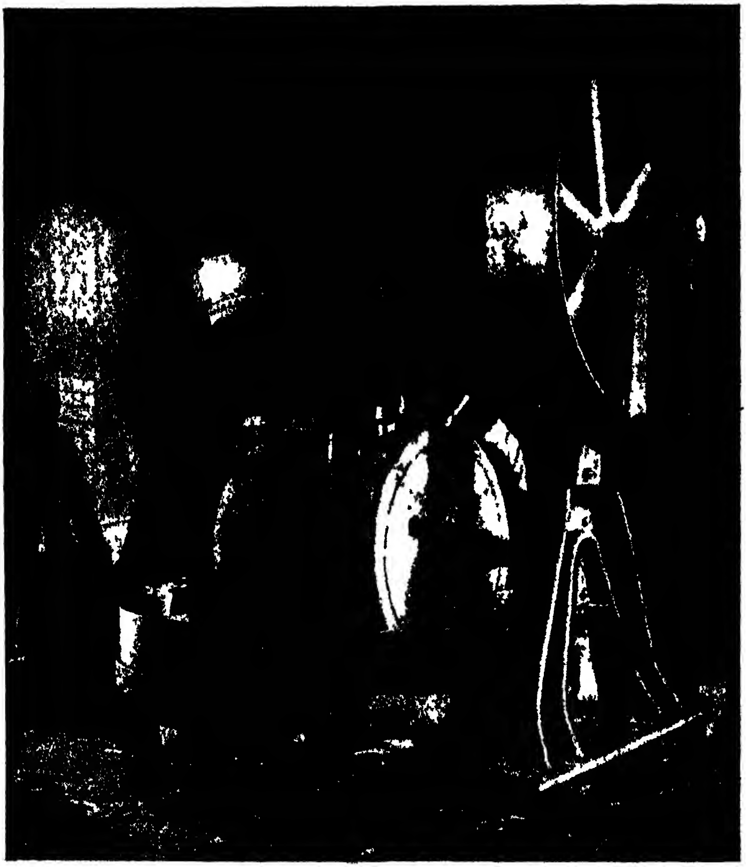


FIG. 237.—Edge Runner. (*See p. 720.*)

In some cases the clay is ground and water is added to it before mixing with the marl.

The mixed materials must now be ground wet before burning. This is often accomplished in mill stones, consisting of a pair of horizontal

stones the upper one of which revolves upon an upright shaft, or in wet tube mills closely similar to that shown in Fig. 233 on page 716.

From the mills, it may be run into tanks, where it is sampled and its chemical composition exactly determined, and from there pumped into the ends of rotary kilns, which, as stated above, are usually made longer than those used in the dry process.

Centrifugal pumps may be employed for conveying the wet material, or if it is too thick for these to handle, plunger pumps may be resorted to. A more recent system of handling is by compressed air.

After calcination the treatment is similar to that in mills where dry raw materials are used.

Stationary Kilns. Before the introduction of rotary or revolving kilns all cement was burned in stationary kilns. Stationary kilns are of two general types (1) intermittent kilns, which are completely charged and then burned, and (2) continuous kilns, where the fire is maintained continuously and the exhaust heat used to dry and heat the raw materials before burning them.

The most common form of intermittent kiln is the *Dome* or *Bottle Kiln.* This consists of a single shaft into which alternate layers of moist bricks of cement slurry and coke are placed by hand and burned. After cooling, the clinker is drawn out by hand through a door at the bottom, picked over to remove under burned clinker, — which is of a yellowish shade instead of black, — and clinker which has fused to fragments of the firebrick lining.

The *Johnson Kiln* is a more economical form of intermittent kiln. The slurry is placed in chambers, and dried by the exhaust gases from the burning of the previous charge before being placed in the kilns.

Of the continuous kilns, the *Hoffman Ring Kiln* consists of several chambers or furnaces around a central chimney. As the material in one furnace is burned, the heat passes around through the other furnaces so as to raise the temperature of the bricks in them and utilize the exhaust heat.

In the *Schoefer Kiln,* which is also of the continuous type, the bricks and fuel are loaded from time to time into the upper end of the shaft, and pass down, increasing in temperature, through the flame, where the area is contracted, to be cooled below and drawn out at the bottom.

The *Dietzsch Kiln* is of a somewhat similar type of construction, except that hand-labor is required in passing the dried material into the heating chamber.

Comparison of Rotary and Stationary Kilns. Mr. Frederick H. Lewis* compares the three classes of kilns as follows:

Quantity of Fuel

Intermittent kilns	15 to 30 bbls. per day
Continuous shaft kilns	40 to 80 bbls. per day
Rotary kilns	120 to 250 bbls. per day

Fuel in Terms of Clinker Produced

Intermittent kilns require	25 to 35% of fuel (coke)
Continuous shaft kilns require	12 to 16% of fuel (coal)
Rotary kilns require	22 to 40% of fuel (coal)

The chief difference in cost between rotary and stationary kilns is for labor. In a rotary plant one sees the machinery running with only an occasional attendant, as no handling of the materials is required from the time they enter the mill until the cement is packed in bags or barrels for shipment. In the stationary kiln plant, even if brick machines are used for molding the slurry, a great deal of hand labor is required, as the kilns must be loaded and emptied by hand. Mr. Lewis estimates the labor cost with continuous kilns to range from three to five times the cost with rotaries.

NATURAL CEMENT MANUFACTURE

The process of manufacture of Natural cement consists, in brief, of burning a natural argillaceous limestone at low heat and grinding it to powder. The stone used in England is very soft, in fact nearly as disintegrated as marl.

Raw Material. Many of the limestones used for Natural cement contain a high proportion of magnesia and an excess of clay, while others are nearly free from magnesia. It must be calcined at a temperature much below that required for Portland cement or it will fuse to a slag which after grinding has no hydraulic properties. Suitable formations occur in many parts of the United States, one of the most noted being that found in the region of eastern New York where Rosendale cements are made. Sometimes the stone is taken entirely from one ledge, while in other cases mixtures of two strata are employed. Little attention is paid to the analysis of the rock, as there is a wide range in the required chemical composition of the product (see p. 47), and the price at which Natural cement is sold does not warrant great refinement.

Process of Manufacture of Natural Cement. There is less variety in the methods employed for producing Natural cement than for Portland.

**Engineering Record*, Dec. 17, 1898, p. 47, and personal correspondence.

In a typical plant, the stones, of about the size that would be required for a large crusher, are brought from the quarry in carts or cars and dumped directly into the top of the kilns, which are of boiler iron lined with firebrick. They have no chimneys, but are open at the top and of the same size throughout. Thick layers of stone are alternated with thin layers of pea coal. The clinker is drawn out at the bottom as it is burned.

In the older plants the burned clinker is crushed and then ground between mill stones, while the newer mills use grinding machinery similar to that in Portland cement plants. When burnt, Natural cement rock is more readily powdered than Portland cement clinker.

PUZZOLAN CEMENT MANUFACTURE*

Puzzolan cement is made in the United States from blast furnace slag mixed with slaked lime. In Europe, natural puzzolanic materials have been employed.

The process of manufacture consists essentially of cooling the slag mixing it with slaked lime, and grinding to a very fine powder.

Slag for Puzzolan Cement. For making pig iron a blast furnace is charged with a mixture of iron ores, fluxes (consisting of limestone, either calcite or dolomite) and fuel, in the proper chemical proportions to produce, after reduction by heat, products of definite chemical composition. These resulting products are pig iron and slag. Any one unacquainted with metallurgy naturally thinks of blast furnace slag as a compound of iron. This is incorrect, as iron forms only a very small impurity.

All slags are not suitable for Puzzolan cement, as they ordinarily contain too high a percentage of magnesia and are often too high in alumina. The specifications for slag used in the manufacture of Steel Portland cement are as follows:†

Slag must analyze within the following limits :

	Per cent.
Silica plus alumina, not over	49
Alumina	13 to 16
Magnesia, under	4

Slag must be made in a hot furnace and must be of light gray color.

Slag must be thoroughly disintegrated by the action of a large stream of cold water directed against it with considerable force. This contact should be made as near the furnace as is possible."

Mr. E. Candlot says‡ "The slag must be basic; according to Mr. Tet-

*An investigation of the manufacture and properties of Puzzolan cement is given in Report of Board of Engineers, U. S. A., 1900, on Steel Portland cement.

†Report of Board of Engineers, U. S. A., 1900, on Steel Portland Cement.

‡Ciments et Chaux Hydrauliques, 1898, p. 157.

majer, when the ratio $\frac{CaO}{SiO_2}$ falls below unity the slag is useless; the ratio of alumina to silica must be between 0.45 and 0.50. According to Mr. Prost, the composition of slags habitually used in the manufacture of Puzzolan cements must be nearly represented by the formula 2 SiO_2, Al_2O_3, 3 CaO."

Mr. E. C. Eckel* gives the following analyses of slag and slag cement:

Analyses of Slags in Actual Use and Composition of Slag Cements

CONSTITUENT.	SLAG			CEMENT		
	Choindez, Switzerland	Saulnes, France	Chicago, Ill	Choindez, Switzerland.	Saulnes, France	Chicago, Illl
SiO_2	26.24	31.50	32.20	19.5	22.45	28.95
Al_2O_3	24.74	16.62	15.50	17.5	13.95	11.40
FeO	0.40	0.62			3.30	0.54
CaO	46.83	46.10	48.14	54.0	51.10	50.29
MgO	0.88		2.27		1.35	2.96
CaS	0.59					
$CaSO_4$	0.32					
S						1.37
SO_3					0.35	
Loss on ignition					7.50	3.39
$\frac{CaO}{SiO_2}$	1.78	1.46	1.49			
$\frac{Al_2O_3}{SiO_2}$	0.93	0.52	0.48			

Process of Manufacture of Puzzolan Cement. No kilns are required except for burning the lime. Molten slag as it flows from the blast furnace is granulated by coming in contact with a stream of cold water. This renders the product more strongly hydraulic, and most of the sulphur is removed as it strikes the water. As sent to the cement plant, it usually contains from 30% to 40% of water, and the first operation is to pass it through a dryer. The dried slag may or may not have a preliminary grinding before adding the slaked lime.

The lime is produced by burning a pure limestone, and then slaking it with water to which has been added a small percentage of caustic soda or other similar material, to make the resulting cement quicker setting. After drying, the slaked lime is mixed with the slag and ground in ball mills and tube mills, or in other forms of fine grinding machinery, and is ready for packing in bags or barrels for shipment.

*Mineral Resources of the United States, 1901.

CHAPTER XXXI

REFERENCES TO CONCRETE LITERATURE

While this chapter is not a complete bibliography of concrete literature, it presents a comprehensive list of valuable books and articles relating to the subject.

Under General References the names of authors are arranged alphabetically. The various subject headings under Subject References are also arranged alphabetically, and the references are printed in order of dates, the latest first. Articles are usually described by their subject-matter instead of giving their titles verbatim. In the case of similar articles printed in two or more periodicals, preference is generally given to the one bearing the earlier date. For references to this treatise see the Index.

ABBREVIATIONS

The following abbreviations (most of which correspond to those adopted by the Engineering Index) are employed:

Ann. de Ponts et Chauss.—Annales des Ponts et Chaussées. m. Paris.
Arch. Rec.- Architectural Record. New York.
Beton u. Eisen.—Beton und Eisen. Vienna.
Can Eng.—Canadian Engineer. Montreal, Canada
Cement and Eng. News.—Cement and Engineering News. Chicago.
Comptes Rendus—Comptes Rendus de l'Académie des Sciences. Paris.
Con. Eng.—Concrete Engineering. Cleveland, Ohio.
Deutsche Bau.—Deutsche Bauzeitung. Berlin.
Eng. Contr.—Engineering Contracting. New York.
Eng. Mag. — Engineering Magazine. New York & London.
Eng. News. — Engineering News. New York.
Eng. Rec. — Engineering Record. New York.
Gen. Civ. — Génie Civil. Paris.
Ins. Eng. — Insurance Engineering. Boston.
Int. Eng. Cong. — International Engineering Congress, St. Louis, 1904.
Jour. Am. Chem. Soc. — Journal American Chemical Society. Easton, Pa.
Jour. Assn. Eng. Socs. — Journal of the Association of Engineering Societies, Philadelphia.
Jour. Fr. Inst. — Journal Franklin Institute. Philadelphia.
Jour. W. Soc. Engs. — Journal of the Western Society of Engineers, Chicago.
Munic. Engng. — Municipal Engineering. Indianapolis.
Oest. Monatschr. f. d. Oeff. Baudienst. — Oesterreichische Monatsschrift für den Oeffentlichen Baudienst. Vienna.

Pro. Am. Soc. Civ. Engs. — Proceedings of the American Society of Civil Engineers. New York.
Pro. Am. Soc. Test. Mat. — Proceedings of American Society for Testing Materials. Philadelphia.
Pro. Assn. Ry. Supts. — Proceedings of the American Association of Railway Superintendents of Bridges and Buildings. New York.
Pro. Engs. Club of Phila. — Proceedings Engineers' Club. Philadelphia.
Pro. Engs. Soc. of W. Penn. — Proceedings of Engineers' Society of Western Pennsylvania. Pittsburgh.
Pro. Inst. Civ. Engs. — Proceedings of the Institution of Civil Engineers. London.
Ry. & Eng. Rev. — Railway & Engineering Review. Chicago.
R. R. Gaz. — Railroad Gazette. New York.
Rept. Chief of Engs., U. S. A. — Report of Chief of Engineers, U. S. A.
Rept. Eng. Dept. — Report of Engineering Department, Washington, D. C.
Rept. Met. W. & S. Board. — Report of Metropolitan Water & Sewerage Board, Massachusetts.
Revue Gen. des Chemins de Fer. — Revue Générale des Chemins de Fer. Paris.
Rev. Tech. — Revue Technique. — Paris.
Schw. Bauz. — Schweizerische Bauzeitung. Zürich.
Tech. — Technograph. University of Illinois. Champaign, Ill.
Tech. Qr. — Technology Quarterly. Boston.
Trans. Am. Soc. Civ. Engs. — Transactions American Society of Civil Engineers. New York.
Trans. Am. Soc. Mech. Engs. — Transactions American Society of Mechanical Engineers. New York.

GENERAL REFERENCES

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Bridges

Location	Max. span ft.	Max. rise ft.	Crown thickness ft.	Reinforcement	Authority
Switzerland	259	87′	4′	Longitudinal & transverse bars	*Eng. News*, Aug., 1909, p. 133.
42d St., Phila.	250	53	3	Double steel arch ribs	*Eng. News*, May, 1909, p. 540.
C. B. & Q. R. R. Trestles					*Eng. News*, May, 1909, p. 546.
Delaware River D. L. & W. R. R.	150	40	6		*Eng. Rec.*, Apr., 1909, p. 542.
Paulins Kill D. L. & W. R. R.	120	60	6		*Eng. Rec.*, Apr., 1909, p. 541.
Grand River L. S. & M. S. Ry.	160	71½	7	Longitudinal & transverse bars	*Eng. Rec.*, Apr. & May, 1909.
Cumberland Valley Ry.	100	32	5	None	*Eng. News*, Apr., 1909, p. 377.
Wyoming Ave., Phila.	90	28	2½	Horizontal longitudinal rods in spandrel walls. No other reinforcement	*Eng. Rec.*, Feb., 1909, p. 233.
Harrisburg, Pa. Viaduct					*Eng. Rec.*, Aug., 1908, p. 228.
Maumee, Waterville, Ohio	90	25	2	Longitudinal & transverse rods	*Cement*, Aug., 1908, p. 116.
Sandy Hill, N. Y.	60	8½	1½	Ribs, angle bars, latticed	*Trans. Am. Soc. Civ. Engrs.*, Vol. *LIX*, p. 195.
Walnut Lane Phila.	233	70	5½	None	*Eng. News*, Jan., 1907, p. 117.
Paterson, N. J.,	54	2.5	1.8	11 ribs about 4 ft. apart	*Eng. Rec.*, Sept., 1904, p. 303
Plainwell, Mich.,	54	8	1.25	4-inch 6-lb. channels 1.9 ft. apart	*Eng. News*, May, 1904, p. 456
Waterloo, Iowa,	72	7.2	1.18	Steel Ribs	*Eng. Rec.*, Feb., 1904, p. 185
Yellowstone River,	120	15	2.0	Lattice girders	*Eng. News*, Jan., 1904, p. 25

*An asterisk precedes the references which are especially noteworthy.

Location.	Max. span ft.	Max. rise ft.	Crown thickness ft.	Reinforcement.	Authority
Plano, Ill.,	75	38½	3	¾" and ½" corrugated bars	*Eng. Rec.*, Jan., 1904, p. 18
3rd St., Dayton, Ohio,	110	14.25	2.1	Melan, 4 angles, latticed	Edwin Thacher, 1904
Newark, N. J.,	132	16.2	3	Melan, 4 angles. latticed	Edwin Thacher, 1904
Kankakee, Ill.,	73	8.4	1.33	Thacher. rods near top and bottom	Edwin Thacher, 1904
Mishawaka, Ind.,	110	14	2	Melan, 4 angles, latticed	Edwin Thacher, 1903
Prospect Ave., N. Y,	85	8½	2.25	Corrugated bars	*Eng. News*, Dec., 1903, p. 588
Riverside, Cal.,	87	36 9	3.5	None	*Eng. News*, Oct., 1903, p. 353
Leominster, Mass.,	45	6.25	1.1	Round rods anchored	J. R. Worcester, 1903
Des Moines River,	100	28	1.67	Melan	*Cement*, July, 1902, p. 200
Zanesville, Ohio,	122	11.5	2.5	¾" x 5" bars	*Eng. News*, March, 1902, p. 261
Concord, Mass.,	66	7	1.1	None	J. R. Worcester, 1901
Lansing, Mich.,	120	23	2	Melan, 4 angles, latticed	Edwin Thacher, 1901
South Bend, Ind.,	100	11	2.5	Melan, 4 angles, latticed	Edwin Thacher
Chatellerault, France,	164	15.7	1.7	Hennebique	Revue Gen. des Chemins de Fer, Dec., 1901
Kirchheim, Germany,	124.6	18	2 6	None	*Eng. News*, Oct., 1899, p. 246
Germany,	132	14.7	0 82	Monier	*Eng. News*, Sept., 1899, p. 179
Switzerland,	128	11	0.56	Monier	*Eng. News*, Sept., 1899, p. 179
Southern Ry., Austria,	32.8	3.3	0.5	Monier	*Eng. News*, Sept., 1899, p. 179
Topeka, Kan.,	125	12	1.8	Melan beams	*Eng. Rec.*, April 16, 1898
Inzigkofen, Germany,	140	14.5	2.3	33 000 lb. cast iron	*Eng. News*, Sept., 1896, p. 178
Munderkingen, Germany,	164	16.4	3.3	None	Inst Civ. Engs., V. 119, p. 224
Cincinnati, Ohio,	70	10	1.25	Melan beams	*Eng. News*, Oct., 1895, p. 214
Maryborough, Queensl'd	50	4	1.25	Steel rails	*Engng.*, London, May 10, 1895, p 395
Neuhäusel, Hungary,	55.78	3.7	0.82	Skeleton girders	Inst. Civ. Engs., V., 114, p. 402
Philadelphia, Penn ,	25.39	6.5	3	1½" mesh, ½" wire netting	*Eng. News*, Sept., 1893, p. 189

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APPENDIX I

METHOD SUGGESTED FOR THE ANALYSIS OF LIMESTONES, RAW MIXTURES, AND PORTLAND CEMENTS BY THE COMMITTEE ON UNIFORMITY IN TECHNICAL ANALYSIS OF THE AMERICAN CHEMICAL SOCIETY, WITH THE ADVICE OF W. F. HILLEBRAND.

Solution: One-half gram of the finely powdered substance is to be weighed out and, if a limestone or unburned mixture, strongly ignited in a covered platinum crucible over a strong blast for 15 minutes, or longer if the blast is not powerful enough to effect complete conversion to a cement in this time. It is then transferred to an evaporating dish, preferably of platinum for the sake of celerity in evaporation, moistened with enough water to prevent lumping, and 5 to 10 c. c of strong HCl added and digested, with the aid of gentle heat and agitation, until solution is completed. Solution may be aided by light pressure with the flattened end of a glass rod.* The solution is then evaporated to dryness, as far as this may be possible on the steam bath.

Silica: The residue, without further heating, is treated at first with 5 to 10 c. c. of strong HCl which is then diluted to half strength or less, or upon the residue may be poured at once a larger volume of acid of half strength. The dish is then covered and digestion allowed to go on for 10 minutes on the bath, after which the solution is filtered and the separated silica washed thoroughly with water. The filtrate is again evaporated to dryness, the residue, without further heating, taken up with acid and water and the small amount of silica it contains separated on another filter paper. The papers containing the residue are transferred wet to a weighed platinum crucible, dried, ignited, first over a Bunsen burner until the carbon of the filter is completely consumed, and finally over the blast for 1 minute and checked by a further blasting for 10 minutes or to constant weight. The silica, if great accuracy is desired, is treated in the crucible with about 10 c. c. of HFl and four drops of H_2SO_4 and evaporated over a low flame to complete dryness. The small residue is finally blasted, for a minute or two, cooled and weighed. The difference

*If anything remains undecomposed it should be separated, fused with a little Na_2CO_3, dissolved and added to the original solution. Of course a small amount of separated non-gelatinous silica is not to be mistaken for undecomposed matter.

between this weight and the weight previously obtained gives the amount of silica.*

Al_2O_3 and Fe_2O_3: The filtrate, about 250 c.c., from the second evaporation for SiO_2, is made alkaline with NH_4OH after adding HCl, if need be, to insure a total of 10 to 15 c.c. strong acid, and boiled to expel excess of NH_3, or until there is but a faint odor of it, and the precipitated iron and aluminum hydrates, after settling, are washed once by decantation and slightly on the filter. Setting aside the filtrate, the precipitate is dissolved in hot dilute HCl, the solution passing into the beaker in which the precipitation was made. The aluminum and iron are then re-precipitated by NH_4OH boiled, and the second precipitate collected and washed on the same filter used in the first instance. The filter paper, with the precipitate, is then placed in a weighed platinum crucible, the paper burned off and the precipitate ignited and finally blasted 5 minutes, with care to prevent reduction, cooled and weighed as $Al_2O_3 + Fe_2O_3$.†

Fe_2O_3: The combined iron and aluminum oxides are fused in a platinum crucible at a very low temperature with about 3 to 4 grams of $KHSO_4$, or, better, $NaHSO_4$, the melt taken up with so much dilute H_2SO_4 that there shall be no less than 5 grams absolute acid and enough water to effect solution on heating. The solution is then evaporated and eventually heated till acid fumes come off copiously. After cooling and redissolving in water the small amount of silica is filtered out, weighed, and corrected by HFl and H_2SO_4.‡ The filtrate is reduced by zinc, or preferably by hydrogen sulphide, boiling out the excess of the latter afterwards while passing CO_2 through the flask, and titrated with permanganate.§ The strength of the permanganate solution should not be greater than .0040 gr. Fe_2O_3 per c.c.

CaO: To the combined filtrate from the $Al_2O_3 + Fe_2O_3$ precipitate a few drops of NH_4OH are added, and the solution brought to boiling. To the boiling solution 20 c.c. of a saturated solution of ammonium oxalate is added, and the boiling continued until the precipitated CaC_2O_4 assumes a well-defined granular form. It is then allowed to stand for 20 minutes,

*For ordinary control work in the plant laboratory this correction may, perhaps, be neglected, the double evaporation never.

†This precipitate contains TiO_2, P_2O_5, Mn_3O_4.

‡This correction of Al_2O_3 Fe_2O_3 for silica should not be made when the HFl correction of the main silica has been omitted, unless that silica was obtained by only one evaporation and filtration. After two evaporations and filtrations 1 to 2 mg. of SiO_2 are still to be found with the Al_2O_3 Fe_2O_3.

§In this way only is the influence of titanium to be avoided and a correct result obtained for iron.

or until the precipitate has settled, and then filtered and washed. The precipitate and filter are placed wet in a platinum crucible, and the paper burned off over a small flame of a Bunsen burner. It is then ignited, redissolved in HCl, and the solution made up to 100 c.c. with water. Ammonia is added in slight excess, and the liquid is boiled. If a small amount of Al_2O_3 separates, this is filtered out, weighed, and the amount added to that found in the first determination, when greater accuracy is desired. The lime is then re-precipitated by ammonium oxalate, allowed to stand until settled, filtered, and washed,* weighed as oxide by ignition and blasting in a covered crucible to constant weight, or determined with dilute standard permanganate.†

MgO: The combined filtrates from the calcium precipitates are acidified with HCl, and concentrated on the steam bath to about 150 c.c., 10 c.c. of saturated solution of $Na(NH_4)HPO_4$ are added, and the solution boiled for several minutes. It is then removed from the flame and cooled by placing the beaker in ice water. After cooling, NH_4OH is added drop by drop with constant stirring until the crystalline ammonium-magnesium ortho-phosphate begins to form, and then in moderate excess, the stirring being continued for several minutes. It is then set aside for several hours in a cool atmosphere and filtered. The precipitate is redissolved in hot dilute HCl, the solution made up to about 100 c.c., 1 c.c. of a saturated solution of $Na(NH_4)HPO_4$ added, and ammonia drop by drop, with constant stirring until the precipitate is again formed as described and the ammonia is in moderate excess. It is then allowed to stand for about 2 hours when it is filtered on a paper or a Gooch crucible, ignited, cooled and weighed as $Mg_2P_2O_7$.

K_2O and Na_2O: For the determination of the alkalies, the well-known method of Prof. J. Lawrence Smith is to be followed, either with or without the addition of $CaCO_3$ with NH_4Cl.

SO_3: One gram of the substance is dissolved in 15 c.c. of HCl, filtered and residue washed thoroughly.‡

The solution is made up to 250 c.c. in a beaker and boiled. To the boiling solution 10 c.c. of a saturated solution of $BaCl_2$ is added slowly drop by drop from a pipette and the boiling continued until the precipitate is well formed, or digestion on the steam bath may be substituted for the

*The volume of wash water should not be too large. *Vide* Hillebrand.

†The accuracy of this method admits of criticism, but its convenience and rapidity demand its insertion.

‡Evaporation to dryness is unnecessary, unless gelatinous silica should have separated and should never be performed on a bath heated by gas. *Vide* Hillebrand.

boiling. It is then set aside over night, or for a few hours, filtered, ignited, and weighed as $BaSO_4$.

Total Sulphur: One gram of the material is weighed out in a large platinum crucible and fused with Na_2CO_3 and a little KNO_3, being careful to avoid contamination from sulphur in the gases from source of heat. This may be done by fitting the crucible in a hole in an asbestos board. The melt is treated in the crucible with boiling water and the liquid poured into a tall, narrow beaker and more hot water added until the mass is disintegrated. The solution is then filtered. The filtrate contained in a No. 4 beaker is to be acidulated with HCl and made up to 250 c.c. with distilled water, boiled, the sulphur precipitated as $BaSO_4$ and allowed to stand over night or for a few hours.

Loss on Ignition: Half a gram of cement is to be weighed out in a platinum crucible, placed in a hole in an asbestos board so that about $\frac{3}{5}$ of the crucible projects below, and blasted 15 minutes, preferably with an inclined flame. The loss by weight, which is checked by a second blasting of 5 minutes, is the loss on ignition.

May, 1903:

Recent investigations have shown that large errors in results are often due to the use of impure distilled water and reagents. The analyst should, therefore, test his distilled water by evaporation and his reagents by appropriate tests before proceeding with his work.

APPENDIX II

FORMULAS FOR REINFORCED CONCRETE BEAMS*

Direct working formulas suited to all ordinary cases of reinforced concrete design are presented in Chapter XXI. The analytical methods of deduction, however, are omitted there in order to make the book handier for every day use and are presented in this Appendix.

These formulas cover all the usual conditions occurring in practice and in theoretical treatment of beam design, as follows:

(1) Rectangular beams with steel in bottom, assuming that concrete bears no tensile stress. (See page 751.)

(2) T-shaped section of the beam, for use in combined beam and slab construction. (See p. 754.)

(3) Beam with steel in both top and bottom, for use in connection with the design of a continuous beam at the supports and other special cases. (See p. 757.)

(4) Beam with steel in bottom and concrete assumed to bear tensile stress, for theoretical use in determining accurate stresses at early stages of loading. (See p. 760.)

(5) Beam with compressive stress varying as a parabola, to illustrate a method of computation occasionally used. (See p. 762)

The first three of these analyses are for common use and follow the recommendations of the Joint Committee on Concrete and Reinforced Concrete. This fact has necessitated no changes in the analyses in the first edition of this treatise except in the adoption of the new standard of notation.

As stated in Chapter XXI, the straight line theory,—that is, the theory in which the modulus of elasticity of concrete in compression is assumed to be constant within usual working limits,—is adopted as the standard and the concrete is assumed to bear no tension.

The various other rational formulas† which have been advanced by

*The authors are indebted to Prof. Frank P. McKibben for the formulas in this Appendix which have been especially prepared by him for this Treatise.

†See Christophe's Béton Armé and Morel's Ciments Armé, 1902.

different mathematicians are based upon the same analytical methods of treatment, but on different assumptions of stress. Many have complicated their equations by taking moments about the neutral axis instead of about the centers of tension or compression, but the general principles of the deduction are the same and in accordance with the analyses given below.

It is possible to evolve by calculus a general formula which satisfies all of the various hypotheses,* but the treatment is omitted here and only the more practical demonstrations are given

NOTATION

The same notation is adopted in this Appendix as in Chapter XIV.

h = height of beam.

t = thickness of slab, *i. e.*, thickness of T-flange.

b = breadth of rectangular beam or breadth of flange of T-beam.

b' = breadth of web of T-beam.

p = ratio of cross-section of steel in tension to cross-section of beam above this steel.

p' = ratio of cross-section of steel in compression to cross-section of beam above the steel in tension.

f_c = unit compressive stress in outside fiber of concrete.

f_c' = unit tensile stress, or pull, in outside fiber of concrete.

f_s = unit tensile stress, or pull, in steel.

f_s' = unit compressive stress in steel.

E_c = modulus of elasticity of concrete in compression.

E_c' = modulus of elasticity of concrete in tension.

E_s = modulus of elasticity of steel.

$n = \dfrac{E_s}{E_c}$

d = distance from outside compressive fiber to center of gravity of steel.

k = ratio of depth of neutral axis to depth of steel in tension.

kd = distance from outside compressive surface to neutral axis in beam in which the depth to steel in tension is d.

z = depth of resultant compression below top.

j = ratio of lever arm of resisting couple to depth d.

jd = $d-z$ = arm of resisting couple.

e = extra thickness of concrete below steel in tension.

d' = depth to center of compressive steel.

M = moment of resistance or bending moment in general.

*See Burr's Materials of Engineering, 1903, p. 633.

ANALYSIS OF RECTANGULAR BEAM

We may represent the stresses in the beam by the diagram shown in Fig. 238, page 751. At any vertical section through the beam the concrete in the upper portion resists the forces which tend to compress it, and the steel in the lower part of the beam resists the forces which tend to stretch and break it in tension. The compressive resistance acts in one direction and the tensile resistance in another direction, as designated by the large arrows in the diagram. The center of tension in the steel is at the center of the bar, or, if there is more than one tier of bars, at the center of gravity of the set of bars. The center of pressure of the concrete passes through the center of gravity of the triangle which represents the compressive stresses.

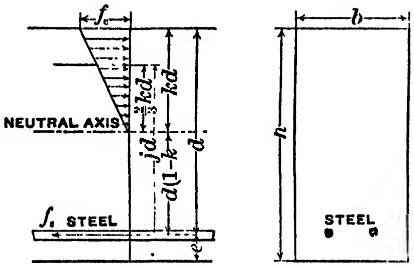


FIG. 238.—Resisting Forces in a Reinforced Concrete Beam. (*See p.* 751.)

The internal resisting forces may be replaced by two forces: the total compression acting in the center of gravity of the triangle, having for its base f_c and its height *kd*, and the total pull acting in the center of gravity of steel. For equilibrium the sum of all forces must equal zero or the total compression must equal the total pull, so that the forces form a couple. If either tension or compression exceeds its maximum strength, the beam fails. These conditions are assumed to be true only after the point of loading is reached at which the tension is transferred to the steel, as otherwise the tension would be made up of two forces, the tension in the steel and the tension in the concrete, as discussed on page 760 in this Appendix.

The moment of resistance of the couple must be equal to or greater than the bending moment produced by the live and dead loads.

Since it is assumed that a plane section before bending remains a plane section after bending, we have the proportion

$$\frac{\text{deformation in steel}}{\text{deformation in outside compressive concrete fibers}} = \frac{d\,(1-k)}{kd}$$

And since deformation $= \dfrac{\text{stress per square inch}}{\text{modulus of elasticity}}$ we have

$$\frac{\dfrac{f_s}{E_s}}{\dfrac{f_c}{E_c}} = d\frac{(1-k)}{kd} \quad \text{or} \quad \frac{f_s}{nf_c} = \frac{1-k}{k} \tag{1}$$

From which

$$k = \frac{\quad}{1 + \dfrac{f_s}{nf_c}} \tag{2}$$

Solving formula (1) for f_c

$$f_c = f_s \frac{k}{n\,(1-k)} \tag{3}$$

Now, as stated above, for equilibrium the total tension in the steel must be equal and opposite to the total compression in the concrete. The total tension in the steel is its unit tension, f_s, multiplied by the area of the steel, pdb, and the total compression in the concrete is represented by the area of the pressure triangle, $\frac{1}{2}f_c kd$ times the breadth of the beam, b. Equating these two stresses and cancelling out the db which occurs in both,

$$pf_s = \frac{f_c k}{2} \tag{4}$$

If the value of k in formula (2) be substituted for the k in formula (4), we have

$$p = \frac{1}{2\dfrac{f_s}{f_c}\left(1 + \dfrac{f_s}{nf_c}\right)} \tag{5}$$

For any given percentage of steel the values of f_s and f_c cannot be assumed independently, as they bear a constant ratio to each other.

Substituting the value of f_c in formula (3) for f_c in formula (4), we have

$$p = \frac{k \quad k}{2\,(1-k)n}$$

Solving this quadratic equation and adopting the positive sign before the square root,

$$k = -np + \sqrt{2np + (np)^2}. \tag{6}$$

We thus have k in terms of n and p, and from formula (6) the location of the neutral axis may be calculated with any percentage of steel for concrete and steel having known moduli of elasticity.

The moment of resistance is obtained from the couple by taking moments about the center of compression in the concrete, using for the force the total tension in the steel, which, as above, is $pf_s bd$ times the arm (see Fig. 238, p. 751), jd

or

$$M = pf_s jbd^2 \text{ and } f_s = \frac{M}{pjbd^2} \tag{7}$$

The moment of resistance may also be expressed in terms of compression in the concrete by combining equations (4) and (7), or, more directly, by taking moments about the center of the tension in the steel, thus

$$M = \frac{f_c kjbd^2}{2} \text{ and } f_c = \frac{2\,M}{kjbd^2} \tag{8}$$

Values for k with various percentages of steel and moduli of elasticity are given in table 12 on page 521.

The value of the moment of resistance, M, may also be expressed without using k by substituting in formulas (7) and (8) the value of p from formula (5) and the value of k from (2), thus giving

$$M = bd^2\left[\frac{f_s}{\frac{2f_s}{f_c}\left(1+\frac{f_s}{nf_c}\right)}\left(1-\frac{1}{3\left(1+\frac{f_s}{nf_c}\right)}\right)\right] \tag{9}$$

or

$$M = bd^2\left[\frac{f_c}{2\left(1+\frac{f_s}{nf_c}\right)}\left(1-\frac{1}{3\left(1+\frac{f_s}{nf_c}\right)}\right)\right] \tag{10}$$

Formula (10) is apparently more complex than (7) and (8), but as the latter require the determination of k, formula (10) is more readily solved unless the table on page 521 is employed.

In the use of formula (10), f_s and f_c must be corresponding values and cannot be assumed independently of each other, since for any given percentage of steel the ratio of f_s to f_c is a constant. (See formula (5), p. 752).

For a given quality of concrete and steel the values of f_s, and f_c and n are constant, so that the term in brackets may be replaced by a constant $\frac{1}{C^2}$

We may thus write in place of formulas (9) and (10) the formula

$$M = \frac{bd^2}{C^2} \tag{11}$$

where C is a constant for any given concrete and steel. Values of C under different conditions are tabulated on pp. 519 and 520.

Following directly from formula (11)

$$d = C\sqrt{\frac{M}{b}} \tag{12}$$

In the above formula M represents the bending moment which must be equal to or smaller than the moment of resistance. Also, since in fig. 238, p. 751, $d = h - e$, the formula may be written

$$h = C\sqrt{\frac{M}{b}} + e \tag{13}$$

from which the required height of the rectangular beam or slab may be directly obtained.

T-SHAPED SECTION OF BEAM

When a reinforced concrete floor slab and beam are built as one piece the slab adds to the strength of the beam by increasing the area which is in compression.

The working formulas for this shape of beam termed a T-beam are given in Chapter XXI, page 423, in sufficient detail for the ordinary design where

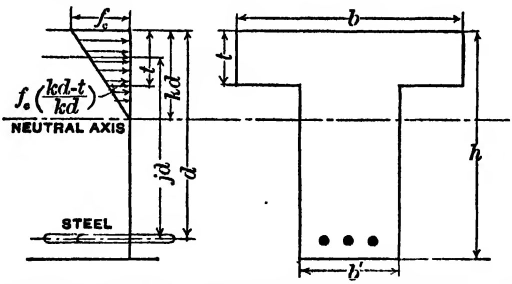


FIG. 239.—Resisting Forces in T-shaped Section of Beam. (*See p.* 755.)

the beam and the slab are assumed to act as a unit. The method of analysis and the formulas deduced are presented below.

These are based upon the assumption that the intensity of the compression in the concrete does not diminish from the web outward towards the

edges of the flange. For a section having a narrow flange, this is practically correct, but with a wide flange, it is probable that the intensity of the compression in the flange diminishes from the web outward so that the breadth of slab should be limited, as indicated on page 424. If this pressure is assumed to decrease either uniformly or otherwise, the formulas may be modified accordingly.

Assuming the compression to be distributed as shown in the diagram, and the steel to take all the tension, the formulas given below may be deduced as in the preceding cases.

Case I. *Neutral Axis Below Flange, $kd > t$.*

Neglect the slight amount of compression in the web below the intersection of the web and flange.

As in the previous case using notation on page 750 and referring to Fig. 239.

$$k = \frac{1}{1 + \frac{f_s}{n f_c}}$$

By equating the forces acting on the section

$$A_s f_s = f_c \frac{2kd - t}{2kd} b t$$

Solving the two above equations for kd and eliminating f_c and f_s

$$k d = \frac{2 n d A_s + b t^2}{2 n A_s + 2 b t} \qquad (14)$$

The position of the resultant compression lies in the center of gravity of the trapezoid, the parallel sides of which are equal to f_c and $f_c \frac{kd - t}{kd}$ and the height to t.

The distance of this center of compression from upper surface of beam is

$$z = \frac{3kd - 2t}{2kd - t} \frac{t}{3} \qquad (15)$$

The arm of resisting couple

$$jd = d - z$$

hence

$$M = A_s jd f_s \ (16) \text{ and } \quad M = \frac{2kd - t}{2kd} btjdf_c \qquad (17)$$

or
$$f_s = \frac{M}{A_s jd} \quad (18) \quad \text{and} \quad f_c = \frac{Mkd}{bt(kd - \frac{1}{2}t)\,jd} \qquad (19)$$

From the figure, taking similar triangles, the relation between f_s and f_c is found to be

$$f_c = \frac{f_s}{n}\,\frac{k}{1-k} \qquad (20)$$

The approximate moment arm of resisting couple may be taken as

$$jd = d - \frac{t}{2} \qquad (21)$$

which changes formula (19) to

$$f_s = \frac{M}{A_s\left(d - \frac{t}{2}\right)} \quad \text{(approximate)} \qquad (22)$$

This formula gives for ordinary cases correct and safe results, but should not be used when the flange is small as compared with the stem.

In the above formulas the compression in the stem is neglected. In large beams, where the stem forms the larger part of the compressive area the following formulas derived by the same principles used in derivation of formulas in the previous analysis should be used,

$$kd = \sqrt{\frac{2nd\,A_s + (b-b')\,t^2}{b'} + \left(\frac{n\,A_s + (b-b')\,t}{b'}\right)^2} - \frac{n\,A_s + (b-b')\,t}{b'} \qquad (23)$$

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)\,b + \left[(kd - t)^2\left(t + \frac{1}{3}(kd - t)\right)\right]b'}{t\,(2kd - t)\,b + (kd - t)^2\,b'} \qquad (24)$$

Arm of resisting couple

$$jd = d - z \qquad (25)$$

Moment of resistance

$$M = A_s jd f_s \quad (26) \quad M = \frac{f_c}{2kd}\left[(2kd - t)bt + (kd - t)^2 b'\right]jd \qquad (27)$$

Fiber stresses

$$f_s = \frac{M}{A_s jd} \quad (28) \quad \text{and} \quad f_c = \frac{2Mkd}{\left[(2kd - t)bt + (kd - t)^2 b'\right]jd} \qquad (29)$$

Case II. *Neutral Axis in Flange or at Underside of Flange, $kd \leqq t$*

In this case use the rectangular beam formula, considering the T-beam as a rectangular beam of the same depth, the breadth of which is the breadth of the flange. The percentage is then based on the total area bd

STEEL IN TOP AND BOTTOM OF BEAM, NO TENSION IN CONCRETE

Although the use of steel in the compressive portion of the beam is generally uneconomical, its introduction there is sometimes a necessity for practical reasons. In the ends of a continuous beam the steel in the bottom is usually carried through into the supports, and if the length is enough to provide bond its value in compression may be taken as assisting to resist the negative bending moment.

It is possible to reduce the working formulas to extremely simple form by introducing constants which vary with different conditions, as outlined on page 427, the values for the constants being given in table 8, page 516

The treatment of a beam subjected to bending and direct stress with the steel in compression is presented in connection with the design of arches on page 563, and these formulas may also be used in other cases of eccentric thrusts.

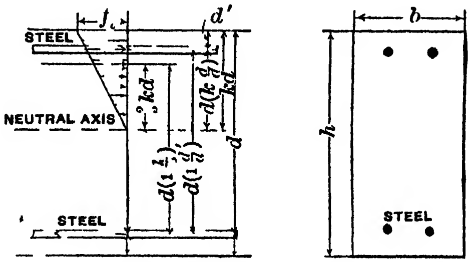


FIG 240—Resisting Forces with Steel in Top and Bottom of Beam.
(See p 757)

The analytical treatment of the design of an ordinary beam adopting as usual the assumption of a constant modulus of elasticity and no tension in the concrete, but assuming that the compressive stresses are partially borne by the steel in the compression portion of the beam, is as follows

FORMULAS.

Deformations, as usual, are assumed to vary directly as distance from neutral axis, hence from Fig 240, using notation on page 750

$$\frac{\frac{f_s}{E_s}}{\frac{f_c}{E_c}} = \frac{d(1-k)}{dk} = \frac{1-k}{k} \qquad \text{Whence } k = \frac{1}{1+\frac{f_s}{nf_c}} \qquad (30)$$

Also,

$$f_s' = f_s \frac{kd - d'}{d - kd} \quad (31) \quad \text{and} \quad f_s' = nf_c \frac{kd - d'}{kd} \quad (32)$$

$$f_s = nf_c \frac{1 - k}{k} \quad (33) \quad \text{and} \quad f_c = \frac{f_s}{n} \frac{k}{1 - k} \quad (34)$$

Equating the horizontal forces acting on the cross-section of the beam we have:

$$bd\left(\frac{f_c k}{2} + p'f_s'\right) = bdpf_s$$

Whence $p = \frac{1}{f_s}\left(\frac{f_c k}{2} + p'f_s'\right) = \frac{1}{f_s}\left(\frac{f_s}{2n} \cdot \frac{k^2}{1-k} + p'f_s \frac{kd - d'}{d - kd}\right)$

Hence

$$p = \frac{k^2}{2n(1 - k)} + p' \frac{k - \frac{d'}{d}}{1 - k} \quad (35)$$

Solving equation (35) for k,

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \quad (36)$$

Taking moments about the center of pull in the steel, we have

$$M = \frac{bf_c kd}{2}\left(d - \frac{kd}{3}\right) + f_s' p'bd(d - d')$$

$$M = bd^2\left[\frac{f_c k}{2}\left(1 - \frac{k}{3}\right) + f_s' p'\left(1 - \frac{d'}{d}\right)\right]$$

or by eliminating f_s' by means of equation (32),

$$M = f_c bd^2\left[\frac{k}{2}\left(1 - \frac{k}{3}\right) + \frac{np'\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k}\right] \quad (37)$$

Taking moments about the center of compresssive stress the steel, we have

$$M = bd^2\left[f_s p\left(1 - \frac{d'}{d}\right) - \frac{f_c k}{2}\left(\frac{k}{3} - \frac{d'}{d}\right)\right]$$

or by eliminating f_c,

$$M = f_s bd^2 \left[p\left(1 - \frac{d'}{d}\right) - \frac{k^2}{2n(1-k)}\left(\frac{k}{3} - \frac{d'}{d}\right) \right] \quad (38)$$

Then taking moments about center of compression in concrete:

$$M = bd^2 \left[f_s p\left(1 - \frac{k}{3}\right) + f'_s p'\left(\frac{k}{3} - \frac{d'}{d}\right) \right]$$

or by eliminating f_s,

$$M = f'_s bd^2 \left[p\frac{1-k}{k - \frac{d'}{d}}\left(1 - \frac{k}{3}\right) + p'\left(\frac{k}{3} - \frac{d'}{d}\right) \right] \quad (39)$$

The values in the square brackets in formulas (37), (38) and (39) are constant for any combination of n, p, p' and $\frac{d'}{d}$.

Substituting

$$C_c = \frac{k}{2}\left(1 - \frac{k}{3}\right) + \frac{np'\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k} \quad (40)$$

$$C_s = p\left(1 - \frac{d'}{d}\right) - \frac{k^2}{2n(1-k)}\left(\frac{k}{3} - \frac{d'}{d}\right) \quad (41)$$

$$C'_s = p\frac{1-k}{k - \frac{d'}{d}}\left(1 - \frac{k}{3}\right) + p'\left(\frac{k}{3} - \frac{d'}{d}\right) \quad (42)$$

$$M = bd^2 f_c C_c \ (43) \quad \text{and } f_c = \frac{M}{bd^2 C_c} \quad (44)$$

$$M = bd^2 f_s C_s \ (45) \quad \text{and } f_s = \frac{M}{bd^2 C_s} \quad (46)$$

and

$$M = bd^2 f'_s C'_s \ (47) \quad \text{and } f'_s = \frac{M}{bd^2 C'_s} \quad (48)$$

and

Values of C_c, C_s, and C'_s for different combinations of n, p, p' and $\frac{d'}{d}$ are given in table 8, page 516.

STEEL IN BOTTOM OF BEAM, CONCRETE BEARING TENSION

In the earlier stages of loading of reinforced concrete beams, the deformation curves (see fig. 130, p. 409) indicate that the concrete actually bears a portion of the pull. Although it is not good practice to consider this pull in the design of beams, but, instead, it is customary to take the working strength as a factor of the ultimate, or nearly the ultimate strength of the beam, the following formulas are useful for determining the actual stresses and for calculating deflections at the earliest stages of loading.

Formulas. Since elongation of steel and concrete at the same point

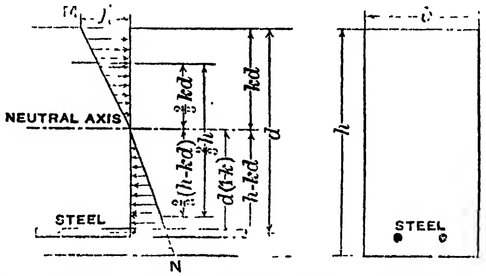


FIG. 241.—Resisting Forces with Concrete Bearing Tension. *(See p. 760.)*

must be equal, and since cross-sectional planes are assumed to remain plane during bending, we have from Fig. 241, the following equations:

$$\frac{\dfrac{f_s}{E_s}}{\dfrac{f_c'}{E_c'}} = \frac{d-kd}{h-kd} \text{ hence } f_s = \frac{E_s}{E_c'} f_c' \frac{d-kd}{h-kd} \qquad (49)$$

$$f_c = \frac{E_c}{E_c'} f_c' \frac{kd}{h-kd} \qquad (50)$$

$$f_s = \frac{E_s}{E_c} f_c \frac{1-k}{k} \quad (51) \qquad \text{also } f_c' = \frac{E_c'}{E_c} f_c \frac{h-kd}{kd} \qquad (52)$$

Equating horizontal forces on the section we have

$$\frac{b f_c kd}{2} = p f_s bd + \frac{f_c' b (h-kd)}{2} \qquad (53)$$

The elimination of f_s and f_c' from (53) gives

$$\frac{kd}{2} = pd \frac{E_s}{E_c} \frac{1-k}{k} + \frac{E_c'}{E_c} \frac{(h-kd)^2}{2\,kd} \qquad (54)$$

From which

$$p = \frac{1}{2(1-k)}\left[\frac{E_c}{E_s}k^2 - \frac{E_c'}{E_s}\left(\frac{h-kd}{d}\right)^2\right] \tag{55}$$

Solving equation (55) for k,

$$k = \sqrt{\frac{2p + \dfrac{E'_c h^2}{E_s d^2}}{\dfrac{E_c}{E_s} - \dfrac{E'_c}{E_s}} + \left[\frac{p + \dfrac{E'_c}{E_s}\dfrac{h}{d}}{\dfrac{E_c}{E_s} - \dfrac{E'_c}{E_s}}\right]^2} - \frac{p + \dfrac{E'_c}{E_s}\dfrac{h}{d}}{\dfrac{E_c}{E_s} - \dfrac{E'_c}{E_s}} \tag{56}$$

Taking moments about the center of the pull in the concrete, the center of compression in the concrete and the center of pull in the steel respectively, we have the three following equations for the moment of resistance:

$$M = f_s\, p\, bd\left(d - \frac{kd}{3} - \frac{2h}{3}\right) + \frac{f_c\, bkd}{2}\,\frac{2h}{3}$$

$$= f_s\, bd\left[p\left(d - \frac{kd}{3} - \frac{2h}{3}\right) + \frac{E_c}{E_s}\,\frac{hk^2}{3(1-k)}\right] \tag{57}$$

or

$$M = f_s\, p\, bd\left(d - \frac{kd}{3}\right) + \frac{f_c'\, b\,(h-kd)}{2}\,\frac{2h}{3}$$

$$= f_c'\, b\left[pd^3\left(1 - \frac{k}{3}\right)\frac{E_s}{E'_c h}\,\frac{1-k}{kd} + \frac{h}{3}(h-kd)\right] \tag{58}$$

or

$$M = \frac{f_c\, bkd}{2}\left(d - \frac{kd}{3}\right) - \frac{f_c'\, b\,(h-kd)}{2}\left(d - \frac{kd}{3} - \frac{2h}{3}\right)$$

$$= \frac{f_c\, b}{2}\left[kd^2\left(1 - \frac{k}{3}\right) - \frac{E'_c}{E_c}\,\frac{(h-kd)^2}{kd}\left(d - \frac{kd}{3} - \frac{2h}{3}\right)\right] \tag{59}$$

If now $E_c' = E_c$, that is, if the modulus of elasticity of concrete is the same in tension as in compression, the line MN becomes straight.

Equation (55) then becomes, letting $\dfrac{E_s}{E_c} = n$

$$p = \frac{1}{2}\,\frac{h}{nd^2}\cdot\frac{2kd - h}{1-k} \tag{60}$$

From which

$$k = \frac{h^2 + 2\,pnd^2}{2\,dh + 2\,pnd^2} \tag{61}$$

Equation (57) is not changed
Equation (58) simply has E_c instead of E_c'
Equation (59) becomes

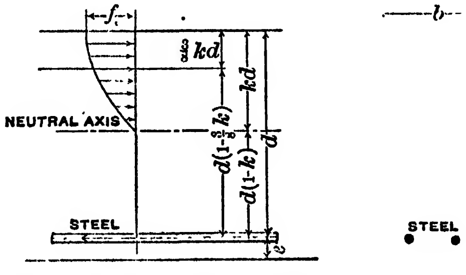
$$M = \frac{f_c b}{2}\left[kd^2\left(1 - \frac{k}{3}\right) - \frac{(h - kd)^2}{kd}\left(d - \frac{kd}{3} - \frac{2h}{3}\right)\right]$$

or

$$M = \frac{f_c bh}{2}\left[2\,d - \frac{h}{k} - h + \frac{2\,h^2}{3\,kd}\right] \tag{62}$$

COMPRESSIVE STRESS AS A PARABOLA, STEEL IN BOTTOM OF BEAM, NO TENSION IN CONCRETE.

Many experiments upon the compression of concrete show a gradually decreasing modulus of elasticity as the load increases. From the form of the stress deformation curve of these specimens, the stress on the compression side of a beam is sometimes assumed to vary as a parabola instead of as a straight line. This method was first suggested in the United States by Prof. W. Kendrick Hatt.* The formulas which follow present this method of analysis, and permit the comparison† of results by this assump-



242.—Resisting Forces with Pressure Varying as a Parabola. (*See p.* 762.)

tion, with results of the straight line theory adopted by the authors in chapter XXI.

* Proceedings American Society for Testing Materials, 1902.
† See p. 407 for comparative values by the two theories.

Formulas. As in preceding cases, from Fig. 242, we have

$$\frac{\frac{f_s}{E_s}}{\frac{f_c}{E_c}} = \frac{d\,(1-k)}{kd} = \frac{1-k}{k}$$

hence

$$k = \frac{1}{1+\dfrac{f_s}{nf_c}}, \tag{63}$$

from which

$$f_c = \frac{f_s}{n}\cdot\frac{k}{1-k} \tag{64}$$

Equating horizontal forces on the section of the beam we have

$$pbdf_s = \frac{2\,bf_c\,kd}{3}\text{, or more simply, } pf_s = \frac{2f_ck}{3} \tag{65}$$

Substitute the value of k from (63) and we have:

$$p = \frac{2}{3\dfrac{f_s}{f_c}\left(1+\dfrac{f_s}{nf_c}\right)} \tag{66}$$

which gives the ratio of steel required for any consistent values of f_s, f_c, E_s, E_c. The position of the neutral axis is dependent upon the per cent of steel and the moduli of elasticity of steel and concrete, and the value of k may be found by substituting in (65) the value of f_s from equation (64). Thus

$$\frac{2f_ck}{3} = pf_cn\frac{1-k}{k} \quad \text{or,} \quad p = \tfrac{2}{3}\,\frac{k^2}{(1-k)\,n}$$

Solving this quadratic equation and using the positive sign after taking the square root,

$$k = \sqrt{\tfrac{3}{2}np + (\tfrac{3}{4}np)^2} - \tfrac{3}{4}np$$

or in another form,

$$k = \tfrac{3}{4}np\left[\sqrt{\frac{8}{3np}+1} - 1\right] \tag{67}$$

The moment of resistance may be found by taking moments about the center of compression in the concrete, thus,

$$M = f_s pbd^2 \left(1 - \tfrac{3}{8} k\right) \tag{68}$$

or by taking moments about the center of pull in the steel,

$$M = \tfrac{2}{3} f_c kbd^2 \left(1 - \tfrac{3}{8} k\right) \tag{69}$$

Eliminating k from these equations by substituting its value from equation (63), and also substituting the value of p from equation (66), we have

$$M = \tfrac{2}{3} f_s bd^2 \frac{1}{\dfrac{f_s}{f_c}\left(1 + \dfrac{f_s}{nf_c}\right)} \left[1 - \frac{3}{8\left(1 + \dfrac{f_s}{nf_c}\right)}\right] \tag{70}$$

or

$$M = \tfrac{2}{3} f_c bd^2 \cdot \frac{1}{1 + \dfrac{f_s}{nf_c}} \left[1 - \frac{3}{8\left(1 + \dfrac{f_s}{nf_c}\right)}\right] \tag{71}$$

VERTICAL AND INCLINED REINFORCEMENT IN BEAMS.

In any beam the horizontal shear is the same as the vertical shear produced by the loads (see p. 448). Concrete, however, is so strong to resist *direct* shear (see p. 382) that, in ordinary rectangular or T-beams, no appreciable direct shear comes upon the vertical or inclined bars, the concrete alone being capable of resisting it. The shear stress is there, however, and tends to resolve itself into components of tension and compression, one of which acts in the line of least resistance. Since concrete is weak in resistance to pull, the tension component tends to pull apart and crack the concrete in the typical diagonal lines (see p. 443). This may be resisted by inclined steel bars at right angles to the cracks, or by vertical stirrups, or both, as illustrated on page 445.

The action of stirrups in resisting this pull may be illustrated by considering the beam acting like a truss in which the horizontal steel is the lower chord, the concrete in the top of the beam the upper chord, the stirrups the vertical tension bars, and the concrete web between the stirrups the compression diagonals. (See Fig. 242*a*, p. 764*a*.) Tests of deformation in reinforced concrete beams show that compression and tensile stresses of this character are all actually taking place when a beam is loaded.

This is the ordinary Howe truss action. If the stirrups are inclined, as bent bars, the action is similar but corresponds to the Pratt truss, the steel diagonals being in tension and the vertical members, *i.e.*, the concrete section, being in compression.

To show how the stress in any stirrup is measured by the shear, vertical stirrups in a reinforced concrete beam may be considered as spaced a distance apart equivalent to the effective depth of the beam, jd, thus giving the diagonal compressive concrete between 2 adjacent stirrups an angle of 45°, so that the horizontal and vertical components of this diagonal are equal.

Considering the joint *a*, in Fig. 242*a*, from simple mechanics the tension, or pull, in the vertical at this point must be equal in magnitude to

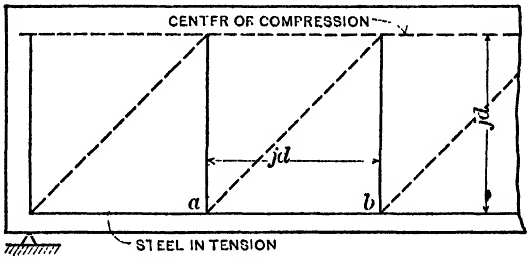


Fig. 242*a*. Illustration of Truss Action. (See page 764*a*).

the vertical component of the compression diagonal in panel *ab* and, since the horizontal and vertical components of stress in a 45° diagonal are equal, the tension, or pull, in the stirrup at *a* is equal in magnitude to the horizontal component of the compressive stress in the diagonal. *This horizontal component of the compressive stress in the diagonal, which, as just stated, is the equivalent in magnitude of the tension or pull in the vertical stirrup, is equal, in a Howe truss, to the difference between the tensile stress, or pull, in the horizontal chord just to the left of a and the tensile stress or pull in the horizontal chord just to the left of b.*

In a given beam under load, *the stress or pull in the horizontal chord at any point is directly proportional to the bending moment.* This is illustrated in formula (7), page 420. From this formula it is seen that the actual stress in the horizontal steel equals the bending moment divided by the effective depth of the beam, or $\frac{M}{jd}$.

Considering now any two points on the horizontal chord that are an infinitesimal distance apart, the difference between the bending moments at these two points is equal to the external vertical shear at this place in the beam times the infinitesimal distance.* Hence, if the two points are a *definite* distance apart, as a and b in Fig. 242*a*, and the shear is assumed to be constant between these two points, then *the difference between the bending moment at a and the bending moment at b is equal to the external shear, V, at this place in the beam multiplied by the length ab.*

It follows from the second preceding paragraph that *the difference between the tensile stress, or pull, in the chord at point a and the pull at point b, as in any simple truss, is equal to the difference between the moments at these two points divided by the effective depth, jd.*

Since, now, this difference in chord stress has been shown to be equal to the tensile stress or pull in the vertical stirrup, $A_s f_s$, and the difference in bending moment has been shown to be equal to the external shear times the length ab, it follows directly that the stress in the vertical stirrup, $A_s f_s$, is equal to the external shear times the length ab divided by the effective depth jd, or $\frac{Vab}{jd}$. That is, *when the stirrups are spaced jd apart, so that ab = jd, the tensile stress or pull, $A_s f_s$, in the vertical stirrup equals the external shear, V.*

For any stirrup spacing, s, the action is that of a multiple truss and the stress in the stirrup, $A_s f_s$, equals, without appreciable error, V times the ratio $\frac{s}{jd}$. In practice stirrups always should be spaced closer together than jd (see p. 450).

In this demonstration, for simplicity, the stirrup is assumed to take all of the vertical tensile stress. If the concrete is assumed to take part of this, the formulas given on page 449 result.

*This follows from the fundamental principle in mechanics that the first differential coefficient of the bending moment, which is of course the rate of change of that moment, is equal to the shear.

APPENDIX III

FORMULAS FOR REINFORCED CONCRETE CHIMNEY AND HOLLOW CIRCULAR BEAM DESIGN

The analysis which follows is based upon the several fundamental assumptions adopted in reinforced concrete beam design with the additional assumption that, since the concrete is usually thin as compared to the diameter of the chimney, no appreciable error is involved in assuming all material as concentrated on the mean circumference of the shell. An analysis for shear is also given together with an example of chimney design and review.

The principles involved in the demonstration of the thickness of steel and concrete are taken by permission from the analysis by Messrs. C. Percy Taylor, Charles Glenday, and Oscar Faber.*

The principal formulas given below are quoted in the text, where the general subject of concrete chimneys is discussed, and tables are presented there with the values of constants for use in design.

NOTATION

W = weight in pounds of the chimney above the section under consideration.

M = moment in inch pounds of the wind about that section.

P = total compression in concrete.

T = total tension in steel.

n = $\frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete

f_c = maximum compression in concrete in pounds per square inch (measured at the mean circumference)

f_s = maximum tension in the steel in pounds per square inch.

D = mean diameter of shell in inches.

r = mean radius of shell in inches.

t = total thickness of shell in inches.

t_c = thickness in inches of concrete only.

* Engineering (London), Mar. 13, 1908

t_s = thickness in inches of an imaginary steel shell of mean radius r. and having a cross-sectional area equivalent to the total area of reinforcing bars.

A_s = total cross-sectional area, in square inches, of reinforcing bars in the section under consideration.

k = ratio of distance of neutral axis, from mean circumference on compression side, to diameter D.

j, z, C_P and C_T = constants for any given value of k. (Tables 1 and 2, pp. 635 and 636.)

jD = distance between center of compression and centre of tension.

zD = distance from center of compression to center of force due to weight.

Referring to Fig. 243, if f_c is the maximum intensity of stress in the concrete at the mean circumference on the compression side, then the intensity of compression in the steel at that point is nf_c. Since f_s is the maximum intensity of stress in the steel at the mean circumference on the tension side, then the variation of the stress in the steel, across the section cd, is represented by the straight line ab which cuts the line cd at e, thus locating the neutral axis or the line of zero stress. Having assumed a constant value for the modulus of elasticity of the concrete in compression, it therefore follows that, at any point of a given section, the stress in either the concrete or the steel is directly proportional to the distance of that point from the neutral axis.

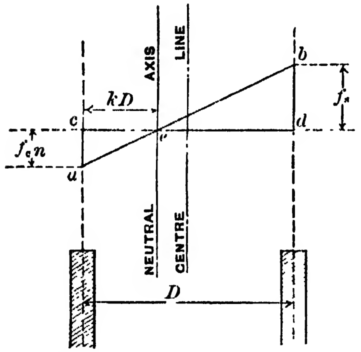


FIG. 243.—Resisting Forces in a Reinforced Chimney. (See p. 766.)

Calling kD the distance of the neutral axis from the mean circumference on compression side as shown in Fig. 243, we have by similar triangles

$$\frac{kD}{D} = \frac{nf_c}{f_s + nf_c}$$

whence

$$k = \frac{1}{1 + \dfrac{f_s}{nf_c}} \qquad (1)$$

By this formula the position of the neutral axis may be determined for any combinations of f_c, f_s, and n.

If now, as shown in Fig. 244, α represents half the angle subtended at the center by the portion in compression, we have

$$\cos\alpha = (1 - 2k)$$

from which, for any given value of z, $\cos\alpha$ becomes known as well as α and $\sin\alpha$. Thus having located the neutral axis for any given combinations of f_c, f_s and n and bearing in mind that the stress at any point of the shell is proportional to the distance of that point from the neutral axis, it is now possible to determine the total force on the compression side, the total force on the tension side, and also the location of the center of compression and the center of tension.

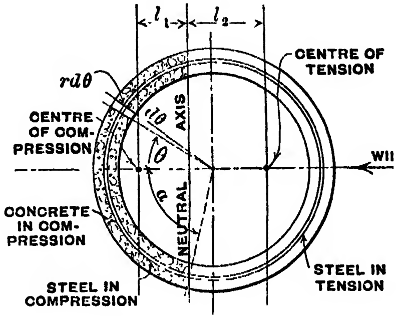


FIG. 244.—Distribution of Stresses in the Steel of a Reinforced Chimney. (See p. 767.)

Considering a small radial element subtending an angle $d\theta$, as shown in Fig. 244, we have in this element, since the length of an arc is its radius times the angle,

$$\text{area of concrete} = t_c r d\theta$$

$$\text{area of steel} = t_s r d\theta$$

The distance of the element from the neutral axis is $r(\cos\theta - \cos\alpha)$, while the distance from the neutral axis to the point of extreme stress f_c is $r(1 - \cos\alpha)$. Therefore the intensity of stress on this elemental area is

$$f_c \frac{r(\cos\theta - \cos\alpha)}{r(1 - \cos\alpha)} \text{ in the concrete}$$

and

$$f_c\, n \frac{r(\cos\theta - \cos\alpha)}{r(1 - \cos\alpha)} \text{ in the steel.}$$

Assuming these intensities at the mean circumference to represent the average for the entire element, we have the total force on the elemental area (concrete and steel)

$$d\,P = (t_c + n\,t_s)\,r\,d\,\theta\,\frac{f_c\,r\,(\cos\theta - \cos\alpha)}{r\,(1 - \cos\alpha)}$$

The total force P on the compression side of the section is therefore

$$P = (t_c + n\,t_s)\,2\int_0^{\alpha}\frac{f_c\,r\,(\cos\theta - \cos\alpha)}{(1 - \cos\alpha)}\,d\,\theta$$

Integrating this expression, gives

$$P = f_c\,r\,(t_c + n\,t_s)\,\frac{2}{(1 - \cos\alpha)}\,(\sin\alpha - \alpha\cos\alpha)$$

Since any given position of the neutral axis determines α, as shown above, this equation may take the form

$$P = C_P f_c r\,(t_c + n\,t_s) \tag{2}$$

in which C_P is a constant for a given position of the neutral axis. (See Table 1, page 635.)

Having determined the magnitude of P, its location, with respect to the neutral axis, may best be found by taking its moment about that axis and dividing by P, thus giving the distance from the neutral axis to the center of compression l_1, as shown in Fig. 244.

As before, the compressive force on an elemental area is

$$d\,P = (t_c + n\,t_s)\,r\,d\,\theta\,\frac{f_c\,r\,(\cos\theta - \cos\alpha)}{r\,(1 - \cos\alpha)}$$

The distance of this force from the neutral axis being $r(\cos\theta - \cos\alpha)$, we have as its moment about that axis

$$d\,M_c = (t_c + n\,t_s)\,r\,d\,\theta\,\frac{f_c\,r^2\,(\cos\theta - \cos\alpha)^2}{r\,(1 - \cos\alpha)}$$

while the moment of the total compressive force P is

$$M_c = (t_c + n\,t_s)\,2\int_0^{\alpha} r\,\frac{f_c\,r\,(\cos\theta - \cos\alpha)^2}{(1 - \cos\alpha)}\,d\,\theta$$

$$= (t_c + n\,t_s)\ \frac{2 f_c r^3}{(1 - \cos\alpha)} \left[\int_0^{\alpha} \cos^2\theta\, d\theta \right.$$

$$\left. - 2\cos\alpha \int_0^{\alpha} \cos\theta\, d\theta + \cos^2\alpha \int_0^{\alpha} d\theta \right]$$

Integrating, we have

$$M_c = (t_c + n\,t_s) f_c r^3 \frac{2}{(1 - \cos\alpha)} [(\alpha \cos^2\alpha. - \tfrac{3}{2} \sin\alpha \cos\alpha + \tfrac{1}{2}\alpha)]$$

Dividing M_c by P we have

$$l_1 = \frac{M_c}{P} = \frac{(\alpha\cos^2\alpha - \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}\alpha)}{(\sin\alpha - \alpha\cos\alpha)}\, r \qquad (3)$$

Following a similar method of procedure it is possible to determine the total tension and the location of the center of tension

In accordance with our assumption that the concrete is to take no tensile stress it is evident that in considering the forces on the tension side of the section we are concerned merely with the steel. On the tension side a small element therefore has an area $= t_s\, r\, d\,\theta$

The intensity of stress on this element, being proportional to its distance from the neutral axis, is

$$f_s \frac{r\,(\cos\theta + \cos\alpha)}{r\,(1 + \cos\alpha)}$$

while the total tension on the small element is

$$d\,T = t_s\, r\, d\,\theta\, f_s \frac{(\cos\theta + \cos\alpha)}{(1 + \cos\alpha)}$$

The total force T on the tension side of the section is therefore

$$T = 2 \int_0^{(\pi-\alpha)} t_s\, r\, f_s \frac{(\cos\theta + \cos\alpha)}{(1 + \cos\alpha)}\, d\theta$$

Integrating, we have

$$T = f_s\, r\, t_s \frac{2}{(1 + \cos\alpha)} (\sin\alpha + (\pi - \alpha)\cos\alpha)$$

Since, as before, any given position of the neutral axis determines α, this equation may take the form

$$T = C_T f_s\, r\, t_s \qquad (4)$$

in which C_T is a constant for a given position of the neutral axis (see Table 1, page 635). By a method similar to that used in considering the force on

the compression side we may write the moment, about the neutral axis, of the force on a small element on the tension side as

$$d\,M_T = t_s\,r\,d\,\theta\,f_s\,\frac{r\,(\cos\theta + \cos\alpha)^2}{(1 + \cos\alpha)}$$

while the moment of the total tensile force T about this axis is

$$M_T = 2\int_0^{(\pi-\alpha)} t_s\,r\,f_s\,\frac{r\,(\cos\theta + \cos\alpha)^2}{(1 + \cos\alpha)}\,d\,\theta$$

Integrating, we have

$$M_T = t_s\,r^2 f_s\,\frac{2}{(1 + \cos\alpha)}\left[(\pi - \alpha)\cos^2\alpha + \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}(\pi - \alpha)\right]$$

Dividing M_T by T we have as the distance of the center of tension from tne neutral axis

$$l_2 = \frac{\left((\pi - \alpha)\cos^2\alpha + \tfrac{3}{2}\sin\pi\cos\alpha + \tfrac{1}{2}(\pi - \alpha)\right)}{(\sin\alpha + (\pi - \alpha)\cos\alpha)}\,r \qquad (5)$$

From formulas (3) and (5) it is evident that the distance between the total force in compression and the total force in tension (i. e., $l_1 + l_2$) may, for any given position of the neutral axis, be expressed as a constant times the diameter D. Thus $l_1 + l_2 = jD$ as shown in Fig. 245. Likewise, as shown in Fig. 245, zD may represent the distance of the center of compression from the center of the chimney, z also being a constant for any given position of the neutral axis.

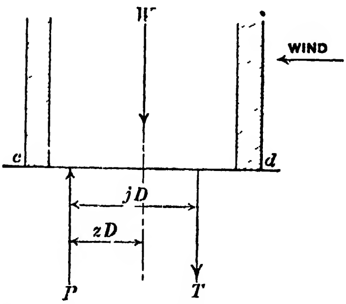


FIG. 245.—External and Internal Forces Acting upon a Chimney. (*See p.* 770.)

In a chimney the tensile and compressive stresses which we have been considering are produced by a combination of wind pressure and the weight of the chimney. Thus, on any horizontal section cd, as shown in Fig. 245, the forces external to that section are: the horizontal pressure of the wind, causing a moment M about the section, and a central vertical load W representing the weight of that portion of the chimney above the section under consideration. These forces are resisted, and held in equilibrium, by the forces P and T which represent the compressive and tensile stresses in the concrete and steel.

The system of forces as shown in Fig. 245 must be in equilibrium. Hence, taking moments about the force P, we may write

$$TjD = M - WzD$$

But

$$T = C_T f_s r t_s$$

Therefore

$$C_T f_s r t_s j D = M - WzD$$

Whence

$$rt_s = \frac{M - WzD}{C_T f_s j D}$$

The total area of steel $A_s = 2\pi r t_s$

Therefore

$$A_s = \frac{2\pi (M - WzD)}{C_T f_s j D} \qquad (6)$$

From Table I, page 635, it may be seen that the constant j changes but slightly for a considerable variation in the position of the neutral axis. Taking $\frac{2\pi}{j} = 8$ for all cases, equation (6) may be

$$A_s = \frac{8 (M - WzD)}{C_T f_s D} \qquad (7)$$

While this formula is not exact, the error involved is inappreciable for almost any case so that formula (7) may always be used instead of formula (6).

Applying now the condition that the summation of all vertical forces must be zero, we have

$$P - T = W$$

Substituting values of P and T as previously found, the equation becomes

$$C_P f_c r (t_c + n t_s) - C_T f_s r t_s = W$$

Transposing and solving for t_c we obtain

$$t_c = \frac{W + (C_T f_s - C_P f_c n) r t_s}{C_P f_c r}$$

The total thickness of the shell is

$$t = t_c + t_s$$

whence

$$t = \frac{W + (C_T f_s - C_P f_c n) r t_s}{C_P f_c r} + t_s$$

For convenience in use, after having determined A_s by the formula given above, by substituting $r = \frac{D}{2}$ and $t_s = \frac{A_s}{\pi D}$, this formula for t may best be written

$$t = \frac{2W + (C_T f_s - C_P f_c n)\frac{A_s}{\pi}}{C_P f_c D} + \frac{A_s}{\pi D} \qquad (8)$$

In view of the fact that formulas (6), (7) and (8) contain the constants z, j, C_T and C_P, which, as has been shown, are dependent for their value solely upon the location of the neutral axis, it is evident that, for any specific values of f_c, f_s, and n, which in turn will determine the position of the neutral axis, the expressions for A_s and t will admit of a further simplification. For given values of f_c, f_s and n, the necessary thickness of shell and area of reinforcement may be expressed merely in terms of the moment of the wind M, the weight W, and the mean diameter D. The expressions, as given, however, seem best adapted to general use, and when supplemented by the tables given on pages 635, 636, are rendered quite simple of solution for specific values

In Table 2, page 636, is given values of k, the location of the neutral axis, for various combinations of f_c, f_s and n, while Table 1, page 635, gives the corresponding values of the constants C_P, C_T, z and j for various positions of the neutral axis.

Shear or Diagonal Tension. Having determined the necessary thickness of shell and vertical reinforcement, the size and spacing of the circular steel hoops must be considered. The external forces produce shear and diagonal tension which may be analyzed similarly to like stresses in rectangular beams, and the reinforcement necessary to resist the diagonal tension, which is a function of the vertical tension, may be determined Usually this reinforcement is not so great as that which it is advisable to insert for the proper distribution of temperature stresses, but nevertheless it should be determined to be sure that it is sufficient in quantity.

The concrete should never be relied upon to carry any tension or vertical shear because the expansion from the heat may cause vertical cracks in the concrete. These need not be considered dangerous if sufficient horizontal reinforcement is provided any more than the vertical cracks in a brick or tile chimney. Considering the stresses due to vertical shear, it may be easily shown that at any horizontal section of a chimney the vertical shear per inch of height is the total horizontal shear on that section divided by the distance between centers of tension and compression, jD. With this as a

basis there may be developed a formula for practical use in determining the necessary area and spacing of horizontal steel hoops at any given section.

Thus let

h_l = height, in feet, of chimney above section under consideration.
F = effective wind pressure against chimney in pounds per square foot.
f_s = allowable tensile stress in pounds per square inch in steel hoops.
D = mean diameter of shell in inches.
P_0 = ratio of area of steel hoop to area of concrete.

At any horizontal section of a chimney the total shear on that section is equal to

$$\frac{D}{12} h_l F$$

while the maximum shear per inch of height is therefore

$$\frac{D}{12} \frac{h_l F}{jD}$$

Having seen that for all positions of the neutral axis j remains practically constant, and giving j an average value of, say, 0.783, the expression for the maximum vertical shear per inch of height becomes

$$0.106\ hF$$

while the shear or diagonal tension in one foot of height is $12 \times 0.106\ h_l F$.

The area of steel in one foot of height of chimney will be $12\ bp_0$ and the stress the hoops in this height are capable of sustaining on their two sections is

$$2 \times 12\ tp_0 f_s$$

Equating these we have

$$12 \times .106\ h_l F = 2 \times 12\ tp_0 f_s$$

whence

$$p_0 = \frac{h_l F}{18.8\ f_s t}$$

This ratio of steel is for shear or diagonal tension only. To provide for temperature stresses or rather to distribute the strains so as to prevent the localization of cracks an additional amount of horizontal steel is needed. This may be provided for arbitrarily by assuming 0.25% steel or rather

0.0025 for temperature stress in addition to the steel for shear. Expressing this as a formula for ratio of steel gives

$$p_0 = \frac{h_1 F}{18.8 f_s} + 0.0025 \qquad (9)$$

Small rods spaced 6 to 10 inches apart except in the upper part of the stack where the spacing may be greater are advised.

The spacing of hoops in many of the chimneys already built has been 18 inches to 36 inches, but as such chimneys have frequently cracked quite seriously, more recent designs have called for 8 or 9 inch spacing through the entire stack.

Design of Hollow Circular Beams. The analysis of a hollow circular reinforced concrete beam whose thickness, compared relatively with its diameter, is small, is similar in principle to that of a chimney. In this case the weight of the member acts in the same direction as the external forces, so that in formulas (7) and (8) W the weight in the axial direction, is zero. The forces of compression, P, and tension, T, are equal. The area of steel and the thickness of shell are therefore obtained from formulas (7) and (8), pages 771 and 772, by making $W = O$.

Note on Slim Chimneys. Since, in designing a chimney the selection of certain allowable working stresses in the concrete and in the steel will fix the position of the neutral axis, it is evident that the ratio of these working stresses limits the compressive area of the section. Hence, for a very high chimney in which there is a large compression in the lower sections, it is possible that the selection of an ordinary working stress in the steel of 14000 or 16000 pounds per square inch together with the customary working stress in the concrete of, say, 500 pounds per square inch, would locate the neutral axis so near the compression side of the section as to make it impossible to obtain sufficient compression area to withstand the compressive forces without exceeding the allowable unit stress in the concrete.

If, therefore, the thickness of shell as computed from formula (2), page 634, should work out materially larger than the assumed thickness, recomputation should be made on the basis of a smaller working stress in the steel, thus changing the position of the neutral axis so as to allow a larger proportion of the section to carry compression. In such a case it may be necessary to make a series of trials with different working stresses in the steel until the computed thickness checks with the assumed thickness. In high chimneys of small diameter it may be impossible to utilize a working stress in the steel greater even than 7000 or 8000 pounds per square inch.

APPENDIX IV

METHOD OF COMBINING MECHANICAL ANALYSIS CURVES

In Chapter XI the method of forming mechanical analysis curves is discussed, and approximate rules are given for combining individual curves to form the curve of the mixture More exact methods, which also illustrate the principles, are given in the following pages, taking up first simple cases and then the more complicated ones

Case I Curves which meet, but do not overlap In Fig 246 are shown three curves, No 1, No 2, and No 3, representing ideal grades of sand and stone, which may be combined in such proportions that the curve of the mixture will be of the ideal form required The problem requires the determination of the percentages of each of the three materials which when combined will form a mixture whose curve is nearly the ideal In order to prove that the percentages found will produce the resultant curve, and also to illustrate the theory of the mixture, the resultant curve will be first plotted and described in a very elementary manner, and afterwards by the method of ratios which would be employed in practice

Curve No 3 represents a material all of whose particles will pass through a sieve having holes 2 00 inches diameter and all of whose particles will be retained on a sieve having holes 0 75 inch diameter Stone represented by curve No 2 lies between diameters 0 75 and 0 25 inch, while the material of curve No 1 is all finer than 0 25 inch, that is, is all under ¼ inch Curves No 3₁ and No 3 are referred to later

The curve *OebA* is first plotted* as a parabola Although the latest tests indicate that the best curve is a combination of an ellipse and a straight line,† the parabola will illustrate the principle of combination as well as any other, and so for this problem we may assume now that the required theoretical mix of materials lies in this parabolic curve This is equivalent to saying that the desired theoretical mixture of materials is such, that at any ordinate

* CONSTRUCTION OF THE PARABOLA

D largest diameter of stone
d = any given diameter
P = per cent of mixture smaller than any given diameter

The equation of the parabola is

$$d = \frac{P^2 D}{10000}$$

The parabola can be constructed in any of the numerous ways given in text-books, the writer finding it easiest to use a slide rule Set D on the B scale of the rule opposite 100 on D scale, read any value of d on the B scale opposite any corresponding value of P on the D scale

† "Laws of Proportioning Concrete," by William B. Fuller and Sanford E Thompson, Transactions American Society of Civil Engineers

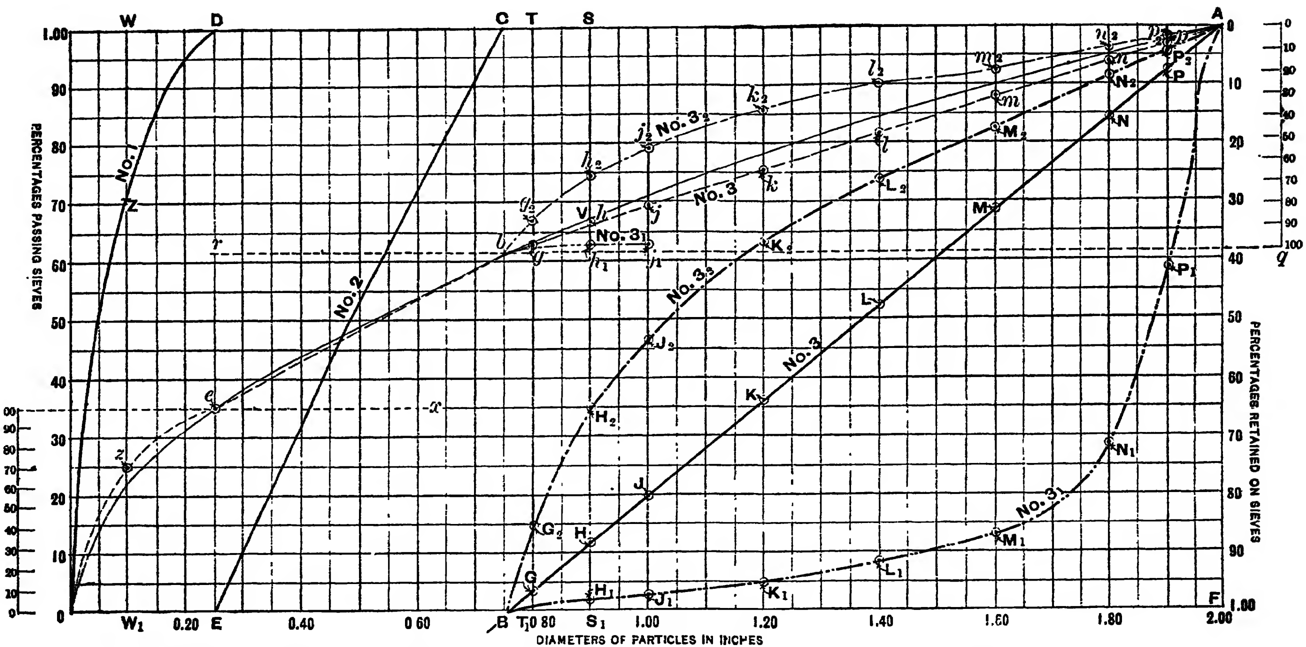


FIG. 246.—Diagram Illustrating Method of Combining Curves which do not Overlap. (*See pp. 775 to 782.*)

or vertical line cutting the parabola, the proportion or percentage of the ordinate below the intersection represents the percentage by weight of the mixed materials which passes a sieve the diameter of whose openings corresponds to the given ordinate, and the percentage above the curve represents that percentage which is too large to pass through this sieve. The parabola shows, for example, that 87% of the mixture of materials should pass a 1.50-inch sieve, 71% should pass a 1-inch sieve, 49% a ½-inch sieve, and so on.

We may now take up the stone curves in succession to determine what percentage by weight of each should be used, so that when they are combined, the mixture will be as nearly as possible like that called for in the parabola.

The chief difficulty in the method of determining the percentages of each material lies in combining the individual curves so as to form a single curve which represents the mixture. This involves drawing on the same piece of paper two different lines, each of which exactly represents the composition of the same lot of stone, that is, the exact per cent. of each size of stone in the lot. For example, as is explained below, on Fig. 246, lines *BKA* and *bkA*, each accurately represents the percentage composition of the same batch of stone, namely, No. 3, and the full meaning and value of these diagrams cannot be understood until it is clear how the same values can be accurately represented on the same diagram by two such totally different curves.

In the first place it is seen that the ordinates, that is, the vertical lines in the diagram, are divided into 100 parts representing percentages. It is clear, therefore, as the divisions are relative, that the diagram would accomplish the same results and curves could be drawn accurately representing the percentages passed and retained by the different sieves, whether the distance from 0 to 100 on the ordinates were, say, three times as large as it is, or whether it were only ¼ or ⅛ of the present length. All that is needed is to divide these vertical lines, whether they are long or short, into 100 parts and let each division represent 1%.

Referring now to Fig. 246, the percentage composition of the No. 3 lot of stone is represented by line *BKA*. This lot of stone contains no stone smaller in diameter than 0.75 inch and none larger than 2.00 inches. Running vertically upward from *B* on the 0.75-inch line to *b* where it crosses the parabola, we see that the parabola from *b* to *A* also represents a lot of stone none of which is smaller than 0.75 inch and none larger than 2.00 inches, and we can look upon this lot of stone for the moment as entirely separated from the rest of the mixture which the whole parabola represents. If we wish to find the exact percentages of the various sizes

of stone which are in the portion or lot represented by the portion of the parabola from b to A, all that is necessary is to draw the horizontal line rq through the point b, then divide the vertical distance from A to rq into 100 parts, so as to obtain a new set of horizontal lines or abscissas representing percentages. Now if we start at the base line rq and follow up any one of the vertical lines or ordinates until it meets the parabola, and then follow horizontally to the right along the line which intersects the parabola at the same vertical line or ordinate point, the reading on the new smaller percentage scale will give us the per cent. of stone in the lot bA which is larger than the diameter represented by this ordinate, etc. For example, taking intersection of 1.00 ordinate with the parabola and running across we find that 75% of the lot is coarser than 1 inch diameter.

It is desirable to see how nearly the stone in lot No. 3 agrees with the theoretical lot of stone called for by section bA of the parabola. In practice, the comparison may be made most readily by ratios with the aid of the slide rule, as is described more fully below, but the reasoning will be more clearly understood if the plan described in the last paragraph is followed.

Taking first curve No. 3 we may redraw it on the same smaller scale as the portion of the parabola bA is drawn, that is, it may be constructed on rbq as a base line instead of on the zero coordinate BF. Since the vertical per cent. line between q and A has been divided into 100 parts, this section of the diagram may be used instead of the original per cent. divisions extending from A to F. A piece of paper the length of Aq may be divided into 100 parts and placed with its upper or 0 end in line with the upper line CA of the diagram. The vertical distance from the line CA to the various points G, H, J, K, etc., may be read by the eye and replotted,— with the assistance of the small scale,—as g, h, j, k, etc.

It is evident then that the broken line $bghjkA$ represents (referring to the small percentage scale Aq) lot No. 3 of stone as accurately as line $BGHJKA$ represents the same lot of stone referring to the larger percentage scale AF.

Stone curve No. 3, however, would never, in actual practice, be an absolutely straight line from A to B. It would be in all practical cases an irregularly curved line, similar, for instance, to some of the actual stone curves shown in Fig. 71, p. 199, or it might be either convex like the curve No. 3_2, Fig. 246, or concave like No. 3_1. These curves may be redrawn in exactly the same way as curve No. 3, and if the lower end of each is assumed to start at point b where the new base line or bq crosses the parabola, we should have for No. 3_2 the new curve $bg_2h_2j_2$, etc., and for No. 3_1 the curve whose beginning is shown by bh_1j_1, etc. Thus again

it is seen that the stone curves No. 3_2 and No. 3_1 on the original full-size diagram are accurately represented also by the curves $bg_2h_2j_2$, etc., bh_1j_1, etc., drawn to the smaller scale on the same piece of paper.

Thus far only the principles involved in understanding the curves and replotting them have been considered. The result at which we are aiming is the determination of the percentage of each material which will be required in the final mixture of the aggregates. Let us first take for this curve No. 3. The curve of stone No. 3 ends at B, which indicates that all of this stone is larger in diameter than 0.75 inches (although about 4% of it, for instance, is smaller than 0.80 inches in diameter). Now following up from B on the vertical line which represents 0.75 inches in diameter until we come to the parabola at point b, we see that the parabola demands that $\frac{bB}{CB}$ or $\frac{61}{100}$ or 61% of all the stone and sand in the entire mixture of stone and sand shall be smaller than 0.75 inches in diameter, and conversely that $\frac{bC}{CB}$ or $\frac{39}{100}$ or 39% of the mixture shall be larger than 0.75 in diameter. No. 3 stone is the only one of the three lots of stone which is larger in diameter than 0.75 inches, and therefore 39% of this grade of stone should be used in making up the mixture.

These ratios give us a clue to the method of plotting the curves to the smaller scale with the aid of the slide rule, instead of employing the longer method of actually dividing the spaces into 100 equal parts. The principle in each case is exactly the same. By the method of ratios the curve bkA would be plotted from the knowledge that $\frac{Cb}{CB} = \frac{Tg}{TG} = \frac{Sh}{SH}$, etc. The distances Tg, Sh, etc., may be read directly from the slide rule or from the equation which follows from the preceding, viz., that $Tg = \frac{TG \times Cb}{CB} = \frac{96 \times 39}{100} = 37\%$, and so on.

This actual plotting of the curves may be unnecessary, in fact, it is usually unnecessary for an experienced calculator, as the percentages can be obtained and the general direction of the curve estimated by inspection.*

*It is evident that neither of the two batches or lots of materials shown by curves No 3_2 and No. 3_1 are so well adapted to form a parabola as curve No 3. Curve No. 3_2 would more nearly fit the parabola than it now does if its new curve were plotted slightly lower so that it would cross the parabola at a different point and a larger percentage of it would be required for the mixture. If it crossed the parabola at V, the percentage of it to use could be found by plotting it in this new location and taking for the percentage the vertical distance from C to the end of the curve, or what is the same thing, taking the percentage as $\frac{SV}{SH_2} = \frac{33}{65} = 51\%$.

The next curve in order is No. 2. We note that this lot of stone is the only one of the three whose particles lie between 0.25 inches diameter and 0.75 inches, and that therefore all of the stone called for by the parabola between these two sizes must be supplied from No. 2 lot. Following down from the upper end, C, of No. 2 to the parabola at b and up from the lower end E to the parabola at e and drawing horizontal line ex, we see that the proportion of No. 2 stone which is called for by the parabola is represented by the distance between the lines rq and ex or by line re, and we have the ratio $\frac{re}{DE} = \frac{26}{100} = 26\%$, as the percentage of the weight of the No. 2 material to the total weight of the mixture.

Plotting curve No. 2 in its new location as a part of the mixture we have the dotted line eb as representing the No. 2 material after it becomes a part, that is, 26%, of the mixture. The upper end must join the line bA because we are now plotting a curve which represents a mixture of the two materials, No. 3 and No. 2, and the mixture must be represented by one single, continuous curve. We may consider rb and ex as two base lines, divide the vertical distance between them into 100 parts, and then plot the percentages downward from rb, equivalent on the small scale to the percentages downward from DC to the original No. 2 curve CE, as described on page 198, or we may take ratios, as described on page 200, and using the slide rule set DE (100) on De (65) and on any vertical distance from DC to the line CE, we may read the distance from rb to the resultant curve eb. In practice, the line rb need not be plotted, but each ratio as it is obtained may be added to the per cent. already found for the No. 3 material to obtain the distance down on the ordinate for the final curve of the mixture, as shown on page 787.

The required percentage of material No. 1 may be obtained by deducting the sum of the percentages of No. 2 plus No. 3 from 100, or by inspection of the parabola and the curve of the portion of the final mixture already plotted, $ebkA$. From the location of the point e it is evident that 35% of the total mixture of the material must pass a 0.25-inch sieve. Since No. 1 is the only material whose particles are finer than this, it is evident that this percentage of the total mixture must be entirely formed by No. 1. In other words, the percentage of No. 1 to the total mixture of 100 parts is 35%. To plot the curve OD as a part of the mixture, we may divide the distance eE into 100 parts, and plot the percentages, or we may take the slide rule and set Ee on DE, that is, 35 on 100, and read the correspond-

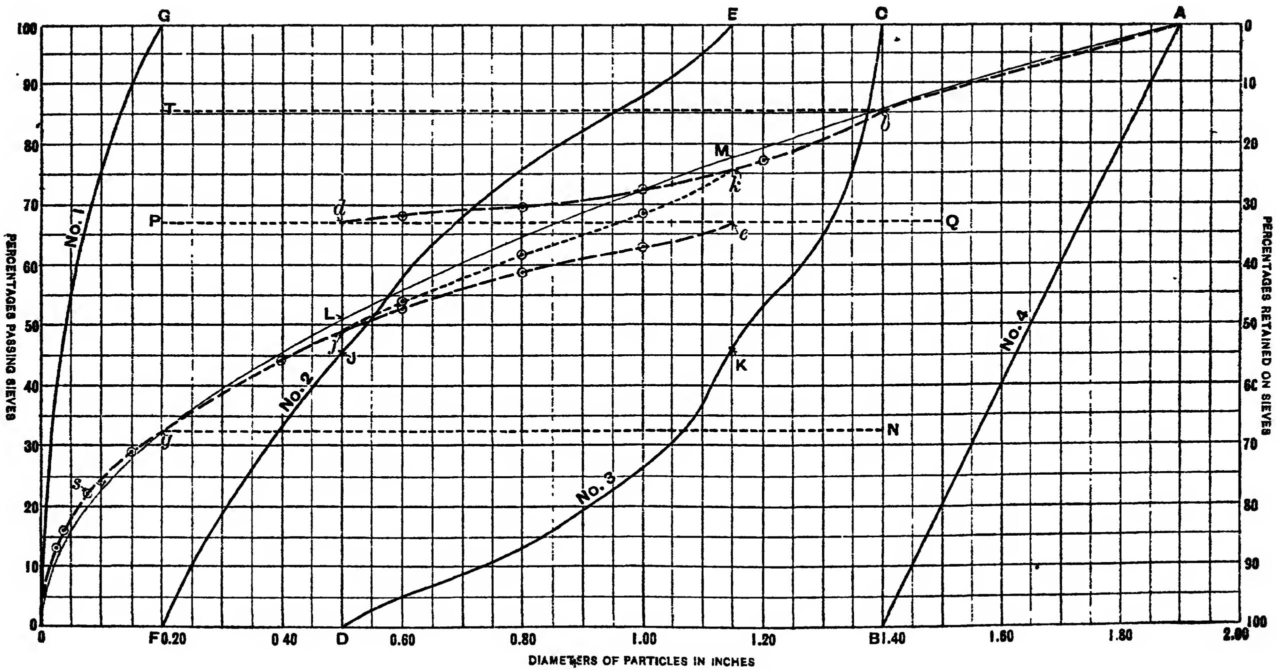


FIG. 247.—Diagram Illustrating Method of Combining Curves which Overlap. (*See pp.* 782 *to* 784.)

ing ratios for the other ordinates. Thus, at ordinate 0.10, $DE : eE = ZW_1 : zW_1$, or $100 : 35 = 71 : zW_1$, hence $zW_1 = 25$.

The final curve of the mixture of materials No. 3, No. 2, and No. 1 in proportions represented by the percentages obtained is represented by the dotted line *AkbezO*.

To illustrate how simply such a diagram as Fig 246 is solved in practice without really going through the processes described, we may determine the percentage by weight of each material to the weight of the final mixture as follows:

For material No. 3, $\frac{Cb}{CB} = \frac{39}{100} = 39\%$

For material No. 2, $\frac{re}{DE}$ or $\frac{De - 39}{DE} = \frac{26}{100} = 26\%$

For material No. 1, $\frac{Ee}{ED} = \frac{35}{100} = 35\%$

We have thus the percentages of each aggregate material which must be contained in the total mixture of aggregate The actual proportions of the concrete expressed in parts are obtained in the same manner as is described for example 2 on page 788.

Case II. Curves which overlap. Fig 247 shows a more complicated combination of materials than Case I. Curves of four materials are drawn

From the foregoing it is clear that the percentage for material No. 4 is represented by *Cb* or 14%. Since curves No 2 and No 3 overlap each other, their values are less easily determined, and we may leave them and first take No 1 Curve No. 1 is determined and may be plotted in the same way as curve No. 1 in diagram Fig. 246, p. 776, giving the curve *Osg*, and the percentage $\frac{gF}{GF} = \frac{33}{100} = 33\%$ the percentage by weight of No. 1 in the final mixture.

Having found the per cent. of No. 1 sand to use and also of No. 4 stone, namely, 33% for No. 1 and 14% for No. 4, we have left 53% of the total mixture which must be made up from No. 2 and No 3 lots.

On curve *FE* the portion from *E* to *J* is overlapped by that part of the *DC* curve extending from *D* to *K*. We note first that about 20% of the material in the parabola (that portion extending from *g* to *L*) must be supplied with stone from the No. 2 lot, while about 10% of the material of the parabola (the portion extending from *b* to *M*) must come from the No. 3, or *DC* curve. There is left then 53% — (20% + 10%) = about

23% of the parabola which must be supplied from the overlapping portions of the two curves. Judging from the general appearance of the two curves it would appear that No. 2 curve contained stone more nearly corresponding to the needs of the parabola than *DC*.

For a trial, therefore, we will give a larger proportion to No. 2 than to No. 3 stone, say, 14% of the remaining 23% to No. 2 and 9% to No. 3. No. 2 stone must then furnish 20 + 14 = 34% of the final mixture and No. 3 must furnish 10 + 9 = 19% of the final mixture. Through *g* draw a base line *gN* on which to construct the new curve for *FE*. 34% higher up draw line *PQ* which forms the upper limit for new curve to represent *FE* and the lower limit for new curve to represent *DC* Then 19% higher up draw line *bT*, which forms the upper base line for new curve to represent *DC*.

Now, by dividing the vertical distance between the lines *gN* and *PQ* into 100 equal parts, — or else by ratios, taking the slide rule and setting *Pg* on *GF* and reading from the ordinates of *FE*, the distances from the base line *gN* to the points which locate the curve *ge*, — we can readily transfer curve *FE* into the new curve indicated by the dotted line *ge* which is assumed to supply 34% of the stone still needed by the parabola, and in the same way by dividing the vertical distance between the lines *PQ* and *Tb* into 100 equal parts, — or else by taking ratios, — the new *db* curve can be laid down.

The curve from *g* to *j* and from *b* to *k* remains as it is.

With a pair of dividers transfer the distance at each ordinate from base line *PQ* up to curve *db* down to curve *ge*, and add it to the curve. These new points will give the dotted curve *jk* as the exact location of the two batches of stone No. 2 and No. 3 combined, 34% of the one being used and 19% of the other.

The resultant curve, *jk*, may be found in another manner after selecting the percentages of the different materials by adding on any ordinate the percentages of each material in the final mixture. For example, on 1 00 diameter, 26% of No 3 stone passes a 1 inch sieve, but since No. 3 actually occupies only 19% of the mixture, the percentage of No. 3 stone passing the 1-inch sieve in terms of the weight of the total mixture (which is 100%) would be 19% of 26% = 5% Similarly, the percentage of the portion of the No. 2 stone in the final mixture which passes a 1-inch sieve is 34% of 88% or 30%. All of the No. 1 material (33%) passes the 1-inch sieve, so this too must be added to the others, and we have 5% + 30% + 33% = 68% as the percentage of the final mixture which will pass a 1-inch sieve.

An inspection of this dotted line *jk* resulting from combining these

curves leads us to the conclusion that we should have done rather better to have taken more of No. 2 stone, say, 38% instead of 34%, and 15% of No. 3 instead of 19%, in which case the combined curve would have more nearly corresponded with the parabola. We would have, therefore, as a result of our study the required percentages of material as 14% of No. 4, 15% of No. 3, 38% of No. 2, and 33% of No. 1.

Practical Examples of Proportioning. Having taken up in a very elementary fashion the principles by which curves are drawn and combined, we may take two examples of other combinations of materials liable to be met with in practise.

Example I.—Curves of two materials. Suppose we have for concrete

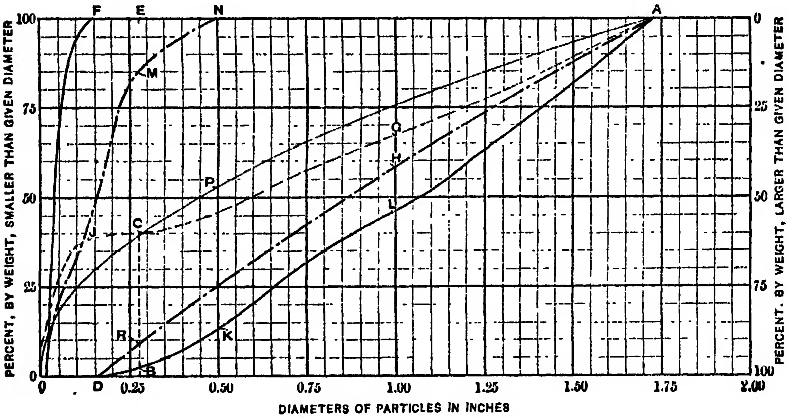


FIG. 248.—Method of Proportioning Two Aggregates. (*See p.* 784.)

the fine sand of Fig. 200, p. 198, to use with the crushed stone of Fig. 70, p. 192, what proportions of each should be employed and how could the mixture be improved?

Solution.—The curves of the two materials are plotted to the same scale in Fig. 248 as OF and $DBLA$, and then the theoretical curve OCA drawn for convenience as a parabola by the method previously described.

The curve indicates that for a theoretical mix of sizes of aggregate up to 1¾ inches, 93% of the mixture should pass a 1½-inch sieve, 76% should pass a 1-inch sieve, 53% a ½-inch sieve and so on.

Where, as in this case, the materials to be mixed are represented by only two curves, no combination of which will make a curve as close to the theoretical as is desirable, there is another limiting condition which was brought

out by the experiments, viz., that for the best results the combined curve shall intersect the theoretical on the 40% line, at C, and that the finer material shall be assumed to include the cement.

In this case, therefore, where the stone and sand curves do not overlap each other, to determine the best proportions of stone and sand, we have merely to take such proportions of each that the resultant curve will pass through the ideal curve at the point C where it crosses the 40% abscissa. This percentage is obtained by taking the ratio $\frac{EC}{EB} = \frac{60}{98} = 61\%$. The percentage by weight of sand plus cement to total aggregate will be 100% — 61% = 39%. The curve of the mixture may now be drawn by replotting the curve $DBLA$ in its new location $JCGA$ and the curve OF in its new location OJ, thus making the combined curve $OJCGA$.

Now decide upon the amount of cement to use in the mix to give the required strength of concrete, say, one cement to eight aggregate (the proportion of aggregate being based on measurement before mixing together the sand and stone), which will make the cement one-ninth or 11% of the total materials. Deducting this from the sand plus cement, we have 39% — 11% = 28% sand, and our best proportions for a 1:8 mixture will be 11 parts cement: 28 parts sand: 61 parts stone, which is equivalent to 1: 2.5: 5.5. If the proportions are required by volume and the relative weights of the sand and stone differ from the relative volumes, the proportions should be corrected accordingly.

Plotting the analysis curves of the two materials, as described above, shows immediately how to improve the mix. If, for instance, the crushed stone had been better proportioned so as to contain more particles of 0.5 and 1.0 inch diameter, — see curve DHA, — a curve much nearer the parabola could have been constructed. In this case the ratio would have been $\frac{EC}{ER} = \frac{60}{91} = 66\%$ of stone, and the proportions of cement, sand, and stone for a 1:8 mixture, 11: 23: 66 or 1: 2:6, a stronger and a more impermeable mix. A still better mixture would have resulted with the use of coarser sand having a curve similar to the broken line OMN, which with the first material, $DBLA$, would have brought the continuous line of the mixture very much nearer the ideal curve, by using the ratio $\frac{MC}{MB} = \frac{45}{83} = 54\%$ of curve $DBLA$ and 46% of curve OMN. This method thus shows not only the best proportions for given materials, but also the defects in the materials and how to remedy them.

The most valuable use of the method of proportioning by mechanical analysis is in cases where the character of the work warrants employing several grades, that is, several sizes, of stone and sand. Such mixtures are being increasingly employed as engineers and contractors more fully appreciate the necessity of so economically proportioning the materials as to produce a mixed aggregate of the greatest possible density, — that is, with the fewest possible voids, — thereby reducing the quantity of cement and at the same time improving the quality of the concrete, in other words, making both a better and a cheaper concrete.

The process of determining the percentages of each material is more complicated than where only two aggregates, sand and stone, are used, but it is not very difficult in practice, especially if one is familiar with the slide rule, and, as illustrated in Example 2, the method is more exact than

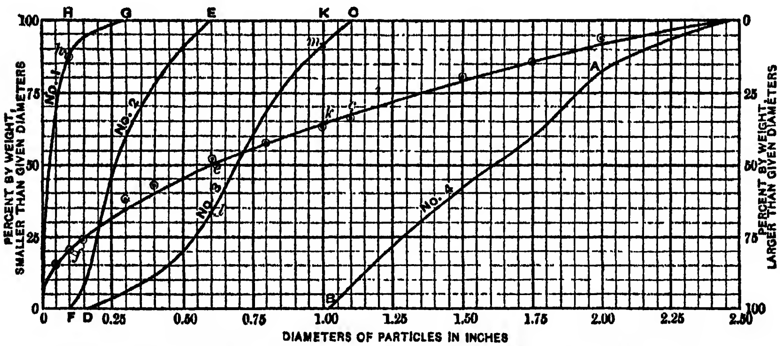


FIG. 249. — Method of Proportioning a Graded Mixture. (*See p.* 786.)

with two materials, for the reason that the resulting curve can be made to more nearly approach the parabola.

Example 2. — *Graded Materials.* Given the medium sand, represented by curve in Fig. 72, page 200 and the three sizes of crushed stone represented by the curves in Fig. 71, page 198, find what percentage of each will best combine to make the strongest and densest concrete.

Solution. — Since mechanical analysis of each material has already been made, we will immediately replot the four curves on the same scale in Fig. 249 and draw parabola passing through point O and the point at which curve No. 4 reaches 100%. We see at once that percentage of No. 4 stone required is $\frac{Kk}{KB} = \frac{36}{100} = 36\%$. (To be sure, about 8% of No. 4 is overlapped by No. 3, but this is so slight it need not here be considered.)

Let us determine sand curve No. 1 at 0.10 diameter ordinate, since it can be seen by inspection that the portion *oh* of curve No. 1 very nearly fits the parabola and grains smaller than 0.10 diameter must be supplied wholly from this curve, while the larger grains represented by portion *hG* are found also in No. 2 curve. Accordingly, we have the percentage $\frac{Ff}{Fh} = \frac{20}{88} = 23\%$.

A part of No. 3 curve, that portion extending from D to l, is overlapped by nearly the whole of No. 2 curve. We can see, however, that No. 3 curve alone must supply 14% of the material in the parabola (that portion extending from e to k). This leaves $100 - (36 + 23 + 14) = 27\%$ of the mixture to be furnished by the overlapping portions of No. 3 and No. 2 in such ratio as best fits the parabola.

From a study of the two curves, we find by inspection and trial plottings that most of the material required would be better supplied by No. 2 curve, since it contains stone corresponding very well to the needs of that part of the parabola extending from f to e. Let us consider 23% as the proper amount of the final mixture to be furnished by No. 2 curve, which would leave $14 + 4 = 18\%$ as the total portion which must be supplied by No. 3 curve.

Now, on any of the ordinates, we can locate points through which a curve may be drawn which represents a mixture of the given sand and stone in the proportions just found, for example:

Ordinate.		% Retained
1.75	$40 \times 36\%$	= 14
1.50	$57 \times 36\%$	= 20
1.10	$92 \times 36\%$	= 26
1.00	$(100 \times 36\%) + (8 \times 18\%) = 36 + 1$	37
0.80	$36 + (31 \times 18\%) = 36 + 6$	= 42
0.60	$36 + (66 \times 18\%) = 36 + 12$	= 48
0.40	$36 + (88 \times 18\%) + (21 \times 23\%) = 36 + 16 + 5$	= 57
0.30	$36 + (93 \times 18\%) + (40 \times 23\%) = 36 + 17 + 9$	= 62
0.15	$36 + 18 + (92 \times 23\%) + (6 \times 23\%) = 36 + 18 + 21 + 1$	= 76
0.05	$36 + 18 + 23 + (30 \times 23\%) = 36 + 18 + 23 + 7$	= 84

These percentages are plotted on the diagram as small circles. The same points would have been obtained if we had begun at the left of the diagram and calculated the percentages passing the sieve.

We find that a curve drawn through these points satisfies the parabola sufficiently well to assume that 23% of sand, 23% of finest stone, No. 2, 18% of medium stone, No. 3, and 36% of the largest stone, No. 4, would make the best concrete mixture out of the given materials.

If 1 : 7 concrete is wanted there would be* $\frac{100}{7} = 14.3$ parts cement, and the proportions would be 14 : 23 : 23 : 18 : 36 or 1 : 1.6 : 1.6 : 1.3 : 2.5 by weight. This would give very nearly an ideal mix, and the resultant concrete would be impermeable and very strong.

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ERRATA

The following slight corrections are given so that they may be entered in ink on the proper pages and this slip destroyed.

Page 61. In second tabulation "Fe_2O_2" should read Fe_2O_3.

Page 88. In last footnote "formula (2)" and "formula (3)" should read *formula (1)* and *formula (2)*.

Page 184. In 7th line from bottom "on page 211" should read *on page 206.*

Page 227. In 6th line from top "on page 258" should read *on page 376.*

Page 227. First footnote, "See page 261" should read *See page 378.*

Page 233. Under 45% void column, 5th value from bottom, "39.4" should read *29.4.*

Page 356. Footnote should be added: **See page 93.*

Page 430. "z" in formula (22) should be *x*.

Page 446. Last footnote, "Bulletin No. 14" should read *Bulletin No. 4.*

Page 475. In 6th line from bottom "three equal parts" should read *four equal parts.*

Page 494. In Fig. 153, the values at the bottom of the table: "0.0005" should read *0.005;* "0.0010" should read *0.010;* "0.0015" should read *0.015.*

Page 572. In Fig. 179: "Values of k, ratio of depth of neutral axis to depth of steel below compressed surface of beam" should read, *Values of k, ratio of depth of neutral axis to depth of beam.*

Page 573. In Fig. 180: "Values of C_a in Formula (" should read *Values of C_a in Formula (59).*

Page 693. In 5th and 6th lines from bottom, "by formula (1)" should read *by formula (2).*

Page 693. In 3d line from bottom, "formula (2)" should read, *formula (1).*

Page 743. In 4th line from bottom, "p. 40" should read *p. 401.*

Page 757. Formula (30) "$\dfrac{\frac{f_c}{E_s}}{\frac{f_c}{E}}$" should read $\dfrac{\frac{f_s}{E_s}}{\frac{f_c}{E_c}}$

Page 788. Footnote should be added: *See page 208.

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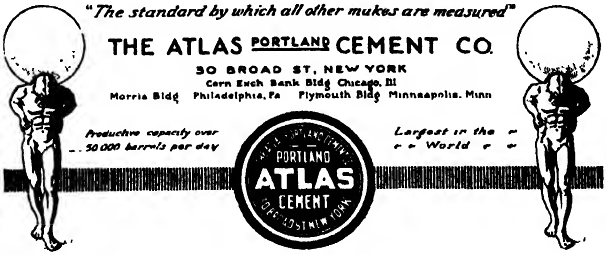
 See pages 154d & 312.

THERE are some words in the English language that are spelled the same, but the meaning is different. It's that way with the word cement—when it has the prefix "Atlas" before it, it means



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